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CMA INTER (G2 PAPER-9)

OPERATION MANAGEMENT

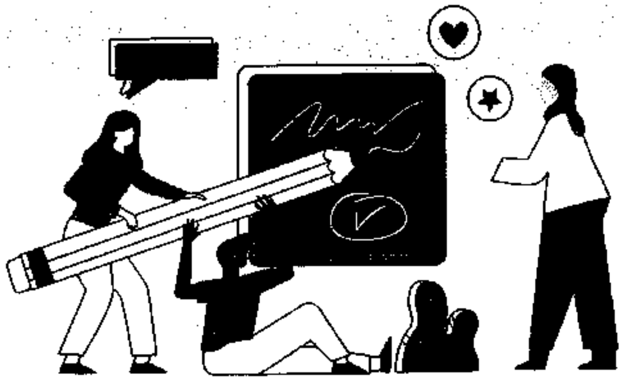
MAIN BOOK

HIGHLIGHTS OF THIS BOOK:

- EXHAUSTIVE COVERAGE OF MODULE
- COMPLETE COVERAGE OF NEW SYLLABUS
- SIMPLE AND CONCISE LANGUAGE FOR EASY UNDERSTANDING
- COVERS THEORY & SUMS WHICH WILL HELP YOU TO TACKEL WITH YOUR CMA EXAM.

APPLICABLE FOR JUNE 25 AND ONWARDS EXAMINATION

INDEX



OPERATIONS MANAGEMENT

CH: OPERATIONS MANAGEMENT - INTRODUCTION

1.1. SCOPE

Operations Management (OM) encompasses all organizational activities that acquire the raw form of materials (input), process or convert into a consumable products and services as required to meet the needs of the end customers. OM deals with both tangible product and intangible services

Example 1 (Product Centric)

Suppose, you require a smartphone. OM deals with procuring all raw materials such as chip, motherboard, battery, lens, speakers etc.; assemble and mount all components; test the performance of finished good; quality check; maintenance; storage and distribution for making the smartphone available to you.

Example 2 (Service Centric)

Suppose, you take a subscription of Netflix to watch a movie. OM covers all activities that includes dealing with movie-makers to get transmission right; make the movie available in the database, arranging for live streaming or recorded version transmission over spectrum and so on.

To better understand OM, let us have a simple diagram

Input (Raw Form) → **Conversion through process** → **Finished Product/ Service**

Examples of input include, raw materials, machines, electricity, manpower, facilities, storage space etc.

In some cases, product and service are required both. For example, if you visit a retail shop. You not only require the availability of the products you want but also you expect courtesy of the salesperson, ambience, convenience of buying etc which are services. In fact, goods (tangible) and services (intangible) follow a trade off relationship over a continuum ranging from pure product (for example, study material) to pure service (for example, teaching).

QUESTION 1.

Objectives of Operations Management

ANSWER:

Objectives of operations management can be categorized into (i) Customer service and (ii) Resource utilization.

(i) Customer service

The first objective is the customer service which means the service for the satisfaction of customer wants. Customer service is therefore a key objective of operations management.

The Operations Management must provide something to a specification which can satisfy the customer in terms of cost and timing. Thus, primary objective can be satisfied by providing the 'right thing at the right price at the right time'.

These three aspects of customer service - specification, cost and timing - are described in a little more detail for the four functions in Table 1. They are the principal sources of customer satisfaction and must, therefore, be the principal dimension of the customer service objective for operation managers.

Table 1: Aspects of Customer Service

Principal customer wants		
Principal function	**Primary consideration**	**Other consideration**
Manufacture	Goods of a given, requested or acceptable specification	Cost i.e. purchase price or cost of obtaininggoods Timing, i.e. delivery delay from order orrequest to receipt of goods
Transport	Movement of a given, requested or acceptable specification	Cost, i.e. cost of movement, Timing ,i.e. (i) duration or time to move (ii) wait, or delay from requesting to its commencement
Supply	Goods of a given, requested or acceptable specification	Cost, that is purchase price or cost obtaining goods Timing, i.e. delivery delay from order or request to supply, to receipt of goods
Service	Treatment of a given, requested or acceptable specification	Cost, i.e. cost of treatmentTiming, i.e. (i) Durationor timing requiredfortreatment (ii) wait, or delay from requesting to its commencement

Generally an organization will aim reliably and consistently to achieve certain standards, or levels, on these dimensions, and operations managers will be influential in attempting to achieve these standards.
Hence, this objective will influence the operations manager's decisions to achieve the required customer service.

(ii) Resource Utilization

Another major objective is to utilize resources for the satisfaction of customer wants effectively, i.e., customer service must be provided with the achievement of effective operations through efficient use of resources. Inefficient use of resources or inadequate customer service leads to commercial failure of an operating system.

Operations management is concerned essentially with the utilization of resources, i.e., obtaining maximum effect from resources or minimizing their loss, under utilization or waste. The extent of the utilization of theresources' potential might be expressed in terms of the proportion of available time used or occupied, spaceutilization, levels of activity, etc. Each measure indicates the extent to which the potential or capacity of suchresources is utilized. This is referred as the objective of resource utilization.

Operations management is also concerned with the achievement of both satisfactory customer service and resource utilization. An improvement in one will often give rise to deterioration in the other. Often both cannot be maximized, and hence a satisfactory performance must be achieved on both objectives. All the activities of operations management must be tackled with these two objectives in mind, and many of the problems will be faced by operations managers because of this conflict. Hence, operations managers must attempt to balance these basic objectives.

Below **Table 2** summarizes the twin objectives of operations management. The type of balance established both between and within these basic objectives will be influenced by market considerations, competitions, the strengths and weaknesses of the organization, etc. Hence, the operations managers should make a contribution when these objectives are set.

Table 2 : The twin objectives of operations management

The customer service objective.	The resource utilization objective.
To provide agreed/adequate levels of customer service (and hence customer satisfaction) by providing goods or services with the right specification, at the right cost and at the right time.	To achieve adequate levels of resource utilization (or productivity) e.g., to achieve agreed levels of utilization of materials, machines and labour.

QUESTION 2.

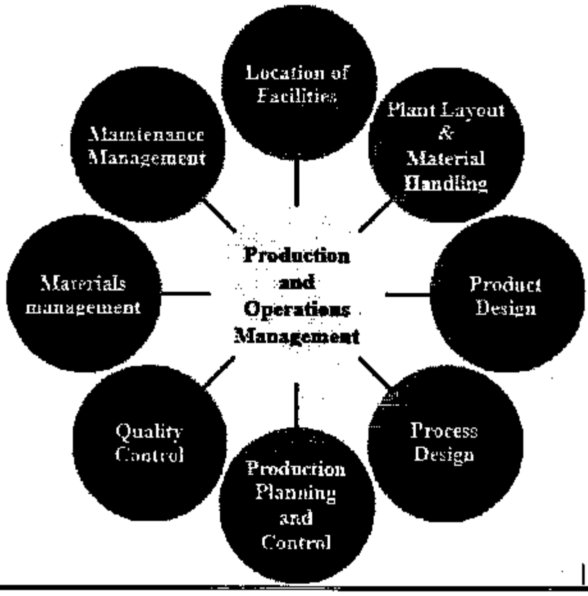
Describe the Scope of Operation Management?

ANSWER:

Operations Management concerns with the conversion of inputs into outputs, using physical resources, so as to provide the desired utilities to the customer while meeting the other organizational objectives of effectiveness, efficiency and adoptability. It distinguishes itself from other functions such as personnel, marketing, finance, etc. by its primary concern for 'conversion by using physical resources'. Following are the activities, which are listed under Production and Operations Management functions:

1. Location of facilities.
2. Plant layouts and Material Handling.
3. Product Design.
4. Process Design.
5. Production Planning and Control.
6. Quality Control.
7. Materials Management.
8. Maintenance Management.

Figure 1.1 : Scope of Production and Operations Management



Let us take an example of a product manufacturing company xyz Ltd.
The xyz Ltd requires to take few important decisions. The first question comes into picture is: "What to produce?" This question is linked with the basic existence of the company It talks about the product that xyz Ltd. is
manufacturing. Here, the organization needs to understand that what is the need of the customers in terms of product attributes/Features & quality. In other words, it talks about the competitive positioning of the company, its products acceptability at the market place this decision is based on the input received from market intelligence team and often is a part of the product design process later on we will study an important concept related to product design, such as QFD. In this regard, one important point to be noted that, many a times the organizations need to forecast about product life cycle & related requirement of the technology. Forecasting we will discuss separately.
One the company is aware that what it needs to produce, the second question comes: "How much to produce?"
This question is an ongoing questions, as the organization is engaged in estimating the quantity ("How much") on a daily, weekly, monthly, quarterly & yearly basis. Again this information is obtained from marketing team. Based on the information received, the planning team (as a part of supply chain's planning section) provides a forecast of demand. Hence, here deal with an important aspect of operational planning known as Demand Forecasting.
The next question is: "Where to produce?".
This question leads to facility location selection problem after this, a series of questions need to be answered that lead to a member of decision areas such as "
Q: "How to produce?" (Process selection & Layout)
Q: "When to produce?" (Aggregate Planning inventory Master Production decision schedule)
Q: "Do we have materials to produce?" (MRP, Inventory Management)
It also deals with Sourcing
Q: "Are we producing right things?" (Quality Management)
Q: "Are our machines able to provide desired results?" (Maintenance Management) Q: "How to reach the products to the customers?"(Distribution or Delivery planning)
It includes transportation decision, warehousing, materials handling ets. Logistics issues
In case the organization is practicing sustainability then another important decision area is reverse Logistics i.e., taking returns

1.3 Recent Trends in Production and Operations Management

QUESTION 5.
How is Modern Operations management characterised?
(a) Technological development
(b) Shorter product life cycle
(c) Changing needs and preferences of the customers
(d) Disruptions (market and product) and pressure for innovation
(e) Globalization
(f) Requirement for supreme service at an affordable price
(g) Pressure for optimization of operational cost

QUESTION:
Compare Production Management vs Operations Management.
Answer:
There are two points of distinction between production management and operations management. First, the term production management is more used for a system where tangible goods are produced. Whereas, operations management is more frequently used where various inputs are transformed into intangible services. Viewed from this perspective, operations management will cover such service organisations as banks, airlines, utilities, pollution control agencies, super bazaars, educational institutions, libraries, consultancy firms and police departments, in addition, of course, to manufacturing enterprises. The second distinction relates to the evolution of the subject. Operations management is the term that is used nowadays. Production management precedes operations management in the historical growth of the subject.
Recent trends in production and operations management relate to global competition and the impact it has on
manufacturing firms. Some of the recent trends are :

1. **Global Market Place :** Globalisation of business has compelled many manufacturing firms to have operations in many countries where they have certain economic advantage. This has resulted in a steep increase in the level of competition among manufacturing firms throughout the world.
2. **Production/Operations Strategy :** More and more firms are recognising the importance of production/ operations strategy for the overall success of their business and the necessity for relating it to their overall business strategy.
3. **Total Quality Management (TQM) :** TQM approach has been adopted by many firms to achieve customer satisfaction by a never-ending quest for improving the quality of goods and services.
4. **Flexibility :** The ability to adapt quickly to changes in volume of demand, in the product mix demanded, and in product design or in delivery schedules, has become a major competitive strategy and a competitive advantage to the firms. This is sometimes called as agile manufacturing.
5. **Time Reduction :** Reduction of manufacturing cycle time and speed to market for a new product provide competitive edge to a firm over other firms. When companies can provide products at the same price and quality, quicker delivery (short lead times) provide one firm competitive edge over the other.

6. **Technology :** Advances in technology have led to a vast array of new products, new processes and new materials and components. Automation, computerisation, information and communication technologies have revolutionised the way companies operate. Technological changes in products and processes can havegreat impact on competitiveness and quality, if the advanced technology is carefully integrated into the existing system.
7. **Worker Involvement :** The recent trend is to assign responsibility for decision making and problem solvingto the lower levels in the organisation. This is known as employee involvement and empowerment. Examplesof worker involvement are quality circles and use of work teams or quality improvement teams.
8. **Re-engineering :** This involves drastic measures or break-through improvements to improve the performance of a firm. It involves the concept of clean-slate approach or starting from scratch in redesigning the business processes.
9. **Environmental Issues :** Today's production managers are concerned more and more with pollution controland waste disposal which are key issues in protection of environment and social responsibility. There is increasing emphasis on reducing waste, recycling waste, using less-toxic chemicals and using biodegradable materials for packaging.
10. **Corporate Downsizing (or Right Sizing) :** Downsizing or right sizing has been forced on firms to shed their obesity. This has become necessary due to competition, lowering productivity, need for improved profit andfor higher dividend payment to shareholders.
11. **Supply-Chain Management :** Management of supply-chain, from suppliers to final customers reduces the cost of transportation, warehousing and distribution throughout the supply chain.
12. **Lean Production :** Production systems have become lean production systems which use minimal amountsof resources to produce a high volume of high quality goods with some variety. These systems use flexible manufacturing systems and multi-skilled workforce to have advantages of both mass production and job production (or craft production).

CMA STUDENTS AND TEAM AAC CELEBRATING SUCCESS OF CMA INTER CLEARED STUDENT

THAT IS WHY STUDENTS SAYS

AAC=CMA

.....NEXT CAN BE YOU.

2.1. DEMAND FORECASTING

- Demand" is in a simpler way defined as the requirement and desire of consumers to purchase products and services and willingness and abilities to pay for availing the same. Example of product: household durable products like television, daily use products like soap. Example of service: Pathological tests by medical service providing laboratories like Drs Ray & Trivedi Lab. The products/services demanded are of two categories such as industrial purpose products like machines and consumers specific products like confectionaries.
- "Forecasting" is the process of making prediction about future happenings and/or requirements based on available information and/evidences. Example, forecasting of product requirements, forecasting of weather, forecasting of fashion trends, forecasting of tourist inflow, forecasting of patient admission, forecasting of technology etc. In this segment we shall restrictour discussions mainlyon forecasting of demand of products.

QUESTION 1
Why do we need to forecast the demand?

ANSWER:
From a holistic perspective any organization is described in terms of its supply chain, also sometimes called value chain which gets into existence from the moment demand is created and/or gets generated. In other word, it is the demand that decides the existence of any business. With the available demand forecasting the organizations perform production/ service planning, take inventory decisions, decide on facility selection and process design, and select appropriate technology, plan for fund requirement and manpower planning. Hence, a reasonably accurate forecasting of demand can make a company while absence of the same lead to breaking of organizations' competition is either on products pace and/or on service space where effectiveness and efficiency of forecasting is a critical success factor.

QUESTION 2
Source of Information (used for forecasting)

ANSWER:
There are a number of sources from where past information and/or evidences are gathered to facilitate forecasting of demand such as

- Market Report
- Sales force opinion
- Experts' views
- Industry report
- Point of Sales data
- Structured customer survey
- Field report etc.

However, all these above sources provide mostly structured data and information thereof. With the development in Information and communication technology, today forecasting is significantly controlled by unstructured and semi structured high volume of data in terms of quantified data, video, audio, image, multimedia message, social media post to name a few.

QUESTION 3
Define Range of period.

ANSWER:
Forecasting is done on short, medium and long term basis. The underlying objectives are explained below.
The period of forecasting, that is the time range selected for forecasting depends on the purpose for which the forecast is made. The period may vary from one week to some years. Depending upon the period, the forecast can be termed as 'Short range forecasting', medium range forecasting' and 'Long range forecasting'. 'Short range forecasting period may be one week, two weeks or a couple of months. Medium range forecasting period may vary from 3 to 6 months. Long range forecasting period may vary from one year to any period. The objective of above said forecast is naturally different.
In general, short term forecasting will be more useful in production planning. The manager who does short range forecast must see that they are very nearer to the accuracy.

• **Short run forecasting** (example, highly innovative products with shorter life cycle like smartphones; usually spanning over 6-8 months)
In case of short-term forecast, which extends from few weeks to three- or six-months Example smart phones usually Spanning are 6-8 months the following purposes are generally served:
(i) To estimate the inventory requirement,
(ii) To provide transport facilities for despatch of finished goods,
(iii) To decide work loads for men and machines,
(iv) To find the working capital needed,
(v) To set-up of production run for the products,
(vi) To fix sales quota,
(vii) To find the required overtime to meet the delivery promises

• **Medium run forecasting** (example, consumer durable products, medicines)
In case of medium range forecasting the period may extend over to one or two years. Example– Consumer Durable products, Medicines The purpose of this type of forecasting is:
(i) To determine budgetary control over expenses,
(ii) To determine dividend policy,
(iii) To find and control maintenance expenses,
(iv) To determine schedule of operations,
(v) To plan for capacity adjustments.

In long range forecast, the normal period used is generally 5 years. Example–Daily used routine household product like Aata. In some cases it may extends to 10 to 15 years also. The purpose of long range forecast is:
(i) To work out expected capital expenditure for future developments or to acquire new facilities,
(ii) To determine expected cash flow from sales,

(iii) To plan for future manpower requirements,
(iv) To plan for material requirement,
(v) To plan for Research and Development. Here much importance is given to long range growth factor.

QUESTION 4
State the Steps in Forecasting.

ANSWER:
Whatever may be the method used for forecasting, the following steps are followed in forecasting.

a) Determine the objective of forecast: What for you are making forecast? Is it for predicting the demand?Is it to know the consumer's preferences? Is it to study the trend? You have to spell out clearly the use of forecast.
b) Select the period over which the forecast will be made? Is it long-term forecast or medium-term forecastor short-term forecast? What are your information needs over that period?
c) Select the method you want to use for making the forecast. This method depends on the period selectedfor the forecast and the information or data available on hand. It also depends on what you expect from the information you get from the forecast. Select appropriate method for making forecast.
d) Gather information to be used in the forecast. The data you use for making forecasting to produce the result, which is of great use to you. The data may be collected by:
    a. Primary source: This data we will get from the records of the firm itself.
    b. Secondary source: This is available from outside means, such as published data, magazines, educationalinstitutions etc.
e) Make the forecast: Using the data collected in the selected method of forecasting, the forecast is made.

QUESTION 5
Forecasting Methods (How to forecast demand?)

ANSWER:
There are two types of approaches such as
A) Qualitative
B) Quantitative

A) Qualitative Methods

- Survey of buyer's intentions or the user's expectation method
- Collective opinion or sales force composite method
- Group executive judgement or executive judgement method
- Experts' opinions
- Market test method

1) **Survey of buyer's intentions or the user's expectation method**: Under this system of sales forecasting actual users of the product of the concern are contacted directly and they are

asked about their intention to buy the company's products in an expected given future usually a year. Total sales forecasts of the product then estimated on the basis of advice and willingness of various customers. This is most direct method of sales forecasting. The chief advantages of this method are:

- Sales forecast under this method is based on information received or collected from the actual userswhose buying actions will really decide the future demand. So, the estimates are correct.
- It provides a subjective feel of the market and of the thinking behind the buying intention of the actualuses. It may help the development of a new product in the market.
- This method is more appropriate where users of the product are numbered and a new product is to be introduced for which no previous records can be made available.
- It is most suitable for short-run forecasting.

2. **Collective opinion or sales force composite method**: Under this method, views of salesmen, branch manager, area manager and sales manager are secured for the different segments of the market. Salesmen, being close to actual users are required to estimate expected sales in their respective territories and sections. The estimates of individual salesmen are then consolidated to find out the total estimated sales for the coming session. These estimates are then further examined by the successive executive levels in the lightof various factors like proposed changes in product design, advertising and selling prices, competition etc. before they are finally emerged for forecasting.
3. **Group executive judgement or executive judgement method:** This is a process of combining, averagingor evaluating, in some other way, the opinions and views of top executives. Opinions are sought from the executives of different fields i.e., marketing; finance; production etc. and forecasts are made.

Experts' opinions: Under this method, the organization collects opinions from specialists in the field outside the organization. Opinions of experts given in the newspapers and journals for the trade, wholesalers and distributors for company's products, agencies or professional experts are taken. By analyzing these opinions and views of experts, deductions are made for the company's sales, and sales forecasts are done

4. **Market test method:** Under this method seller sells his product in a part of the market for sometimes and makes the assessment of sales for the full market on the bases of results of test sales. This method is quite appropriate when the product is quite new in the market or good estimators are not available or where buyers do not prepare their purchase plan.

B) Quantitative Statistical Method

This approach takes into account historical data and uses statistical models to forecast the demand.
There are broadly two types of approach
Causal/Regression analysis
Time Series Analysis

Causal Model

The causal model is expressed at $D_T = f(F_1, F_2...F_N)$
Where, D_T = Demand for period t
$F_1, F_2, F_3...F_n$ are the factors responsible.
For, example, suppose we want to forecast the demand (sales) for period t
F_1 = Sales budget

F_2 = Price
F_3 = Promotional Budget
F_4 = Technology etc.
Now, these Fi may also be a f(t)
Therefore, we need to take into account the 'time'. However, the causal/regression analysis model sometimes is treated as a useful method as it considers various perspectives or influencers into the analysis.
In this context, it may be noted with regard to the forecasting period. We sometimes fail to capture sudden change in given conditions over the study period.
For example,
After spread of covid-19, an organization (lets take an example of a FMCG company) that made forecasting of demand in 2020 and beyond, had undergone an abrupt initial variations in the demand.
Therefore, Forecasting is never absolutely accurate. There is an error in the forecasting process. This concept shall be more clearly explained in the next section on time series analysis.

QUESTION 6
Define Time Series Analysis And its components?

ANSWER:
Time Series Analysis
let, Y_t = Demand at time t
y_t = Realized demand at time t
$y_t \subset Y_t$
(sample) (population)
Therefore, y_t = f(t) and yt = f(t)
For any time series data, there are four components
Trend(T)
Seasonality(S)
Cyclical
Irregular(I) or Random
Time series models are of two kinds of forms
Additive y_t = T+S+C+I
Multiplicative y_t = TSCI Like y_t, T,S,C all are f(t).

Next, Let us define these components
- **Trend:** An indicator of long term movements i.e., it is the tendency of the data to move upward or downward over a considerably longer period of time. Sometimes, the data remains unchanged in time that implies trend = 0 (Stable data)
  For example, Population trend, no of students admitted, no of infected cases of covid-19
- **Seasonality:** Rhythmic, regular & periodic variations over a span of less than 1 year. This type of variations may be recorded daily, weekly, monthly, quarterly or yearly. Example: Consumer Durable products may have seasonal variations during festive times & special occasions every year.

- **Cyclic variation:** A kind of oscillatory movement generally spanning over more than 1 year. Example: Business Cycle (Prosperity, recession, depression, recovery)

- **Irregular variation:** Random, unpredictable, uncontrollable, erratic. They are referred to as noise.

In the following we shall discuss time series model based analysis for both additive and multiplicative model. However, in our discussion we restrict ourselves not to considering cyclical component for the time being.

Additive Model

$Y_t = T+S+I$

We consider linear trend for our discussion.

To find trend we use following models -

$y_t = T+S+I$

We consider linear trend for our discussion.

To find trend we use following models –

1. $y_t = a + bt$ a, b are coefficients (Linear)

2. $y_t = a + bt + ct^2$ (linear in coefficients)

   This is also called intrinsially linear (Parabolic Form)

   Suppose, $t^2 = T$ then the equation become

   $y_t = a + bt + cT$

3. $y_t = ab^t$ (exponential form)

   This can also made in the form of (1)

   $Log_{10}(y_t) = Log_{10}(a) + t\ lot_{10}(b)$

   $\Rightarrow y'_t = A + Bt$ $A = \log_{10}(a)$

   $B = \log_{10}(b)$

$\Rightarrow y'_t = A + Bt \quad A = \log_{10}(a)$

$B = \log_{10}(b)$

Using these models we forecast the demand

Let $\hat{y}_t$ is the predicted demand.

The model's effectiveness depends on how less is the MSE (Mean Square Error)

Observed $\rightarrow y_t = \hat{y}_t + \epsilon \leftarrow$ Error

$\uparrow$

Predicted

$\Rightarrow \epsilon = y_t - \hat{y}_t$

$\Rightarrow Ess = \Sigma\epsilon^2 = \Sigma(y_t - \hat{y}_t)^2$

$$MSE = \frac{ESS}{\text{No. of time periods}}$$

After getting the trend (T) the equation becomes

$(y_t - T) = S + I$

In fact, $y_t - T - S = C + I$

Therefore, now the question is how to find S (seasonal variation)
For this purpose we use

I. Moving average
II. Weighted moving average
III. Simple exponential smoothing
IV. Double exponential smoothing
V. Hot-winter model

Moving Average (MA)

Captures impulsive changes
Dynamic in nature, ability to handle outliers more than arithmetic means which is static in nature
Continuous movement of average values.

MA
- odd term (2n + 1) Generalization
  Ex : 3 period MA; 2n + 1 = 3 ⊠ 2.1 + 1 = 3
  therefore, we forgo one period both at the beginning & end
- Even term (2n) Generalization

The advantage of odd term MA is its simplicity in computation. However, it has to forgo n periods at the beginning & end. Hence, it loses some fluctuations and also it is applicable for long period. For a longer period, possibility of error increases. Another limitation is that, odd term MA is a one time average calculation method.

Further, if we increase n value it misses more number of periods. This is one of the reason that we calculate seasonality index

Solution with time series analysis

Let us take the example of an Insurance company. The data we record is the policy sale (quarter wise for last 4 years). Therefore, we have 16 data points such as d1, d2......d16.

Refer to the following table :-

Year	Quarter	Sales
1	Q_1	d_1
	Q_2	d_2
	Q_3	d_3
	Q_4	d_4
2	Q_1	d_5
	Q_2	d_6
	Q_3	d_7
	Q_4	d_8
3	Q_1	d_9
	Q_2	d_{10}
	Q_3	d_{11}
	Q_4	d_{12}
4	Q_1	d_{13}
	Q_2	d_{14}
	Q_3	d_{15}
	Q_4	d_{16}

Now we want to find out

I. Trend

II. Seasonality

III. Seasonal index

Note that we do not consider cyclic variations for the time being. Seasonal Index allows to calculate error value for each quarter each year.

Steps

Plot the data graphically.

Apply least square approximation method to get the normal equations (2 nos) solving which we find outcoefficient values (a and b).

Our equation is

$Y_t = a + b_t$...(1)

Coefficient for a = 1

Coefficient for b = t

Multiply the equation (1) by coefficient of 'a' and summing up both side we get

$\Sigma y_t = na + b\Sigma t$...(2)

Again multiplying the equation (1) by the coefficient of 'b' i.e. t and summing up both side we get

$\Sigma ty_t = a\Sigma t + b\Sigma t^2$...(3)

In the time series, there is one extra constraint for solving its parameters easily, i.e.,

$\Sigma t = \Sigma t_3 = 0$

Therefore, from equation (2) we gets

$$\boxed{a = \frac{\sum y_t}{n} = \bar{y}_t} \ldots(4)$$

Again from (3) $\Sigma ty_t = a\Sigma t + b\Sigma t^2$

$$\Rightarrow \boxed{b = \frac{\sum ty_t}{\sum t^2}} \ldots(5)$$

Now let $\hat{y}_t$ is the predicted value

Therefore $\hat{y}_t = \hat{a} + \hat{b}t$...(6)

Where, $\hat{a}$ & $\hat{b}$ are known values

In this context, the question is how to get Σt = 0 for odd period and even period. Let us try to explain with a simple example.

T		Y_T	$t = \frac{T - Midyear}{Width}$	y_t	ty_t	t^2
Width = 1	2016	Y_1	-2	✓	✓	4
	2017	Y_2	-1	✓	✓	2
Mid year	2018	Y_3	0	✓	✓	0
	2019	Y_4	+1	✓	✓	1
	2020	Y_5	+2	✓	✓	4
			$\Sigma t = 0$			$\Sigma t^2 = 0$

Therefore, our next step is to find out MSE

y_t	$\hat{y}_t$	$(y_t - \hat{y}_t)$	$(y_t - \hat{y}_t)^2$
y_1	$\hat{y}_1$		
y_2	$\hat{y}_2$		
y_3	$\hat{y}_3$		
.	.		
.	.		
.	.		
			$\Sigma(y_t - \hat{y}_t)^2$

$$MSE = \frac{\sum\left(y_t - \hat{y}_t\right)^2}{n}$$

Let us now use model 2 (parabolic) for the same odd term MA.

$y = a + bt + ct^2 ...(7)$

Following the same procedure as we did in case of y = a+bt. We get three normal equation as follows

$\Sigma y_t = na + b\Sigma t + c\Sigma t^2 ...(8)$

$\Sigma ty_t = a\Sigma t + b\Sigma t^2 + c\Sigma t^3 ...(9)$

$\Sigma t^2 y_t = a\Sigma t^2 + b\Sigma t^3 + c\Sigma t^4 ...(10)$

Now we know

$\Sigma t = \Sigma t^3 = ... = 0$

Therefore from equation (9) we get

$$b = \frac{\sum ty_t}{\sum t^2}$$

From equation (8) we get

$\Sigma y_t = na + c\Sigma t^2 ...(11)$

From equation (10) we get

$\Sigma t^2 y_t = a\Sigma t^2 + c\Sigma t^4 ...(12)$

Solving (11) & (12) we get the values of a and c.

Next we follow the usual process for getting $\hat{y}$& MSE subsequently.

We now show the procedures for

$y_t = ab^t$

The equation can be formed as

$\text{Log}_{10}(y_t) = \log_{10}(a) + t\log_{10}(b)$

$\Rightarrow y_t = A + Bt$...(13)

Now the equation resembles the same form of equation (1)

$A = \log_{10}(a) \Rightarrow a = 10^A$

$B = \log_{10}(b) \Rightarrow b = 10^B$

Therefore we in a similar way proceed to find $\hat{y}_t$ (predicted value) & MSE.

So far we have talked about trend & MSE. Now we shall follow to capture seasonality & proceed to discuss about Moving Average (MA).

MA(odd term) Generalization is $(2n + 1)$

We Know $y_t = T + S + I$

After doing MA we get $\hat{y}_t = T + I'$

$I' < I$

$y_t - \hat{y}_t = S + (I - I')$

$= S + I'' \quad I'' < I' < I$

Year	Quarter	Sales (y_t)	3 term Movingtotal	3 term MA (y_t)	(y_t - y_t)
1	Q_1	d_1			
	Q_2	d_2	D1	D1/3	d_2 - D1/3
	Q_3	d_3	D2	D2/3	d_3 - D2/3
	Q_4	d_4	D3	D3/3	
2	Q_1	d_5	D4	D4/3	
	Q_2	d_6	D5	D5/3	
	Q_3	d_7	D6	D6/3	
	Q_4	d_8	D7	D7/3	
3	Q_1	d_9	D8	D8/3	
	Q_2	d_{10}	D9	D9/3	
	Q_3	d_{11}	D10	D10/3	
	Q_4	d_{12}	D11	D11/3	
4	Q_1	d_{13}	D12	D12/3	
	Q_2	d_{14}	D13	D13/3	
	Q_3	d_{15}	D14	D14/3	d_{15} - D14/3
	Q_4	d_{16}			

Example :

$D_1 = d_1 + d_2 + d_3 : D_8 = d_8 + d_9 + d_{10}$

Usually seasonality index is expressed in base value of 100. Any calculated figure >100 :seasonality↑

We now will show how to derive adjusted seasonality index in the following table.

