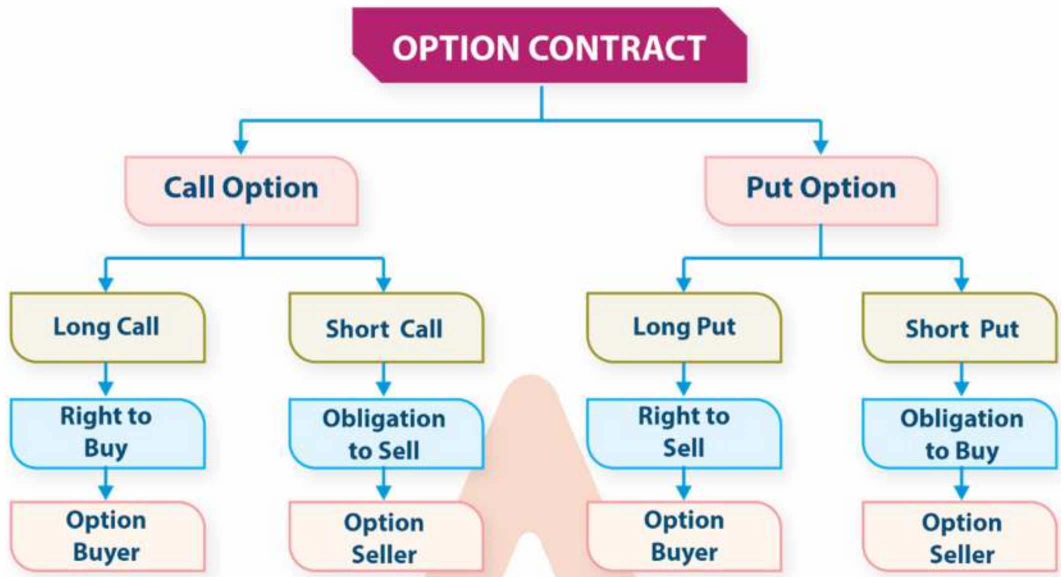
Derivatives Analysis & Valuation (Options)

Study Session 7

LOS 1 : Introduction



Definition of Option Contract:

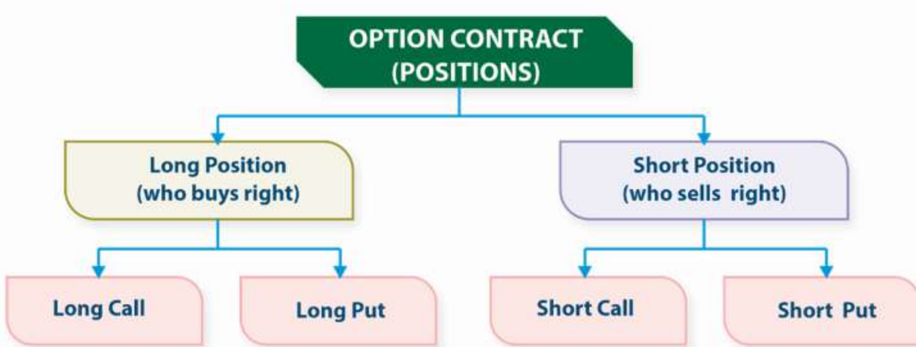
An option contract give its owner the right, but not the legal obligation, to conduct a transaction involving an underlying asset at a pre-determined future date(the exercise date) and at a pre-determined price (the exercise price or strike price)

There are four possible options position

1) Long call : The buyer of a call option → has the right to buy an underlying asset.
2) Short call : The writer (seller) of a call option →has the obligation to sell the underlying asset.
3) Long put : The buyer of a put option → has the right to sell the underlying asset.
4) Short put : The writer (seller) of a put option → has the obligation to buy the underlying asset.

Note:

Meaning of Long position & Short position under Option Contract



Note:

- If question is silent always assume Long Position.
- Exercise Price/ Strike Price:

The fixed price at which buyer of the option can exercise his option to buy/ sell an underlying asset. It always remain constant throughout the life of contract period.

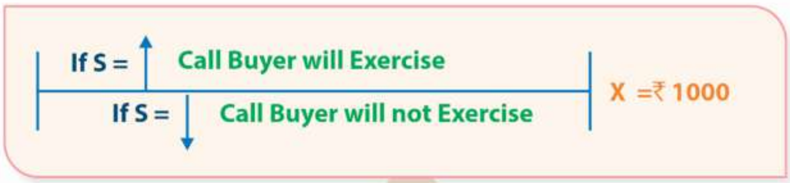
- Option Premium:
  - To acquire these rights, owner of options must buy them by paying a price called the Option premium to the seller of the option.
  - Option Premium is paid by buyer and received by Seller.
  - Option Premium is non-refundable, non-adjustable deposit.

Note:

- The option holder will only exercise their right to act if it is profitable to do so.
- The owner of the Option is the one who decides whether to exercise the Option or not.

LOS 2 : Call Option

When Call Option Contract are exercised:



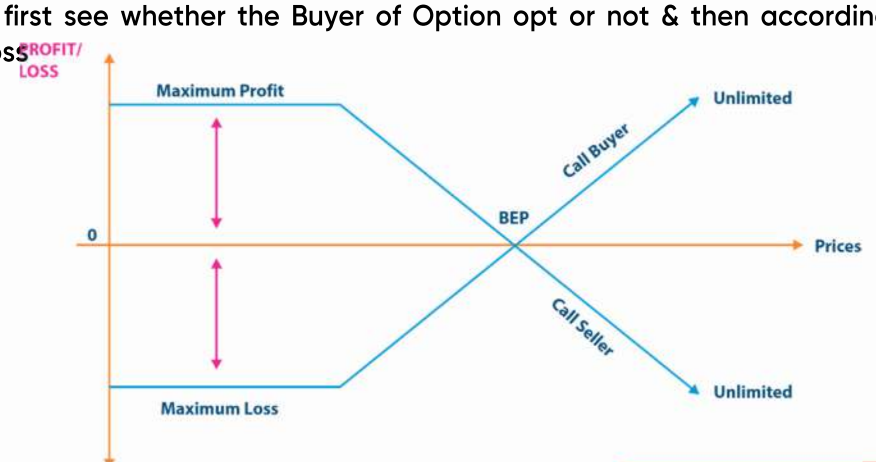
- When CMP > Strike Price → Call Buyer Exercise the Option.
- When CMP < Strike Price → Call Buyer will not Exercise the Option.

Right to Buy reliance share @ 1000 after 3 months	Obligation to Sell reliance share @ 1000 after 3 months if buyer approaches to do so.
LONG CALL	SHORT CALL
OP Paid	OP Received

Note :

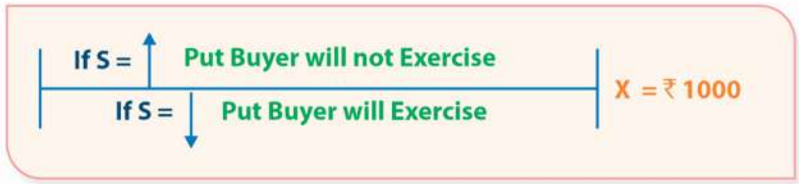
- The call holder will exercise the option whenever the stock's price exceeds the strike price at the expiration date.
- The sum of the profits between the Buyer and Seller of the call option is always Zero. Thus, Option trading is ZERO-SUM GAME. The long profits equal to the short losses.
- Position of a Call Seller will be just opposite of the position of Call Buyer.
- In this chapter, we first see whether the Buyer of Option opt or not & then accordingly we will calculate Profit & Loss

PAY-OFF DIAGRAM



LOS 3 : Put Option

When Put Option Contract are exercised:



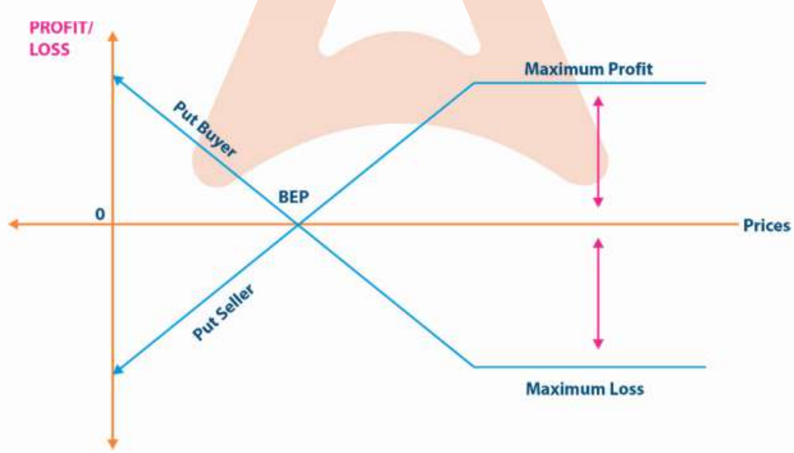
- When CMP > Strike Price → Put Buyer will not Exercise the Option.
- When CMP < Strike Price → Put Buyer will Exercise the Option.