	Quarters			
Year	Q_1	Q_2	Q_3	Q_4
Y_1	–	S_{12}	S_{13}	S_{14}
Y_2	S_{21}	S_{22}	S_{23}	S_{24}
Y_3	S_{31}	S_{32}	S_{33}	S_{34}
Y_4	S_{41}	S_{42}	S_{43}	–
Avg	AV_1	AV_2	AV_3	AV_4

S_{12}, S_{11} er are seasonality values

$$AV_1 = \frac{S_{21} + S_{31} + S_{41}}{3}$$

$$AV_2 = \frac{S_{12} + S_{22} + S_{32} + S_{42}}{4}$$

$$AV_3 = \frac{S_{13} + S_{23} + S_{33} + S_{43}}{4}$$

$$AV_4 = \frac{S_{14} + S_{24} + S_{34}}{3}$$

Now we know that seasonality has a base value 100.

Here, $AV_1 + AV_2 + AV_3 + AV_4 = \tau$ (let us take)

Therefore adjusted seasonality index for Q_1 to Q_4 is calculated as

$$S_1^{adj} = \frac{AV_1}{\tau} \times 400$$

$$S_2^{adj} = \frac{AV_2}{\tau} \times 400$$

$$S_3^{adj} = \frac{AV_3}{\tau} \times 400$$

$$S_4^{adj} = \frac{AV_4}{\tau} \times 400$$

$$\sum_{i=1}^{4} S_i^{adj} = 400 \text{ if } S_i^{adj} > 100 \Rightarrow \text{More seasonality}$$

$$\text{If} \sum_{i=1}^{4} S_j^{adj} > 400 \text{ say } 400.20$$

Then we proceed for further adjustment

as $S_i^{*adj} = S_i^{adj} - \left(\frac{0.20}{4}\right)$

So that $\sum_{i=1}^{4} S_i^{*adj} = 100$

Therefore we get

Therefore we get

y_t	$\hat{y}_t$	SI	I

This is called decomposition $y_t - y_t' = S + I$

Now let us discuss about MA for even term. This is very important & mostly used as it provides less error. The following table depicts the calculation for 4 term MA

Year	Quarter	Sales (y_t)	4 term Moving total	4 term MA	2 term MA (y_t)	(y_t - y_t)
1	Q_1	d_1				
	Q_2	d_2				
			D1	D1/4		
	Q_3	d_3			D_1''	$d_3 - D_1''$
			D2	D2/4		
	Q_4	d_4			D_2''	$d_4 - D_2''$
2	Q_1	d_5				
	Q_2	d_6				
	Q_3	d_7				
	Q_4	d_8				
3	Q_1	d_9				
	Q_2	d_{10}				
	Q_3	d_{11}				
	Q_4	d_{12}				
4	Q_1	d_{13}				
	Q_2	d_{14}				
	Q_3	d_{15}				
	Q_4	d_{16}				

$$D_1 = d_1 + d_2 + d_3 + d_4$$
$$D_2 = d_2 + d_3 + d_4 + d_5$$
$$D_1'' = \left(\frac{D_1}{4} + \frac{D_2}{4}\right)/2$$

So for 4 term MA we actually find the result by dividing with 8. After preparing this table follow the same process as odd term MA. Therefore we complete the discussion on Additive models

Multiplicative Models

General form $y_t = T \times S \times I$ (discarding cyclic component for time being)

$$y_t' = T \times I'$$
$$\frac{y_t}{y_t'} = S \times \left(\frac{I}{I'}\right) = S \times I''$$
$$I'' = \frac{I}{I'} < I' < I$$

Now we follow the same process as we underwent in the case of additive models to find a trend, adjusted seasonality index and decomposition.

It may be noted that for the short-term we normally use additive models whereas for long term we go for multiplicative model.

Illustration 1

From the following time series data of sale project the sales for the next three years.

Year	2015	2016	2017	2018	2019	2020	2021
Sales (000 units)	80	90	92	83	94	99	92

ANSWER:

Computation of Trend Values

Years	Time Deviation from 2004 X	Sales in (`000 units) Y	Squares oftime dev. X^2	Product of time deviations and salesXY
2015	-3	80	9	-240
2016	-2	90	4	-180
2017	-1	92	1	-92
2018	0	83	0	0
2019	+1	94	1	+94
2020	+2	99	4	+198
2021	+3	92	9	+276
n = 7	$\Sigma X = 0$	$\Sigma Y = 630$	$\Sigma X^2 = 28$	$\Sigma XY = +56$

Regression equation of Y on X

Y = a + bX

To find the values of a and b

$$a = \frac{\sum Y}{n} = \frac{630}{7} = 90$$

$$b = \frac{\sum XY}{\sum X^2} = \frac{56}{28} = 2$$

Hence regression equation comes to Y = 90 + 2X. With the help of this equation we can project the trend values for the next three years, i.e. 2022, 2023 and 2024.

Y2008 = 90 + 2(4) = 90 + 8 = 98 (000) units.

Y2009 = 90 + 2(5) = 90 + 10 = 100 (000) units.

Y2010 = 90 + 2(6) = 90 + 12 = 102 (000) units.

Illustration 2

With the help of following data project the trend of sales for the next five years:

Years	2016	2017	2018	2019	2020	2021
Sales (in lakhs)	100	110	115	120	135	140

ANSWER:

Computation of trend values of sales

Year	Time deviations from the middle of 2004 and 2005 assuming 6 months = 1 unit	Sales (in lakh)	Squares of time deviation	Product of time deviation and sales
	X	Y	X^2	XY
2016	-5	100	25	-500
2017	-3	110	9	-330
2018	-1	115	1	-115
2019	+1	120	1	+120
2020	+3	135	9	+405
2021	+ 5	140	25	+700
n = 6	$\Sigma X = 0$	$\Sigma Y = 720$	$\Sigma X^2 = 70$	$\Sigma XY = 280$

Regression equation of Y on X:

Y = a + bX

To find the values of a and b

$$a = \frac{\sum Y}{n} = \frac{720}{6} = 120$$

$$b = \frac{\sum XY}{\sum X^2} = \frac{280}{70} = 4$$

Hence regression equation comes to Y = 120 + 4X

Sales forecast for the next years, i.e., 2022-26 Y_{2022} = 120 + 4 (+7) = 120 + 28 = ` 148 lakhs

Y_{2023} = 120 + 4 (+9) = 120 + 36 = ₹ 156 lakhs

Y_{2024} = 120 + 4 (+11) = 120 + 44 = ₹ 164 lakhs.

Y_{2025} = 120 + 4 (+13) = 120 + 52 = ₹ 172 lakhs.

Y_{2026} = 120 + 4 (+15) = 120 + 60 = ₹ 180 lakhs.

Illustration 3

An investigation into the demand for colour TV sets in 5 towns has resulted in the following data:

Population of the town (in lakhs)	X:	5	7	8	11	14
No of TV sets demanded (in thousands)	Y:	9	13	11	15	19

Fit a linear regression of Y on X and estimate the demand for CTV sets for two towns with a population of 10 lakhs and 20 lakhs.

ANSWER:

Computation of trend values

Population (in lakhs)	Sales of CTV (in thousands)	Squares of the population	Product of population and sales of color TV
X	Y	X^2	XY
5	9	25	45
7	13	49	91
8	11	64	88
11	15	121	165
14	19	196	266
$\Sigma X =$ 45	$\Sigma y =$ 67	$\Sigma X^2 =$ 455	$\Sigma XY =$ 655

Regression equation of Y on X

$Y = a + bX$

To find the values of a and b, the following two equations are to be solved

$\Sigma Y = na + b\Sigma X$ ... (i)

$\Sigma XY = a\Sigma X + b\Sigma X^2$ ... (ii)

By putting the values we get

$67 = 5a + 45b$ ... (iii)

$655 = 45a + 455b$ ... (iv)

Multiplying equation (iii) by 9 and putting it as no. (v) we get,

$603 = 45a + 405b$ ... (v)

By deducting equation (v) from equation (iv); we get $52 = 50b$

$b = \frac{52}{60} = 1.04$

By putting the value of b in equation (iii), we get

$67 = 5a + 45 \times 1.04$

or, $67 = 5a + 46.80$

or, $67 - 46.80 = 5a$

or, $5a = 20.20$

or, $a = \frac{20.20}{5}$

or $a = 4.04$

Now by putting the values of a and b the required regression equation of Y on X, is

$Y = a + bX$ or, $Y = 4.04 + 1.04X$

When X = 10 lakhs than

$Y = 4.04 + 1.04 (10)$

or, Y = 4.04 + 10.40 or 14.44 thousand CTV sets.
Similarly for town having population of 20 lakhs, by putting the value of X = 20 lakhs in regression equation Y = 4.04 + 1.04 (20)
= 4.04 + 20.80 = 24.84 thousands CTV sets.
Hence expected demand for CTV for two towns will be 14.44 thousand and 24.84 thousand CTV sets.

Illustration 4

An investigation into the use of scooters in 5 towns has resulted in the following data: Population in town

Population in town (in lakhs)	(X)	4	6	7	10	13
No. of scooters	(Y)	4,400	6,600	5,700	8,000	10,300

Fit a linear regression of Y on X and estimate the number of scooters to be found in a town with a population of 16 lakhs.

ANSWER:

Computation of trend value

Population (in lakhs) X	No. of scooters demanded Y	Squares of populationX^2	Product of population andNo. of scooters demanded XY
4	4,400	16	17,600
6	6,600	36	39,600
7	5,700	49	39,900
10	8,000	100	80,000
13	10,300	169	1,33,900
$\Sigma X = 40$	$\Sigma Y = 35,000$	$\Sigma X^2 = 370$	$\Sigma XY = 3,11,000$

Regression equation of Y on X

Y = a + bX

To find the values of a and b we will have to solve the following two equations

$\Sigma Y = na + b\Sigma X$ ... (i)

$\Sigma XY = a\Sigma X + b\Sigma X^2$ ....(ii)

By putting the values, we get

35,000 = 5a + 40b ... (iii)

3,11,000 = 40a + 370b ... (iv)

By multiplying equation no. (iii) by 8 putting as equation (v) we get,

2,80,000 = 40a + 320b ... (v)

By subtracting equation (v) from equation (iv), we get 31,000 = 50b

or, 50b = 31,000

or, $b = \frac{310}{50} = 620$

By substituting the value of b in equation no. (iii), we get

35000=5a+40b

Or, 35000=5a+40x620

Or, 35000=5a+24800

Or, 10200=5a

Or, a=10200/5=2040

Now putting the values of a and b the required regression equation of Y on X, is

$Y = a + bX$ or,	$Y = 2040 + 620X$
When X = 16 lakhs then	$Y = 2040 + 620(16)$
or	$Y = 2040 + 9920$
or	$Y = 11{,}960$

Hence, the expected demand of scooters for a town with a population of 16 lakhs will be 11,960 scooters.

2.2. CAPACITY PLANNING

QUESTION 7
Capacity Planning

ANSWER:
The effective management of capacity is the most important responsibility of production and operations management. The objective of capacity management i.e., planning and control of capacity, is to match the level of operations to the level of demand.
Capacity planning is concerned with finding answers to the basic questions regarding capacity such as:

(i) What kind of capacity is needed?
(ii) How much capacity is needed?
(iii) When this capacity is needed?

Capacity planning is to be carried out keeping in mind future growth and expansion plans, market trends, sales forecasting, etc. Capacity is the rate of productive capability of a facility. Capacity is usually expressed as volume of output per period of time.
Capacity planning is required for the following:

- Sufficient capacity is required to meet the customers demand in time,
- Capacity affects the cost efficiency of operations,
- Capacity affects the scheduling system,
- Capacity creation requires an investment,
- Capacity planning is the first step when an organization decides to produce more or new products.

Capacity planning is mainly of two types:
(i)Long-term capacity plans which are concerned with investments in new facilities and equipments. These plans cover a time horizon of more than two years.

(ii) Short-term capacity plans which takes into account work-force size, overtime budgets, inventories etc.
Capacity refers to the maximum load an operating unit can handle. The operating unit might be a plant, a department, a machine, a store or a worker. Capacity of a plant is the maximum rate of output (goods or services) the plant can produce.
The production capacity of a facility or a firm is the maximum rate of production the facility or the firm is capable of producing. It is usually expressed as volume of output per period of time (i.e., hour, day, week, month, quarter etc.). Capacity indicates the ability of a firm to meet market demand - both current and future.

Effective Capacity can be determined by giving due consideration to the following factors:
Facilities - design, location, layout and environment.
Product - Product design and product-mix.
Process - Quantity and quality capabilities of the process or to be followed.
Human factors - Job content, Job design, motivation, compensation, training and experience of labour, learning rates and absenteeism and labour turn over.

Operational factors - Scheduling, materials management, quality assurance, maintenance policies, and equipment break-downs.
External factors - Product standards, safety regulations, union attitudes, pollution control standards.

QUESTION 8
Measurement of capacity

ANSWER:
Capacity of a plant is usually expressed as the rate of output, i.e., in terms of units produced per period of time (i.e., hour, shift, day, week, month etc.). But when firms are producing different types of products, it is difficult to use volume of output of each product to express the capacity of the firm. In such cases, capacity of the firm is expressed in terms of monetary value (production value) of the various products produced put together.

QUESTION 9
Capacity Planning Decisions

ANSWER:
Capacity planning involves activities such as:
(i) Assessing the capacity of existing facilities.
(ii) Forecasting the long-range future capacity needs.
(iii) Identifying and analysing sources of capacity for future needs.
(iv) Evaluating the alternative sources of capacity based on financial, technological and economical considerations.
(v) Selecting a capacity alternative most suited to achieve strategic mission of the firm.

Capacity planning is necessary when an organisation decides to increase its production or introduce new products into the market or to increase the volume of production to gain the advantages of economies of scale. Once the existing capacity is evaluated and a need for new or expanded facilities is determined, decisions regarding the facility location and process technology selection are undertaken.

When the long-range capacity needs are estimated through long-range forecasts for products, a firm may find itself in one of the two following situations:
(i)A capacity shortage situation where present capacity is not enough to meet the forecast demand for the product.
(ii) An excess or surplus capacity situation where the present capacity exceeds the expected future demand

QUESTION 10
Factors affecting determination of plant capacity

ANSWER:

(i) Capital investment required,
(ii) Changes in product design, process design, market conditions and product life cycles,
(iii) Flexibility for capacity additions,
(iv) Level of automation desired,
(v) Market demand for the product,
(vi) Product obsolescence and technology obsolescence and
(vii) Type of technology selected.

QUESTION 11
Forms of capacity planning:

ANSWER:

Based on time-horizon

i. Long-term capacity planning and
ii. Short-term capacity planning

Based on amount of resources employed

(i) Finite capacity planning and
(ii) Infinite capacity planning

QUESTION 12
Factors Affecting Capacity Planning

ANSWER:

Factors Affecting Capacity Planning: Two kinds of factors affecting capacity planning are:

(i) Controllable Factors: amount of labour employed, facilities installed, machines, tooling, shifts of work per day, days worked per week, overtime work, subcontracting, preventive maintenance and number of production set ups.

(ii) Less Controllable Factors: absenteeism, labour performance, machine break-downs, material shortages, scrap and rework, strike, lock-out, fire accidents, natural calamities (flood, earthquake etc.) etc.

Capacity Requirement Planning : Capacity requirement planning (CRP) is a technique which determines what equipment and labour/personnel capacities are required to meet the production objectives (i.e., volume of products) as per the master production schedule and material requirement planning (MRP-I).

QUESTION 13

Capacity Requirement Planning Strategies:

ANSWER:

Two types of capacity planning strategies used are:

(i) **"Level capacity" plan and**

(ii) **"Matching capacity with demand" plan.**

"Level capacity" plan is based in "produce-to-stock and sell" approaches wherein the production systems are operated at uniform production levels and finished goods inventories rise and fall depending upon whether production level exceeds demand or vice versa from time period to time period (say every quarter).

Matching capacity with demand" Plan: In this plan, production capacity is matched with the demand in each period (weekly, monthly or quarterly demand). Usually, material flows and machine capacity are changed from quarter to quarter to match the demand. The main advantages are low levels of finished goods inventory resulting in lesser inventory carrying costs. Also, the back-ordering cost is reduced. The disadvantages are high labour and material costs because of frequent changes in workforce (hiring, training and lay-off costs, overtime or idle time cost or subcontracting costs).

Optimum Plant Capacity: Plant capacity has a great influence on cost of production with increasing volume of production, economies of scale arises which results in reduction in average cost per unit produced.

For a given production facility, there is an optimum volume of output per year that results in the least average unit cost. This level of output is called the "best operating level" of the plant.

As the volume of output increases outward from zero in a particular production facility, average unit costs fall. These declining costs are because of the following reasons: (i) Fixed costs are spread over more units produced, (ii) Plant construction costs are less, (iii) Reduced costs of purchased material due to quantity discounts for higher volume of materials purchased and (iv) Cost advantages in mass production processes. Longer production runs (i.e., higher batch quantity of products produced) have lesser setup cost per unit of product produced, lesser scrap etc., resulting in savings which will reduce the cost of production per unit. This is referred to as "economies of scale". But this reduction in per unit cost will be only upto certain volume of production. Additional volumes of outputs beyond this volume results in ever-increasing average unit production cost. This increase in cost per unit arise from increased congestion of materials and workers, which decreases efficiency of production, and due to other factors such as difficulty in scheduling, damaged products, reduced employee morale due to excessive work pressure, increased use of overtime etc., resulting in "diseconomies of scale". Hence, the plant capacity should be such that the optimum level of production which gives the minimum average cost of production per unit should be possible. This plant capacity is referred to as optimum plant capacity.

Balancing the Capacity: In firms manufacturing many products (a product line or a product-mix) the load on different machines and equipments vary due to changes in product-mix. When the output rates of different machines do not match with the required output rate for the products to be produced, there will be an imbalance between the work loads of different machines. This will result in some machine or equipment becoming a "bottleneck work centre" thereby limiting the plant capacity which will in-turn increase the production costs per unit.

To overcome problem of imbalance between different machines, additional machines or equipments are added to the bottleneck work-centre to increase the capacity of the bottle-neck work centre to match with the capacity of other work centres. Adding new machines or equipments to bottleneck work centres to remove the imbalance in capacity between various work centres is found to be economical than giving excessive overtime to workers working in bottle-neck centres which increases production costs. Another method to remove imbalance is to subcontract excess work load of bottleneck centres to outside vendors or subcontractors. Another way to balance capacities is to try to change the productmix by manipulating the sales for different products to arrive at a suitable product-mix which loads all work centres almost uniformly.

QUESTION 14
Implications of Plant Capacity

ANSWER:
There are two major cost implications of plant capacity:

(i) Changes in output of an existing plant of certain installed capacity affect the production costs.
(ii) Changes in the plant capacity by changing the size of a plant have significant effects on costs.

QUESTION 15
Factors influencing Effective Capacity

ANSWER:
The effective capacity is influenced by - (1) Forecasts of demand, (2) Plant and labour efficiency, (3) Subcontracting, (4) Multiple shift operation, (5) Management policies.

Forecasts of demand: Demand forecast is going to influence the capacity plan in a significant way. As such, it is very difficult to forecast the demand with accuracy as it changes significantly with the product life-cycle stage, number of products. Products with longer lifecycle usually exhibit steady demand growth compared to one with shorter life-cycle. Thus the accuracy of forecast influences the capacity planning.

Plant and labour efficiency: It is difficult to attain 100 per cent efficiency of plant and equipment. The efficiency is less than 100 percent because of the enforced idle time due to machine breakdown, delays due to scheduling and other reasons. The plant efficiency varies from equipment to equipment and from organisation to organisation. Labour efficiency contributes to the overall capacity utilisation. The standard time set by industrial engineer is for a representative or normal worker. But the actual workers differ in their speed and efficiency. The actual efficiency of the labour should be considered for calculating efficiency. Thus plant and labour efficiency are very much essential to arrive at realistic capacity planning.

Subcontracting: Subcontracting refers to off loading, some of the jobs to outside vendors thus hiring the capacity to meet the requirements of the organisation. A careful analysis as to whether to make or to buy should be done. An economic comparison between cost to make the component or buy the component is to be made to take the decision.

Multiple shift operation: Multiple shifts are going to enhance the firm's capacity utilisation. But especially in the third shift the rejection rate is higher. Specially for process industries where investment is very high it is recommended to have a multiple shifts.

Management policy: The management policy with regards to subcontracting, multiplicity of shifts (decision regarding how many shifts to operate), which work stations or departments to be run for third shift, machine replacement policy, etc., are going to affect the capacity planning.

Factors favouring over capacity and under capacity

It is very difficult to forecast demand as always there is an uncertainty associated with the demand. The forecasted demand will be either higher or lower than the actual demand. So always there is a risk involved in creating capacity based on projected demand. This gives rise to either over capacity or under capacity.

The over capacity is preferred when:

(a) Fixed cost of the capacity is not very high.
(b) Subcontracting is not possible because of secrecy of design and/or quality requirement.
(c) The time required to add capacity is long.
(d) The company cannot afford to miss the stipulated delivery date and cannot afford to lose the customer.
(e) There is an economic capacity size below which it is not economical to operate the plant.

The under capacity is preferred when:

(a) Fixed cost of the capacity is very high.
(b) Shortage of products does not affect the company (i.e., lost sales can be compensated).
(c) The technology changes fast, i.e., the rate of obsolescence of plant and equipment are high.
(d) The cost of creating the capacity is prohibitively high.

Illustration 5

A department works on 8 hours shift, 250 days a year and has the usage data of a machine, as given below:

Product	Annual demand (units)	Processing time (standard time in hours)
X	300	4.0
Y	400	6.0
Z	500	3.0

Determine the number of machines required

ANSWER:

Step 1: Calculate the processing time needed in hours to produce product x, y and z in the quantities demanded using the standard time data.

Product	Annual demand(units)	Standard processing timeper unit (Hrs.)	Processing time needed (Hrs.)
X	300	4.0	300 x 4 = 1200 Hrs.
Y	400	6.0	400 x 6 = 2400 Hrs.

Product	Annual demand (units)	Standard processing timeper unit (Hrs.)	Processing time needed (Hrs.)
Z	500	3.0	500 × 3 = 1500 Hrs.
			Total = 5100 Hrs

Step 2 : Annual production capacity of one machine in standard hours = 8 × 250 = 2000 hours per year

Step 3 : Number of machines required

= $\frac{\text{Work load per year}}{\text{Production capacity per}}$ = $\frac{5100}{2000}$ = 2.55 machines = 3 machines.

Illustration 6

A steel plant has a design capacity of 50,000 tons of steel per day, effective capacity of 40,000 tons of steel per day and an actual output of 36,000 tons of steel per day. Compute the efficiency of the plant and its utilisation.

ANSWER:

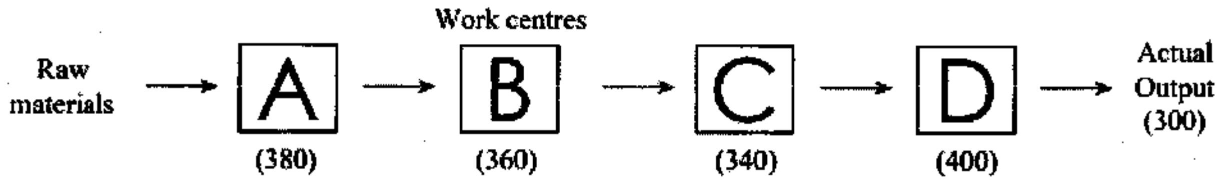
Actual output

$$\text{Efficiency of the plant} = \frac{\text{Actual output}}{\text{Effective Capacity}} = \left(\frac{36000}{40000}\right) \times 100 = 90\%$$

$$\text{Utilisation} = \left(\frac{\text{Actual output}}{\text{Design Capacity}}\right) = \left(\frac{36000}{50000}\right) \times 100 = 72\%$$

Illustration 7

A firm has four work centres, A, B, C & D, in series with individual capacities in units per day shown in the figure below



(i) **Identify the bottle neck centre.**

(ii) **What is the system capacity?**

(iii) **What is the system efficiency?**

ANSWER:

(i) **The bottle neck centre is the work centre having the minimum capacity. Hence, work centre 'C' is the bottleneck centre.**

(ii) **System capacity is the maximum units that are possible to produce in the system as a whole. Hence, system capacity is the capacity of the bottle neck centre i.e., 340 units.**

(iii) System efficiency = $\frac{\text{Actual output}}{\text{System Capacity}}$

= $\frac{300}{340}$ × 100 (i.e., maximum possible output) = 88.23%

Illustration 8

A manager has to decide about the number of machines to be purchased. He has three options i.e., purchasing one, or two or three machines. The data are given below.

Number of machine	Annual fixed cost	Corresponding range of output
One	12,000	0 to 300
Two	15,000	301 to 600
Three	21,000	601 to 900

Variable cost is 20 per unit and revenue is 50 per unit

(a) Determine the break-even point for each range

(b) If projected demand is between 600 and 650 units how many machines should the manager purchase?

ANSWER:

(i) Break-even point

Let Q be the break even point.

FC = Fixed cost, R = Revenue per unit, VC = Variable cost

At, BEP, TR = FC + TVC

or, Revenue p.u × Q = FC + VCp.u. × Q

Q (R - VC) = FC

$Q=\frac{EC}{R-VC}$

Let Q1 be the break-even-point for one machine option

Then, $Q1=\frac{1200}{(50-20)}=\frac{1200}{30}==$ 400 units

(Not within the range of 0 to 300)

Let Q2 be the break-even-point for two machines option.

Then, $Q2 =\frac{1500}{(50-20)}=\frac{1500}{30}=$ *500 units*

(within the range of 301 to 600)

Let Q3 be the break-even-point for three machines option.

Then, $Q3 =\frac{21000}{(50-20)}=\frac{21000}{30}=$ *700 units*

(with in the range of 601 to 900)

(ii) The projected demand is between 600 to 650 units.

The break even point for single machine option (i.e., 400 units) is not feasible because it exceeds the range of volume that can be produced with one machine (i.e., 0 to 300).

Also, the break even point for 3 machines is 700 units which is more than the upper limit of projected demand of 600 to 650 units and hence not feasible. For 2 machines option the break even volume is 500 units and volumerange is 301 to 600.

Hence, the demand of 600 can be met with 2 machines and profit is earned because the production volume of 600 is more than the break even volume of 500. If the manager wants to produce 650 units with 3 machines, therewill be loss because the break even volume with three machines is 700 units. Hence, the manager would choose two machines and produce 600 units.

2.3. FACILITY LOCATION AND LAYOUT

QUESTION 16

General Introduction of Facility Location and Layout

ANSWER:

In this module we shall discuss about Facility Location selection issue. Before we proceed further, Let us first take an example of a product manufacturing company XYZ Ltd.

The XYZ Ltd requires to take few important decisions. The first question comes into picture is: "What to produce?"

This question is linked with the basic existence of the company It talks about the product that XYZ Ltd. Is manufacturing. Here, the organisation needs to understand that what is the need of the customers in terms of product attributes/Features & quality. In other words, it talks about the competitive positioning of the company, its products acceptability at the market place this decision is based on the input received from market intelligence team and often is a part of the product design process later on we will study an important concept related to product design, such as QFD. In this regard, one important point to be noted that, many a times the organisations need to forecast about product life cycle & related requirement of the technology. Forecasting we will discuss separately.

One the company is aware that what it needs to produce, the second question comes: "How much to produce?"

This question is an ongoing questions, as the organisation is engaged in estimating the quantity ("How much") on a daily, weekly, monthly, quarterly & yearly basis. Again this information is obtained from marketing team. Based on the information received, the planning team (as a part of supply chain's planning section) provides a forecast of demand. Hence, here deal wiith an important aspect of operational planning known as Demand Forecasting

The next question is: "Where to produce?".

This question leads to facility location selection problem after this, a series of questions need to be answered that lead to a member of decision areas such as"

Q: "How to produce?" (Process selection & Layout)

Q: "When to produce?" (Aggregate Planning inventory Master Production decision schedule) Q: "Do we have materials to produce?" (MRP, Inventory Management)

It also deals with Sourcing Q

Q: "Are we producing right things?" (Quality Management)

Q: "Are our machines able to provide desired results?" (Maintenance Management) Q: "How to reach the products to the customers?" (Distribution or Delivery planning)

It includes transportation decision, warehousing, materials handling ets. Logistics issues

In cast the organisation is practicing sustainability then another important decision area is reverse Logistics i.e., taking returns

Therefore, in summary the major decision areas are:

1. Product selection
2. Facility Location Selection
3. Demand Forecasting
4. Process selection & Layout decision
5. Capacity planning

6. Aggregate Planning, Master production schedule
7. Materials Requirement Planning (MRP)/Manufacturing Resource Planning (MRP I)/ Distribution Resource Planning (DRP)/Enterpnse Resource Planning (ERP)

Inventory Management
Supplier Selection/Sourcing
Process Management
Quality Management
Maintenance
Warehousing/Transporation
Reverse Logistics
In Addition, an operations manager is also responsible for working capital management, skill-management etz

A. Definition of a Facility and Facility Location Decision

In a simpler term, facility means any type of set-up that an organisation requires to run its operations and produce required products and/or delivers intended services to the customers. Location, on the other hand is the place or region where the above-mentioned facility may be established.
Therefore, the decision of facility location involves following decision:

Where to establish the set-up?

B. Need for an appropriate facility location

The need for selecting an appropriate facility location stems from the following concerns of an organisation such as

(i) When an organisation wants to start a new business and/or expand the existing business by entering into a new market and/or increases the scale of operations

(ii) When the business faces some complexities and/or difficulties in terms of resource shortage, expiry of lease agreement, socio-cultural problems, legal and political issues and other economic and social issues that force the organisations to change the location.

C. A Good Facility Location

The effectiveness of a selected location depends on the context and its relevance in the business objectives as set by the organisations and finally, on the way the location provides a competitive edge to the organisation. In general, a good facility location provides the following benefits to the organisations

(i) Cost benefit in terms reduced fixed and variable cost, transportation cost.
(ii) Proximity to market and source
(iii) Easy and hassle free transport facility

In short, a good facility location is one that enables the organisations to strike a balance between cost (cost of production and service) and service and intends to maximize the service quality while minimizing the cost to the extent possible and remain competitive at the market place (in terms of visibility, proximity to source and market). It is a strategic decision that organisation takes.

D. Factors affecting location selection decision

The following factors influence the location selection decision.

1. Sourcing
   (i) Availability of raw materials
   (ii) Availability of natural resources, energy and waters
   (iii) Availability of internet connectivity
   (iv) Proximity to the key suppliers
   (v) Connectivity to alternate vendors
   (vi) Opportunity to cross-docking and utilizing milk vans

2. Markets

   (i) Proximity to market

   (ii) Coverage of wide geographical area (with close proximity to target customers) keeping the facility atfocal point

   (iii) Connectivity with a large customer base

   (iv) Lesser time to market

   (v) Connectivity

3. Cost

   (i) Lesser transportation cost and well availability of various transportation modes

   (ii) Lesser lease and/or rental cost

   (iii) Tax, and other duties

   (iv) Other hidden cost

4. Socio-cultural, community and Political issues

   (i) Supportive community

   (ii) Familiarity with language, rituals and culture

   (iii) Level of crime and other disturbances

   (iv) Availability of prospective employees

   (v) Quality of living

   (vi) Statutory and regulatory rules and regulations

   (vii) Availability of medical facilities, fire and police

5. Environmental concerns
6. Availability of skilled labours
7. Competitive pressure

In a nutshell, the location selection depends on

- Additional growth-space and layout;
- Better transportation network;
- Reduction in plant operations and service costs;
- Cordial labor situation;
- Better environment in terms of water, air, and land;
- Better community and public relations;
- Greater inter-plant coordination and centralized control;
- Improved logistics; and
- Increased future plant profitability.

E. Some approaches for facility location selection

An organisation follows certain steps to make a correct location choice.
These Steps are:

1. Decide on the criteria for evaluating location alternatives
2. Identify important factors
3. Develop location alternatives
4. Evaluate the alternatives
5. Make a decision and select the location

Some of the popular approaches are:

(i) Factor Rating Method
(ii) Centre of Gravity Technique
(iii) Transportation Model
(iv) Optimization and Heuristic Models

Let us consider some situations of facility location selection problems

Illustration 9

Suppose, an E-Commerce company wants to open Central order fulfilment center in Kolkata South in West Bengal. The possible locations are say L1, L2, and L3. The company form a group of experts. The team identifies say 6 actors such as F1, F2, F3, and F4 to evaluate L1 to L3.

ANSWER:

This situation can be solved using Factor Rating Method. The steps are:

In the first stage the expert team needs to give weightage to the factors. This can be done in many ways. In the
following one simple way is explained. A possible approach:

Suppose, the experts rate each factor on a scale 1 to 5 (1: least important and 5: Most important)

Factor	Rating						
	E-1	E-2	E-3	E-4	E-5	Row	Weight
F_1	4	3	4	4	3	18	18/68
F_2	5	5	5	5	4	24	24/68
F_3	3	4	4	3	5	19	19/68
F_4	2	1	2	1	1	7	7/68
						68	

There may be other ways (e.g., AHP method). Let us now come back to our problem. Let us assume the factors are following weights

Factors	Weight
F_1	0.3
F_2	0.2
F_3	0.1
F_4	0.4
	Total 1.0

The experts are requested to rate each of the location alternatives with respect to the factors, e.g., 10: Most beneficial and 1: Least beneficial

Factors	Alternatives		
	L_1	L_2	L_3
F_1	10	9	7
F_2	7	3	10
F_3	7	5	10
F_4	6	8	5

So the complete table becomes

Factors	Weight	Alternatives		
		L_1	L_2	L_3
F_1	0.3	10	9	7
F_2	0.2	7	3	10
F_3	0.1	7	5	10
F_4	0.4	6	8	5
	Best Location	7.5	7	7.1

Example of calculation

for L1 : $0.3 \times 10 + 0.2 \times 7 + 0.4 \times 6 = 3 + 1.4 + 0.7 + 2.4 = 7.5$

As per the weighted score

Location L1 is the best locatior

Illustration 10

Suppose, XYZ Ltd wants to open a retail shop in Kolkata, West Bengal.

It first selects the 4 locations such as L1, L2, L3 and L4. The coordinates of the locations (i.e., latitudes and

longitudes) and volume of customers (i.e., average number of customers in a day in '000) are given in the following table

Location	Volume	Coordinates	
		X	Y
L_1	200	30	100
L_2	100	90	120
L_3	100	130	130
L_4	200	60	40

ANSWER:

Find out the best location using Center of Gravity (COG) method.

Loc	V_1	x_i	y_i	$v_i x_i$	$v_i y_i$
L_1	200	30	100	6000	20000
L_2	100	90	120	9000	12000
L_3	100	130	130	13000	13000
L_4	200	60	40	12000	8000
	600		Total	40,000	53,000

Therefore, ΣVi = 600; Σvixi = 40000

Σviyi = 53000

COG location is given by (X, Y)

$$X = \frac{\sum V_i X_i}{\sum V_i}$$

$= 40000/600 = 200/3$

$$Y = \frac{\sum V_i Y_i}{\sum V_i} = 53000/600 = 265/3$$

QUESTION 17

Facility Layout

ANSWER:

Plant Layout, also known as layout of facility refers to the configuration of departments, work-centres and equipment and machinery with focus on the flow of materials or work through the production system.

Plant layout or facility layout means planning for location of all machines, equipments, utilities, work stations, customer service areas, material storage areas, tool servicing areas, tool cribs, aisles, rest rooms, lunch rooms, coffee/tea bays, offices, and computer rooms and also planning for the patterns of flow of materials and people around, into and within the buildings. Layout planning involves decisions about the physical arrangement of economic activity centres within a facility. An economic activity centre can be anything that consumes space, a person or group of people, a machine, a work station, a department, a store room and so on. The goal of layout planning is to allow workers and equipments to operate more effectively.

The questions to be addressed in layout planning are:

- How much space and capacity does each centre need?
- How should each center's space be configured?
- What centres should the layout include?
- Where should each centre be located?

The location of a centre has two dimensions:

- Absolute location or the particular space that the centre occupies within the facility.
- Relative location i.e., the placement of a centre relative to other centers

QUESTION 18
The importance of layout decisions:

ANSWER:

The need for layout planning arises both in the process of designing new plants and the redesigning existing plants or facilities.

Most common reasons for design of new layouts are:

(i) Layout is one of the key decisions that determine the long-run efficiency in operations.

(ii) Layout has many strategic implications because it establishes an organisation's competitive priorities in regard to capacity, processes, flexibility and cost as well as quality of work life, customer contact and image (in case of service organisations).

(iii) An effective layout can help an organisation to achieve a strategic advantage that supports differentiation, low cost, fast response or flexibility.

(iv) A well designed layout provides an economic layout that will meet the firm's competitive requirements.

QUESTION 19
Need for redesign of layout arises because of the following reasons

ANSWER

- Accidents, health hazards and low safety,
- Changes in environmental or legal requirements,
- Changes in processes, methods or equipments,
- Changes in product design/service design,
- Changes in volume of output or product-mix changes,
- Inefficient operations (high cost, bottleneck operations),
- Introduction of new products/services,
- Low employee morale.

QUESTION 20
Good Plant layout- Objectives

ANSWER

- Efficient utilisation of labour reduced idle time of labour and equipments,
- Higher flexibility (to change the layout easily),
- Higher utilisation of space, equipment and people (employees),
- Improved employee morale and safe working conditions,
- Improved flow of materials, information and people (employees),
- Improved production capacity,
- Reduced congestion or reduced bottleneck centers,
- Reduced health hazards and accidents,
- To allow ease of maintenance,
- To facilitate better coordination and face-to-face communication where needed,
- To improve productivity,
- To provide ease of supervision,
- To provide product flexibility and volume flexibility,
- To utilise available space efficiently and effectively.

QUESTION 21
Choices of Layout

ANSWER:
Layout choices can help greatly in communicating an organisation's product plans and competitive priorities. Layout has many practical and strategic implications. Altering a layout can affect an organisation and how well it meets its competitive priorities by:

- Facilitating the flow of materials and information,
- Improving communication,
- Improving employee morale,
- Increasing customer convenience and sales (in service organisations such as retail stores),
- Increasing the efficient utilisation of labour and equipment,
- Reducing hazards to employees.

The type of operations carried out in a firm determines the layout requirements.
Some of the fundamental layout choices available to managers are:

- Whether to plan the layout for the current or future needs?
- Whether to select a single-story or multistory building design?
- What type of layout to choose?
- What performance criteria to emphasize?

QUESTION 22
Factors influencing layout choices

ANSWER:
Primarily the layout of a plant is influenced by the relationship among materials, machinery and men. Other factors influencing layout are type of product, type of workers, the type of industry, management policies etc.
Some of these factors are discussed in detailed below:

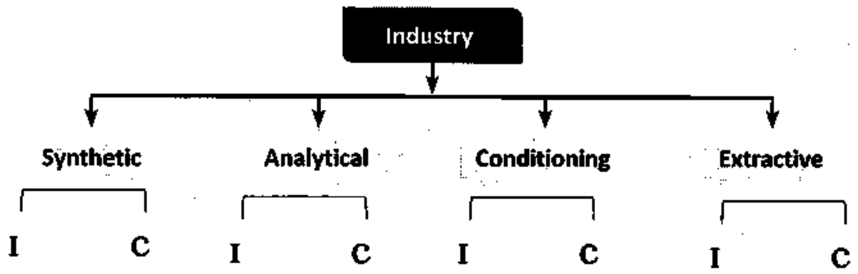
- **Location:** The size and type of the site selected for the plant, influences the type of buildings (single story or multi story) which in turn influences the layout design. Also, the location of the plant determines the mode of transportation from and into the plant (such as by goods trains, truck, or ships) and the layout should provide facilities for mode of transport used. Also, the layout should provide for storage of fuel, raw materials, future expansion needs, power generation requirements etc.
- **Machinery and Equipments:** The type of product, the volume of production, type of processes and management policy on technology, determines the type of machines and equipments to be installed
- **Managerial Policies:** regarding volume of production, provision for future expansion, extent of automation, make-or-buy decisions, speed of delivery of goods to customers, purchasing and inventory policies and personnel policies influence the plant layout design.
- **Materials:** Plant layout includes provision for storage and handling of raw materials, supplies and components used in production. The type of storage areas, racks, handling equipments such as cranes, trolleys, conveyors or pipelines etc., used - all depend on the type of materials used - such as solid, liquid, light, heavy, bulky, big, small etc.
- **Product:** The type of product i.e., whether the product is light or heavy, big or small, liquid or solid etc., it influences the type of layout. For example, Ship building, Aircraft assembly,

Locomotive assembly etc., requires a layout type different from that needed to produce refrigerators, cars, scooters, television sets, soaps, detergents, soft drinks etc. The manufacturing process equipments and machines used and the processing steps largely depend on the nature of the product and hence the layout design depends, very much on the product.

QUESTION 23
Type of Industry

ANSWER:

Type of Industry:



I is intermittent type of industry.
C is continuous type of industry
Whether the industry is classified under (a) Synthetic, (b) Analytical, (c) Conditioning and (d) Extraction industries and again whether the industry has intermittent production or continuous production has a relevance to the type of layout employed

- **Workers** : The gender of employees (men or women), the position of employees while working (i.e., standing or sitting), employee facilities needed such as locker rooms, rest rooms, toilets, canteens, coffee/teabays etc., are to be considered while designing the plant layouts.

QUESTION 24
Plant Layout- Principles?

The layout selected in conformity with layout principles should be an ideal one. These principles are:-

- **Principle of Minimum Travel:** Men and materials should travel the shortest distance between operations so as to avoid waste of labour and time and minimise the cost of materials handling.
- **Principle of Sequence:** Machinery and operations should be arranged in a sequential order. This principle is best achieved in product layout, and efforts should be made to have it adopted in the process layout.
- **Principle of Usage:** Every unit of available space should be effectively utilised.
- **Principle of Compactness:** There should be a harmonious fusion of all the relevant factors so that the final layout looks well integrated and compact.
- **Principle of Safety and Satisfaction:** The layout should contain built in provisions for safety for the workmen. It should also be planned on the basis of the comfort and convenience of the workmen so that they feel satisfied.

- **Principle of Flexibility:** The layout should permit revisions with the least difficulty and at minimum cost.
- **Principle of Minimum Investment:** The layout should result in savings in fixed capital investment, not by avoiding installation of the necessary facilities but by an intensive, use of available facilities.

QUESTION 25

Types of Layout?

ANSWER:

A layout essentially refers to the arranging and grouping of machines which are meant to produce goods. Grouping is done on different lines. The choice of a particular line depends on several factors. The methods of grouping or the types of layout are:

(i) Process layout or functional layout or job shop layout;

(ii) Product layout or line processing layout or flow-line layout;

(iii) Fixed position layout or static layout;

(iv) Cellular manufacturing (CM) layout or Group Technology layout and

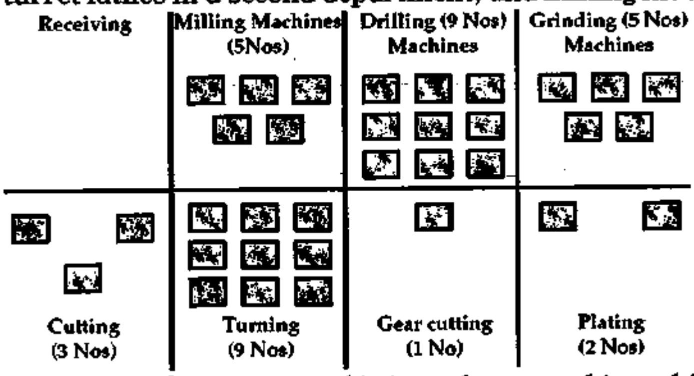
(v) Combination layout or Hybrid layout.

QUESTION 26

Process Layout?

ANSWER:

Also called the functional layout, layout for job lot manufacture or batch production layout, the process layout involves a grouping together of similar machines in one department. For example, machines performing drilling operations are installed in the drilling department; machines performing turning operations are grouped in the turning department; and so on. In this way, there would be an electroplating department, a painting department, a machining departments and the like, where similar machines or equipments are installed in the plants which follow the process layout. The process arrangement is signified by the grouping together of like machines based upon their operational characteristics. For example, centre lathes will be arranged in one department, turret lathes in a second department, and milling machines in a third departments.



A quantity of raw material is issued to a machine which performs the first operation. This machine may be situated anywhere in the factory. For the next operation, a different machine may be required, which may be situated in another part of the factory. The material should be transported to the other machine for the operation. Thus, material would move long distances and along crisscrossing paths. At one stage, the material may be taken to a separate building, say, for heat treatment, and then brought back for grinding. If machines in one department are engaged, the partly finished product awaiting operations may be taken to the store and later reissued for production. Partly finished goods would be waiting for processing in every department, like commuters waiting for buses in a city.

Machines in each department attend to any product that is taken to them. These machines are, therefore, called general purpose machines. Work has to be allotted to each department in such a way that no machine in any department is idle. In a batch production layout, machines are chosen to do as many different jobs as possible, i.e., the emphasis is on general purpose machines. The work which needs to be done is allocated to the machines according to loading schedules, with the objective of ensuring that each machine is fully loaded. The process layout carries out the functional idea of Taylor and from the historical point of view, process layout precedes product layout. This type of layout is best suited for intermittent type of production.

While grouping machines according to the process type, certain principles must be kept in mind. These are:

- Convenience for inspection.
- Convenience for supervision. Process layout may be advantageously used in light and heavy engineering industries, made-to-order furniture industries and the like.
- The distance between departments needs to be as short as possible with a view to avoiding long distance movement of materials.
- Though similar machines are grouped in one department, the departments themselves should be located in accordance with the principle of sequence of operations. For example, in a steel plant, the operations are smelting, casting; rolling etc. These different departments may be arranged in that order to avoid crossovers and backtracking of materials.

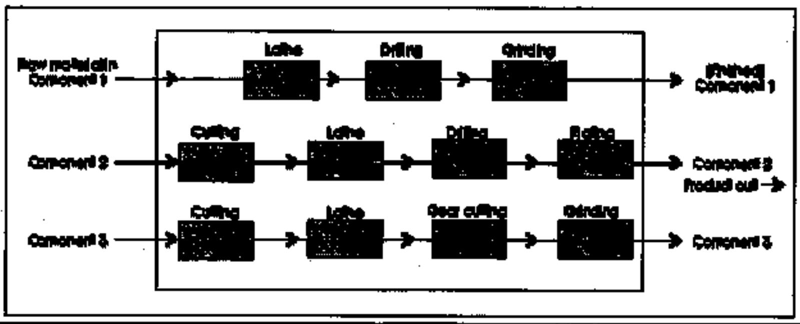
QUESTION 27

Product Layout

ANSWER:

Also called the straight-line layout or layout for serialised manufacture. The product layout involves the arrangement of machines in one line depending upon the sequence of operations. Material is fed into the first machine and finished products come out of the last machine. In between, partly finished goods move from machine to machine. The output of one machine becoming the input for the next. In a sugar mill, sugar cane, fed at one end of the mill comes out as sugar at the other end. Similarly, in paper mill, bamboos are fed into the machine at one end and paper comes out at the other end.

In product layout, if there are more than one, line of production, there are as many, lines of machines. The emphasis here, therefore, is on special purpose machines in contrast to general purpose machines, which are installed in the process layout. Consequently, the investment on machines in a straight line layout is higher than the investment on machines in a functional layout.



The grouping of machines should be done, on product line, keeping in mind the following principles:

- All the machine tools or other types of equipment must be placed at the point demanded by the sequence of operations.
- All the operations, including assembly, testing and packing should be, included in the line.
- Materials may be fed where they are required for assembly but not necessarily all at one point; and
- There should be no points where one line crosses another line;

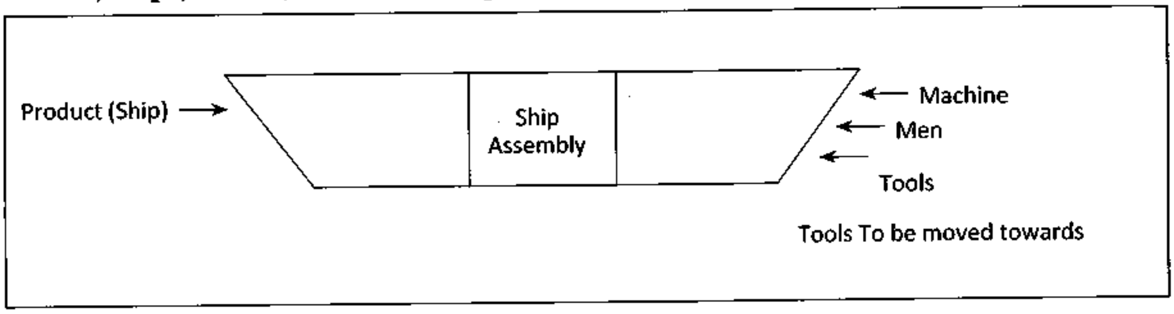
The product layout may be advantageously followed in plants manufacturing standardised products on a mass scale such as chemical, paper, sugar, rubber, refineries and cement industries.

QUESTION 28

Layout in the form of Fixed Position?

As the term itself implies, the fixed position layout involves the movement of men and machines to the product which remains stationary. In this type of layout, the material or major component remains in a fixed location, and tools, machinery and men as well as other pieces of material are brought to this location. The movement of men and machines to the product is advisable because the cost of moving them would be less than the cost of moving the product which is very bulky.

Also called static layout, this type is followed in the manufacture, if bulky and heavy products, such as locomotives, ships, boilers, air crafts and generators.



QUESTION 29

Mixed Layout or Combined Layout

The application of the principles of product layout or process layout in their strict meanings is difficult to come across. A combination of the product and process layouts, with an emphasis on either, is noticed in most industrial establishments. Plants are never laid out in either pure form. It is possible to have both types of layout in an efficiently combined form if the products manufactured are somewhat similar and not complex.

QUESTION 30

Layout of Service Facility?

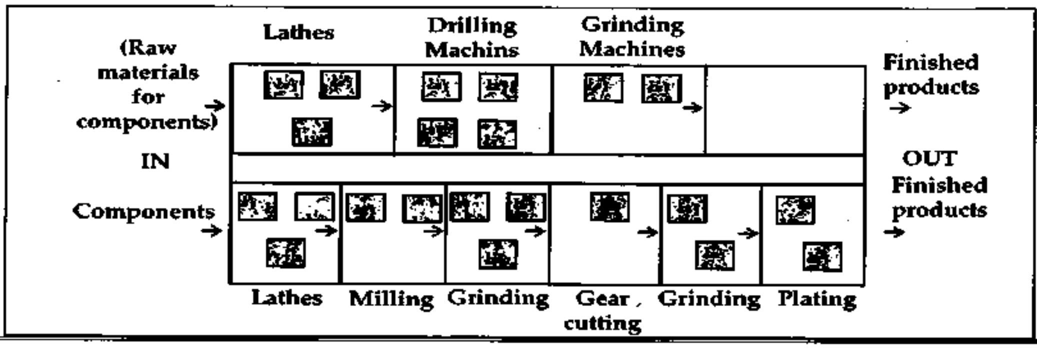
ANSWER

The fundamental difference between service facility and manufacturing facility layouts is that many service facilities exist to bring together customers and services. Service facility layouts should provide for easy entrance to these facilities from freeways and busy thoroughfares. Large, well

organized and amply lighted parking areas and well designed walkways to and from parking areas are some of the requirements of service facility layouts.
Because of different degree of customer contact, two types of service facility layouts emerge, viz., those that are almost totally designed around the customer receiving and servicing function (such as banks) and those that are designed around the technologies, processing of physical materials and production efficiency (such as hospitals).

QUESTION 31
Other facilities with reference to Plant Layout?

A plant layout involves, besides the grouping of machinery, an arrangement for other facilities as well. Such facilities include receiving and shipping points, inspection facilities, employee facilities and storage. Not all the facilities are required in every plant. The requirements depend on the nature of the product which is manufactured in a particular plant.



QUESTION 32
Importance of layout?

ANSWER:
The importance of a layout can be described as under:

- **Avoidance of Bottlenecks:** Bottlenecks refer to any, place in a production process where materials tend to pile up or produced at rates of speed less rapid than the previous or subsequent operations. Bottlenecks are caused by inadequate machine capacity, inadequate storage space or low speed on the part of the operators. The results of bottlenecks are delays in production schedules, congestion, accidents and wastage of floor area. All these may be overcome with an efficient layout.
- **Avoidance of Unnecessary and Costly Changes:** *A* planned layout avoids frequent changes which are difficult and costly. The incorporation of flexibility elements in the layout would help in the avoidance of revisions.
- **Better Production Control:** Production control is concerned with the production of a product of the right type at the right time and at reasonable cost. A good plant layout is a requisite of good production control and provides the plant control officers with a systematic basis upon which to build organisation and procedures.
- **Better Supervision:** A good plant layout ensures better supervision in two ways: (a) Determining the number of workers to be handled by a supervisor and (b) Enabling the supervisor to get a full view of the entire plant at one glance. A good plant layout is, therefore, the first step in good supervision.

- **Economies in Handling:** Nearly 30 per cent to 40 per cent of the manufacturing costs are accounted for by materials handling. Every effort should, therefore, be made to cut down this cost. Long distance movements should be avoided and specific handling operations must he eliminated.
- **Effective Use of Available Area:** Every unit of the plant area is valuable, especially in urban areas. Efforts should therefore, be made to make use of the available area by planning the layout properly.
- **Improved Employee Morale:** Employee morale is achieved when workers are cheerful and confident. This state of mental condition is vital to the success of any organisation. Morale depends on better working conditions; better employee facilities; reduced number of accidents; and increased earnings.
- **Improved Quality Control:** Timely execution of orders will be meaningful when the quality of the output is not below expectations. To ensure quality, inspection should be conducted at different stages of manufacture. An ideal layout provides ample space to carryout inspection to ensure better quality control.
- **Improved Utilisation of Labour:** A good plant layout is one of the factors in effective utilisation of labour. It makes possible individual operations, the process and flow of materials handling in such a way that the time of each worker is effectively spent on productive operations.
- **Minimisation of Production Delays:** Repeat order and new customers will be the result of prompt execution of orders. Every management should try to keep to the delivery schedules by minimising delays in production.
- **Minimum Equipment Investment:** Investment on equipment can be minimised by planned machine balance and location, minimum handling distances, by the installation of general purpose machines and by planned machine loading. A good plant layout provides all these advantages.

Illustration 11

The present layout is shown in the figure. The manager of the department is intending to interchange the departments C and F in the present layout. The handling frequencies between the departments is given. All the departments are of the same size and configuration. The material handling cost per unit length travel between departments is same. What will be the effect of interchange of departments C and F in the layout?

A	C	E
B	D	F

From / To	A	B	C	D	E	F
A	-	0	90	160	50	0
B	-	-	70	0	100	130
C	-	-		20	0	0

From / To	A	B	C	D	E	F
D	-	-	-	_	180	10
E		-	-	-	-	40
F	-	-	-	-	-	-

ANSWER:

The distance matrix of the present layout :

From / To	A	B	C	D	E	F
A		1	1	2	2	3
B			2	1	3	2
C				1	1	2
D					2	1
E						1
F						-

Computation of total cost matrix (combining the inter departmental material handling frequencies and distance matrix.

From / To	A	B	C	D	E	F	Total
A		0	90	320	100	0	510
B			140	0	300	260	700
C				20	0	0	20
D					360	10	370
E						40	40
F							-
Total							1,640

If the departments are interchanged, the layout will be represented as shown below.

C	F	E
B	D	C

The distance matrix and the cost matrix are represented as shown.

From / To	A	B	C	D	E	F
A		1	3	2	2	1
B			2	1	3	2

From / To	A	B	C	D	E	F
C				1	1	2
D					2	1
E						1
F						

Total cost matrix for the modified layout.

From / To	A	B	C	D	E	F	Total
A	-	0	270	320	100	0	690
B			140	0	300	260	700
C				20	0	0	20
D					360	10	370
E						40	40
F							-
Total							1,820

The interchange of departments C and F increases the total material handling cost. Thus, it is not a desirable modification.

Illustration 12

A defense contractor is evaluating its machine shops current process layout. The figure below shows the current layout and the table shows the trip matrix for the facility. Health and safety regulations require departments E and F to remain at their current positions.

E	B	F
A	C	D

Current Layout

From / To	A	B	C	D	E	F
A		8	3		9	5
B		-		3		
C			-		8	9
D				-		3
E					-	3
F						-

Can layout be improved? Also evaluate using load distance (ld) score.

ANSWER:

Keep the departments E and F at the current locations. From the Trip Matrix, C is having maximum no. of trips from E&F. So C must be as close as possible to both E and F, put C between them. Place A directly south of E, and B next to A. All of the heavy traffic concerns have been accommodated. Department D is located in the remaining place. The proposed layout is shown in figure below. The load distance (ld) scores for the existing and proposed layout are shown below. As ld score for proposed layout is less, the proposed layout indicates improvement over existing.

E	C	F
A	B	D

Comparative Analysis : Current and Proposed Layout :-

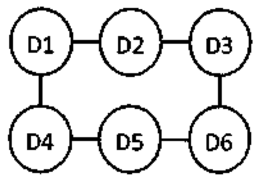
Dept. Pair	No. of Trips (1)	Existing plan		Proposed plan	
		Distance (2)	Load × Distance (1 × 2)	Distance (3)	Load × Distance (1 × 3)
A-B	8	2	16	1	8
A-C	3	1	3	2	6
A-E	9	1	9	1	9
A-F	5	3	15	3	15
B-D	3	2	6	1	3
C- E	8	2	16	1	8
C-F	9	2	18	1	9
D-F	3	1	3	1	3
E-F	3	2	6	2	6
Total			92		67

As 'ld' score of the proposed layout is lower than the existing one, there is an improvement in the new layout.

Illustration 13 (Facility Layout Decision)

Suppose a hospital has 6 major departments namely D1, D2, D3, D4, D5 and D6. The initial layout of the hospital is given below.

Initial Layout



The average traffic movement to and fro each department is given in the following table.

Table - Average traffic flow (Direct)

	D1	D2	D3	D4	D5	D6
D1	-	10	20	0	5	6
D2	8	-	6	10	0	2
D3	10	6	-	20	7	8
D4	0	25	5	-	10	3
D5	15	10	1	20	-	6
D6	0	6	0	3	4	-

The hospital wants to find out an optimum layout.

ANSWER:

We notice quite obviously that from Di to Di (i = 1, 2....6), there is no movement.

From D2 to D1, the average movement is 10 (circle) and from D1 to D2 the average movement is 8 (circle)

Therefore, the combined average traffic movement from D1 to D2 is = (10 + 8) = 18

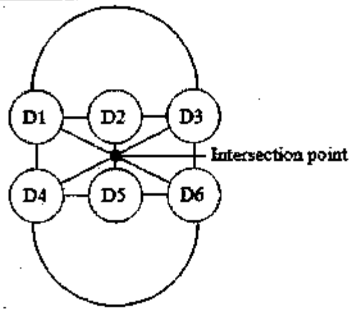
Let us now take another pair, e.g., D4 and D2

Movement	Avg traffic
D4 → D2	10 (red circle)
D2 → D4	25 (Green circle)

Therefore, the combined average traffic movement is 35. Proceeding in the same way, we get the combined average traffic movement for all pairs as follows:

	D1	D2	D3	D4	D5	D6
D1	-	18	30	0	20	6
D2		-	12	35	10	8
D3			-	25	8	8
D4				-	30	7
D5					-	10
D6						-

Let us now draw the initial layout again.



Looking at the diagram we can find the adjacent and non-adjacent pairs.

C	Non-adjacent Pairs
D1 & D2	D1 & D3
D2 & D3	D1 & D6
D3 & D6	D3 & D4
D6 & D5	D4 & D6
D5 & D4	
D2 & D5	
D1 & D4	
D1 & D5	
D3 & D5	
D2 & D4	
D2 & D6	

Let us now concentrate on the non-adjacent pairs

Non-adjacent Pair	Distance
D1 & D3	(D1 → D2: D2 → D3) D1 → D3 : 2 nodal points Hence, distance is 2

Non-adjacent Pair	Distance
D1 & D6	D1 → D6 = D1 → P & P → D6 Distance = 2
D3 & D4	D3 → D4 = D3 → P & P → D4 Distance = 2
D4 & D6	D4 → D6 = D4 → D5 & D5 → D6 Distance = 2

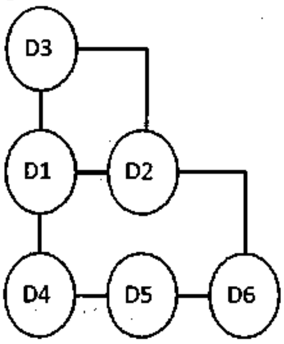
The combined average traffic movement between any two non-adjacent nodes is called the load distance. Our objective is to reduce the load distance.