Right to Sell reliance share @ 1000 after 3 months	Obligation to Buy reliance share @ 1000 after 3 months if buyer approaches to do so.
LONG PUT	SHORT PUT
OP Paid	OP Received

Note:

- Put Buyer will only exercise the option when actual market price is less the exercise price.
- Profit of Put Buyer = Loss of Put Seller & vice-versa. Trading Put Option is a Zero-Sum Game.

PAY-OFF DIAGRAM



Profit or Loss/ Pay off of call Option & Put Option

While calculating profit or loss, always consider option Premium,

Call Buyers (Long Call)	
If S – X > 0 Exercise the option Net Profit = S – X – OP	**If S – X < 0** Not Exercise Loss = Amount of Premium
Put Buyers (Long Put)	
If X – S >0 Exercise the option Net Profit = X – S –OP	**If X – S < 0** Not Exercise Loss = Amount of Premium

Calculation of Maximum Loss, Maximum Gain, Breakeven Point for Call & Put Option

Call Option		
	Maximum Loss	Maximum Gain
Buyer (Long)	Option Premium	Unlimited
Seller (Short)	Unlimited	Option Premium
Breakeven	X + Option Premium	

Put Option		
	Maximum Loss	Maximum Gain
Buyer (Long)	Option Premium	X – Option Premium
Seller (Short)	X – Option Premium	Option Premium
Breakeven	X - Option Premium	

QUESTION NO. 1C

The equity share of VCC Ltd is quoted at ₹ 210. A 3-month call option is available at a premium of ₹ 6 per Share and a 3-month put option is available at a premium of ₹ 5 per share. Ascertain the net pay offs to the option holder of a call option and a put option, given that:

(i) The strike price in both cases is ₹ 220; and

(ii) The share price on the Exercise day is ₹ 200, 210, 220, 230, 240.

Also indicate the price range at which the call and the put options maybe gainfully exercised.

Solution :

Q.1C (8 marks)

X = 220

OP → Call = ₹6
Put = ₹5

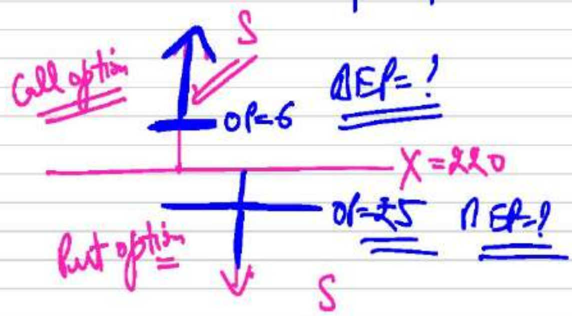
S = 200 to 240
S = Underlying Asset Price
X = Exercise Price

1) For long Call :-

S – X			GP	Net Pay-off NP = GP – OP
200 – 220	< 0	Not Exercise	0	0 – 6 ⇒ –6
210 – 220	< 0	Not Exercise	0	0 – 6 ⇒ –6
220 – 220	= 0	Not Exercise	0	0 – 6 ⇒ –6
230 – 220	> 0	Exercise	10	10 – 6 ⇒ 4
240 – 220	> 0	Exercise.	20	20 – 6 ⇒ 14

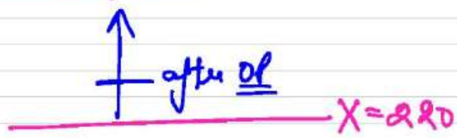
2) Long Put :– OP = ₹5

X – S		GP	Net Pay-off NP = GP – OP
220 – 200 > 0	Exercise	20	20 – 5 ⇒ 15
220 – 210 > 0	Exercise	10	10 – 5 ⇒ 5
220 – 220 = 0	Not Exercise	0	0 – 5 ⇒ –5
220 – 230 < 0	Not Exercise	0	0 – 5 ⇒ –5
220 – 240 < 0	Not Exercise	0	0 – 5 ⇒ –5



(b) Range of Price at which Call & Put option may be gainfully exercised :–

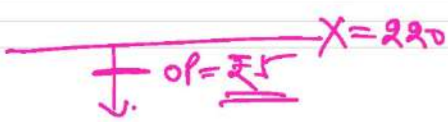
1) For Call option :–



BEP = X + OP
= 220 + 6 ⇒ ₹ 226

If Actual Price on Expiry > ₹226, then the Call option will be gainfully exercised.

(ii) For Put option :–



BEP ⇒ X − OP
= 220 − 5 ⇒ ₹ 215

If Actual Price on Expiry < ₹215, then the Put option will be gainfully exercised.

QUESTION NO. 1G

Mr. X established the following spread on the Delta Corporation's stock

(i) Purchased one 3-month call option with a premium of ₹ 30 and an exercise price of ₹ 550.

(ii) Purchased one 3-month put option with a premium of ₹ 5 and an exercise price of ₹ 450.

Delta Corporation's stock is currently selling at ₹ 500. Determine profit or loss, if the price of Delta Corporation's:

(i) Remains at ₹ 500 after 3 months.

(ii) Falls at ₹ 350 after 3 months.

(iii) Rises to ₹ 600. Assume the size option is 100 shares of Delta Corporation.

Solution :

Q1G

Long Call	Long Put
X = 550	X = 450
OP = 30	OP = 5

Lot size = 100

(i) If on Expiry S = 500 :−

Long Call	Long Put
S − X	X − S
500 − 550 < 0	450 − 500 < 0
Call Buyer will not exercise	Put Buyer will not exercise
GP = 0	GP = 0
NP = 0 − 30	NP = 0 − 5 = −5
⇒ −30 × 100	⇒ −5 × 100

⇒ −3000	⇒ −500

Investor's Position | Net Pay-off
⇒ −3000 − 500
⇒ **−3500**

(ii) If on Expiry S = 350 :−

Long Call	Long Put
S − X	X − S
350 − 550 < 0	450 − 350 > 0
Call Buyer will not exercise	Put Buyer will exercise
GP = 0	GP = 450 − 350
NP = 0 − 30	= 100
= −30 × 100	NP = 100 − 5
⇒ −3000	= +95 × 100
	⇒ +9500

Investor's Position | Net Pay-off
⇒ −3000 + 9500
⇒ **+6500**

(iii) If on Expiry S = 600 :−

Long Call	Long Put
S − X	X − S
600 − 550 > 0	450 − 600 < 0
Call Buyer will exercise	Put Buyer will not exercise.
GP = 600 − 550	GP ⇒ 0
= +50	

N/ = +50 − 30 | NP = 0 − 5
⇒ +20 × 100 | = −5 × 100
⇒ $\underline{\underline{+2000}}$ | = −500

Investor's position (Net pay-off) ⇒ +2000 − 500
= $\underline{\underline{+1500}}$

LOS 4 : Concept of Moneyness of an Option

Moneyness refers to whether an option is *In-the money or Out- of the money.*

Case I : If immediate exercise of the option would generate *a positive pay-off*, it is in the money

Case II : If immediate exercise would result in loss (*negative pay-off*), it is out of the money.

Case III : When current Asset Price = Exercise Price, exercise will generate *neither gain nor loss* and the option is at the money.

	Call Option		Put Option
Case 1	$S - X > 0$	**In-the-Money**	$X - S > 0$
Case 2	$S - X < 0$	**Out-of- the-Money**	$X - S < 0$
Case 3	$S = X$	**At-the-Money**	$X = S$

Note:

Do not consider option premium while Calculating Moneyness of the Option.

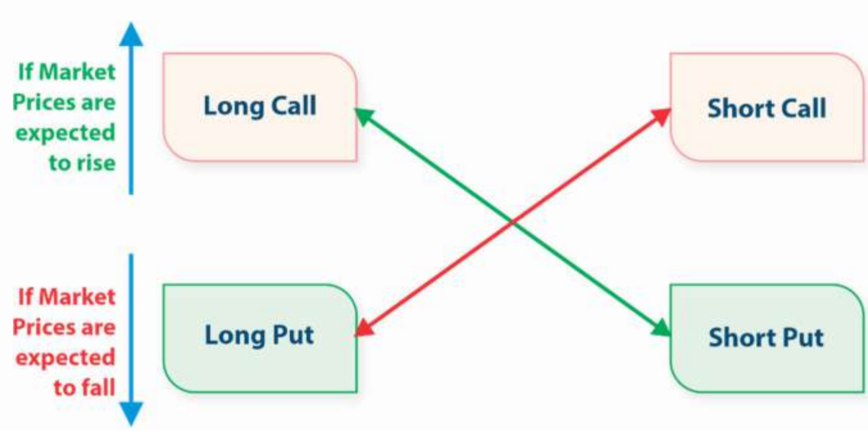
LOS 5 : European & American Options

American Option : American Option may be exercised at any time upto and including the contract's expiration date.

European Option : European Options can be exercised only on the contract's expiration date.

The name of the Option does not imply where the option trades – they are just names.

LOS 6 : Action to be taken under Option Market



LOS 7 : Intrinsic Value & Time Value of Option

Option value (Premium) can be divided into two parts:-

(i) Intrinsic Value

(ii) Time Value of an Option (Extrinsic Value)

Option Premium = Intrinsic Value + Time Value of Option

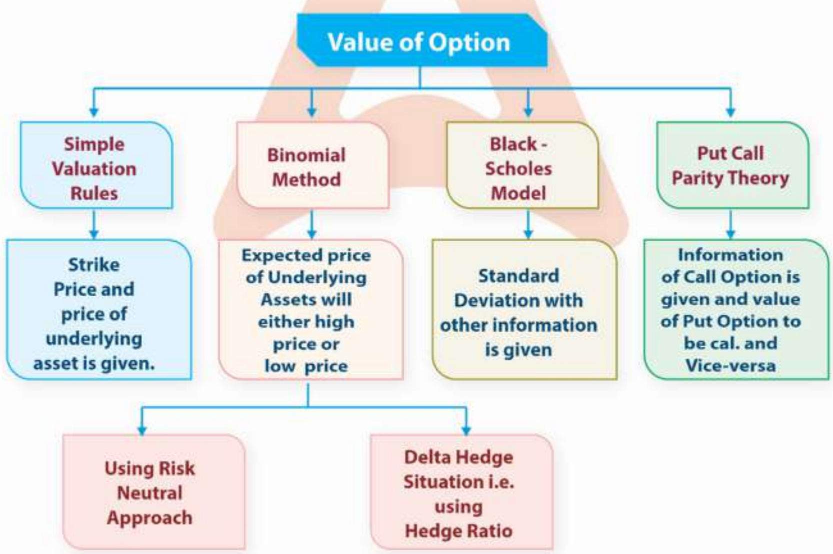
Intrinsic Value:

- An Option's intrinsic Value is the amount by which the option is In-the-money. It is the amount that the option owner would receive if the option were exercised.
- Intrinsic Value is the minimum amount charged by seller from buyer at the time of selling the right.
- An Option has ZERO Intrinsic Value if it is At-the-Money or Out-of-the-Money, regardless of whether it is a call or a Put Option.
- The Intrinsic Value of a Call Option is the greater of (S – X) or 0. That is
  C = Max [0, S –X]
- Similarly, the Intrinsic Value of a Put Option is (X - S) or 0. Whichever is greater. That is:
  P = Max [0, X - S]

Time Value of an Option (Extrinsic Value):

- The Time Value of an Option is the amount by which the option premium exceeds the intrinsic Value.
- Time Value of Option = Option Premium – Intrinsic Value
- When an Option reaches expiration there is no "Time" remaining and the time value is ZERO.
- The longer the time to expiration, the greater the time value and, other things equal, the greater the option's Premium (price).

Option Valuation



LOS 8 : Fair Option Premium/ Fair Value/ Fair Price of a Call on Expiration

Fair Premium of Call on Expiry = Maximum of [(S – X), 0]

Note:

Option Premium can never be Negative. It can be Zero or greater than Zero.

LOS 9 : Fair Option Premium/ Fair Value/ Fair Price of a Put on Expiration

Fair Premium of Put on Expiry = Maximum of [(X – S), 0]

LOS 10 : Fair Option Premium/ Theoretical Option Premium/ Price of a Call before Expiry or at the time of entering into contract or As on Today

Fair Premium of Call $= \left[S - \frac{X}{(1+RFR)^t}, 0\right]$ Max Or $= \left[S - \frac{X}{e^{rt}}, 0\right]$ Max

RFR (r) = Risk-free rate

T = Time to expiration

LOS 11 : Fair Option Premium/ Theoretical Option Premium/ Price of a Put before Expiry or at the time of entering into contract or As on Today

Fair Premium of Put $= \left[\frac{X}{(1+RFR)^T} - S, 0\right]$ Max Or $= \left[\frac{X}{e^{rt}} - S, 0\right]$ Max

LOS 12 : Expected Value of an Option on expiry

Under this approach, we will calculate the amount of Option premium on the basis of Probability.

Expected value of an option at Expiry = ∑ Value of Option at expiry × Probability

QUESTION NO. 7A

You as an investor had purchased a 4 month call option on the equity shares of X Ltd. of ₹ 10, of which the current market price is ₹ 132 and the exercise price ₹ 150. You expect the price to range between ₹ 120 to ₹ 190. The expected share price of X Ltd. and related probability is given below:

Expected Price (₹)	120	140	160	180	190
Probability	.05	.20	.50	.10	.15

Compute the following:

a) Expected Share price at the end of 4 months.

b) Value of Call Option at the end of 4 months, if the exercise price prevails.

c) In case the option is held to its maturity, what will be the expected value of the call option?

Solution :

Q.7A (3 times) (8 marks)

(a) Expected Share Price at the end of 4 months:-

Stock Price on Expiry	Prob.	Expected MPS
120	0.05	120×0.05
140	0.20	140×0.20
160	0.50	160×0.50

180 ✓	0.10	180 × 0.10
190 ✓	0.15	190 × 0.15
	1	₹ 160.50 ✓

(b) Value of Call option after 4 mths, if Exercise Price prevails:-

It means after 4 mths:-

$$\boxed{S = X = 150}$$

Value of Call on Expiry:- (V_c)

$\Rightarrow [S - X, 0]$ Max

$\Rightarrow [150 - 150, 0]$ Max.

$$\boxed{V_c \Rightarrow 0}$$ ✓

(c) Expected Value of Call option on Expiry:-

X = 150

Expected MPS = 160.50

$V_c = [S - X, 0]$ max

$\Rightarrow [160.50 - 150, 0]$ Max.

$\Rightarrow$ 10.50

Value of Call on Expiry. $[S - X, 0]$ Max.	Prob	Exp. Value of Call on Expiry
$[120 - 150, 0]$ Max. $\Rightarrow$ 0	.05	0 × 0.05
$[140 - 150, 0]$ Max. $\Rightarrow$ 0	.20	0 × 0.20

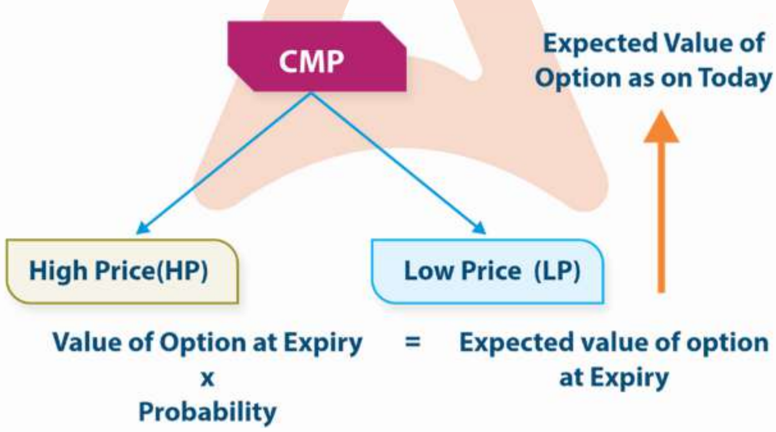
[160 − 150, 0] max. ⇒ 10   .50   10 × .50
[180 − 150, 0] max. ⇒ 30   .10   30 × .10
[190 − 150, 0] max. ⇒ 40   .15   40 × .15
1   14 ✓

Expected Value of Call on Expiry
= ₹14

Note:- If Nothing is mentioned about Call or Put option, than always assume it to be Call option.