Non-adjacent Pair	Load distance
D1 & D3	$30 \times 2 = 60$
D3 & D4	$25 \times 2 = 50$
D1 & D6	$6 \times 2 = 12$
D4 & D6	$7 \times 2 = 14$
	Total = 136

Note that for getting the load values, please refer table (Solution).

To meet our objective, we find the highest load distance, i.e., 60. Therefore, we need to rearrange the nodes.

We notice that from D1 to D3 and back, the highest traffic is involved. Therefore, we need to rearrange their
positions to make them adjacent as follows: First rearrangement



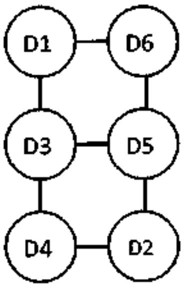
The revised non-adjacent pairs and load distance calculation is given below

Non-adjacent Pair	distance	Load distance
D4 & D6	2	14

Non-adjacent Pair	distance	Load distance
D1 & D6	2	12
D3 & D6	2	16
D3 & D5	2	16
D3 & D4	2	50
		108

We notice that there is an improvement. However, now the pair of D3 and D4 creates the problem. Therefore, we need to make them adjacent through rearrangement as follows:

2nd Arrangement



The revised non-adjacent pairs and load distance (after second arrangement) is given below

Non-adjacent Pair	Load	distance	load-distance
D_1D_4	2	0	0
D_6D_2	2	8	16
D_1D_2	2	18	36
D_6D_4	2	7	14
			66

Through trial and error approach we arrive at a considerable improvement. Therefore, the above layout (2nd Arrangement) is the acceptable one.

2.4. RESOURCE AGGREGATE PLANNING

QUESTION 33
Aggregate Planning

ANSWER:

Aggregate planning is an intermediate term planning decision. It is the process of planning the quantity and timing of output over the intermediate time horizon (3 months to one year). Within this range, the physical facilities are assumed to be fixed for the planning period. Therefore, fluctuations in demand must be met by varying labour and inventory schedule. Aggregate planning seeks the best combination to minimise costs.

Production planning in the intermediate range of time is termed as 'Aggregate Planning'. It is thus called because the demand on facilities and available capacities is specified in aggregate quantities. For example aggregate quantities of number of Automobile vehicles, Aggregate number of soaps etc. Here the total expected demand is specified without regard to the product mix that makes up the specified figure.

While dealing with production problems, the planning process is normally divided in three categories.

(i) Long range Planning which deals with strategic decisions such as purchase of facilities, introduction of new products, processes etc.

(ii) Short term planning which deals with day-to-day work, scheduling and sometimes inventory problems.

(iii) Intermediate Planning or Aggregate Planning, which is in between long range and short term planning, which is concerned in generally acceptable planning taking the load on hand and the facilities available into considerations. In aggregate planning the management formulates a general strategy by which capacity can be made to satisfy demand in a most economical way during a specific moderate time period, say for one year. The aggregate planning is made operational through a master schedule that gives the manufacturing schedule (Products and dates of manufacture). Generally, day-to-day schedules are prepared from master schedule. Facility planning and scheduling has got very close relationship with aggregate planning.

QUESTION 34
Aggregate Planning Strategies

ANSWER:

The variables of the production system are labour, materials and capital. More labour effort is required to generate higher volume of output. Hence, the employment and use of overtime (OT) are the two relevant variables. Materials help to regulate output. The alternatives available to the company are inventories, back ordering or subcontracting of items.

These controllable variables constitute pure strategies by which fluctuations in demand and uncertainties in
production activities can be accommodated.

Vary the size of the workforce: Output is controlled by hiring or laying off workers in proportion to the changes in demand.

Vary the hours worked: Maintain the stable workforce, but permit idle time when there is a 'slack' and permit overtime (OT) when demand is 'peak'.

Vary inventory levels: Demand fluctuations particularly increase in demand can be met by large amount of inventory.

Subcontract: In case of upward shift in demand from low level. Required production rates can be met by using the capacities available with the external vendors. This is also known as subcontracting.

Aggregate planning guidelines:

1. Determine corporate policy regarding controllable variables.
2. Use a good forecast as a basis for planning.
3. Plan in proper units of capacity.
4. Maintain the stable workforce.
5. Maintain needed control over inventories.
6. Maintain flexibility to change.
7. Respond to demand in a controlled manner.
8. Evaluate planning on a regular basis.

QUESTION 35

Properties of Aggregate Planning

ANSWER:

To facilitate the production manager the aggregate planning must have the following characteristics:

(i) Both out put and sales should be expressed in a logical overall unit of measuring. For example, an automobile manufacturing company can say 1000 vehicles per year, without giving the number of each variety of vehicle. Similarly a paint industry can say 10,000 litres of paint and does not mention the quantities of each variety of colour.

(ii) Acceptable forecast for some reasonable planning period, say one year.

(iii) A method of identification and fixing the relevant costs associated with the plant. Availability of alternatives for meeting the objective of the organisation. Ability to construct a model that will permit to take optimal or near optimal decisions for the sequence of planning periods in the planning horizon.

(iv) Facilities that are considered fixed to carry out the objective.

Illustration 14

ABC. Co. has developed a forecast for the group of items that has the following demand pattern

Quarter	Demand	Cumulative demand
1	270	270
2	220	490
3	470	960
4	670	1630
5	450	2080
6	270	2350
7	200	2550
8	370	2920

The firm estimates that it costs 150 per unit to increase production rate 200 per unit to decrease the production rate, 50 per unit per quarter to carry the items in inventory and 100 per unit if subcontracted. Compare the costs of the pure strategies.

ANSWER:

Different pure strategies are

Plan I In this pure strategy, the actual demand is met by varying the work force size. This means that during the period of low demand, the company must fire the workers and during the period of high demand the company must hire workers. These two steps involve associated costs. In this strategy, the production units will be equal to the demand and values in each period. The cost of the plan is computed in the table below,

Quarter	Demand	Cost of increasing Production level	Cost of decreasing Production level	Total cost of plan
1	270	–	–	–
2	220	–	50 × 200 = 10,000	10,000
3	470	250 × 150 = 37,500	–	37,500
4	670	200 × 150 = 30,000	–	30,000
5	450	–	220 × 200 = 44,000	44,000
6	270	–	180 × 200 = 36,000	36,000
7	200	–	70 × 200 = 14,000	14,000
8	370	170 × 150 = 25,500	–	25,500
	Total			1,97,000

Plan II In this plan, the company computes the average demand and sets its production capacity to this average demand. This results in excess of units in some periods and also shortage of units during some other periods. The excess units will be carried as inventory for future use and shortage of units can be fulfilled using future inventory. The cost of the plan II is computed in the table below. The plan incurs a maximum shortage of 255 units during quarter 5. The firm might decide to carry 255 units from the beginning of period 1 to avoid shortage. The total cost of the plan is 96,500.

Quar-ter	Demand forecast	Cumu lative demand	Production level =Av.demand =2920÷8	Cumu. prod. level	Inventory = (Cum. Production - Cum. Demand)	Adjusted inventory with 255 at beginning of period 1	Cost of holding inven-tory
1	270	270	365	365	95	350	17,500
2	220	490	365	730	240	495	24,750
3	470	960	365	1095	135	390	19,500
4	670	1630	365	1460	-170	85	4,250
5	450	2080	365	1825	-255	0	0

Qua r-ter	Demand forecast	Cumu lative dema nd	Product ionlevel =Av. deman d =2920÷ 8	Cumu .prod. level	Inventory = (Cum. Production - Cum. Demand)	Adjusted inventory with 255 at beginning of period 1	Cost of holding inventory
6	270	23 50	365	2190	-160	95	4,750
7	200	25 50	365	2555	5	260	13,000
8	370	29 20	365	2920	0	255	12,750
	Total						96,500

Plan III

Normal Production Capacity is assumed to be 200 units i.e. Minimum of the demand values. The additional demand other than the normal capacity is met by subcontracting. The cost of the plan III amounts to 1,32,000 as shown in table below.

Quarter	Demand forecast	Production units	Subcontract units	Incremental cost@ ` 100/units
1	270	200	70	70 × 100 = 7,000
2	220	200	20	20 × 100 = 2,000
3	470	200	270	270 × 100 = 27,000
4	670	200	470	470 × 100 = 47,000
5	450	200	250	250 × 100 = 25,000
6	270	200	70	70 × 100 = 7,000
7	200	200	0	0
8	370	200	170	170 × 100 = 17,000
			Total	= 1,32,000

The total cost of pure strategies is given below. On observation Plan II (Changing inventory levels) has the least cost.

Plan	Total cost
Plan I	1,97,000
Plan II	96,500
Plan III	1,32,000

Illustration 15

A firm has developed the following forecast (units) for an item which has a demand influence by seasonal factors.

Month	Forecasted Demand	Production Days
Jan	220	22
Feb	90	18
Mar	210	21
Apr	396	22
May	616	22
Jun	700	20
Jul	378	21
Aug	220	22
Sep	200	20
Oct	115	23
Nov	95	19
Dec	260	20

(a) Prepare a chart showing the daily demand requirements.

(b) Determine the production rate required to meet average demand.

(c) Determine the monthly inventory balance required to follow a plan with:

1. Constant workforce
2. No idle time or overtime
3. No Backorder
4. No use of Sub-Contractor
5. No capacity adjustment

(d) The firm has determined that to follow a plan of meeting demand by varying the size of the workforce strategy

Put result in hiring and lay-off cast estimated at 12000. If the unit cost is 100 each to produce, carrying cost per year are 20% of the average inventory value and storage cost (based upon maximum inventory) are 0.90 per unit which plan results in the lower cost, varying inventory or varying employment? [Where Plan 1 indicates varying inventory and Plan 2 indicates varying Employment]

(e) Suppose the firm wishes to investigate two other plans (alternatives). A third plan is to produce at a rate of 10 units per day and sub-contract the additional requirements at a delivered cost of 107 per unit. Any accumulated inventory is carried forward at a 20% carrying cost (No extra Storage cost).

The Fourth Plan is to produce at a steady rate of 10 units per day and use overtime to meet the additional requirement at a premium of ` 10 per unit. Accumulated inventory is again carried forward at a 20% cost.

(f) Compare 4 plans given in Question (d) and (e) and comment which plan gives the minimum cost.

ANSWER:

Chart of Production Requirement

Month	Forecasted Demand	Production Days	Demand/ Day	Cumulative Production Days	Cumulative Demand
Jan	220	22	10	22	220
Feb	90	18	5	40	310
Mar	210	21	10	61	520
Apr	396	22	18	83	916
May	616	22	28	105	1532
Jun	700	20	35	125	2232
Jul	378	21	18	146	2610
Aug	220	22	10	168	2830
Sep	200	20	10	188	3030
Oct	115	23	5	211	3145
Nov	95	19	5	230	3240
Dec	260	20	13	250	3500
Total	3500				

(a) Average Requirement = Total Demand / Total Production Days = 3500/25 = 14units/day

(b) Inventory Balance = ΣProduction – ΣDemand

Showing the ending Inventory Balance and Ending Balance with Negative Shortage.

Month	Production at 14/day	Forecasted Demand	Inventory Change	Ending Inventory Balance	Ending Balance adjusted in the month of Jan
Jan	308	220	88	88	654
Feb	252	90	162	250	816
Mar	294	210	84	334	900
Apr	308	396	-88	246	812
May	308	616	-308	-62	504
Jun	280	700	-420	-482	84
Jul	294	378	-84	-566	0
Aug	308	220	88	-478	88

Month	Production at14/day	Forecasted Demand	Inventory Change	Ending Inventory Balance	Ending Balance adjusted in the month of Jan
Sep	280	200	80	-398	168
Oct	322	115	207	-191	375
Nov	266	95	171	-20	546
Dec	280	260	20	0	566

Month	Demand	Production at 10/day	Inventory to carry	Inventory carried until	No. of Months	Cost at $1.67 per unit month
Initial			150	150 units to April	3	750
Feb	90	180	90	26 units to April	2	87
				64 units to May	3	320
Oct	115	230	115	60 units to Dec	2	200
				55 units to Year End	3	275
Nov	95	190	95	95 units to Year End	2	317
					Total	1952

Therefore, Inventory Cost from above = 1952 Calculating Marginal Cost of Sub-contracting:

The marginal cost of sub-contracting

Number of units = Demand - Production = 3500 - (10 × 250) = 1000 units for sub-contracting

Therefore, Cost per unit = 107 - 100 = 7 per unit Therefore, Marginal Cost = 1000 units × 7 per unit = 7000

The total Cost of Plan 3 = Inventory Cost + Sub-contracting cost = 1952 + 7000 = `8952

Plan 4:

This plan differs from plan 3 only in the marginal cost which is now due to overtime rather than sub-contracting.

So, Inventory cost (same as plan 3) i.e., `1952 and Marginal cost of Overtime = 1000 units × rate of `10 per unit = `10,000

Therefore, total cost of Plan 4 = `10,000 + `1952 = `11952 Table: Comparison of Plans

Plan	Strategy	Cost
Plan 1	Pure Strategy (Vary Inventory)	10010
Plan 2	Pure Strategy (VaryEmployment)	12000
Plan 3	Mixed Strategy (Sub-contractand Vary Inventory)	8952
Plan 4	Mixed Strategy (Overtime andVary Inventory)	11952

2.5. Material Requirements Planning

Material requirement planning (MRP) refers to the basic calculations used to determine component requirements from end item requirements. It also refers to a broader information system that uses the dependence relationship to plan and control manufacturing operations.
MRP is a technique of working backward from the scheduled quantities and needs dates for end items specified in a master production schedule to determine the requirements for components needed to meet the master production schedule. The technique determines what components are needed, how many are needed, when they are needed and when they should be ordered so that they are likely to be available as needed. The MRP logic serves as the key component in an information system for planning and controlling production operations and purchasing. The information provided by MRP is highly useful in scheduling because it indicates the relative priorities of shop orders and purchase orders.
"Materials Requirement Planning (MRP) is a technique for determining the quantity and timing for the acquisition of dependent demand items needed to satisfy master production schedule requirements."
MRP is one of the powerful tools that, when applied properly, helps the managers in achieving effective manufacturing control.

QUESTION 36
State MRP Objectives & Functions?

ANSWER:

1. **Inventory reduction:** MRP determines how many components are required, when they are required in order to meet the master schedule. It helps to procure the materials/components as and when needed and thus avoid excessive build up of inventory.
2. **Reduction in the manufacturing and delivery lead times:** MRP identifies materials and component quantities, timings when they are needed, availabilities and procurements and actions required to meet delivery deadlines. MRP helps to avoid delays in production and priorities production activities by putting due dates on customer job orders.
3. **Realistic delivery commitments:** By using MRP, production can give marketing timely information about likely delivery times to prospective customers.
4. **Increased efficiency:** MRP provides a close coordination among various work centres and hence helps to achieve uninterrupted flow of materials through the production line. This increases the efficiency of production system.

Functions served by MRP

1. **Order planning and control:** When to release orders and for what quantities of materials.
2. **Priority planning and control:** How the expected date of availability is compared to the need date for each component.
3. Provision of a basis for planning capacity requirements and developing a broad business plans.

QUESTION 37

Advantages and Disadvantages of MRP?

ANSWER:

Advantages :

(i) Reduced inventory,
(ii) Reduced idle time,
(iii) Reduced set up time,
(iv) Ability to change the master production schedule,
(v) Ability to price more competitively,
(vi) Better customer service,
(vii) Better response to market demands,
(viii) Reduced sales price.

In addition the MRP system enables the following:

(i) Aids capacity planning,
(ii) Helps managers to use the planned schedule before actual release orders,
(iii) Tells when to expedite or deexpedite,
(iv) Delays or cancels orders,
(v) Changes order quantities,
(vi) Advances or delays order due dates.

Disadvantages :

Even though MRP system has many advantages, there are some problems with MRP systems which make them
fail in many firms. Three major causes for failures of an MRP system are:

(i) Lack of top management commitment. MRP must be accepted by top management as a planning tool with specific reference to profit results. All executives concerned with the implementation of the MRP system must be educated emphasizing the importance of MRP as a closed-loop, integrated strategic planning tool.
(ii) MRP was presented and perceived as a complete and stand-alone system to run a firm, rather than as part of the total system.
(iii) The issue of how MRP can be made to function with just-in-time production system.

MRP also needs a high degree of accuracy for operation, which often requires (i) changing how the firm operates and (ii) updating files.

The major complaint by users of MRP is that MRP is too rigid because when MRP develops a schedule, it is quite difficult to deviate from the schedule if need arises.

2.6. MANUFACTURING RESOURCE PLANNING

Manufacturing Resource Planning (MRP II) has been developed to facilitate manufacturing managers address the planning and controlling of a manufacturing process and all of its related support functions. It encompasses logically correct planning and control activities related to materials, capacity, finance, engineering, sales and marketing. MRP II is universally applicable to any manufacturing organisation regardless of its size, location, product or process.

MRP II is a management process for taking the business plan and breaking it down into specific, detailed tasks that people evaluate, agree upon and are held accountable for. It involves all departments viz., materials department, engineering department that must maintain bill of materials, sales/marketing department that must keep sales plan upto date, purchasing and manufacturing departments that must meet due dates for bought out items and in-house manufactured items respectively.

From MRP I to MRP II : Manufacturing resource planning (MRP II) is a natural outgrowth of Materials Requirement Planning (MRP I) Whereas MRP I focuses upon priorities of materials, CRP is concerned with time. Both material and time requirement are integrated within the MRP system *[i.e.,* MRP I). Beyond this, MRP II has been coined to 'close the loop' by integrating financial, accounting, personnel, engineering and marketing information along with the production planning and control activities of basic MRP systems. MRP II is the heart of corporate management information system for many manufacturing firms.

QUESTION 38
Evolution of MRP II?

The earlier resource requirement planning systems were quite simple and unsophisticated. The MRP technique was used for its most limited capability to determine what materials and components are needed, how many are needed and when they are needed and when they should be ordered so that they are likely to be available when needed. In other words, MRP simply exploded the MPS into the required materials and was conceived as an inventory control tool or a requirements calculator. Later the logic of MRP technique was extended to serve as the key component in an information system for planning and controlling production operation and purchasing. It was helpful to production and operations managers to determine the relative priorities of shop orders and purchase orders. As a manufacturing planning and control system, MRP laid the basic foundation for production activity control or shop-floor control.

2.7. ECONOMIC BATCH QUANTITY

production managers often have to decide what quantity of output must be produced in a batch (known as lot size or batch size). The products are manufactured in lot sizes against the anticipated demand for the products. Often the quantity produced may exceed the quantity which can be sold. (i.e., production rates exceed demand rates). The optimum lot size which is known as economic lot size or Economic Order

Quantity or economic batch quantity or economic manufacturing quantity is that quantity of output produced in one batch, which is most economical to produce, i.e., which results in lowest average cost of production.

QUESTION 39

Determination of Economic Lot Size for Manufacturing:

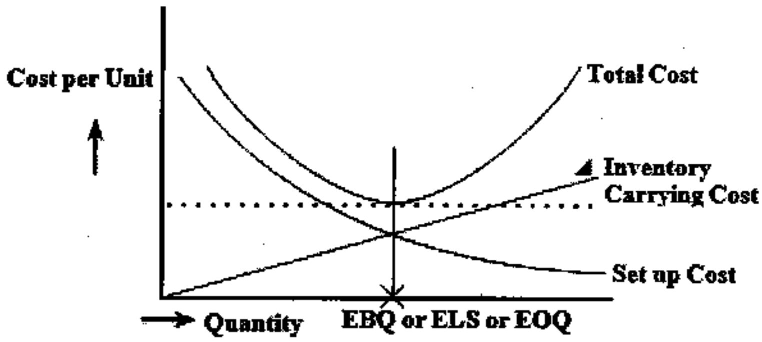
ANSWER:

The factors to be considered in arriving at the economic lot size are:

(i) **Usage rate:** The rate of production of parts should match with the rate of usage of these parts in the assembly line.

(ii) **Manufacturing cost:** Higher the lot size, lower will be the cost per unit produced because of distribution of set up costs for setting up production or machines and preparing paper work (production orders). But the carrying cost (handling and storing costs) will increase with increase in lot size.

(iii) **Cost of deterioration and obsolescence:** Higher the lot size, higher will be the possibility of loss due to deterioration (items deteriorating after shelf life) or obsolescence (due to change in technology or change in product design).



EBQ = Economic Batch Quality

ELS = Economic Lot Size

If S is the set up cost per set up also known as Ordering Cost, 'C' is the production cost per unit produced and I is the inventory carrying or holding changes (%) and A is the annual demand for the item in units, then,

Economic Batch Quantity (EBQ) or Economic Order Quantity (EOQ) or Economic Lot Size (ELS) or Economic Manufacturing Quantity (EMQ)

$$= \sqrt{\frac{2AS}{C}} = \sqrt{\frac{2 \times (\text{Annual demand in unit}) \times (\text{Set up Cost per set up})}{(\text{Production Cost per unit}) \times (\text{Inventory carrying charges (percentage)})}}$$

Economic Run Length: When a firm is producing an item and keeping it in inventory for later use, instead of buying it, the formula used to calculate economic order quantity (EOQ) can be used to calculate the economic production quantity referred to as Economic Run Length (ERL).
If 'p' is the production rate and 'd' is the demand rate (or consumption rate), A is the annual demand for the item in units, I is the inventory carrying charges (percentage), C is the production cost per unit, then

$$\text{Economic Run Length (ERL)} = \sqrt{\frac{2AS}{CI\left(1-\frac{d}{p}\right)}}$$

$$= \sqrt{\frac{2 \times (\text{Annual Demand (in units)}) \times (\text{Setup Cost per setup})}{(\text{Production Cost per unit}) \times (\text{Inventory Carrying charges (Percentage)})\left(1-\frac{\text{Demand Rate}}{\text{Production Rate}}\right)}}$$

Illustration 16

The monthly requirement of raw material for a company is 3000 units. The carrying cost is estimated to be 20% of the purchase price per unit, in addition to 2 per unit. The purchase price of raw material is 20 per unit. The ordering cost is 25 per order. (i) You are required to find EOQ.(ii) What is the total cost when the company gets a concession of 5% on the purchase price if it orders 3000 units or more but less than 6000 units per month. (iii) What happens when the company gets a concession of 10% on the purchase price when it orders 6,000 units or more? (iv) Which of the above three ways of orders the company should adopt?

ANSWER:

We are given that,

A = Annual demand = 3,000 × 12 = 36,000 units per annum ; S = Ordering Cost = ` 25; C = Inventory carrying cost = 2 + 20% of ` 20 = 2 + 4 = ` 6

(i) $EOQ = \sqrt{\frac{2AS}{C}} = \sqrt{\frac{2 \times 36000 \times 25}{6}} = \sqrt{3,00,000} = 548$ units (approx.)

Total cost = Ordering Cost + Cost of purchasing the material + Storage cost

= (36,000 / 548) × 25 + (36,000 × 20) + (548/2) × 6 [⊠ Storage cost = Average Inventory × Inventory carrying cost

= ` 1642.33 + 7,20,000+ 1,644 = ` 7,23,286. $= \frac{eoq}{2} \times 6$]

(i) When the company has an option to order between 3000 and 6000 units, the EOQ should be calculated with a reduction in price by 5% (due to concession); The purchase price = 95% of ` 20 = ` 19.

A = 36,000 units per annum; S = 25; C = 2 + 20% of 19 = 2 + 3.80 = `= ` 5.80

$$EOQ = \sqrt{\frac{2 \times 36000 \times 25}{5.80}} = \sqrt{\frac{18,00,000}{5.80}} = 557 \text{ units app.}$$

Total cost = (36,000/557) × 25 + (36,000 × 19) + (557/2) × 5.80

= ` (1,615.79 + 6,84,000 + 1,615.30) = ` 6,87,231.09

For monthly order quantity being 3000 units or more but less than 6000 units

EOQ = 557 units

$$\text{No. of orders per year} = \frac{\text{Yearly demand}}{\text{EOQ}} = \frac{36000}{557} = N \text{ (let)}$$

$$\text{No. of orders per month} = \frac{N}{12} = \frac{36000/557}{12} = 5.385 = 6 \text{ (say)} = N^*$$

Quantity to be orderd per month = N* x EOQ = 6 ×557 = 3342 units This quantity lies in the range of 3000 to 6000 units

Hence the EOQ (557 units) can be considered to be a feasible quantity for availing 5% discount on Purchase Price.

(i) When the company orders more than 6,000 units purchase price = 90% of ` 20 (because 10% concession)

= ` 18; A = 36,000 units per annum; S = ` 25; C = 2 + 20% of ` 18

= 2 + 3.60 = 5.60

$$EOQ = \sqrt{\frac{2AS}{C}} = \sqrt{\frac{2 \times 36000 \times 25}{5.60}} = 567 \text{ units app.}$$

For monthly order quantity more than or equal to 6000 units

EOQ = 567 units

$$\text{No. of orders per year} = \frac{\text{Yearly demand}}{\text{EOQ}} = \frac{36000}{557} = N \text{ (let)}$$

$$\text{No. of orders per month} = \frac{N}{12} = \frac{36000/557}{12} = 5.385 = 6 \text{ (say)} = N^*$$

This quantity does not lie in the range of 6000 or more units.

Hence the EOQ (567 units) can not be considered as feasible quantity for availing 10% discount on Purchase Price.

To understand the effect of 10% on Total Cost, we consider the minimum value of price break quantity of this

range i.e. 6000 units to be the optimum order quantity and calculate. Total Cost as follows –

TC = Ordering Cost + Cost of Purchasing the material + Storage Cost

$$= \frac{36000}{6000} \times 25 + 36000 \times 18 + \frac{6000}{2} \times 5.60$$

= 150 + 648000 + 16800 = ` 6,64,950

Hence the total cost will be minimum (` 6,64,950) if orders are placed in lot size of 6000 units.

Illustration 17

M/s. Tubes Ltd. are the manufacturers of picture tubes of T.V. The following are the details of their operation during 2001:

Average monthly market demand	2,000 tubes
Ordering cost	100 per order
Inventory carrying cost	20% per annum
Cost of tubes	500 per tube
Normal usage	100 tubes per week
Minimum usage	50 tubes per week
Maximum usage	200 tubes per week
Lead time to supply	6 - 8 weeks

Compute from the above:

(1) Economic order quantity. If the supplier is willing to supply quarterly 1,500 units at a discount of 5%, is it worth accepting?

(2) Maximum level of stock.

(3) Minimum level of stock.

(4) Re-order level of stock.

ANSWER:

(1) Economic Order Quantity:

Annual usage of tubes (A) = Normal usage per week × 52 weeks

= 100 tubes × 52 weeks

= 5,200 tubes.

Ordering cost per order (S) = 100.

Inventory carrying cost per unit per annum (C) = 20% of 500 = 100.

$$EOQ = \sqrt{\frac{2AS}{C}} = \sqrt{\frac{2 \times 200 \text{ units} \times r100}{r100}} = 102 \text{ units (approx.)}.$$

(A) Evaluation of order size of 1,500 units at 5% discount

$$\text{No. of orders} = \frac{5,200 \text{ units}}{1,500 \text{ units}} = 3.46 \text{ or } 4$$ (in case of a fraction, the next whole number is considered).

Ordering cost (No. of order per year at ` 100 per order)	400
Carrying cost of average inventory:	
$\frac{1,500 \text{ units}}{2} \times$ ` $(500 \text{ less } 5\%) \times \frac{20}{100}$	71,250
Total annual cost (excluding item cost)	71,650

Annual cost if EOQ (102 units) is adopted :

Ordering cost: 5,200 ÷ 102 or 51 orders per year at ` 100 per order — 5,100

Carrying cost of average inventory $\frac{102 \text{ units}}{2} \times$ ` 500 $\times \frac{20}{100}$ — 5,100

Total annual cost (excluding item cost) — 10,200

Increase in annual cost by adopting (A) above : ` (71,650 - 10,200) = ` 61,450. Amount of quantity discount: 5% × ` 500 × 5,200, units = ` 1,30,000.

Since the amount of quantity discount (` 1,30,000) is more than the increase in total annual cost (` 61,450), it is advisable to accept the offer. This will result in a saving of ` (1,30,000 - 61,450) or ` 68,550 p.a. in inventorycost.

(2) Maximum Level of Stock:

= Re-order level + Re-order quantity – (Minimum usage × Minimum delivery period)
= 1,600 units + 102units – (50 units × 6 weeks) = 1,402 units.

[Assume that the Reorder quantity is supplied as soon as the Reorder level is reached]

(3) Minimum Level of Stock:

= Re-order level – (Normal usage × Normal delivery period) [see Note] = 1,600 units – (100 units × 7 weeks)

= 900 units. Note: Normal delivery period is taken to be the average delivery period.

(4) Re-order Level of Stock:
= Maximum usage × Maximum delivery period = 200 units × 8 weeks = 1,600 units

Illustration 18

M/s Kobo Bearings Ltd., is committed to supply 24,000 bearings per annum to M/s Deluxe Fans on a steady daily basis. It is estimated that it costs 10 paisa as inventory holding cost per bearing per month and that the setup cost per run of bearing manufacture is ` 324.

(a) What is the optimum run size for bearing manufacture?

(b) What should be the interval between the consecutive optimum runs?

(c) Find out the minimum inventory holding cost.

ANSWER:

(a) Optimum run size or Economic Batch Quantity (EBQ)

$$= \sqrt{\frac{2 \times \text{Annual Output} \times \text{Setup cost}}{\text{Annual Cost of Carrying one unit}}} = \sqrt{\frac{2 \times 24000 \times 324}{0.10 \times 12}} = 3600 \text{ units}$$

(b) Interval between two consecutive optimum runs $= \frac{\text{FBQ}}{\text{Monthly Output}} \times 30$

$$= \frac{3600}{24000 \div 12} \times 30 = 54 \text{ Calendar days}$$

(b) Minimum inventory holding cost = Average inventory × Annual carry-ing cost of one unit of inventory

= (3600 ÷ 2) × 0.10×12 = ` 2,160.

Illustration 19

A company planning to manufacture a household cooking range has to decide on the location of the plant. Three locations are being considered viz., Patna, Ranchi, and Dhanbad. The fixed costs of the three location are estimated to be ` 30 lakh, ` 50 lakh, and ` 25 lakh per annum respectively. The variable costs are ` 300, ` 200 and ` 350 per unit respectively.