Note:- If Position is missing i.e. Long or Short Position, than assume to be Long Position.

LOS 13 : Risk Neutral Approach for Call & Put Option(Binomial Model)



- Under this approach, we will calculate Fair Option Premium of Call & Put as on Today.
- The basic assumption of this model is that share price on expiry may be higher or may be lower than current price.

Step 1: Calculate Value of Call or Put as on expiry at high price & low price

Value of Call as on expiry = Max [(S – X),0]
Value of Put as on expiry = Max [(X – S), 0]

Step 2: Calculate Probability of High Price & Low Price

Probability of High Price $= \frac{\text{CMP } (1+r)^{n} - LP}{\text{HP} - \text{LP}}$ **or** Probability of High Price $= \frac{\text{CMP } (e^{rt}) - \text{LP}}{\text{HP} - \text{LP}}$

Step 3: Calculate expected Value/ Premium as on expiry by using Probability

Step 4: Calculate Premium as on Today

By Using normal Compounding = $\frac{\text{Expected Premium as on expiry}}{(1+r)^{t}}$

By Using Continuous Compounding = $\frac{\text{Expected Premium as on expiry}}{e^{rt}}$

QUESTION NO. 8B

The current market price of an equity share of Penchant Ltd is ₹ 420. Within a period of 3 months, the maximum and minimum price of it is expected to be ₹ 500 and ₹ 400 respectively. If the risk free rate of interest be 8% p.a., what should be the value of a 3 months Call option under the "Risk Neutral" method at the strike rate of ₹ 450?

Solution :

Q.8B (5 marks)

CMP = ₹420

3 mths

HP = 500 LP = 400

X = 450

r = 8% p.a.

Step 1:- Cal. Value of Call option at Expiry

$V_c = [S - X, 0]\max.$

At HP ⇒ $[500 - 450, 0]\max$ ⇒ 50

At LP ⇒ $[400 - 450, 0]\max$ ⇒ 0

Step 2: Cal. Probability of High Price & Low Price:

			Exp. MPS
HP	500	p	500 × p
LP	400	1 − p	400 (1 − p)
		1	500p + 400 − 400p

$$CMP = 420 = \frac{500p + 400 - 400p}{(1 + .08 \times \frac{3}{12})}$$

$$\Rightarrow 420 = \frac{500p - 400p + 400}{(1 + .02)}$$

$$\text{Prob of High Price} = \frac{CMP\, e^{rt} / (1+r)^t - LP}{HP - LP}$$

$$p \Rightarrow \frac{420(1 + .02) - 400}{500 - 400}$$

$$\text{Prob of HP} \Rightarrow \frac{428.40 - 400}{100}$$

$$\Rightarrow \frac{28.40}{100} \Rightarrow \underline{\underline{0.2840}}$$

$$\text{Prob. of LP} \Rightarrow 1 - 0.2840 \Rightarrow 0.7160$$

Step 3:- Cal. Expected Value of Call on Expiry:-

V_c on Expiry		Prob.	Exp Value at Expiry
At HP	50	0.2840	$50 \times .2840$
At LP	0	0.7160	$0 \times .7160$
			14.20

Step 4:- Cal. Expected Value of Call as on Today:-

$$\frac{14.20}{\left(1 + .08 \times \frac{3}{12}\right)}$$

$$V_c \text{ as on Today} \Rightarrow \frac{14.20}{(1 + .02)} \Rightarrow \underline{\underline{13.92}}$$

FOP = 13.92 ⟶ AOP = ?

$\boxed{\text{Exchange}}$

QUESTION NO. 8E

Spot Price is ₹ 60. A One year European Call Option is being quoted in the market at option premium of ₹ 15 with Exercise Price of ₹ 55. Risk Free Rate of return is 12% p.a.c.c. The stock can either rise or fall after a year. If it can fall by 30% by what percentage (%) can it rise?

Solution :

Q.8E (RTP) V.Imp.

OP = 15

SP = 60

X = 55

r = 12% p.a.c.c.

1 year

p = ?

↓30%

1−p = ?

HP = ?

LP = 42

Step 1:- Cal. Value of Call at HP & LP on Expiry:-

$$V_c = [S - X, 0]\ \text{Max.}$$

$$\text{At HP} = [HP - 55, 0]\ \text{Max} \Rightarrow HP - 55$$

$$\text{At LP} \Rightarrow [42 - 55, 0]\ \text{Max} \Rightarrow 0$$

Step 2: Cal. Prob. of HP & LP:-

		Exp. MPS
At HP	p	$HP \times p$
At LP = 42	$1-p$	$42(1-p)$
		$HP \times P + 42(1-p)$

$$CMP = 60 \quad \frac{HP \times p + 42 - 42p}{e^{rt}}$$

$$60 = \frac{HP \times p + 42 - 42p}{e^{.12 \times 1}}$$

$$\Rightarrow 60 \times e^{.12} = (HP - 42)p + 42$$

$$\Rightarrow p \Rightarrow \frac{60 \times e^{.12} - 42}{HP - 42}$$

$\Rightarrow$ Prob. of HP $= \dfrac{60 \times 1.1275 - 42}{HP - 42}$

$\Rightarrow$ $\boxed{P \Rightarrow \dfrac{25.65}{HP - 42}}$ ----(i)

Step 3:- Cal. Exp. Value of Call at Expiry.

V_c at Exp.	Prob	Exp Value of Call at Expiry
HP − 55	P	P(HP − 55)
0	1 − P	(1 − P)(0)
		P(HP − 55)

Step 4: Cal. Expected Value of Call as on today:-

OP = 15 ← 1 year → $\dfrac{P(HP-55)}{e^{rt}}$

$15 = \dfrac{P(HP-55)}{e^{.12 \times 1}}$

$\Rightarrow 15 \times e^{.12} = P(HP-55)$

$\Rightarrow 15 \times 1.1275 = P(HP-55)$

$\Rightarrow 16.9125 = P(HP-55)$

$\Rightarrow$ $\boxed{P = \dfrac{16.9125}{HP-55}}$ ----(ii)

Equate both equations:-

$$\frac{16.9125}{HP-55} = \frac{25.65}{HP-42}$$

$$\Rightarrow 16.9125\,HP - 710.325 = 25.65\,HP - 1410.75$$

$$\Rightarrow 25.65\,HP - 16.9125\,HP = 1410.75 - 710.325$$

$$\Rightarrow 8.7375\,HP = 700.425$$

$$\boxed{HP \Rightarrow 80.16}$$

Put in eq. (ii)

$$\text{Prob. of } HP = \frac{16.9125}{HP-55} = \frac{16.9125}{80.16-55} \Rightarrow 0.6722$$

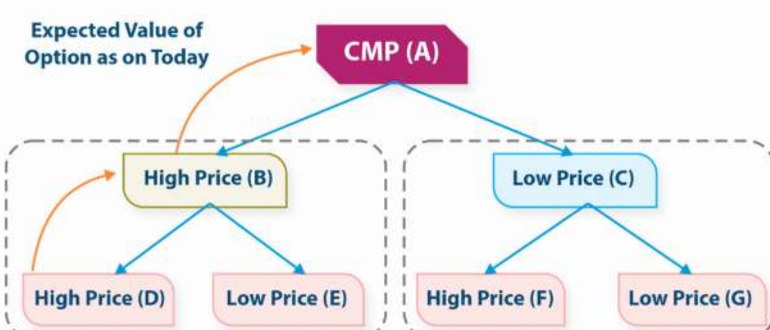
% rise in Price:-

$$\Rightarrow \frac{80.16-60}{60} \times 100$$

$$\Rightarrow 33.60\%$$

LOS 14 : Two Period Binomial Model

We divide the option period into two equal parts and we are provided with binomial projections for each path. We then calculate value of the option on maturity. We then apply backward induction technique to compute the value of option at each nodes.

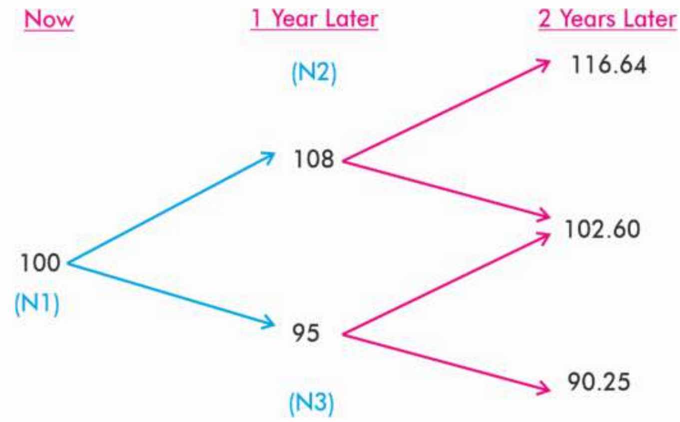


For American Options :

Max (Intrinsic Value, Calculated Value)

QUESTION NO. 9C

A two year tree for a share of stock in ABC Ltd., is as follows:



Consider a two years American call option on the stock of ABC Ltd., with a strike price of ₹ 98. The current price of the stock is ₹ 100. Risk free return is 5 per cent per annum with a continuous compounding and $e^{0.05}$ = 1.05127.

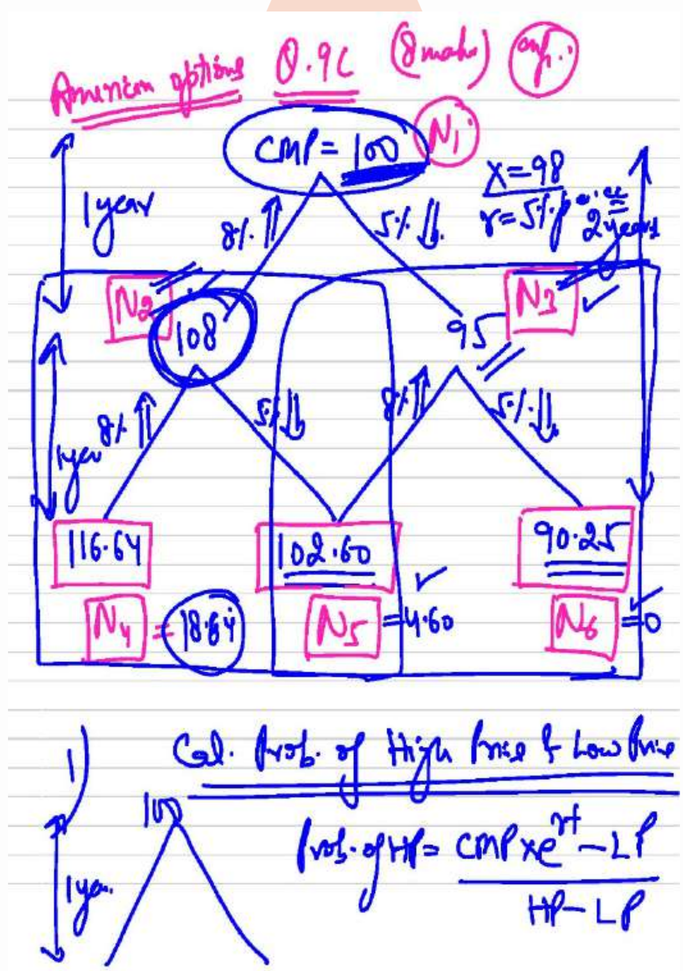
Assume two time periods of one year each.

Using the Binomial Model, calculate:

(i) The probability of price moving up and down;

(ii) Expected pay offs at each nodes i.e. N1, N2 and N3 (round off upto 2 decimal points).

Solution :



108 95 $\Rightarrow \dfrac{100 \times e^{.05 \times 1} - 95}{108 - 95}$

Prob. of HP $\Rightarrow \dfrac{100 \times 1.05127 - 95}{13}$

Prob. of HP $\Rightarrow$ 0.78
Prob. of LP $\Rightarrow 1 - 0.78$
$\Rightarrow$ 0.22

2) At Nodes N_4, N_5 & N_6

V_c at Expiry = $[S - X, 0]$ max.

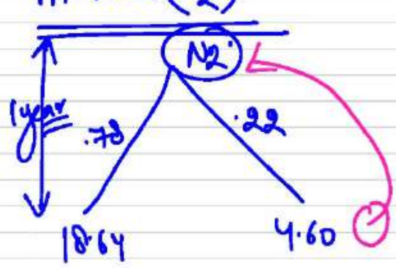
N_4 $[116.64 - 98, 0]$ max. $\Rightarrow$ 18.64

N_5 $[102.60 - 98, 0]$ max. $\Rightarrow$ 4.60

N_6 $[90.25 - 98, 0]$ max $\Rightarrow$ 0

$[CV, IV]$ max.

At Node (N_2)



$\Rightarrow \dfrac{18.64 \times .78 + 4.60 \times .22}{e^{.05 \times 1}}$

$\Rightarrow \dfrac{15.5512}{1.05127} \Rightarrow$ 14.79 $\rightarrow$ N_2

For American option:-

Derivatives Analysis & Valuation (Options)

$[CV, IV]$ max.

$CV = 14.79$ (Calculated Value)

$IV \Rightarrow [S - X, 0]$ max.

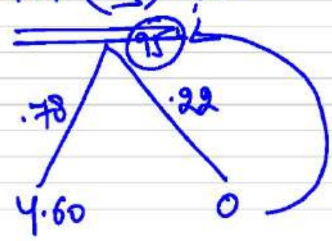
$= [108 - 98, 0]$ max $\Rightarrow \underline{10}$

$[CV, IV]$ max.

$\Rightarrow [14.79, 10]$ max $\Rightarrow \underline{14.79}$

N_2

For (N_3) :-



$$\Rightarrow \frac{4.60 \times .78 + 0 \times .22}{e^{.05 \times 1} = 1.05127}$$

$\boxed{N_3} \Rightarrow \underline{3.41}$

For American option:-

$CV = 3.41$

$IV = [S - X, 0]$ max.

$= [95 - 98, 0]$ max $\Rightarrow 0$

$\boxed{N_3} \Rightarrow [CV, IV]$ max.

$\Rightarrow [3.41, 0]$ max $\Rightarrow \underline{3.41}$

For $\boxed{N_1}$:-

CMP = 100

.78   .22

$\boxed{14.79}$ $\boxed{3.41}$

$$N_1 \Rightarrow \frac{14.79 \times .78 + 3.41 \times .22}{e^{.05 \times 1} = 1.05127}$$

$$\boxed{N_1 \Rightarrow \underline{\underline{11.69}}}$$

For American option:-

$[CV, IV]_{max.}$

$CV = 11.69$

$IV = [S - X, 0]_{max.}$

$\Rightarrow [180 - 98, 0]_{max} \Rightarrow 2$

$N_1 = [CV, IV]_{max.}$

$= [11.69, 2]_{max.}$

$$\boxed{N_1 \Rightarrow 11.69}$$

LOS 15 : Put Call Parity Theory (PCPT)

Put Call Parity is based on Pay-offs of two portfolio combination, a fiduciary call and a protective put.

Fiduciary Call

A Fiduciary Call is a combination of a pure-discount, riskless bond that pays X at maturity and a Call.

Protective Put

A Protective Put is a share of stock together with a put option on the stock.

$$\textbf{PCPT} \rightarrow \textbf{Value of Call} + \frac{X}{(1+RFR)^T} = \textbf{Value of Put} + S$$

Protective Put					
If on Maturity S > X			If on Maturity S < X		
Put option is lapse i.e. pay off	=	NIL	Put option is exercise i.e. pay off	=	X – S
Stock is sold in the Market	=	S	Stock is sold in the Market	=	S
		S			X

Fiduciary Call					
If on Maturity S > X			If on Maturity S < X		
Call option is exercise i.e. pay off	=	S – X	Call option is lapse i.e. pay off	=	NIL
Bond is sold in the Market	=	X	Bond is sold in the Market	=	X
		S			X

Through this theory, we can calculate either Value of Call or Value of Put provided other Three information is given.

Assumptions:

- Exercise Price of both Call & Put Option are same.
- Maturity Period of both Call & Put are Same.