The expected sales price of the cooking range is ` 700 per unit Find out:

(i) The range of annual production/sales volume for which each location is most suitable and

(ii) Which one of the three locations is the best location at a production/sales volume of 18,000 units?

ANSWER:

The total cost of the three locations are:

At Total cost = Fixed cost + Variable cost for a volume "X"

Patna => Total cost = 30,00,000 + 300 × X Ranchi => Total cost = 50,00,000 + 200 × X Dhanbad => Total cost = 25,00,000 + 350 × X

We can compute and plot the total costs per annum at the three different locations for the various cases of

production volume of 5,000, 10,000, 15,000, 20,000 25,000 units.

(i) **Patna**

Volume (x Units)	5,000	10,000	15,000	20,000	25,000
Fixed Cost	30,00,000	30,00,000	30,00,000	30,00,000	30,00,000
Variable Cost(300 x)	300 (5,000)	300 (10,000)	300 (15,000)	300 (20,000)	300 (25,000)
Total Cost *	= ` 45 lakhs	= ` 60 lakhs	= ` 75 lakhs	= ` 90 lakhs	= ` 105 lakhs

(i) **Ranchi**

Volume (x Units)	5,000	10,000	15,000	20,000	25,000
Fixed Cost	50,00,000	50,00,000	50,00,000	50,00,000	50,00,000
Variable Cost(200 x)	200 (5,000)	200 (10,000)	200 (15,000)	200 (20,000)	200 (25,000)
Total Cost*	= ` 60 lakhs	= ` 70 lakhs	= ` 80 lakhs	= ` 90 lakhs	= 100 lakhs

(i) Dhanbad

Volume (x Units)	5,000	10,000	15,000	20,000	25,000
Fixed Cost	25,00,000	25,00,000	25,00,000	25,00,000	25,00,000
Variable Cost(300x)	350 (5,000)	350 (10,000)	350 (15,000)	350 (20,000)	350 (25,000)
Total Cost *	= ` 42.5 lakhs	= ` 60 lakhs	= 77.5 lakhs	=95 lakhs	= 112.5 lakhs

* In all the above tables, Total Cost = Fixed Cost + Variable Cost
If the volume distribution be as follows:

	Up to 10,000 units	Between 10,000 units to 20,000units	Above 20,000 units
Favourable Location	Dhanbad	Patna	Ranchi

For a volume of 18000 units favourable location is Patna which can be substantiated by the followings calculations of Total Cost :-
Patna => 30,00,000 + 300 × 18,000 = ` 84 lakhs
Ranchi => 50,00,000 + 200 × 18,000 = ` 86 lakhs
Dhanbad => 25,00,000 + 350 × 18,000 = ` 88 lakhs.

Illustration 20

Monthly demand for a component is 1000 units. Setting-up cost per batch is ` 120. Cost of manufacture per unit is ` 20. Rate of interest may be considered at 10% p.a. Calculate the EBQ.

ANSWER:

Calculation of EBQ:

$$EBQ = \sqrt{\frac{2 \times \text{Annual Demand} \times \text{Set-upcost}}{\text{Unit Cost} \times \text{Inventory carrying cost per unit per year (r)}}}$$

$$= \sqrt{\frac{(2 \times 12 \times 1000 \times 120)}{(0.1 \times 20)}} = 1200 \text{ units.}$$

Illustration 21

Based on the following data on the exports of an item by a company during the various years fit a straight line, (for the time being, assume that a straight line gives a good fit). Give a forecast for the years 2013 and 2014.

Year	No. of items ('000)
2004	13
2005	20
2006	20
2007	28
2008	30
2009	32
2010	33
2011	38
2012	43

ANSWER:

We can call the years as 'X' and exports as 'Y. In order to use the normal equations for the least square line, we need ΣX, ΣY, ΣXY and ΣX2. If we arrange X in such a way that ΣX = 0, it will simplify our calculations. Therefore, we call the year 2008 as 0, 2007 as -1 and 2009 as + 1 and likewise for the other years in the data.

The rearrangement is shown in the table as follows:

X	Y	X^2	XY
-4	13	16	-52
-3	20	9	-60
-2	20	4	-40
-1	28	1	-28
0	30	0	0
1	32	1	32
2	33	4	66
3	38	9	114
4	43	16	172
$\Sigma X = 0$	$\Sigma Y = 257$	$\Sigma x^2 = 60$	$\Sigma XY - 204$

Let the equation of the best fit straight line to the given data be Y ⊠ a_o ⊠ a_1x

So the normal equations are

$\Sigma Y = aoN + a1\Sigma X$ (1)

$\Sigma XY = a\,\Sigma X + a\,\Sigma X^2$ (2)

As $\Sigma X = 0$, from (1) $\Sigma Y = a\,N$ from (2) $\Sigma XY = a\,\Sigma X^2$ Therefore, $ao = \Sigma Y / N = 257 / 9 = 28.56$ [N = No. of years] $a = \Sigma XY / \Sigma X^2 = 204 / 60 = 3.4$

The equation of a straight line fitting the data is:

$Y = 28.56 + 3.4\,X$

(a) Forecast for 2013, (i.e., X = 5): Y = 28.56 + 3.4 (5) = 45.56 ('000) nos.

(b) Forecast for 2014, (i.e., X = 6): Y = 28.56 + 3.4 (6) = 48.96 ('000) nos.

Illustration 22

Find the economic order quantity and the reorder point, given Annual demand (D) = 1000 units

Average daily demand (d) = 1000/365

Ordering Cost (S) = 5 per order

Holding cost(H) = 1.25 per unit per year.

Lead time (L) = 5 days

Cost per unit (C) = 12.50

What quantity should be ordered?

ANSWER:

$$EOQ = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2 \times 1000 \times 5}{1.25}} = \sqrt{8000} = 89.44 \text{ units}$$

$$\text{Re-order unit} = dL = \frac{1000}{365} \times 5 = 13.7 \text{ units}$$

$$\text{Total Cost} = DC + \frac{D}{Q} \times S + \frac{Q}{2} \times H = 1000 \times 12.5 + (1000/89.44) \times 5 + (89.44/2) \times 1.25$$

= ` 2611.81

Illustration 23

Consider an economic order quantity case where annual demand D=1000 units, economic order quantity Q= 200 units , the desired probability of not stocking out P=0.95 , the standard deviation of demand during lead time 6L

=25units and lead time = L=15 days. Determine the reorder point. Assume the demand is over a 250 week day year.

ANSWER:

d = D/no. week days = 1000/250 = 4

Re-order level(R) = dL + z L = 4×15 + 1.64 × 25 = 101

Illustration 24

Daily demand for a certain product is normally distributed with a mean of 60 and standard deviation of 7. The source of supply is reliable and maintain a constant lead time of six days. The cost of placing the order is ` 10 and annual holding costs are ` 0.50 per unit. There are no stock out costs, and unfilled orders are filled as soon as the order arrives. Assume sales occur over the entire 365 days of the year. Find the order quantity and reorder point to satisfy a 95 percent probability of not stocking out during the lead time

ANSWER:

$$EOQ = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2 \times (60 \times 365)}{0.5}} = \sqrt{876000} = 936 \text{ units}$$

$$\sigma_l = \sqrt{\sum_{i=1}^{l} \sigma_d^2} = \sqrt{6 \times 7^2} = 17.15$$

Re-order leve(R) = dL + z 6L = 60×6 + 1.64 × 17.15 = 188

Illustration 25

Fixed -Time period Model with safety stock

Daily demand for a product is 10 units with a standard deviation of 3 units. The review period is 30 days, and lead time is 14 days. Management has set a policy of satisfying 98% of demand from items in stock. At the beginning of this review period, there are 150 units in inventory.

ANSWER:

$$Q = d(T+L) + z\sigma_{T+L} - I = 10(30+14) + \tau\, \sigma_{T+L} - 150 =$$

$$\sigma_{T+L} = \sqrt{\sum_{i+1}^{T+L} \sigma_d^2} = \sqrt{(T+L)\sigma_d^2} = \sqrt{(30+40) \times 3^2} = 19.90$$

T for P = 0.98 is 2.05

Q = 160 – 150 + 2.05×19.9 = 331 units.

Illustration 26

Average Inventory calculation - Fixed order quantity model

Suppose the following item is being managed using a fixed order quantity model with safety stock

Annual Demand (D) = 1000 units Order quantity (Q) = 300 units

Safety stock (SS) = 40 units

What are the average inventory level and inventory turn for the item?

ANSWER:

Avg. Inventory = Q/2 + SS = 150 + 40 =190

$$\text{Inventory Turn} = \frac{D}{\frac{Q}{2} + SS} = \frac{1000}{190} = 5.263 \text{ tem per year}$$

Illustration 27

Average Inventory calculation - Fixed Time period model

Consider the following item that is being managed using a fixed time period model with safety stock

Weekly demand (d) = 50 units Review cycle (T) = 3 weeks Safety stock (SS) = 30 units

What are the average inventory level and inventory turn for the item?

ANSWER:

Avg. Inventory = dT/2 + SS = (50 × 3)/2 + 30 = 105 units

$$\text{Inventory Turn} = \frac{52d}{\text{Avg Inventory}} = \frac{52 \times 50}{105} = 24.8 \text{ turns per year}$$

Illustration 28

Price Break Problem

Consider the following case, where D = 10000 units (annual demand) S = ` 20 to place order

I = 20 percent of cost (annual carrying cost, storage, interest, obsolescence, etc)

C = Cost per unit (according to the order size: order of 0 to 499 units, ` 5.00 per unit; 500 to 999 units, ` 4.50 per unit; 1000 and up, ` 3.90 per unit)

What quantity should be ordered?

ANSWER:

$$EOQ1 = \sqrt{\frac{2DS}{iC}} = \sqrt{\frac{2 \times 10000 \times 20}{20 \times 5}} = 63.24$$

$$EOQ2 = \sqrt{\frac{2 \times 10000 \times 20}{20 \times 4.5}} = 66.67$$

$$EOQ3 = \sqrt{\frac{2 \times 10000 \times 20}{20 \times 3.9}} = 71.6$$

$$\text{Total Cost1} = DC + \frac{D}{Q} \times S + \frac{Q}{2} \times iC = 56323$$

TC2 = 51000

TC2 = 51000

TC3 = 44585.69

1000 units should be ordered.

Illustration 29

A product is priced to sell at ` 100 per unit, and its cost is constant at ` 70 per unit. Each unsold unit has a salvage value of ` 20. Demand is expected to range between 35 and 40 units for the period. 35 definitely can be sold and no units over 40 will be sold. The demand probabilities and the associated cumulative probability distribution (P) for this situation fo;;ow.

Number of Units Demanded	Probability of this Demand	Cumulative Probability
35	0.10	0.10
36	0.15	0.25
37	0.25	0.50
38	0.25	0.75
39	0.15	0.90
40	0.10	1.00

How many units should be ordered?

ANSWER:

The cost of underestimating the demand is loss of profit (Cu) or 100-70=30/unit. The cost of overestimating demand is the loss incurred when the unit must be sold at salvage value(Co)=70-20=50 The optimal prob. Of not being sold

P<= Cu/Co + Cu = 30/30 + 50 = 0.375

From the data, this corresponds to 37th value.

No. of unit sold

Unit demand	Prob.	35	36	37	38	39	40
35	0.1	0	50	100	150	200	250
36	0.15	30	0	50	100	150	200
37	0.25	60	30	0	50	100	150
38	0.25	90	60	30	0	50	100
39	0.15	120	90	60	30	0	50
40	0.1	150	120	90	60	30	0
Total	1	75	53	43	53	83	125

Illustration 30

1. A company currently has 200 units of a product on hand that it orders every two weeks when the salesperson visits the premises. Demand for the product averages 20 units per day with a standard deviation of 5 units. Lead time for the product to arrive is seven days. Management has a goal of 95 percent probability of not stocking out for this product. The salesperson is due to come in late this afternoon when 180 units are left in stock (assuming that 20 are sold today). How many units should be ordered?

ANSWER:

S.D = $\sqrt{21(5)\times(5)} = 23$

Z = 1.64

q = d×(T + L) + Z×S.D-I

= 20(14 + 7) + 1.64×23-180

= 278 units

Illustration 31

1. Solve the ABC analysis of the following table and show graphically taking Percentage of total list of different stock items as x axis and Percentage of total inventory value along y axis

Annual Usage if Inventory by Value

Item Number	Annual Rupee Usage (`)	Percentage of total value (%)
22	95000	40.69
68	75000	32.13
27	25000	10.71
03	15000	6.43
82	13000	5.57
54	7500	3.21
36	1500	0.64
19	800	0.34
23	425	0.18
41	225	0.10
TOTAL	`233450	100%

ANSWER:

Classification	Item no.	Annual Rupee Usage	% of total
A	22,68	1,70,000	72.9%
B	27,03,82	53,000	22.7%
C	54,36,19,23,41	10,450	4.5%

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3.1 Product Design

Product is an important part of the fundamental marketing mix. Customers pay for the products that best fulfil their requirement and provide them the desired value. The attributes and quality of the product triggers the demand which in turn is a reason for the existence of the value chains and hence organizations. Therefore, designing a product is an important strategic decision. Product design is a collection of interdependent directional activities that are planned and executed in a structured and planned way to develop the value propositions to be offered to the end customers for fulfilling their needs. Therefore, the basic objectives of product design include

- To develop the products as per the needs of the end customer with an objective to provide optimum value
- To minimize the cost, and lead time (i.e., design to market)
- To maximize resource utilization

QUESTION.1
Importance of Product design?

Answer.
(a)To establish detailed characteristics (i.e., core and augmented) of the products in line with market demand and competitions.
(b)To provide the technical requirements for defining the technological requirements and processes
(c)To provide a guideline for production system design
(d)To provide necessary impetus to production and operations strategy
A good product design enables the organizations to stay ahead of the competition and sustain in this VUCA world and helps to build long-term relationship with the end customers. Product design has an indirect impact on employment too. In other words, a distinctive product design stands as an order winning criteria for the organizations. Some of the attributes of a good design are user-friendliness, features, aesthetics, reliability, durability, innovativeness and appropriateness.

QUESTION 2.
What Does Product Design Do?

Answer.
The activities and responsibilities of product design include the following:

(i) Understand and translate the requirements of the customers (Voice of the Customers) into a set of technical requirements (Voice of the Process) for design and execution planning and processes.
(ii) Differentiate the existing products to stretch the product life cycle
(iii) Developing new products
(iv) Providing inputs required for the formulation of the quality goals
(v) Help in cost optimization
(vi) Building and testing model prototypes
(vii) Documentation of the design specifications

The general product design process is depicted in figure 1.

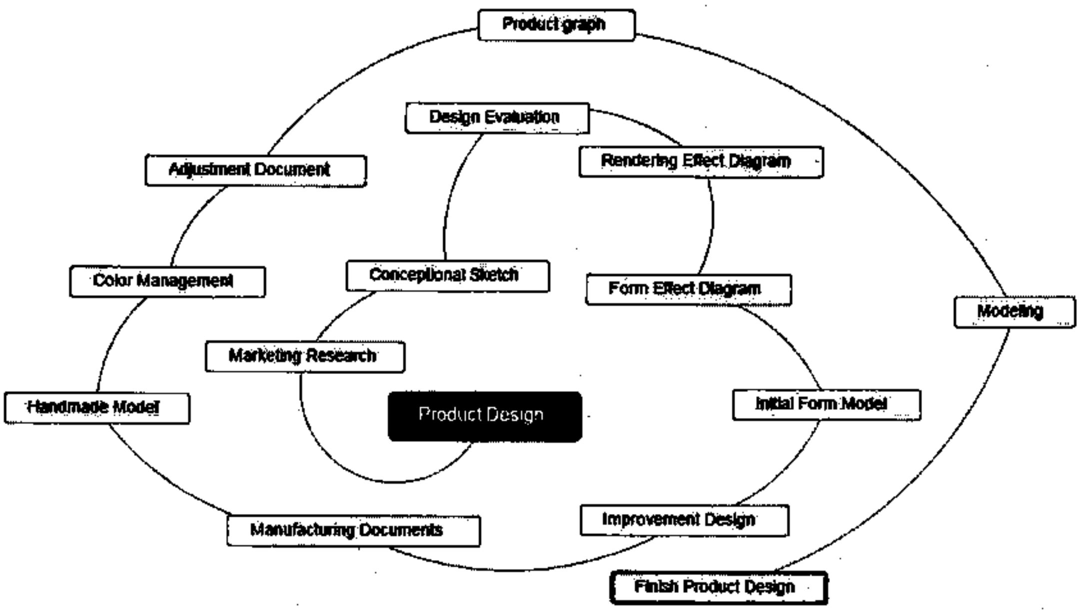


Figure 3.1: Product design process

QUESTION.3
Factors affecting the Product Design?

ANSWER.

(a) True understanding of the stated and unstated needs and requirements of the end customers. Sometimes it is difficult to get an idea of the unstated or latent needs. Modern day designers most often rely on social media analytics for discovering the latent needs.
(b) User-friendliness of the products attracts people from various demographic backgrounds.
(c) Striking a balance among form, function/features and cost. In other words, an appropriate balance between economies of scale and economies of scope is required.
(d) Quality of raw materials or basic ingredients
(e) Selection/design of the processes and layouts
(f) The quality and conditions of the machines/instruments used in the design process
(g) Capability and maturity of the processes
(h) Skilled resource persons
(i) Effect on the existing products
(j) Presentation (e.g., packaging) of the products

QUESTIONS.4
Characteristics of Good Product Design?

ANSWER.

A good product design must ensure the following

(a) Product quality: The product must satisfy the needs of the end customers while providing optimum value. The performance should be at par with the expectations.
(b) The product must be reliable and worthy for paying for the same
(c) The product must be designed at an optimum cost to be offered at an affordable price to the

target customers

(d) The product must be having a shorter design to market lead time

(e) The aesthetics/looks of the product must create an immediate impression in the minds of the customers

(f) The product must be compatible, user-friendly and upgradable with availability of after sales support (e.g.,spare parts)

(g) The product must be easily maintainable and reproducible

(h) The product should balance between standardized basic features and customized augmented features

(i) A detailed specification

(j) The product must be safe to use, error proof and should not harm the environment and users

Process Design encompasses all the activities that are performed to produce the final products as per the specifications in line with the requirements of the customers.

(a) Characteristics/ nature of the product - Type of the product

(b) Variety: the degree of customization and standardization (Make to Order, Make to Stock, Engineer to Order and Assemble to Order)

(c) Volume: the amount to be produced and size of the lot (single piece or batch or continuous production)

(d) Level of involvement of human resource: Accordingly, the process may be automatic (least involvement of manual labor; capital intensive) or semi-automatic or manual (labour intensive).

(e) Resource requirement: Machines (special purpose and/or general), human capital, space, energy, raw materials and others

(f) Expenditure: the cost of operations- fixed and overhead

(g) Decision on the extent of In-house ('Make') or Outsource ('Buy') production

Process choice determines whether resources are organised around products or processes in order to implement the flow strategy. It depends on the volumes and degree of customisation to be provided.

QUESTION.5

These major process decisions are discussed in detail in the following paragraphs?

ANSWER.

These major process decisions are discussed in detail in the following paragraph;

1. Process Choice: The production manager has to choose from five basic process types – (i) job shop, (ii) batch, (iii) repetitive or assembly line, (iv) continuous and (v) project

(i) Job shop process: It is used in job shops when a low volume of high-variety goods are needed. Processing is intermittent, each job requires somewhat different processing requirements. A job shop is characterized by high customization (made to order), high flexibility of equipment and skilled labour and low volume. A tool and die shop is an example of job shop, where job process is carried out to produce one-of-a kind of tools. Firms having job shops often carry out job works for other firms. A job shop uses a flexible flow strategy, with resources organized around the process.

(ii) Batch process: Batch processing is used when a moderate volume of goods or services is required and also a moderate variety in products or services. A batch process differs from the job process with respect to volume and variety. In batch processing, volumes are higher because same or similar products or services are repeatedly provided, examples of products produced in batches include paint, ice cream, soft drinks, books and magazines.

(iii) Repetitive process: This is used when higher volumes of more standardized goods or services are needed. This type of process is characterized by slight flexibility of equipment (as products are standardized) and generally low labour skills. Products produced include automobiles, home appliances, television sets, computers, toys etc. Repetitive process is also referred to as line process as it includes production lines and assembly lines in mass production. Resources are organized around a product or service and materials move in a line flow from one operation to the next according to a fixed sequence with little work-in-progress inventory. This kind of process is suitable to "manufacture-to-stock" strategy with standard products held in finished goods inventory. However, "assemble-to-order" strategy and "mass customization" are also possible

(iv) generally low labour skills. Products produced include automobiles, home appliances, television sets, computers, toys etc. Repetitive process is also referred to as line process as it include production lines and assembly lines in mass production. Resources are organised around a product or service and materials move in a line flow from one operation to the next according to a fixed sequence with little work-in-progress inventory. This kind of process is suitable to "manufacture-to-stock" strategy with standard products held in finished goods inventory. However, "assemble-to-order" strategy and "mass customization" are also possible

(v) Project process: It is characterised by high degree of job customisation, the large scope for each project andneed for substantial resources to complete the project. Examples of projects are building a shopping centre, a dam, a bridge, construction of a factory, hospital, developing a new product, publishing a new book etc. Projects tend to be complex, take a long time and consist of a large number of complex activities. Equipment flexibility and labour skills can range from low to high depending on the type of projects.

Likewise the business organizations and human beings, each product has a life that goes through various phases or cycles. All these cycles during the usable life of a product is collectively called as Product Life Cycle (PLC).

QUESTION 6
Five Stage of Product Life Cycle?

(a) **Introduction phase:** During this phase the product (either completely new product or a new variant of the existing product) gets introduced in the market for the first time. For the introduction of the new products in the market, at this stage, the volume stays low, sales are low and effect of learning curve is not realized. Hence, the return on investment is low. This phase is featured by higher level of expenditure in the promotional campaigns. The pricing depends on the innovativeness of the product, nature of the target customer segment and most often discounts are given to entice the potential customers.

(b) **Growth phase:** In this stage, the company focuses on rapid revenue generation and market growth. During this phase, the product sales intend to cover up the fixed cost and bring down the overhead costs while utilizing the learning in the previous stage. Promotional and advertising strategy is decided according to the level of the growths. The objective is to hold the existing customers and create new customers.

(c) **Maturity phase:** This phase is characterized by saturation in the market place. This is a critical phase for the organizations. In the earlier stage (i.e., growth) the objective of the company is to achieve fast growth while in this stage the company wants to flatten the curve to slow down the movement toward fall down. Further, at this stage the organizations infuse variety and differentiation in the products most often to start a new PLC from hereon for finding out a niche market. At this stage, organizations get engaged in aggressive promotional and pricing programs. Profit margin is comparatively lower at this stage.

(d) **Decline phase:** After maturity, the products start losing their attractiveness in the market and sales get falling down. Profit margin becomes increasingly narrower. The organizations take a call to scrap the product and focus on cost consolidation. Sometimes, organizations come up with revival planning with product differentiation and promotional strategy to improve the sales.

A typical PLC for a FMCG product and high tech product are given in following figures

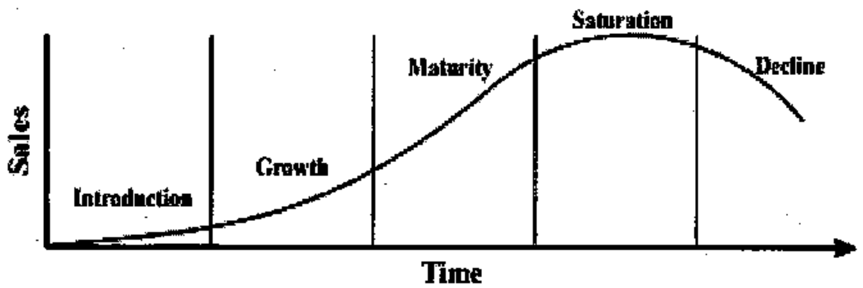


Figure 3.2: PLC for FMCG Product

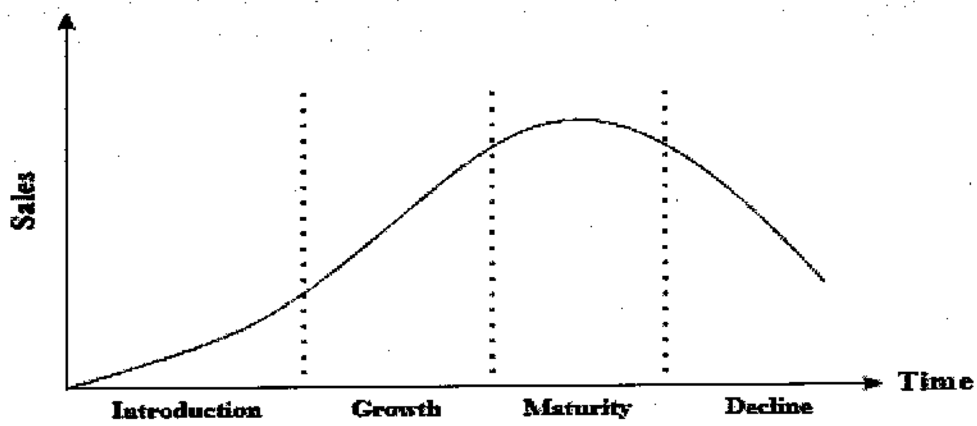


Figure 3.3: PLC for High Tech Product

Process Planning refers to all decisions regarding facility selection, layout planning, design of work systems and defining operating procedures, capacity planning, arrangement of equipment and resources, human resource planning etc which are necessary to facilitate the smooth execution of the activities to produce the intended products as required by the customers.

Process planning depends on variety and volume of outputs of the products, degree of equipment flexibility and flow of activities.

QUESTIONS.7
Define Process Strategy?

ANSWER.
A process strategy is a decision taken by the organization vis-à-vis selection of the processes for converting the input (i.e., resources) into output (i.e., finished products and services as required by the customers) in line with the product specifications. A typical process strategy depends on long-term efficiency and productivity, resource availability, flexibility, cost and benefits, quality of the products and lead time. Accordingly, the process strategy stands on the following premises:

(a) Trade-off between Make (in house conversion, fully or partial) or Buy (outsourcing, fully or partial) decisions
(b) Degree of capital intensity that decides the optimum balance between level of automation and manual operations
(c) The extent of flexibility required in the process (i.e., the flexibility in the positioning and functioning of the machines, works stations and requisite skills for layout decisions)

Accordingly, the facilities are designed while having three focus areas such as

(a) Process focused: The facility is designed in a process centric way. Accordingly, the equipment, machines and work stations are organized. Each process is capable of carrying a wide range of activities (aka intermittent processes) and flexible enough to adopt frequent changes. This type of arrangement allows a higher level of customization, i.e., product flexibility. This type of system is also known as job shop production. Example of products: Aircraft
(b) Product focused: The facility is planned in a product centric way to allow a higher level of standardization. The products in higher volume (with lower variety) are produced to give economies of scale and learning benefits for better facility utilization rate. Examples of products: steel, glass, paper, electric bulbs, chemicals and pharmaceutical products. This type of arrangements is suited for continuous flow and batch production. However, this type of structure incurs a higher amount of fixed cost.
(c) Repetitive Focus: This structure utilizes the benefits of the above-mentioned arrangements. It uses modular production. This type of structure is also known as assembly production. Examples include automobile process, household appliances etc.

QUESTIONS.8
What is Process Layout Selection?

ANSWER.
Process layout aims to identify the necessary arrangement of facilities such as equipment /machines, material, people, and work stations for

(a) facilitating the production efficiently
(b) minimizing unnecessary movements and transportation

(c) efficient material handling
(d) effective design and organizations of the work station
(e) identification and removal of the bottlenecks/ constraints
(f) effective utilization of the spaces

The underlying objective is to provide the value added products and services to the end customers while minimizing the waste in the process and hence, optimizing the operational cost and resource utilization.

The classical way of categorization includes four types of layouts
(a) process layout
(b) product layout
(c) Group layout (combination layout)
(d) Fixed position layout

(a) **Process layout or functional layout:** It organizes the work stations in such a way that similar type of machines and services (i.e., facilities) are located together. Therefore, each such sub-facility is specialized in performing a particular activity of the whole conversion process. This type of layout is suitable for low volume, high variety products produced by job shop, batch production and other non-repetitive processes. Examples: Furniture, restaurants etc.

(b) **Product layout or line layout:** In this type of layout, the facility is organized as per the logical/sequential flow of the activities performed to produce the products. This type of layout is used for high volume and continuous production where level of customization is low. Typical examples include assembly line or mass production used in consumer electronics, automobile sectors etc.

(c) **Group (combination) layout:** This combines the features of both the previously mentioned layouts. In this layout the individual processes are replicated at multiple cells wherein each cell is equipped with all facilities to complete the corresponding process. This type of layout is suitable for cellular manufacturing that minimizes the cost of transportation and material handling

(d) **Fixed position or Project layout:** In this type of layouts, main facilities are fixed at specified locations while the materials, people and work stations move as per the requirements to those locations. This type of layout is of single use and suitable for highly customized (ETO type) products. Examples: Air Craft, Ships

Each production system is uniquely suited to produce a particular mix and volume of products. Each production system provides different levels and a unique set of the manufacturing outputs: cost, quality, performance, delivery, flexibility and innovativeness. One of the tasks of the manufacturing strategy is to select the best production system for each product or product family.