LOS 16 : Put - Call Parity Theory → ARBITRAGE

As per PCPT,

$$\text{Value of Call} + \frac{X}{(1+RFR)^T} \qquad\qquad \text{Value of Put} + S$$

LHS RHS

<u>Case I :</u> If LHS = RHS, arbitrage is not possible.

<u>Case II :</u> If LHS ≠ RHS, arbitrage is possible.

A. If LHS > RHS, Call is Over-Valued & Put is Under-Valued

Option Market		Cash Market	Net Amount
Short Call i.e. Obligation to sell & Option Premium Received	Long Put i.e. Right to sell & Option Premium Paid	Buy i.e. Buy one share	Borrow S + P - C

B. If LHS < RHS, Call is Under-Valued & Put is Over-Valued

Option Market		Cash Market	Net Amount
Long Call i.e. Right to Buy & Option Premium Paid	Short Put i.e. Obligation to buy & Option Premium Received	Sell i.e. Sell one share	Invest S + P - C

QUESTION NO. 11B

The following table provides the prices of options on equity shares of X Ltd. and Y Ltd.
The risk free interest is 9%. You as a financial planner are required to spot any mispricing in the quotations of option premium and stock prices. Suppose, if you find any such mispricing then how you can take advantage of this pricing position.

Share	Time to exercise	Exercise price (₹)	Share price (₹)	Call Price (₹)	Put price (₹)
X Ltd.	6 Months	100	160	56	4
Y Ltd.	3 Months	80	100	26	2

Solution :

Q.11B

X Ltd.

$t = 6$ mths $\quad r = 9\%$ p.a.

$X = 100 \quad S = 160$

$C = 56 \quad P = 4$

As per PCPT :-

$$C + \frac{X}{(1+r)^n} \qquad P + S$$

⇓ L.H.S $\qquad$ ⇓ R.H.S

$$56 + \frac{100}{(1+.09\times\frac{6}{12})} \quad \Big| \quad 4 + 160$$

$$\Rightarrow 56 + 95.69 \quad \Big| \quad 164$$

$$\boxed{151.69 < 164}$$

⇓ ⇓

$$\boxed{L.H.S < R.H.S}$$

⇓ Call $\qquad$ ⇓ Put

Call is Under-valued
&
Put is over-valued

Strategy as on today:-

Options Mkt.		Cash Mkt.	Invest Net Amount
Long Call i.e. Right to Buy ⇓ OP Paid	Short Put (i.e. Obligation to Buy ⇓ OP Rec	Sell 1 share @ 160	160 + 4 − 56 ⇒ 108

@56 ✓	@4 ✓	for 6 mths
X = 100	X = 100	

Arbitrage Profit [On Expiry]
i.e. after 6 mths

Delivery Based Settlement

Case I: If S > X (↑) on Expiry:-

⇒ Call option will be exercise & buy share at X = 100 — 100

⇒ Put option will be lapse NIL

⇒ Received from Bank with Interest:- +112.86

$108 \left(1 + .09 \times \frac{6}{12}\right)$

Arbitrage Profit (+) 12.86

Case II: If S < X i.e. (↓) on Expiry:-

⇒ Call option will be lapse NIL

⇒ Put option will be exercised against us at X = 100 (he sells & we buy) —100

⇒ Received from Bank with Interest +112.86

$108 \left(1 + .09 \times \frac{6}{12}\right)$

Arbitrage Profit (+) 12.86

Y Ltd:

$t = 3$ mths $\quad X = 80$

$r = 9\%$ p.a. $\quad S = 100$

$C = 26 \quad P = 2$

As per PCPT:-

$C + \frac{X}{(1+r)^n}$	$P + S$
$26 + \frac{80}{(1 + .09 \times \frac{3}{12})}$	$2 + 100$
$26 + 78.24$	102

$$\boxed{104.24 > 102}$$

i.e. L.H.S > R.H.S

Call is over-valued & Put is Under-valued

Strategy as on today:-

Opt. Mkt.

Short Call	Long Put	Cash Mkt.	Borrow Net Amt.
i.e. obligation to sell ⇓ of Rec. @26	i.e. right to sell ⇓ of Paid @ 2	Buy 1 Share ⇓ @ 100	100 + 2 − 26 = 76
X = 80	X = 80		

Cal. of Arbitrage Profit:- [on Expiry]

i.e. after 3 mths

Delivery Based Settlement

Case I :- If S > X (↑) on Expiry.

⇒ Call option will be exercised against us at X = 80 (he buys & we sells) +80

⇒ Put option will be lapse NIL

⇒ Repayment to Bank with Interest (−) 77.71

$76\left(1 + .09 \times \frac{3}{12}\right)$

Arbitrage Profit (+) 2.29

Case II :- If S < X (↓) on Expiry

⇒ Call option will be lapse NIL

⇒ Put option will be exercised at X = 80 (i.e. we sell & they buy) +80

⇒ Repayment to Bank with Interest (−) 77.71

$76\left[1 + .09 \times \frac{3}{12}\right]$

Arbitrage Profit (+) 2.29

LOS 17 : Option Strategies

Combination of Call & Put is known as OPTION STRATEGIES.

Types of Option Strategies:

Some important Option Strategies are as follows:

1. Straddle Position
2. Strangle Strategy
3. Strip Strategy
4. Strap Strategy
5. Butterfly Spread

1. Straddle Position :

Straddle may be of 2 types :

Long Straddle	Short Straddle
Buy a Call and Buy a Put on the same stock with both the options having the same exercise price.	Sell a Call and Sell a Put with same exercise price and same exercise date.
Option: Buy One Call and Buy One Put Exercise Date: Same of Both Strike Price / Exercise Price: Same of Both	Option: Sell One Call and Sell One Put Exercise Date: Same of Both Strike Price / Exercise Price: Same of Both
<u>Note:</u> A Long Straddle investor pays premium on both Call & Put.	<u>Note:</u> A Short Straddle investor receive premium on both Call and Put.

Note:

- When an investor is not sure whether the price will go up or go down, then in such case we should create a straddle position.
- If Question is Silent, always assume Long Straddle.

2. Strangle Strategy :

- An option strategy, where the investor holds a position in both a call and a put with different strike prices but with the same maturity and underlying asset is called Strangles Strategy.
- Selling a call option and a put option is called seller of strangle (i.e. Short Strangle).
- Buying a call and a put is called Buyer of Strangle (i.e. Long Strangle).
- If there is a large price movement in the near future but unsure of which way the price movement will be, this is a Good Strategy.

3. Strip Strategy (Bear Strategy)	4. Strap Strategy (Bull Strategy)
Buy Two Put and Buy One Call Option of the same stock at the same exercise price and for the same period.	Buy Two Calls and Buy One Put when the buyer feels that the stock is more likely to rise Steeply than to fall.
Strip Position is applicable when decrease in price is more likely than increase.	Strap Position is applicable when increase in price is more likely than decrease.
Option: Buy Two Put and Buy One Call Exercise Date: Same of Both Strike Price/ Exercise Price: Same of Both	Option: Buy Two Calls and Buy One Put Exercise Date: Same of Both Strike Price/ Exercise Price: Same of Both

5. Butterfly Spread :

In Butterfly spread position, an investor will undertake 4 call option with respect to 3 different strike price or exercise price.

<u>It can be constructed in following manner:</u>

- Buy One Call Option at High exercise Price (S1)
- Buy One Call Option at Low exercise Price (S2)
- Sell two Call Option $\left(\frac{S_1 + S_2}{2}\right)$

LOS 18 : Binomial Model (Delta Hedging / Perfectly Hedged technique) for Call Writer

Under this concept, we will calculate option premium for call option.

It is assumed that expected price on expiry may be greater than Current Market Price or less than Current Market Price.

CMP

High Price(HP) | Low Price (LP)

Steps involved:

Step 1: Compute the Option Value on Expiry Date at high price and at low price

Value of Call as on expiry = Max [(S – X),0]

Step 2: Buy 'Delta' No. of shares 'Δ' at Current Market Price as on Today. Delta 'Δ' also known as Hedge Ratio.

$$\text{Hedge Ratio or '}\Delta\text{'} = \frac{\text{Change in Option Premium}}{\text{Change in Price of Underlying Asset}}$$

OR

$$= \frac{\text{Value of call on expiry at High Price} - \text{Value of call on expiry at Low Price}}{\text{High Price} - \text{Low Price}}$$

Step 3: Construct a Delta Hedge Portfolio i.e. Risk-less portfolio / Perfectly Hedge Portfolio

Sell one call option i.e. Short Call ,Buy Delta no. of shares and borrow net amount.