The PV-LF Matrix is a useful tool for analyzing the similarities and differences among the seven production systems. The PV-LF Matrix has four dimensions:
(a) Number of products produced
(b) Production volume of each product
(c) Layout or arrangement of equipment and processes used to manufacture the products
(d) Flow of material through the equipment and processes

A typical PV-LF diagram is given below

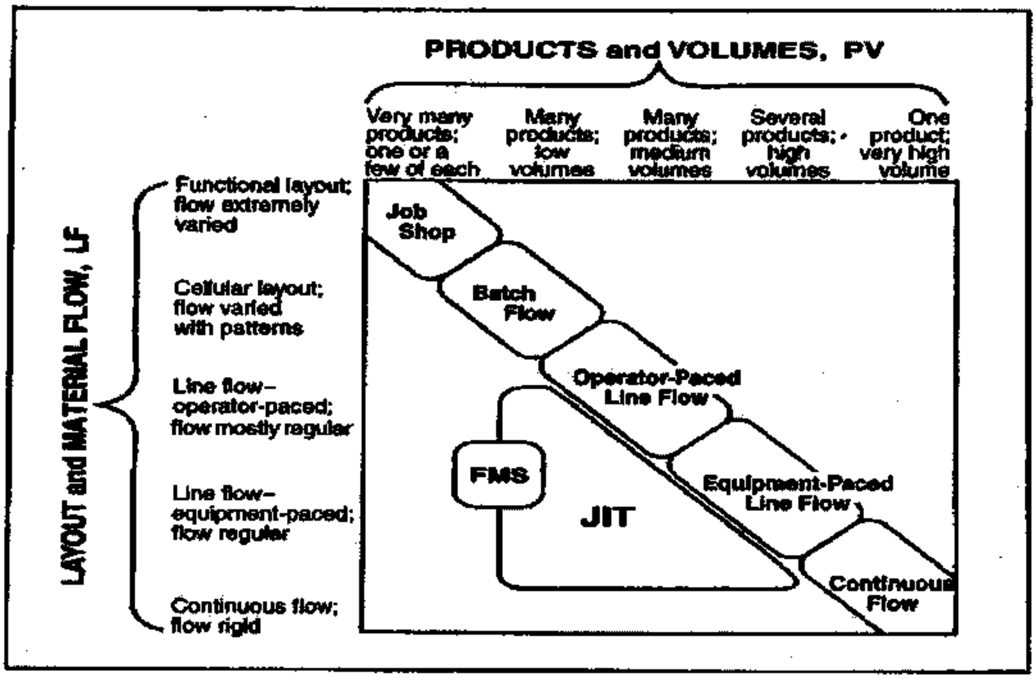


Figure 3.4: PV - LF Diagram

QUESTION.9
What is Design Thinking? Explain the Five Stages of Design Thinking

ANSWER.
Design thinking is a non-linear, iterative process that seeks to understand user's needs, challenge assumptions, redefine problems and create innovative solutions to prototype and test. This is involving five phases–Empathize, Define, Ideate, Prototype and Test–it is most useful to tackle problems that are ill-defined or unknown.

The Five Stages of Design Thinking

The Hasso Plattner Institute of Design at Stanford (aka the d.school) describes design thinking as a five-stage process. Note: These stages are not always sequential, and teams often run them in parallel, out of order and repeat them in an iterative fashion.

Stage 1: Empathize–Research Your Users Needs

Here, you should gain an empathetic understanding of the problem you're trying to solve, typically through user research. Empathy is crucial to a human-centered design process such as design thinking because it allows you to set aside your own assumptions about the world and gain real insight into users and their needs.

Stage 2: Define–State Your Users' Needs and Problems

It's time to accumulate the information gathered during the Empathize stage. You then analyze your observations and synthesize them to define the core problems you and your team have identified. These definitions are called problem statements. You can create personas to help keep your efforts human-centered before proceeding to ideation.

Stage 3: Ideate–Challenge Assumptions and Create Ideas

Now, you're ready to generate ideas. The solid background of knowledge from the first two phases means you can start to "think outside the box", look for alternative ways to view the problem and identify innovative solutions to the problem statement you've created. Brainstorming is particularly useful here...

Stage 4: Prototype–Start to Create Solutions

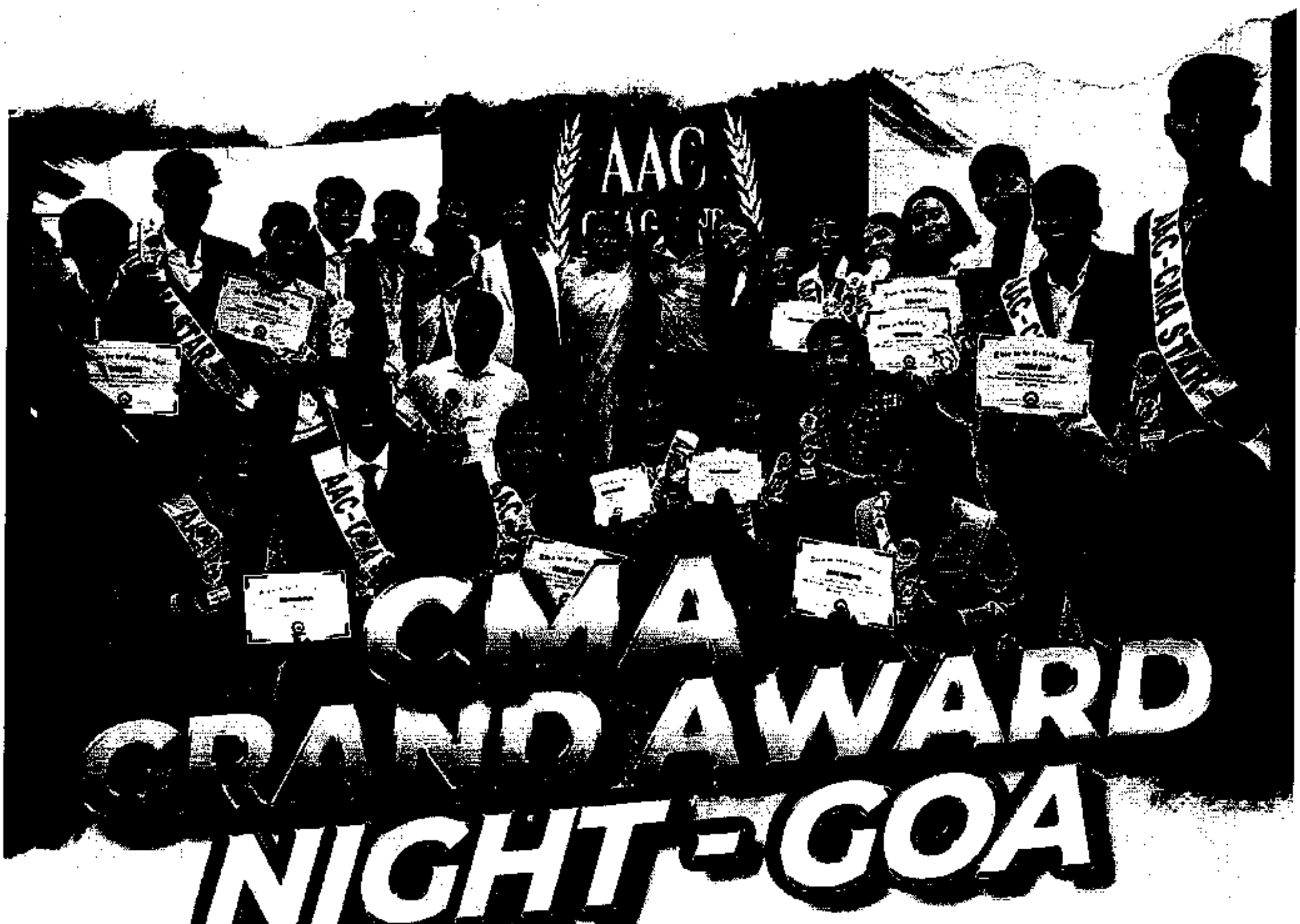
This is an experimental phase. The aim is to identify the best possible solution for each problem found. Your team should produce some inexpensive, scaled-down versions of the product (or specific features found within the product) to investigate the ideas you've generated. This could involve simply paper prototyping.

Stage 5: Test–Try Your Solutions Out

Evaluators rigorously test the prototypes. Although this is the final phase, design thinking is iterative: Teams often use the results to redefine one or more further problems. So, you can return to previous stages to make further iterations, alterations and refinements - to find or rule out alternative solutions.

Overall, you should understand that these stages are different modes which contribute to the entire design project, rather than sequential steps. Your goal throughout is to gain the deepest understanding of the users and what their ideal solution/product would be.

CMA STUDENTS AND TEAM AAC CELEBRATING SUCCESS OF CMA INTER CLEARED STUDENT IN GOA



FIRST TIME IN HISTORY OF CMA......

.....NEXT CAN BE YOU.

4.1 Introduction

Applications of Operations Research

Application of OR is different for various fields. Depending on the specific applications, the appropriate OR technique is used to obtain the results.

QUESTION 1.
What are Applications of Operation Research?

ANSWER.
A few typical applications are listed below:

National planning and budgeting

- Preparation of five-year plans.
- Annual budgets.
- Forecasting income and expenditure.
- Scheduling major projects of national importance.
- Estimating GNP, GDP, population, employment.
- Generation, agricultural yields, etc.

Defense service operations - Military principles lay greater importance on "economy of efforts" and "surprise" Element to enemy to forces. Some applications are given below:

- Development of new technology.
- Optimization of cost and time in defense projects.
- Tender evaluation technology and equipment
- Sitting and layout of defense factories.
- Assessment of "threat-analysis" from the enemy.
- Effective battle "strategies" and "tactics".
- Effective maintenance and replacement of equipments.
- Strategy of defense supplies during war and peace.
- Vendor evaluation of canteen stores departments.
- Inventory control.
- Transportation problems during 'mobilization,' 'operations.' Mock simulation exercises.'
- Ideal locations of "supply depots" to support operational units.

Industrial field OR is applied in a number of cases. Some are given here:

- Plant location and siting .
- Finance planning.
- Product and process planning.
- Facility planning and construction of factory building and layout.
- Purchasing, vendor evaluation and bid evaluation.
- Inventory control.
- Maintenance management and replacement.
- Personnel management, viz. merit rating, incentive payments, etc.

R&D and engineering

- Technology forecasting.
- Technology evaluation.
- Technology management.
- Project management of turnkey projects.
- Systems evaluation.
- Preparation of tender.
- Negotiation.
- Value engineering and selection of components.
- Work/Method study and activity sampling, etc.

Business management and competition

- Taking business decisions under risk and uncertainty.
- Capita investment and returns.
- Selection of business and area of operations.
- Decision-making under competition.
- Business strategy formation.
- Optimum advertisement outlay.
- Optimum sales force and their distribution.
- Market survey and analysis.
- Market research techniques, etc.

Agriculture and irrigation

- Project management of irrigation projects and dams.
- Construction of major dams at minimum cost.
- Optimum distribution of irrigation canals.
- Optimum location of supply points of inputs like seeds and fertilizers to the farmers.
- Optimum location of "collection points" of agricultural outputs.

Education and training

- Optimum number of schools and their locations.
- Optimum mix of students/teachers ratio.
- Optimum financial outlay to meet national objectives.
- Optimum number and location of examination centres and number of students in each centre.
- Location of supply depot of educational inputs.
- Demand and supply of textbooks and stationery, etc.

Public works department

- Time and cost control of roads, bridges and buildings.
- Estimate of time of completion of projects.
- Time estimate of various activities.
- Selection of machines and equipment.
- Maintenance and replacement.
- Preparation of budgets.
- Tender evaluation and selection of bids, etc.

Transportation and communication

- Forecast requirement of public transport.
- Optimum routing of buses to maximize utility
- Estimate/forecast income and expense.
- Project management of railway projects within a time frame and a cost-frame.
- Railway network distribution.
- Estimate/forecast of telephone demand.
- Optimum capacity selection of exchanges.

- Optimum number of "busy-hour-call" of telephone exchanges, etc.

Home management and budgeting

- Control of expenses to maximize saving.
- Optimum number of purchase of provisions.
- Order quantity of provisions at each purchase.
- Time-management.
- Work-study methods in kitchen activity.
- Time-study methods in kitchen activity.
- Preparation of budgets item-wise like 'provisions,' 'petrol,' 'vegetables and meat,' 'dress,' 'entertainment,'
- educational expenditure,' etc.
- Investment of 'surplus income' in the most appropriate manner to earn maximum profit.
- Selection of appropriate investment from alternatives provided to minimize income tax and other taxes.
- Appropriate insurance of life and properties.
- Estimate of depreciation and optimum premium of insurance, etc.

QUESTION 2.

What are Steps in solving a problem through Operations Research?

ANSWER.

There are six steps involved in solving a problem through Operation Research. These are discussed below:

Step 1:

Formation of a problem: From the definition of OR, we have seen that OR is a problem solving technique. Hence, the first step is to formulate the problem. This is an important step, because accuracy of a solution

Depends to a great extent on the correct formulation of the problem in regards to definition of terms, worming Environment, influences of environmental factors and assumptions made, intentions, objectives and constraints. In this step, following are to be clearly defined:

- Input, output, processor, objective, constraints.
- Independent variables, which are controllable variables.
- Decision variables, which are dependent variables.
- Parameters, which are independent variables.

Step 2:

Making a model: We have seen that the models are essential to analyses the problem and identify the various

Factors like input, output, constraints, etc. In addition it will also establish the relationship between input (often referred to as parameters) and output (often referred to as decision variables).

Mathematical models are often built up in this step, which give information on the following:

- Relationship of input and output.
- Constraints under which operations take place.

The objective functions of the operations. For example, the mathematical model takes the following form:

Minimise $Z = f(x)$ (objective function)

Subject to constraints

- $g(X) \leq k$
- $(X) \geq 0$

Step 3:

Solution to problems (derivation): In this step, the mathematical model development in step 2 is solved through scientific methods. A solution must satisfy the objective functions as well as the constraints. There are various scientific methods or techniques available such as LP transportation/assignment algorithm, game theory, queuing theory, statistical method, probability laws, etc.

Step 4:

Validation of model: A model is said to be valid if it gives a reliable result (output) for a set of inputs under the given conditions. Such a validation is possible for a limited period of time. In due course of time. the original assumptions and conditions in which the model was developed also change. Hence, it is essential to check the validity of a model from time to time.

Step 5:

Feedback and control on solution: Solutions are derived from the model for a set of environmental conditions. In this derivation, a number of assumptions are also made inregard to the cost and quality of inputs like labour and materials, performance characteristics and technology of plant and machinery and the market conditions like price and demand. These are called "information variables." When such variables change significantly in the field, the original solution goes out of control. Hence, these variables are applied to make the solution fall within acceptable limits. Control is applied either on the method of solution or on the design of the model itelf.

Step 6:

Implementation: Implementation of the solution is a very important step because the "proof of the pudding is in its eating. "In this step, the OR specialists team must work in closed coordination with the field staff. The OR team must brief the field staff in regard to the operating conditions of the solution and basic assumptions made therein. They must also be briefed in regard to its limitations along with its advantages and capabilities. They must also lay down in clear terms the method of operations and procedure along with precautions and corrective mechanism wherever applicable.

QUESTION 3.

Describe three methods used for OR problems.

ANSWER.

Basically, there are three methods used for OR problems.

- Analytical method
- Trial and error method
- Simulation method

These are briefly explained below:

(a) **Analytical method:** In this method, the OR techniques are developed mostly based on mathematical modeling. The method of solution also depends on classical steps and techniques in mathematics, like the use of differential calculus, integration, sets, matrices, vector algebra and coordinate geometry. Examples are EOQ, graphical solution for product mix through LP. This is a deterministic method.

(b) **Trial and error method:** The analytical method of solution has its limitations. Some problems and models fail to yield a solution through classical, mathematical or graphical methods. Trial and error method is used here.

In this method, a certain algorithm is developed. One starting point is an initial solution, which is the first approximation, The method of solution is repeated with a certain set of rules so that the initial solution is gradually modified at each subsequent solution till an optimal solution is reached. There are certain laid down to check whether the solution has become a optimal solution. The trial solutions are called iterations and the method is linear programming. This is deterministic method.

(c) **Simulation method:** Solution of problem using the principles of statistics, sampling and probability is called simulation method. This method is applied where the data is insufficient, or where the situation is quite uncertain or when it is impossible to generate data by direct measurement. In such situations, samples are created as faithfully as possible to represent the real situation called the "universe", In order to establish the nature of events, the following devices are used:

- Random tables
- Mechanical devices
- Electronic computers

Typical examples are:
Monte-Carlo simulation in queuing problem and simulation of performance of aircraft flight.
These are stochastic models.

QUESTION 4.
List Operations Research (OR) Techniques

ANSWER.
There are various techniques used in OR. Some of these are listed here:

- Probability theory
- Statistical methods
- Frequency distribution
- Graphical solution LP
- Linear programming simplex
- Transportation algorithm
- Assignment of problem
- Game theory
- Decision-tree approach.
- Replacement theory
- Simulation model
- PERT/CPM method
- Break-even analysis (BE)
- Forced decision matrix method
- Discounted Cash Flow (DCF) and Net Present Value (NPV) method
- Trend analysis and time series
- Correlation techniques
- Variance analysis
- Significance analysis
- Statistical quality Control (SQC) techniques
- Dynamic programming, etc.

OR techniques are 'application specific.' Maximum benefit can be derived from selecting the most appropriate technique for each specific area or problem. Appropriate selection of OR is an equally important task. Each technique has its own advantages and limitations. In certain cases computation is easy. Some are amenable to computer applications. In all such cases, the ability of the manager is tested in an appropriate selection of OR technique.

Some examples are listed below in Table. This is, by no means, an exhaustive list.

Sl. No.	Functional Area	Specific Problem	OR Techniques Applicable
1	Production	Siting and location	Location dynamics BE.,
	Management		Routing travel charts
		Factory layout, Productplanning	Profitability analysis, Product line analysis
		Process planning	Line of balance (LOB),
			Routing GT
		Technology selection	Forced decision matrix- bid
			Evaluation
		Facility planning	Capital investment, Selection
			Of plant, BE, DCF.
		Workstation design	Line balancing, Heuristic
			Methods
		Capacity planning	Break-even, Decision tree,
			Statistical
		Selection of plant	BE mathematical, DCF/NPV,
		Product mix	LP simplex, LP graphical
		Production planning & control	BE Sequencing assignment
			Scheduling
2	Materials	Warehouse/stores	Transportation algorithm, EOQ, ABC & VED analysis
	Management	Layout, Inventory	Functional analysis, Forced decision matrix
		Control, Vendor rating	
		Value engineering,	Break-even, Decision tree, DCF
		Make or buy decision	
3	R&D and engineering	Technology selection,System analysis reliability	Forced decision matrix,Statistical probability,Statistical frequency distribution
4	Quality Management	Inwards goods inspection	Statistical, probability, AQL, OC curve.
		Outwards goods	Statistical, probability, AQL
		Inspection	
		Process inspection	Statistical, Frequency distribution, CSP
5	Servicing	Optimum efficiency	Queuing theory
	Industry	Saving of waiting time	Monte-Carlo simulation
6	Maintenance Management	Maximization of utilization and	Replacement theory, Statistical DCF, MAPI

		minimization of cost	
7	Project management	Time estimate	Statistics, Frequency distribution
		Minimisation of time	PERT
		Minimisation of cost	CPMCPM
8	Marketing management	Forecast demand	Trend analysis, Probability, Statistics
		Marketing strategy	Game theory, Markov's chain simulation
9	Finance management	Capital investment costing	DCF, NPV, ROI, PI, Ratio analysis, Varianceanalysis, Signification, Decision tree

QUESTION 5.
What are Nature and characteristics of Operations Research?

ANSWER.
Nature of OR and its characteristics are evident form the definition itself. These are given here:

- Existence of a problem.
- Intention to solve the problem.
- Application of system concept and system analysis to the problem.
- Scientific approach to the problem where research methods are used.
- Formation of a group consisting of different specialists.
- Multi-disciplinary team with common aim.
- Team is constituted by management
- OR assists the management to take decisions.
- OR's role is that of a recommendatory nature.
- Existence of a number of solutions to the problem.
- Solutions must be optimum.
- Solutions must be most appropriate.
- Solutions must meet the objective within the constrains.
- Solutions must be given in quantifiable terms.
- Solutions must be practical, application specific and result-oriented.

QUESTION 6.
Explain scope of Operations Research.

ANSWER.
The scope of OR is not confined to any specific agency like defense services or an industrial field. The scope is wider. It is useful in every field of human activities, where optimization of resources is required. It is perhaps easy to name areas where OR is not applicable. The main fields where OR is extensively used are listed here.
This list is by no means exhaustive, but only illustrative.

- National plans and budgets.
- Defense services and battle field operations.
- Government development and public sector units.
- Industrial establishments and private sector units.
- R&D and engineering divisions.

- Business management and marketing.
- Agriculture and irrigation projects.
- Education and training.
- Public works department and construction of mega projects.
- Transportation and communication.
- Home management and personal budgeting, etc.

QUESTION 7.
Explain Functions of Operations Research.

ANSWER.
In order to achieve the above objectives, the following are the functions of OR.

1. **Provides scientific basis to decision-making:** OR emphasizes scientific methods in the analysis and interpretation of facts and data. This will enable managers to take decisions based on facts scientifically arrived at. This will also improve the objectivity of analysis and assessment of situations.
2. **Reduce complex problems to simple problems:** OR techniques can reduce complex problems into a set of simple problems which can be easily visualized and solved. This will improve clarity of thought and reduce confusions and there by improve the quality of decisions
3. **Bringing in trade-offers:** Management often comes across situations in which it has to satisfy objectives which are in conflict with each other. A typical example is inventory control. A production manager would like to see that all his raw materials and components are stocked in sufficient quantities and well in time, so that there is no fear of stock-out and production 'holdups.' Materials manager on the other hand would recommend a course to order minimum quantity at a time and hold it in stores for the minimum period. Attempts to reduce the quantity of order will increase order costs since more orders will be necessary. If quantity per order is increased to reduce the number of orders and thereby the order costs, it will increase the inventory carrying cost. The optimum solution lies between these two 'objectives' as seen by production manager and material manager separately. This is the 'trade-off' decision OR finds where the trade-off takes place when total cost is the minimum. Accordingly the equation for Economic Order Quantity (EOQ) is derived.
4. **Provides system integration:** OR integrates the system. In the systems approach, the problem is seen as a 'whole' and not as parts. In the wholistic approach, the main system consists of a number of sub-systems. All such sub-systems must be fully integrated into the main system and work as a signal system to meet the same objective. This, in other words, is called "goal congruence." The OR team consists of experts from different disciplines, which are the sub-systems. But all of them work towards the single objective to arrive at an optimum solution to the given problem. This is an integrated approach. In order to achieve optimization of the whole system, "sub-optimisation" of sub- system may be found necessary on "trade-off" consideration.
5. **Optimisation of resources:** Resources are scarce. Conservation and optimisation of the resources are the main objectives of management. OR helps to do this. In addition, OR techniques are used in the functional analysis of materials in value engineering/analysis to find better utilization of materials.
6. **Minimising time:** Modern concepts of management have recognized time as one of the most important resources. Time costs money. OR techniques are developed to minimize the time. Some examples are given below:
   - PERT in project management.
   - Queuing theory to optimize 'waiting' and 'servicing' times.
   - Transportation algorithm, Lp, etc.

7. **Minimising cost:** The primary consideration of the management is to minimise the cost . Various OR techniques have been developed to meet this function. Some of them are:
   - Linear programming.
   - CPM in project management.
   - Assignment algorithm in loading machines.
   - EOQ concept in inventory, etc.
8. **Maximising Profit:** Success and survival of industry and business depends on their ability to make profits.
   The resulting surpluses are put back into industry and business for their growth and continued existence. OR has recognized this function and accordingly developed a number of techniques. In fact many of the cost reduction techniques are applicable here. These are LP, PERT/CPM, transportation, queuing, break- even concepts, decision tree, etc.
9. **Selection of best alternative:** Most of the problems will have a number of solutions. When such alternatives have different parameters, it is very difficult to select the best by conventional methods. A typical example is that of evaluation of global bids. Bidders from different countries offer different terms on technical, commercial and financial bids. Their comparison is difficult unless all these parameters are 'normalised' and brought within the same datum. Similarly, managers find it difficult to make proper decisions on investment without OR techniques. When alternatives are presented, OR techniques are able to rank them in the order of priority as per the objectives given by the management. Some examples are:
   - Game theory on competing tenders.
   - Forced decision matrix in bids evaluation.
   - NPV/DCF method on capital investment.
   - Break-even analysis in capacity and investment decision.
   - Decision tree approach in 'make or buy' decision, etc.
10. **Solution to specific problems:** It is one of the OR functions to provide answers to specific problems posed by the management. Some examples are:
    - Line balancing of plant.
    - Design of workstation.
    - Selection of appropriate machine.
    - Decision of capacity of plant.
    - Number of observations in "time study".
    - Check to see whether a machine has come out of original alignment and accuracy, etc.

OR techniques are available to give answers to all the above problems and many more.

QUESTION 8.
What are Limitations of OR techniques.

ANSWER.
Following are the limitations of OR techniques:

(a) **Information gap:** OR is a specialist's work. It involves a lot of mathematical manipulations, use of various theories like game, statistical, probability, queuing, etc. Unless the manager has a sufficient knowledge of mathematics, he will not be able to appreciate and understand the philosophy and working based on which the OR team has developed the solution. Similarly, a OR specialist is unable to understand many other aspects of business like the goodwill of the customer, the feelings, sentiments and attitude of workers or the expectations of the owners. Many of these facts cannot be quantified. Hence, there is a big gap or "information gap" between the OR specialists and the management. Due to this gap, managers look upon OR as a "theoretical solution" and reject the same without giving it a fair chance to prove its worthiness. This will also enforce the natural tendency to resist changes.

- (b) **Quantification techniques:** OR works on quantifiable terms and figures. All factors like goodwill, attitude, expectations, etc. cannot be quantified. Hence the solution ignores all such factors, which are important but cannot to be expressed in quantifiable terms.
- (c) **Finite variables:** OR deals with finite number of variables. Most of them deal with two sets of variables, viz. independent variable (parameters) and dependent variable (decision variable). In an actual situation, there are a large number of variables which influence the input, the processor and consequently the output. These are not considered.
- (d) **Limited number of constraints:** OR can solve problem having a limited number of constraints. In practice,
  there are unlimited number of constraints.
- (e) **Single objective function:** OR deals with a single objective function. It does not deal with sub-optimization. In practice, more than one objective function exists.

4.2 Production Planning and Control

Introduction

Production planning control can be viewed as the nervous system of a production operation. The primary concern of production planning and control is the delivery of products to customers or to inventory stocks according to some predetermined schedule. All the activities in the manufacturing or production cycle must be planned, coordinated, organized, and controlled to achieve this objective. From a long-term point of view (usually from seven to ten years or more) production planning largely deals with plant construction and location and with product-line, design and development. Short-range planning (from several months to a year) focuses on such areas as inventory goals and wage budgets. In plans projected over a two-to-five-year period, capital-equipment budgeting and plant capacity and layout are the major concern. Production planning and control normally reflects the short-range activities and focuses on the issues and problems that arise in the planned utilisation of the labor force, materials, and physical facilities that are required for manufacturing the products in accordance with the primary objectives of the firm.

Production systems are usually designed to produce a variety of products and are, therefore, complex. In such complex systems, anything can happen and usually it is so. Therefore, it is vital to exercise some kind of control over the production activities. Control is possible only when everything is planned. Production planning and control is thus a very important aspect of production management.

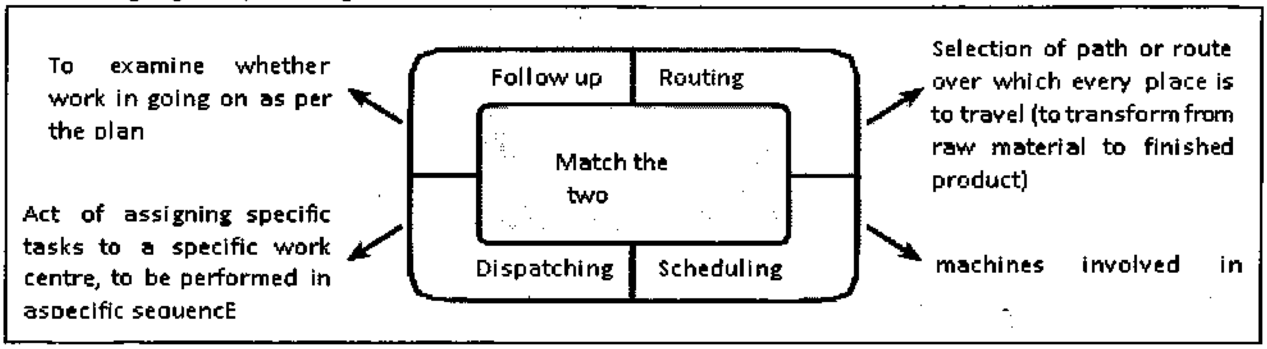
QUESTION 9.
What are Objectives of production planning and control?

ANSWER.
The ultimate objective of production planning and control is to contribute to the profits of the enterprise. This is accomplished by keeping the customers satisfied through the meeting of delivery schedules. Further, the specific objectives of production planning and control are to establish the routes and schedules for work that will ensure the optimum utilisation of raw materials, labourers, and machines to provide the means for ensuring the operation of the plant in accordance with these plans. Production planning and control is essentially concerned with the control of work-in-process. To control work-in-process effectively it becomes necessary to control not only the flow of material but also the utilisation of people and machines.
Production planning and control fulfils these objectives by focusing on the following points:
(i) Analysing the orders to determine the raw materials and parts that will be required for their completion,
(ii) Answering questions from customers and salesmen concerning the status of their orders,
(iii) Assisting the costing department in making cost estimates of orders,
(iv) Assisting the human resource departments in the manpower planning and assignment of men to particular jobs,
(v) Controlling the stock of finished parts and products,
(vi) Determining the necessary tools required for manufacturing,
(vii) Direction and control of the movement of materials through production process,
(viii) Initiating changes in orders as requested by customers while orders are in process,
(ix) Issuing requisitions for the purchase of necessary materials,
(x) Issuing requisitions for the purchase or manufacture of necessary tools and parts,
(xi) Keeping the up-to-date records scheduled and in process,
(xii) Maintaining stocks of materials and parts,
(xiii) Notifying sales and accounting of the acceptance of orders in terms of production feasibility,

(xiv) Preparing the route sheets and schedules showing the sequence of operation required to produce particular products,
(xv) Production of work orders to initiate production activities,
(xvi) Receiving and evaluating reports of progress on particular orders and initiating corrective action, if necessary,
(xvii) Receiving orders from customers,
(xviii) Revising plans when production activities cannot conform to original plans and when revisions in scheduled production are necessary because of rush orders.

QUESTION 10.
Which functions does Production control involves.

ANSWER.
Production control involves the following functions:
(i) Planning the production operations in detail,
(ii) Routing, i.e., laying down the path for the work to follow and the order in which the various operations will be carried out,
(iii) Scheduling, i.e., establishing the quantity of work to be done, and fixing the time table for performing the operations,
(iv) Dispatching, i.e., issuing the necessary orders, and taking necessary steps to ensure that the time targets set in the schedules are effectively achieved,
(v) Follow-up, taking necessary steps to check up whether work proceeds according to predetermined plans and how far there are variances from the standards set earlier,
(vi) Inspection, i.e., conducting occasional check-ups of the products manufactured or assembled to ensure high quality of the production.



Techniques of Production Control

QUESTION 11.
Explain Basic types of production control.

ANSWER.
Production control can be of six types:
(i) Block control
This type of control is most prominent in textiles and book and magazine printing. In these industries it is
necessary to keep things separated and this is the fundamental reason why industries resort to block control.
(ii) Flow control
This type of control is commonly applied in industries like chemicals, petroleum, glass, and some areas of food manufacturing and processing. Once the production system is thoroughly designed, the

production planning and control department controls the rate of flow of work into the system and checks it as it comes out of the system. But, under this method, routing and scheduling are done when the plant is laid out. That is to say, the production line which is established is well balanced and sequenced before production operations begin; this type of control is more prevalent in continuous production systems.

(iii) Load control

Load control is typically found wherever a particular bottleneck machine exists in the process of manufacturing.

(iv) Order control

The most, common type of production control is called order control. This type of control is commonly employed in companies with intermittent production systems, the so-called job-lot shops. Under this method, orders come into the shop for different quantities for different products. Therefore, production planning and control must be based, on the individual orders.

(v) Special project control

Special production control is necessary in certain projects like the construction of bridges, office buildings, schools, colleges, universities, hospitals and any other construction industries. Under this type of control, instead of having sets of elaborate forms for tooling and scheduling, a man or a group of men keeps in close contact with the work.

(vi) Batch control

Batch control is another important, type of production control which is frequently found in the food processing industries. Thus, production control in batch-system of control operates with a set of

Production planning and control in continuous-production systems involve two activities:

(i) Assuring that supply of raw materials and supplies are on hand to keep the production system supplied and assuring that finished products are moved from the production-system,

(ii) Maintaining a constant rate of flow of the production, so that the system can operate near capacity in some case or can meet the quantity requirements of the production.

Production planning in intermittent production systems:

The intermittent production systems are characterized by the following:

(i) General purpose production machines are normally utilised and process layout is favoured.

(ii) Materials handling equipment is typically of the varied path type such as hand trucks and forklift trucks.

(iii) Relatively high cost, skilled labour is needed to turn out the various quantities and types of products.

(iv) The company generally manufactures a wide variety of products; for the majority of items, sales volumes and consequently production order sizes are small in relation to the total production.

QUESTION 12.

Production planning and control in continuous-production systems.

Production systems may be continuous or intermittent. The continuous production systems are characterised by:

(i) Fixed-path material handling equipment,

(ii) High volume of production,

(iii) Product layouts,

(iv) Production of standardised products,

(v) Production to stock or long-range orders,

(vi) The use of special-purpose machines or automation.

QUESTION 13.
Explain Time Study & Motion Study.

ANSWER.

Time study is defined to be a searching scientific analysis of the methods and equipment used or planned in doing a piece of work, development in practical detail of the best manner of doing it and determination of the time required.

Operation analysis is the study of the entire process to determine whether operations can be eliminated, combined or the sequence changed. Operation analysis aims to determine the one best way and can be applied to method, materials, tools equipment layout, working conditions and human requirements of each operation.

Job standardization consists in determining the one best way of performing a job under the means at command of recording the exact method along with the time for each element of operation and establishing means to maintain the standard conditions.

Another term connected with time and motion study is the job analysis. Job analysis is the determination of
essential factors in a specific kind of work and of the qualifications of a worker necessary for its performance.

Time study aims at determining the best manner of doing a job and timing the performance of the job when done in the best manner.

In motion study the work is divided into fundamental motions and in time study work is divided into elements of operations. In both cases attempts are made to remove useless motions and improve combination and sequences of motions and operations. In motion study the best way of doing a work is determined by motion analysis and operators are trained to follow the method so determined but in time study the best method is determined by analysis of the methods and equipment, used and motions only roughly considered and that too indirectly. In time study, setting of production standards, standards for cost purposes and wage incentives are emphasized. The measurement of human effort is a difficult job which can only be solved by using scientific method and industrial experience combined with knowledge of psychology.

The use of scientific method involves experiment measurement and elimination of variables connected with a job.

The variables connected with a job are the method of manufacture, tools and equipment's, material, working conditions, worker concerned and time required to perform the job. In order to measure the last variable time, the other variables must be eliminated by standardizing. In going to proceed for time study, it is first necessary to standardize the method and conditions of work and to define what an average worker is. Time study has two sides, mechanical and human.

Before commencing the time study, the time study man should ensure and ascertain the following:

(i) That motion studies have been carried out so that planning of work, work places and appliances are satisfactory.

(ii) That the operations can be performed in the correct; sequence without interruption.

(iii) That the human effort involved is minimum.

QUESTION 14.
Explain Work Study.

ANSWER.

It is a general term for the techniques: methods study and work measurement which are used in the examination;of human work in all its contexts and systematically investigate all factors leading to improvement of efficiency.

Work study aims at finding the best and most efficient way of using the available resources–men, materials, money and machinery. Once the method study has developed an improved procedure for doing a work the work measurement or time study will study the time to complete a job.

QUESTION 15.
Explain Method Study.

ANSWER.

It is the systematic investigation of the existing method of doing a job in order to develop and install an easy, rapid, efficient, effective and less fatiguing procedure for doing the same job and at minimum cost. This is achieved by eliminating unnecessary motions involved in a certain operation or by changing the sequence of operation or the process itself.

Methods study can be made by the help of both motion study and time study. The methods study programme must include the following features:-

(a) Uniform application,
(b) Established standard practice,
(c) Continuous review,
(d) Credit distribution.

A new and improved method developed in one department should be spread out to the entire plant preferably with further improvements.

A new method must not be forgotten between orders as it happens sometimes in batch production methods. Department should always aim at improved and better ways of doing jobs.

For successful control of methods study, the enthusiastic cooperation of every employee is required. To gain employee cooperation, distribution of credit is essential. It has been correctly said that a good methods department rarely takes credit for an original idea. Its success lies in getting new ways and methods adopted promptly, universally, continuously and cooperatively towards the improvement of productivity.

4.4 Optimum Allocation of Resources - LPP

Introduction

Linear Programming is an optimization technique. It is "**a technique for specifying how to use limited resources or capacities of a business to obtain a particular objective, such as least cost, highest margin or least time, when those resources have alternate uses**".

The situation which require a search for "**best**" values of the variables, **subject to** certain constraints, are amenable to programming analysis. These situations cannot be handled by the usual tools of calculus or marginal analysis. The calculus technique can only handle **exactly equal** constraints, while this limitation does not exist in case of linear programming problem.

A linear programming problem has two basic parts.

- The first part is the objective function, which describes the primary purpose of the formulation - to maximize some return (for example, profit) or to minimize some cost (for example, production cost or investment cost).
- The second part is the constraint set. It is the system of equalities and/or inequalities, which describes the restrictions (conditions or constraints) under which optimization is to be accomplished.

QUESTION 16.
State Definition of Linear Programming.

ANSWER.
According to Kohlar "A method of planning and operation involved in the construction of a model of a real situation containing the following elements: (a) variables representing the available choices, and (b) mathematical expressions (i) relating the variables to the controlling conditions, and (ii) reflecting the criteria to be used in measuring the benefits derivable from each of the several possible plans, and (iii) establishing the objective. The method may be so devised as to ensure the selection of the best of a large number of alternatives".

Samuelson, Dorfman and Solow defines LP as "The analysis of problems in which a linear function of a number of variables is to be maximized (or minimized) when those variables are subject to a number of restraints in the form of linear inequalities".

In the words of Loomba, "LP is only one aspect of what has been called a system approach to management wherein all programmes are designed and evaluated in terms of their ultimate affects in the realization of business objectives".

QUESTION 17.
What are Application Areas of Linear Programming?

ANSWER.
In practice linear programming has proved to be one of the most widely used technique of managerial decision making in business, industry and numerous other fields.

1. Industrial Applications
Linear programming is extensively used to solve a variety of industrial problems. In each of these applications, the general objective is to determine a plan for production and procurement in the time period under consideration. It is necessary to satisfy all demand requirements without violating any of the constraints. Few examples of industrial applications are as follows:
(a) Product Mix-Problem.
(b) Production Scheduling.
(c) Production Smoothing Problem.
(d) Blending Problems.
(e) Transportation Problems.
(f) Production distribution problems.
(g) Trim Loss.
(h) Linear programming is also used by oil refineries to determine the optimal mix of products to be produced by the refinery during a given period.
(i) Communication Industry. LP methods are used in solving problems involving facilities for transmission, switching, relaying etc.
(j) Rail Road Industry: An LP model for optimal programming of railway freight, and train movements has been formulated to handle scheduling problems as found at large terminal switching rail points.

2. Management Applications:
(a) Portfolio Selection.
(b) Financial Mix Strategy.
(c) Profit Planning.
(d) Media Selection.
(e) Travelling Salesmen Problem.
(f) Determination of equitable salaries.
(g) Staffing problem.

3. Miscellaneous Applications:
The additional application of Linear Programming are as follows:
(a) Form planning.
- The particular crops to be grown or cattle to keep during a period
- The acreage to be devoted to each, and
- The particular production methods to be used.

(b) Airline routine.
(c) Administration, Education and Politics have also employed linear programming to solve their problems.
(d) Diet Problems. The diet problem, one of the earliest applications of linear programming was originally used by hospitals to determine the most economical diet for patients.

4. Administrative applications of Linear Programming:
Linear programming can be used for administrative applications. Administrative applications of Linear Programming are concerned with optimal usage of resources like men, machine and material.

5. Non-Industrial applications of linear programming:
Linear programming techniques/tools can be applied in the case of non-industrial applications as well.
Examples of the use of L.P techniques for non-industrial applications are given below:

- Agriculture.
- Environmental Protection.
- Urban Department.
- Facilities Location.

6. Further applications of Linear Programming are:

- In structural design for maximum product.
- In balancing assembly lines.
- In scheduling of a military tanker fleet.
- In determining which parts to make and which to buy to obtain maximum profit margin.
- In selecting equipment and evaluating methods improvements that maximize profit margin.
- In planning most profitable match of sales requirements to plant capacity that obtains a fair share of the market.
- In design of optimal purchasing policies.

Formulation of Linear Programming Problem:

The formulation of linear programming problem as a mathematical model involves the following basic steps:

Step 1: Find the key-decision to be made from the study of the solution. (In this connection, looking for variables

helps considerably).

Step 2: Identify the variables and assume symbols x_1, x_2 ...... for variable quantities noticed in step 1.

Step 3: Express the possible alternatives mathematically in terms of variables. The set of feasible alternatives generally in the given situation is:

$[(x1, x2); x1 > 0, x2 > 0]$

Step 4: Mention the objective quantitatively and express it as a linear function of variables.

Step 5: Express the constraints also as linear equalities/inequalities in terms of variables.

Some definitions:

(a) Solution:
Values of decision variables xj ($j = 1, 2 \ldots\ldots, n$) which satisfy the constraints of a general L.P.P., is called the solution to that L.P.P.
(b) Feasible Solution:
Any solution that also satisfies the non-negative restrictions of the general L.P.P., is called a feasible solution.
(c) Basic Solution:
For a set of m simultaneous equations in n unknowns ($n>m$), a solution obtained by setting ($n-m$) of the variables equal to zero and solving the remaining m equations in m unknowns is called a basic

solution. Zero variables (n-m) are called non basic variables and remaining m are called basic variables and constitute a basic solution.

(d) Basic Feasible Solution:

A feasible solution to a general L.P. problem which is also basic solution is called a basic feasible solution.

(e) Optimal Feasible Solution:

Any basic feasible solution which optimize (maximize or minimize) the objective function of a general L.P.P. is called an optimal feasible solution to that L.P. problem.

(f) Degenerate Solution:

A basic solution to the system of equations is called degenerate if one or more of the basic variables become equal to zero.

QUESTION 18.
What are Limitations of Linear Programming?

ANSWER.

Although linear programming is a very useful technique for solving optimization problems, there are certain
important limitations in the application of linear programming. Some of these are discussed below:

1. Firstly, the linear programming models can be applied only in those situations where the constraints and the objective function can be stated in terms of linear expressions.
2. In linear programming problems, coefficients in the objective function and the constraint equations must be completely known and they should not change during the period of study.
3. Yet another important limitation of linear programming is that it may give fractional valued answers.
4. Linear programming will fail to give a solution if management have conflicting multiple goals.
5. Linear programming problem requires that the total measure of effectiveness and total resource usage resulting from the joint performance of the activities must equal the respective sums of these quantities resulting from each activity being performed individually.
6. Many real-world problems are so complex, in terms of the number of variables and relationships constrained in them, that they tax the capacity of even the largest computer.
7. Other limitations of LP includes:-
   - Does not take into consideration the effect of time and uncertainty.
   - Parameters appearing in the model are assumed to be constants but in real-life situations they are frequently neither known nor constants.

Formulation of Mathematical Model - At a glance

Definitions

(a) **Linear programming** Linear programming is a versatile mathematical technique in Operations Research and a plan of action 5 solve a given problem involving linearly related variables in order to achieve the laid down objective in the form of minimising or maximizing the objective function under a given set of constraints.

(b) **Basic solution** There are instances where number of unknowns 'p' are more than then umber of linear equations 'q' available. In such cases, we assign zero values to all surplus unknowns. There will be 'N' such unknowns. With these values, we solve 'q' equations and get values of 'q' unknowns. Such solutions are called basic solutions.

(c) Basic variables The variables whose value is obtained from the basic solution is called basic variables.

(d) Non- basic variables The variables whose values are assumed as zero in the basic solution are known as non-basic variables.

(e) Solution A solution to LPP is the set of values of the variables which satisfies the set of constraints of the problem.

(f) Feasible solution A feasible solution to a LPP is the set of values of the variables which satisfies the set ofconstraints as well as the non-negative constraints of the problem.

(g) Basic feasible solution A feasible solution to a LPP in which the vectors associated with the non-zerovariables are linearly independent is called basic feasible solution.

Note: Linearly independent variables x_1, x_2, x_3 .....are said to be linearly independent if

$k_1 x_1 + k_2 x_2 + ... + k_n x_n = 0$; $k_1 = 0$, $k_2 = 0$.....

(h) Optimum (optimal) solution: A feasible solution of a LPP is said to be the optimum (optimal) solution, if it also optimizes the objective function of the problem.

(i) Slack variables linear equations are solved through the equality form of equations. Normally, constraints are given in the "less than or equal to" (⊠) form. In such cases, we add the appropriate variable to make it an"equality" (=) equation. These variables added to the constraints to make it an equality equation in LPP arecalled slack variables and are often denoted by the letter S.

> **Example:** $2x_1 + 3x_2 \leq 500$
>
> $2x_1 + 3x_2 + S_1 =$ where S_1= slack variable

(j) Surplus variables sometimes, constraints are given in the "more than or equal to" (≥) form. In such cases,we subtract an appropriate variable to make it into the "equality" (=) form. Hence, variable subtracted to theconstraints to make an equality equation LPP is called surplus variable and is often represented by the letter S.

> **Example:** $3x_1 + 4x_2$ ⊠ 100
>
> $3x_1 + 4x_2 - S_2 = 100$ where S_2 = surplus variable

(k) Artificial variable Artificial variables are fictitious variables. These are introduced to help computation and solution of equations in LPP these are also used when constraints are given in "greater than or equal to" (⊠)form. As discussed, surplus variables are subtracted in such cases to convert inequality to "equality" form. In certain cases, even after introducing surplus variables, the simplex tableau may not contain an "identity matrix" or unit vector. Thus in a LPP artificial variable are introduced in order to get a unit vector in the simplex tableau to get a feasible solution. Normally, artificial variables are represented by the letter A. Problems where artificial variables are introduced can be solved by two methods, viz. Big -M method and Two- phase method.

(l) Big -M method Big -M method is a modified simplex method for solving LPP when high penalty cost (orprofit) has been assigned to the artificial variable in the objective function.

This method is applicable for minimising and maximizing problems

(m) Two- phase method LPP where artificial variables are added can be solved by two-phase method. This is amodified simplex method. Here the solution takes two phase as follows:

Phase I : Basic feasible solution Here apply simplex method to a specifically constructed LPP called Auxiliary

LPP and obtain basic feasible solution.

Phase II: Optimum basic solution Form basic feasible solution, we obtain optimum basic feasible Solution.

(n) Simplex tableau: This is a table prepared to show and enter the value obtained for a basic variable at eachstage of iteration. This is the derived value at each stage of calculation.

QUESTION 19.
What are three forms of representing a LPP?

ANSWER.

There are three forms of representing a LPP. These are:

- General form of LPP
- Canonical form of LPP
- Standard form of LPP

These are written in 'Statement' form or in 'Matrix' form as explained in the subsequent paragraphs

General Form of LPP

(a) Statement form: This is given as:

Find the value of $x_1, x_2, \ldots\ldots x_n$ which values optimize $Z = c_1x_1 + c_2 x_2 + \ldots + c_n x_n$

Subject to:

$$A_{11}x_1 + a_{12} x_2 + \ldots + a_{1n} x_n \leq (\text{or} = \text{or} \geq) b_1$$

$$A_{21}x_1 + a_{22} x_2 + \ldots + a_{2n} x_n \leq (\text{or} = \text{or} \geq) b_2$$

$$A_{m1}x_1 + a_{m2} x_2 + \ldots + a_{mn} x_n \leq (\text{or} = \text{or} \geq) b_m$$

Where all coefficients (c_j, a_{ij}, b_I) are constants and all x_j's are variables.

I =1,2,......m

J =1,2,.......n

(b) Matrix form: This is stated as:

Find the value of $x_1, x_2, \ldots\ldots x_n$ to maximise $Z = c_1x_1 + c_2 x_2 + \ldots + c_n x_n$

Let Z be a linear function on R^n defined by

(i) $Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$

where (c_j, a_{ij}, b_i) are constant. Let a_{ij} be an mxn matrix and let $\{b_1, b_2 \ldots b_m\}$ be a set of constants such that

$$\begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n} x_n \leq (\text{or} = \text{or} \geq) b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n} x_n \leq (\text{or} = \text{or} \geq) b_2 \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn} x_n \leq (\text{or} = \text{or} \geq) b_m \end{bmatrix}$$

(ii) and let $x_j \geq 0$, j= 1,2...n, The problem of determining an n-tuple ($x_1, x_2 \ldots x_n$) which make Z a minimum (or maximum) is called general linear programming problem.

Canonical Form of LPP

(a) Statement form: This form is given as:

Maximise $Z = c_1x_1 + c_2x_2 + ... + c_nx_n$

Subject to constraints

$a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n \le b_j$ (I= 1,2,…m)

$x_1, x_2 \dots x_n \ge 0$

The characteristics of canonical form are:

1. Objective function is of the "maximization" type.

Note: Minimisation of function f (x) is equivalent to maximization of function {-f(x) }.

Minimise {f(x)} = Maximise{-f(x)}

2. All constraints are of the type "less than or equal to," except the non-negative restrictions.

Note: An inequality of greater than can be replaced by less than type by multiplying both sides by -1and vice versa.

e.g. $2x_1 + 3x_2 \ge 100$ can be written as $-2x_1 - 3x_2 \le -100$.

3. All variables are non-negative, viz. $x_j \ge 0$

(b) Matrix form This is stated as:

Maximise $Z = C_X$

Subject to constraints $A_X \le b$; $X \ge 0$

Where $X = (x_1, x_2 \dots x_n)$; $C = (c_1, c_2 \dots c_n)$; $b = (b_1, b_2 \dots b_m)$; $A = (a_{ij})$

Where I= 1,2,…m; j = 1,2,…n

The Standard Form of LPP

(a) Statement form This is stated as:

Maximise $Z = c_1 x_1 + c_2 x_2 + ... + c_n x_n$

Subject to the constrains $a_{i1}x_1 + a_{i2}x_2 + ... + a_{in} x_n \le b_j$ (I= 1,2,…m)

$x_1, x_2 \dots x_n \ge 0$

The characteristics of standard form of LPP are:

1. Objective function is of the "maximization" type.
2. All constraints are expressed in the function of equality form except the restrictions.
3. All variables are non-negative.

Note: Constraints given in the form of "less than or equal to" can be converted to the equality form by adding 'slack' variables. Similarly, those given in "greater than or equal to" can converted to the equality form by subtracting 'surplus' variables.

(b) Matrix notations This is given as:

Maximise Z = CX

Subject to the constraints AX = b; $b \ge 0$; $X \ge 0$

Where $X = (x_1, x_2 \dots x_n)$; $C = (c_1, c_2 \dots c_n)$; $b = (b_1, b_2 \dots b_m)$; $A = (a_{ij})$

Where I=1,2,3…m; j=1,2,3…n

Note: Coefficients of slack and surplus variables in objective function are always assumed to be zero. The theoretical background of LP is given in matrix vector algebra, discussed in any standard book on the subject. We have also defined certain terms like basic solution in chapter2. These are now recalled in matrix vector form:

(a) Basic solution

(i) In a system of m equations and n unknowns where n > m

$AX = B$ and $X^t \in R^n$

When A is an $m \times n$ matrix of rank m.

Let B be any X sub-matrix formed by linearly independent columns of A, then the solution is obtained
by setting (n-m) variables, not associated with the columns of B, equal to zeroes and solving the resultant system; this is called a basic solution to the given system of equations.
(ii) The value of non-zero m variable obtained by solving this system of equations is called basic variables.
(iii) The (m×n) non-singular matrix B, of the given system of equations, is called a basic matrix.
(iv) Column of B is basic vectors.
(v) B is the basic sub-matrix. Then the basic solution to the system is
$X_B = B^{-1}b$
Notes:
1. $X^T_B \in R$ and so as such cannot be called a solution of the system of equations. Therefore, in strict sense,if X_B is a basic solution, then the solution to the given system of equation is $[X_B^T, 0]$ when $X_B^T \in R^m$ and $0 \in R^{n-m}$
2. However, we follow the convention and call X_B as the basic solution of the system equation, all the same, remembering the above fact in mind.

(b) Degenerate solution A basic solution of the system is called degenerate if one or more of the basic variables become zero.

(c) Basic feasible solution A feasible solution to a LPP which is also a basic solution to the problem is called basic feasible solution (BfS) to the LPP.
In matrix vector form, the above term is defined as follows:
Given $Z = CX$; $C, X^T \in R^n$
Subject to the constraints $AX = b$; $X \geq 0$
Then X_B is a basic feasible solution (BSF) to the problem if B is a m × m non-singular sub-matrix of A and $BX_B = b$; $X_B \geq 0$

(d) Cost vector Let X be a basic feasible solution (BSF) of LPP.
Maximise $Z= CX$
Subject $AX = b$; $X \geq 0$
Then, the vector $C = C_{Bi}$, I= 1, 2...q are components of C associated with the basic variables, is called the cost vector associated with BFS of X_B.
Objective function of BFS is $Z_B = C_B X_B$

(e) Improved basic feasible solution (IBFS) If X_{B-p} X_{B-II} are two BFS to a standard LPP, X_{B-II} is said to be improved BFS as compared to X_{B-I} if
or $Z_{B-II} \geq Z_{B-I}$
$C_{B-II} X_{B-II} \geq C_{B-I} X_{B-I}$
Where C_{B-I} and C_{B-II} are the cost components associated with X_{B-I} and X_{B-II}.

(f) Optimum basic feasible solution (OBFS) A basic feasible solution (BSF), X_B to LPP.
Maximise $Z = CX$
Subject to $AX = b$ and $X \geq 0$
Is called an optimum BFS if
$Z_o = C_B X_B \geq Z$
Where Z is the value of the objective function for any feasible solution.

QUESTION 20.
Explain theorems are applied for solution of LPP by simplex method?

ANSWER.
Following theorems are applied for solution of LPP by simplex method.
(a) Reduction theorem If an LPP has a basic solution, then it has a basic feasible solution.
(b) Finite solution There exists only a finite number of basic feasible solutions to an LPP.
(c) Replacement theorem If we substitute one of the basis vectors with a non-basis vector in the basis set of LPP having a BFS, then the new solution of the new LPP is also a BFS.
(d) Improving BFS theorem Let X_{B-I} be a BFS to an LPP.
Maximise Z =CX
Subject to AX = b, X ≥ 0
Let XB-II be another BFS obtained by admitting a non basis column vector a in the basis for which net evaluation $\Delta I = C_j - Z_j$ is positive, then X_{B-II} is an improved basic feasible solution.
$C_{B-II} X_{B-II} > C_{B-I} X_{B-I}$ and $\Delta_{I-II} > \Delta_{I-I}$
or $\Delta_{I-II} - \Delta_{I-I} > 0$ (positive)
(e) Sufficient condition theorem A sufficient condition for a BFS to a LPP to become an optimum BFS (OBFS) is that $Cj - Zj \leq 0$ for all j's for which the column vector $aj \notin B$.
(f) Optimality theorem No. 1 when there is an optimum basis feasible solution (OBFS) to a LPP with $Cj - Zj \leq 0$ for all j's for which $ai \notin B$, a necessary and sufficient condition for some other BFS to become OBFS is $Xj > 0$, $Cj - Zj = 0$ for all j's for which $ai \notin B$.
(g) Optimality theorem No. II Any convex combination of 'm' different optimum solution to an LPP is again an optimum solution to the problem.
(h) Minimax theorem Let f(X) be a linear function of n variables such that $f(X_i)$ is its minimum value for some point X_1^T where $X_1^T \notin Rn$, then it is said - f(X) attain its maximum value at point X_1^T where
$X_1^T \in Rn$. In other words
mini f(X) = - max {- f(X)}
(i) Undounded theorem Let there exist a BFS to a given LPP if for at least on j
$C_j - Z_j > 0$; I = 1,2,3...
Then, there does not exist any optimum solution to this LPP.
(j) Extreme point theorem A basic feasible solution (BFS) to an LPP must correspond to an extreme point of the set of all feasible solutions.

QUESTION 21.
Explain steps and scope of simplex solution.

ANSWER.
Steps of simplex solution
Step 1: Formulation of LPP.
Step 2: Convert constraints into equality form.
Step 3: Construct the starting simplex tableau.
Step 4: Test optimality by analysis.
Step 5: Find "incoming" and "outgoing" variables and rewrite the tableau as per given set of rules.
Step 6: Repeat form step 4 onwards again till the optimum basic feasible solution (OBFS) is obtained.

Scope of Simplex Solution
Following are the types of problems solved by simplex method.
1. Maximise Z with inequalities of constraints in "≤" form.
2. Minimise Z with inequalities of constraints in "≤" form.
3. Maximise Z with inequalities of constraints "≥" form.

4. Maximise or minimise Z with inequalities of constraints in "≤" or "=" or "≥" form.

Solution by Two-phase Method

Linear programming problems where artificial variables are introduced can be solved by a modified simplex method known as two-phase method. Here, we obtain solution as follows:

Phase I: Design an auxiliary LPP as per a set of rules and obtain a basic feasible solution (BFS).

Phase II: Restate the LPP with auxiliary LPP of BFS as the starting tableau and obtain optimum basic feasible solution.

The following are the steps involved in two-phase method.

Phase I

Step 1: Formulation of Problems

(a) All "minimization problems" must be first converted to that of "maximizing problems." This is done by multiplying both sides by- 1.

(b) Rewrite objective function by assigning "-1" value to coefficients of artificial variables, instead of M, as in Big -M method and "0" values to all other variables.

Step 2: Initial tableau

(a) Convert inequality form of constraints to equality form by use of artificial, surplus or slack variables.

(b) Assign zero values to all variables whose coefficients are not unit vectors.

(c) Based on (a) and (b) above, develop the initial tableau.

Step 3: Iteration

(a) Examine optimality condition by checking up whether there is any positive value in (C- Z) row.

(b) If (C-Z) row has positive values, identify "optimum column" and "replacing row" as per standard rules given earlier.

(c) Obtain subsequent tableau by process of iteration as per standard rules given earlier.

Step 4: Basic feasible solution

(a) Obtain BFS by repetitive iterations as given in step 3.

(b) The BFS at (a) above is called auxiliary LPP.

Phase II

Step 1:

(a) Restate the objective function without the use of artificial variables.

(b) Use constraints as obtained in auxillary LPP in phase 1.

Step 2: Initial tableau

Use auxillary LPP obtained in phase I as the initial tableau by replacing the zero values of coefficients of decision variable with actual values as per the revised objective function given in step I above.

Step 3: Obtain optimum basic feasible solution (OBFS) by iterative process.

This method is reproduced below.

QUESTION 22.
Explain Inverse Matrix Using Simplex.

ANSWER.

Inverse Matrix Using Simplex

We have matrix notation of standard form of LPP as:
Maximise Z = CX
Subject to constraints AX = b; $X \geq 0$
The solution obtained is $X = A^{-1}b$
Thus, we see that simplex solution obtained on the tableau which is declared as declared as Optimum Basic Feasible Solution (OBFS) contains inverse of matrix A.
Following are the steps involved in finding inverse of a given matrix A.

Step 1:
Objective function (Z) Introduce a fictitious objective function.

$$Z = CX \text{ where } C = 0,\ X = \begin{bmatrix} X1 \\ X2 \end{bmatrix}$$

Introduce artificial variables $A_1 = -1$ and $A_2 = -1$.
$Z = 0x_1 + 0x_2 - A_1 - A_2$

Step 2:
Constraints with given vector Introduce artificial variables A_1 and A_2 and assign values of –1 to each as per method discussed in 2-phase method with the given matrix as coefficients of constraints.

Step 3:
Column vector (b) Introduce a dummy non-zero column vector b. Assume any positive integers.

Step 4:
Simplex solution Solve the above problem by 2-phase simplex method and obtain optimum solution as per phase I of computation given in the last chapter. This may be feasible or not feasible.

Step 5:
Inverse of A Follow 2-phase-method of simplex solution and carry out phase I computation and obtain a basis which contains all the variables of X. Then the inverse of A is directly obtained form the simplex tableau and consists of those column vector which are present in the initial basis.

QUESTION 23.
Explain Case of Infeasibility.

ANSWER.

Case of Infeasibility

This is the situation in which we get optimum solution even before all variables in X find a place in the basis viz. under column "B" of the tableau. This solution is called optimum basic infeasible solution (OBIS). When optimum is reached in which the basis contain all variables of X, viz. x1, x2,... we call it an optimum basic feasible solution (OBFS).
The following problem gives an OBIS.
Maximise $Z = 4x_1 + 3x_2$
Subject to constraints $3x_1 + 4x_2 \leq 6$
$5x_1 + 6x_2 \geq 15$
$x_1 + x_2 \geq 0$
Students may take this as an exercise and satisfy themselves. In such situations, one or two of the basic variables are artificial variables. In other words, in Big-M method 2, all artificial variables do not disappear form final simplex tableau. Here, optimum value of Z can be expressed only in terms of M.

QUESTION 24.
Explain Case of Unbounded Solutions.

ANSWER.

Case of Unbounded Solutions

When Lpp doed not give finite value of variables X viz. x1, x2... such a solution is called unbounded. Here, variable X can take very high values without violating conditions of constraints. In such cases, the final tableau shows all ratios viz. R values are negative so that no- minimum ratio condition can be applied.

Students may try the following problem and establish the unbound condition.