Step 4: Borrow the net Amount required for the above steps

$$B = \frac{1}{1+r}\left[\Delta \times \text{HP} - V_C\right]$$

Or

$$B = \frac{1}{1+r}\left[\Delta \times \text{LP} - V_C\right]$$

Where r = rate of interest adjusted for period

Step 5: Calculate Value of call as on today

Borrowed Amount = Amount required to purchase of share – Option Premium Received

B = Δ × CMP – OP

Or

(Option Premium = Δ × CMP – Borrowed Amount)

Note: Calculation of Cash flow Position/ Value of holding after 1 year

- If on Maturity Actual Market Price is HP

  Cash Flow = Δ × HP – V_C

- If on Maturity Actual Market Price is S_2

  Cash Flow = Δ × LP – V_C

  Cash Flow at HP and LP will always be same.

Meaning of perfectly hedge position under binomial model

Perfectly hedge position means Profit or Loss will be Zero or NIL. It can be achieved by buying Δ shares, sell one call option and borrowing the required amount.

Delta is the number of shares which makes the portfolio perfectly hedged i.e. whether the stock price on maturity goes up or decline, the value of portfolio doesn't vary i.e. our profit and loss position will be Zero.

QUESTION NO. 13D

AB Ltd.'s equity shares are presently selling at a price of ₹ 500 each. An investor is interested in purchasing AB Ltd.'s shares. The investor expects that there is a 70% chance that the price will go up to ₹ 650 or a 30% chance that it will go down to ₹ 450, three months from now. There is a call option on the shares of the firm that can be exercised only at the end of three months at an exercise price of ₹ 550.

Calculate the following:

(i) If the investor wants a perfect hedge, what combination of the share and option should he select ?

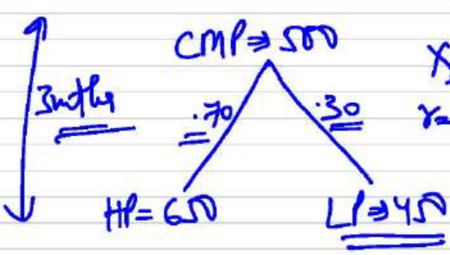
(ii) Explain how the investor will be able to maintain identical position regardless of the share price.

(iii) If the risk-free rate of return is 5% for the three months period, what is the value of the option at the beginning of the period ?

(iv) What is the expected return on the option?

Solution :

Q.13D ✓ (8marks)
2times



X = 550
r = 5% for 3mths

Soln:- W.No.1

1) Cal. Value of Call at Expiry at HP & at LP:

[S − X, 0] Max.

At HP:- [650 − 550, 0] Max ⇒ 100

At LP:- [450 − 550, 0] Max ⇒ 0

2) Cal. Delta / Hedge Ratio:-

$$\Delta = \frac{V_c \text{ at } HP - V_c \text{ at } LP}{HP - LP}$$

$$\Rightarrow \frac{100 - 0}{650 - 450} \Rightarrow \frac{100}{200}$$

$$\Delta \Rightarrow 0.50$$

(i) ⇒ Investor should sell one Call & Buy 0.50 shares for perfect hedging.

Option Mkt	Cash Mkt	Net Amt
Short Call ⇓ OP Rec.	Buy Δ = .50 No. of Shares @ CMP = 500	Borrow

$$\boxed{B = \Delta \times CMP - OP\ Rec./V_c}$$

W.No.2:- Cal. of Borrowed Amount as on today:-

$B_0 \Rightarrow$

At HP ⇒ $B = \Delta \times HP - V_c$

$= .50 \times 650 - 100$

$B \Rightarrow 225$

or

At LP ⇒ $B = \Delta \times LP - V_c$

$= 0.50 \times 450 - 0$

$\Rightarrow 225$

Borrowed Amt. as on today:-

3 mths

$$\frac{225}{(1 + .05)}$$

$B_0 \Rightarrow 214.29$

Value of Call as on today:-

$B \Rightarrow \Delta \times CMP - OP\ Pre.\ /\ V_c$

$214.29 = 0.50 \times 500 - V_c$

Value of Call as on today $\Rightarrow \boxed{35.71}$

(ii) Investor will be able to maintain his position if he purchase 0.50 Shares for 1 Call written:-

(a) If Price on Expiry goes upto ₹650:- $X = 550$

Call Buyer will exercise against OS:-

$S - X > 0$ $\Rightarrow -100$

$650 - 550 > 0$

Pay-off (Loss on Short Call)

Sell Share in the Market:- $\Rightarrow +325$

$650 \times .50$

Cashflow Position after 3 mths +225

(b) If Price on Expiry falls to ₹450:- $X = 550$

Call option will be lapse   NIL

Sell Stock in the market   +225

0.50×450

Cash flow position after 3 mths +225

(iii) Value of Call at the Beginning of the period

⇒ ₹ 35.71 (W.No.2)

(iv) Cal. of Expected Return of an option:-

V_c ⇒ ₹ 35.71

Expected V_c at Expiry

	V_c	Prob	Exp Value
At HP	100	.70	100×.70
At LP	0	.30	0×.30
			70

% Return:-

35.71 — 70

$$\Rightarrow \frac{70-35.71}{35.71} \times 100$$

⇒ 96.02% for 3 mths

% Return p.a.

$$\Rightarrow \frac{96.02\%}{3} \times 12$$

⇒ 384.08% p.a.

LOS 19 : Black & Scholes Model

The BSM Model uses five variables to value a call option:

1. The price of the Underlying Stock (S)
2. The exercise price of the option (X)
3. The time remaining to the expiration of the option (t)

4. The riskless rate of return (r)
5. The volatility of the underlying stock price (σ)

Assumptions of BSM Model :

- The price of underlying asset follows a log normal distribution
- Markets are frictionless. There is no taxes, no transaction cost, no restriction on short sale.
- The option valued are European options.
- Risk Free continuous compounding interest rate is known and constant.
- Annualized volatility of the stock is known and constant.
- The underlying asset has no cash flow as dividend, coupons etc.

For Call:

$$\text{Value of a Call Option/ Premium on Call} = S \times N(d_1) - \frac{X}{e^{rt}} \times N(d_2)$$

Calculation of d_1 and d_2

$$d_1 = \frac{l_n\left[\frac{S}{X}\right] + [r + 0.50\sigma^2] \times t}{\sigma \times \sqrt{t}}$$

$$d_2 = d1 - \sigma\sqrt{t}$$

Or

$$d_2 = \frac{l_n\left[\frac{S}{X}\right] + [r - 0.50\sigma^2] \times t}{\sigma \times \sqrt{t}}$$

where

S = Current Market Price
X = Exercise Price
r = risk-free interest rate
t = time until option expiration
σ = Standard Deviation of Continuously Compounded annual return

For Put:

$$\text{Value of a Put Option/ Premium on Put} = \frac{X}{e^{rt}} \times [1 - N(d_2)] - S \times [1 - N(d_1)]$$

Calculation of $N(d_1)$ & $N(d_2)$

$N(d_1)$ and $N(d_2)$ can be calculated by using 2 steps:

1. Calculate the value of d_1 and d_2
2. Calculate $N(d_1)$ and $N(d_2)$ by using

Method 1: $N(d_1)$ and $N(d_2)$ table
Method 2: Z- table or Normal Distribution Curve
Method 1:

Example 1:	Example 2:
$d_1 = 0.70$, $d_2 = 0.50$	$d_1 = -1.31$, $d_2 = -1.49$
$N(d_1) = N(0.70) = 0.758036$	$N(d_1) = N(-1.31) = 0.095098$
$N(d_2) = N(0.50) = 0.691462$	$N(d_2) = N(-1.49) = 0.068112$

Example 3:

d1 = 0.4539
0.45 = 0.673645
0.46 = 0.677242

When d1 increases by 0.01, the value increases by 0.003597

When d1 increases by 1, the value increases by $\frac{0.003597}{.01}$

When d1 increases by 0.0039, the value increases by $\frac{0.003597}{.01} \times 0.0039 = 0.00140283$

$N(d_1) = N(0.4539)$

= 0.673645 + 0.00140283

= 0.675047

Method 2: Using Normal Distribution Table or Z-Table

Example1: d1 = 0.70 , d2 = 0.50 d1 = 0.70	Example2: d1 = - 1.31 , d2 = - 1.49 d1 = - 1.31
Z-value of 0.70 (Through table) = 0.258036 N(d1) = N(0.70) = 0.50 + 0.258036 = 0.758036	Z-value of 1.31 (Through table) = 0.404902 N(d1) = N(- 1.31) = 0.50 - 0.404902 = 0.095098
Z-value of 0.50 = 0.191462 N(d2) = N(0.50) = 0.50 + 0.191462 = 0.691462	d2 = - 1.49 Z-value of 1.49 (Through table) = 0.431888 N(d2) = N(- 1.49) = 0.50 - 0.431888 = 0.068112

Calculation of Natural log (l_n)

Example1:	Example2:
0.75 Natural log (0.75) ln (0.75) = -0.28768	1.24 ln (1.24) = 0.21511

QUESTION NO. 14B

From the following data for certain stock, find the value of a call option:

Price of stock now	=	₹ 80
Exercise price	=	₹ 75
Standard deviation of continuously compounded annual return	=	0.40
Maturity period	=	6 months
Annual interest rate	=	12%

Given

Number of S.D. from Mean, (z)	Area of the left or right (one tail)
0.25	0.4013
0.30	0.3821
0.55	0.2912
0.60	0.2743
$e^{0.12 \times 0.5}$ = 1.062	
ln 1.0667 = 0.0646	

Solution :

Q.14B (RTP) V.V.Imp.

$S = 80$ $X = 75$ $\sigma = 0.40$

$r = 12\%$ p.a.c.c $t = 6$ mths

Soln. Value of call option as on today using BSM:-

$$\Rightarrow S\,N(d_1) - \frac{X}{e^{rt}} N(d_2)$$

$$d_1 = \frac{\ln\left[\frac{S}{X}\right] + [r + .50\,\sigma^2]\,t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

W.No.1 Cal. of d_1 & d_2

$$d_1 = \frac{\ln\left[\frac{S}{X}\right] + [r + .50\,\sigma^2]\,t}{\sigma\sqrt{t}}$$

$$\Rightarrow \frac{\ln\left[\frac{80}{75}\right] + [.12 + .50 \times .40^2]\frac{6}{12}}{.40\sqrt{\frac{6}{12}}}$$

$$\Rightarrow \frac{\ln[1.0667] + [0.12 + 0.08]\,.50}{.40 \times 0.7071}$$

$$\Rightarrow \frac{0.0646 + 0.10}{0.2828} \Rightarrow 0.5820$$

$$\boxed{d_1 = 0.5820}$$

$d_2 = d_1 - \sigma\sqrt{t}$

$\Rightarrow 0.5820 - .2828$

$\Rightarrow \underline{\underline{0.2992}}$

W.No. Cal. of N(d1) & N(d2)

$N(d_1) = N(0.5820)$

Note: To Cal. Cumulative Values :-

(*) 1 − One tail Value

Cumulative

0.25	1 − .4013 ⇒	0.5987
0.30	1 − .3821 ⇒	0.6179
0.55	1 − .2912 ⇒	0.7088
0.60	1 − .2743 ⇒	0.7257

Hint: If two tail Values are given :-

.25 ⇒ .4013 × 2 ⇒ .8026 ÷ 2

.30 ⇒ .3821 × 2 = .7642 ÷ 2

Finally: Cal. $N(d_1) \Rightarrow N(0.5820)$

.0320 Extra Val

0.55 ⇒ .7088

0.60 ⇒ .7257

0.05   0.0169

$1 \Rightarrow \frac{0.0169}{0.05} \times 0.0320$

$\Rightarrow \underline{\underline{0.0108}}$

$N(0.5820) \Rightarrow 0.7088 + 0.0108$

$N(d_1) \Rightarrow 0.7196$

$N(d_2) \Rightarrow N(0.2992)$

$.25 \Rightarrow .5987 \rightarrow .0492$

$.30 \Rightarrow 0.6179$

$0.05 \qquad 0.0192$

$1 \Rightarrow \frac{0.0192}{0.05} \times .0492$

$\Rightarrow 0.0189$

$N(d_2) \Rightarrow 0.5987 + 0.0189$

$\boxed{N(d_2) \Rightarrow 0.6176}$

Final Answer:-

$S > X$

$V_c = \left[S\,N(d_1) - \frac{X}{e^{rt}} N(d_2) \right]$

$\Rightarrow 80 \times 0.7196 - \frac{75}{e^{.12 \times \frac{6}{12}}} \times .6176$

$V_c \Rightarrow 57.568 - \frac{75}{1.062} \times .6176$

$\Rightarrow 57.568 - 43.62$

$\Rightarrow ₹13.95$

Extra Part:-

$V_p \Rightarrow \frac{X}{e^{rt}}[1 - N(d_2)] - S[1 - N(d_1)]$

$\Rightarrow \frac{75}{e^{.12 \times \frac{6}{12}}}[1 - .6176] - 80 \times [1 - .7196]$

$\rightarrow 1.062$

$V_P \Rightarrow 70.62 \times .3824 - 80 \times 0.2807$

$V_P \Rightarrow 27.01 - 22.43$

$V_P \Rightarrow 4.58$

OR $V_P \rightarrow$ Using PCPT

$$C + \frac{X}{e^{rt}} = P + S$$

$$13.95 + \frac{75}{e^{.12 \times \frac{6}{12}}} = P + 80$$

$\to 1.062$

$\Rightarrow 13.95 + 70.62 = P + 80$

$\Rightarrow$ $\boxed{\text{Value of Put} \Rightarrow ₹4.57}$

LOS 20 : BSM → when dividend amount is given in the question

Adjust Spot Price (S) or CMP as [Spot Price – PV of Dividend Income]

Value of a Call Option = $[S - \text{PV of Dividend Income}] \times N(d_1) - \frac{X}{e^{rt}} \times N(d_2)$

$$d_1 = \frac{l_n\left[\frac{S - \text{PV of Dividend Income}}{X}\right] + \left[r + 0.50\sigma^2\right] \times t}{\sigma \times \sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

LOS 21 : Put-Call Ratio

$$\text{Put- Call Ratio} = \frac{\text{Volume of Put Traded}}{\text{Volume of Call Traded}}$$

The ratio of the volume of put options traded to the volume of Call options traded, which is used as an indicator of investor's sentiment (bullish or bearish)

The put-call Ratio to determine the market sentiments, with high ratio indicating a bearish sentiment and a low ratio indicating a bullish sentiment.

LOS 22 : Option Greek Parameters

Option price depends on 5 factors:

Option Price = f [S, X, t, r, σ], out of these factors X is constant and other causing a change in the price of option.

We will find out a rate of change of option price with respect to each factor at a time, keeping others constant.

1. Delta: It is the degree to which an option price will move given a small change in the underlying stock price. For example, an option with a delta of 0.5 will move half a rupee for every full rupee movement in the underlying stock.

   The delta is often called the hedge ratio i.e. if you have a portfolio short 'n' options (e.g. you have written n calls) then n multiplied by the delta gives you the number of shares (i.e. units of the underlying) you would need to create a riskless position - i.e. a portfolio which would be worth the same whether the stock price rose by a very small amount or fell by a very small amount.

2. Gamma: It measures how fast the delta changes for small changes in the underlying stock price i.e. the delta of the delta. If you are hedging a portfolio using the delta-hedge technique described under "Delta", then you will want to keep gamma as small as possible, the smaller it is the less often you will have to adjust the hedge to maintain a delta neutral position. If gamma is too large, a small change in stock price could wreck your hedge. Adjusting gamma, however, can be tricky and is generally done using options.

3. Vega: Sensitivity of option value to change in volatility. Vega indicates an absolute change in option value for a one percentage change in volatility.

4. Rho: The change in option price given a one percentage point change in the risk-free interest rate. It is sensitivity of option value to change in interest rate. Rho indicates the absolute change in option value for a one percent change in the interest rate.

5. Theta: It is a rate change of option value with respect to the passage of time, other things remaining constant. It is generally negative.

Notes