Maximise $Z = 2x_1 + x_2$

Subject to constraints $x_1 - x_2 \leq 10$

$2x_1 - x_2 \leq 40$

$x_1, x_2 \geq 0$

QUESTION 25.
Explain Case of More Than One Optimum Solution

ANSWER.

Case of More Than One Optimum Solution

This is the case where LPP given solutions which are optimum but not unique. This means more than one optimum solution is possible. Following are the characteristics of the final tableau in such cases.

1. Solution is optimum.
2. (C-Z) values of all basic variables are zero.
3. (C-Z) values of non-basic variables may or may not be zero.
4. (C-Z) values of at least one non-basic variable will be zero.
5. More than one optimum solution is possible.
6. Value of objective function Z remains the same for all possible solutions.
7. Basis in final tableau contains in addition to basic variable X viz. x1, x2...non-basic variables S viz.s1, s2...

QUESTION 26.
Explain Case of Degeneracy

ANSWER.

Case of Degeneracy

A basic feasible solution of LPP is said to degenerate if at least one of the basic variable is zero.

Types of degeneracy

Type 1 First iteration Degeneracy can occur right in the first (initial tableau). This normally happens when the number of constrains is less than number of variables in the objective function. Problem can be overcome by Trial and Error method.

Type 2 Subsequent iteration Regenercy can also occur in subsequent iteration. This is due to the fact that minimum ratio values are the same for two rows. This will make the choice difficult in selecting the "replacing row" The basis logic of simplex is to improve the solution at each iteration. Rules are accordingly formed. When the subsequent iteration shows deterioration or limitation of existing rule to further, we say 'regeneracy' has occerd. At each iteration, we select a new basis set to replace the existing set. At each phase, there is a replacement, viz. replacement of existing vector which is called 'outgoing' with an 'incoming' one. Incoming is determined by (C-Z) value viz.max. positive value in maximizing problem. There is little problem in deciding incoming vector. The real problem arises in selecting the outgoing vector,viz. the replacing row.

This decision on replacing row becomes very difficult under the following cases :

(a) Initial tableau has zero values for one or more basic vectors.

(b) Value of minimum ratio is the same for two or more of the basis vectors.

In case (a), it is difficult to obtain optimum solution because; by replacement of vector, Z value does not change. In case of (b), one method is to remove the "tied vectors" from the basis. This will resuli in reduction of rows in subsequent iterations.

QUESTION 27.
Explain Case of Cycling.

ANSWER.
Case of Cycling
We have seen that when degeneracy exists the replacement of vector in the basis does not improve the objective function. Another difficulty encountered in degeneracy is that the simplex tableau gets repeated without getting optimum solution. Such occurrences in LPP is called cycling. Fortunately such occurrences in LPP's are very rare. Also in such cases, certain techniques are developed to prevent cycling and reach optimum solutions.

QUESTION 28.
Explain Case of Duality.

ANSWER.
Case of Duality
Every LPP is associated with another LPP which is called the dual of original LPP The original LPP is called the primal. If the dual is stated as a given problem, the primal becomes its dual and vice versa. The solution of one contains the other. In other words, when the optimum solution of primal is known, the solution of dual is also obtained from the very same optimum tableau.
Characteristics
(a) If primal is a maximization problem, the dual will be a minimization problem and vice versa.
(b) Constants of the RHS of the constraints equation on the primal becomes coefficients of objective function in the dual and vice versa.

$C^* = b^r$ and $b^* = C^T$

(c) Coefficients of the constraints of dual are the transpose of coefficients of constraints of the primal and vice versa.

$A^* = A^r$ and $A = (A^*)^r$

(d) In maximization problem, constraints and in less than or equal to form whereas in minimization problem, these are in more than or equal to form.

QUESTION 29.
Define primal and dual.

ANSWER.

Primal LPP "An LPP to evaluate the value of X ⊠R^n so as to
Maximise Z = CX, where C $\in R^n$
Subject to the constraints AX ≤ b where $b^r \in R^m$ and X ≥o
When A is a (m x n) real matrix, A is called a primal LPP".

Dual LPP "An LPP to evaluate the value of $Y^r \in R^n$ so as to
Minimize $Z^* = b^TY$ where $b^r \in R^m$
Subject to the constraints $A^TY \geq C^r$ where $C^r \in R^n$ and Y ≥ o
When A is a (mxn) real matrix, A is called a primal LPP."

User There are many user of this concept. Some of these are not within the scope of this text. However, one of the most important use is to solve difficult LPP. Many times, a given LPP is complicated having a large number of constraints. Dual is now subjected to solution by simplex method. The optimum tableau of dual will have the solution of primal, form the following rule.
Rule "Solution of primal is given by the negative values of (C-Z) row under slack surplus variables of the optimum solution tableau of the dual."
A similar rule applies to the solution of dual in case the solution of primal is known from the rule.
Rule "Solution of the dual is given by the negative values of (C-Z) row under slack/surplus variables of the optimum solution tableau of the primal.

Fundamental Properties of Dual

Case 1 : Dual problem when primal is given in standard form Here the constraints are given in the standard from or in the canonical from viz. constraints are either in equality form or in less than or equal to form. This has two more conditions:
(a) Primal variables are restricted.
(b) Primal variables are unrestricted.
(a) Primal variables are restricted X ≥ o primal is given as follows
Determine $X^r \in R^n$ so as to
Maximise f(x) = CX; C $\in R^n$
Subject to AX = b; X ≥o; $b^t \in R^n$
Where A is a (m×n) matrix
Now AX = b can be written in a set of simultaneous equations.
AX ≤b and AX ≥b
If Y(y1,y2) are the dual variables associated with equation (4.1)the dual is given as follows:
"Determine $Y^T \in R^m$ so as to
Minimize g(Y) =b^TY; $YX^T \in R^n$
Subject to $A^TY \geq C^T$; Y is unrestricted and Y = y_1-y_2"
(b) The primal variables are unrestricted. Here primal is given as follows:
"Determine $X^T \in R^n$ so as to
Maximise f(X) = CX;C $\in R^n$
Subject to AX = b; X is unrestricted"
Here we set X = x_1-x_2
where $x_1 \geq$ o,x_1^r, $x_2^r \in R^n$
The corresponding dual is written as follows:
"Determine $Y^r \in R^m$ so as to
Minimise g(Y) =b^TY; $Y^T \in R^m$
Subject to $A^TY = C^T$; Y is unrestricted and Y = y^1-y^2"

Note: From (a) and (b) we find that if primal is standard form then dual variables will be always in unrestricted form.

Case 2 : Dual when primal is given in mixed form This is the case where the constraints are a mixture of less than equal to or equality form. Here primal is given as

"Determine $X^T \in R^n$ so as to

Maximise $f(X) = CX$; $C \in R^n$

Subject to $A_1 X \leq b_1$

$A_2 X = b_2$; $X \geq 0$

Where A_1 and A_2 are (p×n) and [(m-p) × n] real matrices and b_1 and b_2 are (p×1) and [(m-p) ×1] real vectors respectively."

The constants $A_2 X = b_2$ can be written as two inequalities viz. $A_2 X \leq b_2$, $A_2 X \geq b_2$ so that constraints of primal equation are written in canonical form viz.

$A_1 X \geq b_1$

$A_2 X \leq b_2$

$- A_2 X \geq - b_2$

The corresponding dual is given as follows:

"Determine Y [$y_1 \in R^p$, $y_2, y_3, y_4 \in R^{m-p}$ so] as to

Minimise $g(y_1, y_4) = b_1^T y_1 + b_2^T y_4$

Subject to $A_1^T y_1 + A_2^T y_4 \geq C^T$; $y_1 \geq 0$; y_4 is unrestricted Where $y_4 = y_2 - y_3$ "

Note: For the above we may also notice that if the ith primal constraint is an equation with an equality sign, that its dual variable will be unrestricted.

All these can be easily visualized by working out problems using numerical quantities. Hence, these cases are now illustrated in problems given below.

Illustration 1.

A Chemical Company produces two compounds A and B. The following table gives the units of ingredients C and D per kg of compounds A and B as well as minimum requirements of C and D and costs/kg of A and B. Write drawn the problem mathematically for minimisation of cost.

		Table Compound		Minimum requirement
		A	B	
Ingredient	C	1	2	80
	D	3	1	75
Cost per kg.		4	6	

Solution:

Let x_1 be the no. of units of A

Let x_2 be the no. of units of B

Objective function: Min.$Z = 4x_1 + 6x_2$

Subject to Constraints:

$x_1+2x_2 \geq 8$ (Constraint on requirement of Ingradient C)

$3x_1+x_2 \geq 75$ (Constraint on requirement of ingradient D)

And $x_1, x_2 \geq 0$ (No negativity constraint)

Illustration 2.

A pension fund manager is considering investing in two shares A and B. It is estimated that:

(i) Share A will earn a dividend of 12% per annum and share B 4% per annum.

(ii) Growth in the market value in one year of share A will be 10 paise per ` 1 invested and in B 40 paise per ` 1 invested.

He requires investing the minimum total sum which will give:

Dividend income of at least ` 600 per annum and growth in one year of at least ` 1,000 on the initial investment.

You are required to:

State the mathematical formulation of the problem which will facilitate computation of the minimum sum to be invested to meet the manager's objective.

Solution:

Shares	Dividend	Growth in `
A	12%	10/100 = 0.1
B	4%	40/100 = 0.4
Min-income	600	1000

Let x_1 be the amount invested on share ALet x_2 be the amount invested on share BObjective function: Min. $Z = x_1 + x_2$

Subject to constraints:

$0.12\,x_1 + 0.04\,x_2 \geq 600$ (Dividend income constraint)

$0.1\,x_1 + 0.4\,x_2 \geq 1000$ (Growth constraint)

And $x_1, x_2 \geq 0$. (Non negativity constraint)

Illustration 3.

A company possesses two manufacturing plants each of which can produce three products x, Y and Z from a common raw material. However, the proportions in which the products are produced are different in each plant and so are the plant's operating costs per hour. Data on production per hour costs are given below, together with current orders in hand for each product.

	Product			Operating cost/hour in `
	X	Y	Z	
Plant A	2	4	3	9
Plant B	4	3	2	10
Orders on hand	50	24	60	

You are required to formulate the problem to find the number of production hours needed to fulfill the orders on hand at minimum cost.

Solution:

Let α be no. of hours of plant A in use

Let β be no. of hours of plant B in use

Objective function: Min $Z = 9\alpha + 10\beta$

Subject to constraints:

$2\alpha + 4\beta \geq 50$ (Constraint relating to Product X)

$4\alpha + 3\beta \geq 24$ (Constraint relating to Product Y)

$3\alpha + 2\beta \geq 60$ (Constraint relating to Product Z) And

$\alpha, \beta \geq 0$ (Non negativity constraint)

Illustration 4.

The products P, Q and R are being produced in a plant having profit margin as ` 3, ` 5 and ` 4 respectively. The raw materials A, B and C are of scarce supply and the availability is limited to 8, 15 and 10 units respectively. Specific consumption is indicated in the table below:

	P	Q	R	Available units
A	2	3	-	8
B	3	2	4	15
C	-	2	5	10
	3/-	5/-	4/-	

Write down the problem mathematically for maximization of profit margin.

Solution:

Let x_1 be the no. of units of product P

Let x_2 be the no. of units of product Q

Let x_3 be the no. of units of product R

Objective function: Max. $Z = 3x_1+5x_2 + 4x_3$

Subject to constraints:

$2x_1+3x_2 \leq 8$ (Constraint on availability of Raw Material 'A')

$3x_1+2x_2 + 4x_3 \leq 15$ (Constraint on availability of Raw Material 'B')

$2x_2+5x_3 \leq 10$ (Constraint on availability of Raw Material 'C')

And $x_1, x_2, x_3 \geq 0$ (Non negativity constraint)

Illustration 5.

A Bank is in the process of formulating its loan policy. Involving a maximum of ` 600 Million. Table below gives the relevant types of loans. Bad debts are not recoverable and produce no interest receive. To meet competition from other Banks the following policy guidelines have been set. At least 40% of the funds must be allocated to the agricultural and commercial loans. Funds allocated to housing must be at least 50% of all loans given to personal, car, Housing. The overall bad debts on all loans may not exceed 0.06.

Formulate a linear program Model to determine optimal loan allocations.

Type of loan	Interest rate %	Bad debts (Probability)
Personal	17	0.10
Car	14	0.07
Housing	11	0.05
Agricultural	10	0.08
Commercial	13	0.06

Solution:

Let x_1 be the amount allocated for personal loan

Let x_2 be the amount allocated for car loan

Let x_3 be the amount allocated for Housing loan

Let x_4 be the amount allocated for agricultural loan

Let x_5 be the amount allocated for Commercial loan

Objective Function: Max Z

$= 0.17x_1 + 0.14x_2 + 0.11x_3 + 0.1x_4 + 0.13x_5 - (0.10x_1 + 0.07x_2 + 0.05x_3 + 0.08x_4 + 0.06x_5)$

$= (0.17 - 0.10)x_1 + (0.14 - 0.07)x_2 + (0.11 - 0.05)x_3 + (0.10 - 0.08)x_4 + (0.13 - 0.06)x_5$

$= 0.17x_1 + 0.07x_2 + 0.06x_3 + 0.02x_4 + 0.07x_5$

Subject to constraints

(i) $x_1 + x_2 + x_3 + x_4 + x_5 \le 600$ Millions (Constraint on total loan amount)

(ii) $x_4 + x_5 \ge 0.4 (x_1 + x_2 + x_3 + x_4 + x_5)$ (Constraint due to policy set for Agricultural and Commercial Loan)

(iii) $x_3 \ge 0.5 (x_1 + x_2 + x_3)$ (Constraint due to policy set for Housing Loan)

(iv) $0.1x_1 + 0.07x_2 + 0.05x_3 + 0.08x_4 + 0.06 \times 5 \le 0.06$ Million (Constraint on limit of overall bad debt)

(v) $x_1, x_2, x_3, x_4, x_5 \ge 0$ (Non negativity constraint)

Illustration 6.

The annual hand-made furniture show and sales occurs next month and the school of vocational studies is planning to make furnitures for sale. There are three wood working classes – I year, II year, III year at the school and they have decided to make three styles of chairs A, B and C. Each chair must receive work in each class and the time in hours for each chair in each class is given.

Chair	I year	II year	III year
A	2	4	3
B	3	3	2
C	2	1	4

In the next month there will be 120 hours available in first year class, 160 hours in the second year class and 100 hours in the third year class to produce chairs. The teacher of the wood working class feels that a maximum of 40 chairs can be sold at the show. The teacher has determined that the profit from each type of chair will be A – `40, B – `35 and C – `30.

Formulate a linear programming model to determine how many chairs should be produced to maximize profit.

Solution:

Let x_1 be the chairs produced of A type

x_2 be the chairs produced of B type

x_3 be the chairs produced of C type

Objective function

Maximise $Z = 40x_1 + 35x_2 + 30x_3$

Subject to constraints:

$2x_1 + 3x_2 + 2x_3 \le 120$ (Constraint on available time of 1st year class)

$4x_1 + 3x_2 + x_3 \le 160$ (Constraint on available time of 2nd year class)

$3x_1 + 2x_2 + 4x_3 \le 100$ (Constraint on available time of 3rd year class)

$x_1, x_2, x_3 \ge 0$ (Non negativity constraint)

Illustration 7.

A company produces three products P, Q and R from three raw materials A, B and C. One unit of product P requires 2 units of A and 3 units of B. One unit of product Q requires 2 units of B and 5 units of C and one unit of product R requires 3 units of A, 2 units of B and 4 units of C. The company has 8 units of material A, 10 units of material B and 15 units of material C available to it. Profits per unit of products P, Q and R are Rs. 3, Rs. 5 and Rs. 4 respectively.

Formulate the question mathematically to maximize the profit.

Solution:

DATA SUMMARY CHART

Decision variables	Products	Type of raw material			Profit per unit (`)
		A	B	C	
x_1 x_2 x_3	P Q R	2 - 3	3 2 2	- 5 4	3 5 4
Units of material available:		8 Maximum	10 maximum	15 maximum	

x_1 = number of units of Product P

x_2 = number of units of Product Q

x_3 = number of units of Product R

The given Q is formulated as the LP model as follows: Maximize $Z = 3x_1 + 5x_2 + 4x_3$

Subject to the constraints:

$2x_1 + 3x_3 \leq 8$ (Constraint due to availability of Material A)

$3x_1 + 2x_2 + 2x_3 \leq 10$ (Constraint due to availability of Material B)

$5x_2 + 4x_3 \leq 15$ (Constraint due to availability of Material C)

$x_1, x_2, x_3, \geq 0$ (Non negativity constraint)

Illustration 8.

A city hospital has the following minimal daily requirement for nurses:

Period	Clock time (24 hours day)	Minimal Number of NursesRequired
1	6 a.m. - 10 a.m.	2
2	10 a.m. - 2 p.m.	7
3	2 p.m. - 6 p.m.	15
4	6 p.m. - 10 p.m.	8
5	10 p.m. - 2 a.m.	20
6	2 a.m. - 6 a.m.	6

Nurses report to the hospital at the beginning of each period and work for 8 consecutive hours. The hospital wants to determine the minimal number of nurses to be employed so that there will be sufficient number of nurses available for each period.

Formulate this as a Linear Programming question by setting up appropriate constraints and objective function.

Solution:

$x_2 + x_3 \geq 15$, $x_3 + x_4 \geq 8$, $x_4 + x_5 \geq 20$, $x_5 + x_6 \geq 6$, and $x_6 + x_1 \geq 2$.

Since, the objective is to minimize the total number of nurses employed in the hospital,

$Z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6$.

Obviously, we must have $x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$.

Illustration 9.

A marketing manager wishes to allocate his annual advertising budget of ` 20,000 in two media vehicles A and B. The unit cost of a message in media A is ` 1,000 and that of B is ` 1,500. Media A is a monthly magazine and not more than one insertion is desired in one issue. At least 5 messages should appear in media B. The expected effective audience for unit messages in the media A is 40,000 and for media B is 55,000.

(i) Develop a mathematical model

Solution:

Step 1. The appropriate mathematical formulation of the given Q. . is as follows:

Maximize (total effective audience) $Z = 40{,}000\,x_1 + 55{,}000\,x_2$

Subject to the constraints

$1{,}000x_1 + 1{,}500x_2 \leq 20{,}000$ (Budget constraint)

$x_1 \leq 12$ (Constraint on annual no. of insertions in Media A)

$x_2 \geq 5$ or $-x_2 \leq -5$ (Constraint on annual no. of insertions in Media B)

$x_1, x_2 \geq 0$ (Non negativity constraint)

where

x_1 = annual number of insertions/messages for media A.

x_2 = annual number of insertions/ messages for media B.

Illustration 10.

One unit of product A contributes ` 7 and requires 3 units of raw material and 2 hours of labour. One unit of product B contributes ` 5 and requires one unit of raw material and one hour of labour. Availability of raw material at present is 48 units and hence there are 40 hours of labour.

i. Formulate it as a linear programming problem.
ii. Write its dual.

Solution:

i. The mathematical formulation of the linear programming problem is

Maximise $Z = 7x_1 + 5x_2$

Subject to

$3x_1 + x_2 \leq 48$

$2x_1 + x_2 \leq 40$

$x_1, x_2 \geq 0$

Where x_1 and x_2 denote the number of units of product A and B respectively.

ii. The dual of the above problem is: Minimize

$Z^* = 48y_1 + 40y_2$

$3y_1 + 2y_2 \geq 7y_1 +$

$y_2 \geq 5$ $y_1, y_2 \geq 0$

Where y_1 and y_2 are the dual variables indicating the shadow prices of raw material and labour respectively.

Illustration 11.

A Company produces the products P, Q and R from three raw materials A, B and C. One unit of product P requires 2 units of A and 3 units of B. A unit of product Q requires 2 units of B and 5 units of C and one unit of product R requires 3 units of A, 2 unit of B and 4 units of C. The Company has 8 units of material A, 10 units of B and 15 units of C available to it. Profits/unit of products P, Q and R are Rs.3, Rs.5 and Rs.4 respectively.

(a) Formulate the problem mathematically,

(b) Write the Dual problem.

Solution:

Raw Materials	x_1	x_2	x_3	Available units
	P	Q	R	
A	2	-	3	8
B	3	2	2	10
C	-	5	4	15
	3	5	4	

Profits 3/- 5/- 4/-

Let x_1 be the no. of units of P

Let x_2 be the no. of units of Q

Let x_3 be the no. of units of R

Objective function: Max. $Z = 3x_1 + 5x_2 + 4x_3$

Subject to constraints:

$2x_1 + 3x_2 \leq 8$ (Constraint on availability of Raw Material 'A')

$3x_1 + 2x_2 + 2x_3 \leq 10$ (Constraint on availability of Raw Material 'B')

$5x_2 + 4x_3 \leq 15$ (Constraint on availability of Raw Material 'C')

And $x_1, x_2, x_3 \geq 0$. (Non negativity constraint)

Primal

Max. $Z = 3x_1 + 5x_2 + 4x_3$

Subject to

$2x_1 + 3x_2 \leq 8$

$3x_1 + 2x_2 + 2x_3 \leq 10$

$5x_2 + 4x_3 \leq 15$

And $x_1, x_2, x_3 \geq 0$

Dual

Min. $Z = 8y_1 + 10y_2 + 15y_3$

Subject to

$2y_1 + 3y_2 \geq 3$

$3y_1 + 2y_2 + 5y_3 \geq 5$

$2y_2 + 4y_3 \geq 4$

And $y_1, y_2, y_3 \geq 0$

$2x_1 + 3x_2 + S_1 = 8$

$3x_1 + 2x_2 + 2x_3 + S_2 = 10$

$5x_2 + 4x_3 + S_3 = 15$

Max $Z = 3x_1 + 5x_2 + 4x_3 + 0.S_1 + 0.S_2 + 0.S_3$

$\therefore x_1 = 23/20 \quad x_2 = 19/10 \quad x_3 = 11/8$

$Z = 18.45$

Illustration 12.

Four Products A,B,C and D have ` 5, ` 7, ` 3 and ` 9 profitability respectively. First type of material (limited supply of 800 kgs.) is required by A,B,C and D at 4 kgs., 3 kgs, 8 kgs, and 2 kgs. respectively per unit.

Second type of material has a limited supply of 300 kgs. and is for A,B,C and D at 1 kg, 2 kgs, 0 kgs, and 1 kg per unit. Supply of the other type of materials consumed is not limited. Machine hrs. available are 500 hours and the requirements are 8,5,0 and 4 hours for A,B,C and D each per unit. Labour hours are limited to 900 hours and requirements are 3,2,1 and 5 hours for A,B,C and D respectively.

How should the firm approach so as to maximize its profitability? Formulate this as a linear programming problem. You are not required to solve the LPP.

Solution:

Let x_1 be the no. of units of product A

Let x_2 be the no. of units of product B

Let x_3 be the no. of units of product C

Let x_4 be the no. of units of product D

Objective function Maximize $Z = 5x_1+7x_2+3x_3+9x_4$

	A	B	C	D	Supply in Kgs.
I type material	4	3	8	2	800
II type material	1	2	0	1	300
Machine	8	5	0	4	500
Labour	3	2	1	5	900
Profit	5	7	3	9	

Subject to constraints

$4x_1+3x_2+8x_3+2x_4 \le 800$ (Constraint on availability of Material type I)

$x_1+2x_2+0.x_3+x_4 \le 300$ (Constraint on availability of Material type II)

$8x_1+5x_2+0.x_3+4x_4 \le 500$ (Constraint on Machine Hours available)

$3x_1+2x_2+x_3+5x_4 \le 900$ (Constraint on Labour Hours available)

and $x_1, x_2, x_3, x_4 \ge 0$. (Non negativity constraint)

Illustration 13.

Mutual Fund has cash resources of ` 200 million for investment in a diversified portfolio. Table below shows the opportunities available, their estimated annual yields, risk factor and term period details.

Formulate a Linear Program Model to find the optimal portfolio that will maximize return, considering the following policy guidelines:

- **All the funds available may be invested**
- **Weighted average period of at least five years as planning horizon.**
- **Weighted average risk factor not to exceed 0.20.**
- **Investment in real estate and speculative stocks to be not more than 25% of the monies invested in total.**

Investment type	Annual yield (percentage)	Risk factor	Term period (years)
Bank deposit	9.5	0.02	6
Treasury notes	8.5	0.01	4
Corporate deposit	12.0	0.08	3
Blue-chip stock	15.0	0.25	5
Speculative stocks	32.5	0.45	3
Real estate	35.0	0.40	10

Solution:

Let x_1, x_2, x_3, x_4, x_5 and x_6 represent the six different investment alternatives, i.e., x_1 is bank deposit, x_2 is treasurynote. X_3 corporate deposit, X_4 blue chip stock, X_5 speculative stock and x6 real estate. The objective is to maximizethe annual yield of the investors (in number of units) given by the linear expression.

Maximize $Z = 9.5x_1 + 8.5x_2 + 12.0x_3 + 15.0x_4 + 32.5x_5 + 35.0x_6$

Subject to the Constraints:

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \leq 1$ **(Investment decision)**

$0.02x_1 + 0.01x_2 + 0.08x_3 + 0.25x_4 + 0.45x_5 + 0.40x_6 \leq 0.20$ **(Constraint on weighted average risk of the portfolio)**

$6x_1 + 4x_2 + 3x_3 + 5x_4 + 3x_5 + 10x_6 \geq 5$ **(Constraint on weighted average length of period of investment)**

$x_5 + x_6 \leq 0.25$ **(Constraint on investment in real estate and speculated stock)**

$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$ **(non-negativity condition)**

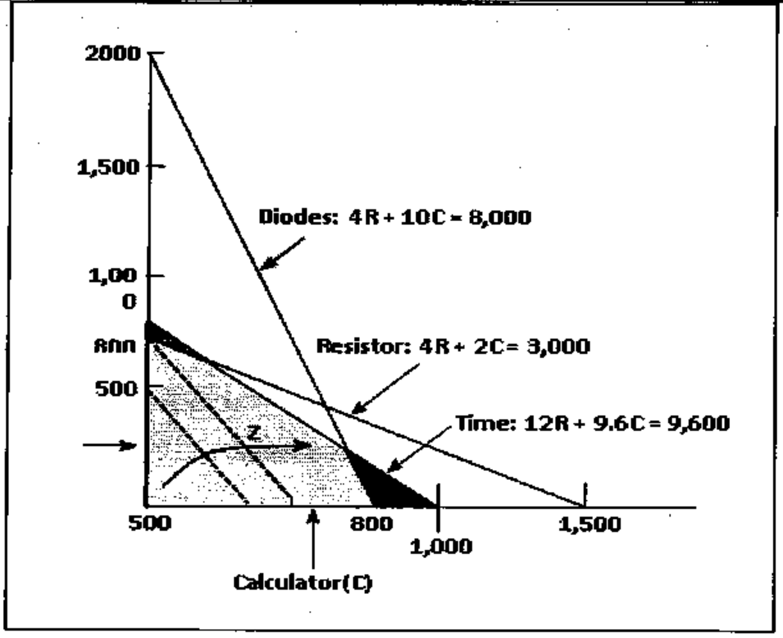
Illustration 14.
What is the slope of the objective function Max Z = 15*X* + 45 *Y*?

Solution:
The slope form is $Y = mX + b$ where m = slope Rearranging,
$45Y = -15X + Z$
$Y = -\frac{15X}{45} + \frac{Z}{45}$
Slope is –15/45 or – 1/3

Illustration 15.
An electronic goods manufacture has distributors who will accept shipments of either transistor radios or electronic calculators to stock for Christmas inventory Whereas the radios contribute ` 10 per unit and the calculator ` 15 per unit to profits, both products use some of the same components. Each radio requires each of diodes and resistors, while each calculator requires 10 diodes and 2 resistors. The radio takes 12.0 minutes and the calculators take 9.6 minutes of time on the company's electronic testing machine, and the production manager estimates that 160 hours of test time is a available. The firm has 8,000 diodes and 3,000 resistors in inventory. What product of mix of products should be selected to obtain the highest profit?

Solution:
The decision variables are radios, R, and calculators, C, and we must determine how many of each should be produced to maximize profit, Z.
(1) Objective function
Max Z= 10R + 15C
Constraints
Diodes (8,000 available): Radios require 4 each, and calculators require 10 each.
$\therefore 4R + 10C \le 8{,}000$
Resistors (3,000 available): Radios require 4 each, and calculators require 2 each.
$\therefore 4R + 2C \le 3{,}000$
Testing (9,600 minutes available): Radios require 12.0 minutes, and calculators require 9.6 minutes.
$\therefore 12R + 9.6C \le 9{,}600$

(2) Graph 'bf variables and constraints
Plotting each of the constraints inequality as an equality, we have: For Diodes: 4R + 10C = 8000
If R = 0, then C = 800
If C = 0, then R = 2,000
For Resistors: 4R + 2C = 3,000
If R = 0, then C = 1500 If C = 0, then R = 750
For Testing: 12R + 9.6C = 9,600
If R = 0, then C = 1,000
If C = 0, then R = 800
Note: The resulting graph establishes a feasible region bounded by the time, diode, and resistor constraints that R ≥ 0 and C ≥ 0.



(3) Slope of the objective function. We can express our objective function in slope intercept from, where the Y axis corresponds to R and the X axis to C.

$Z = 10R + 15C$

Or, $10R = -15C + Z$

$\therefore R = -\frac{15}{10}C + \frac{Z}{10} = -\frac{3}{2}C + \frac{Z}{10}$

∴ Slope = – 3/2, which means that for every 3-unit decrease in Y there is a 2 increase in *X*. This slope is plotted as a dotted line in the graph by marking off 3 units (negative) in *R* for each 2 units (positive) in *C*.

(4) Move objective function to optimize. The slope of the objective function hi to objective line) is moved away from the origin until constrained. In this case the binding constraints are the diode inventory supply and testing machine time availability.

(5) Read solution values. The arrows point to the approximate R and C coordinates o the constraining intersection.

Number of radios = 240

Number of calculators = 700

Note: That the simultaneous solution of the two binding constraint equations would lend more accuracy to the answer:

$$(4R + 10C = 8{,}000) \times (-3) = -12R - 30C = -24{,}000$$
$$12R + 9.6C = 9{,}600$$
$$-20.4C = -14{,}400$$
$$C = 705 \text{ calculators}$$

Substituting to solve for R:

$4R + 10(705) = 8{,}000$

$\therefore R = \frac{8{,}000 - 7{,}050}{4} = 237$ radios

Comment: We had two decision variables (that is, products) to choose from and established a profit function, Z, and constraints and optimized the function by moving it away from the origin. The

graph of this example showed that the resistor supply was not constraining, so only two constraints (diodes and test time) were binding. Similarly, there were two decision variables in the solution, that is, we ended up producing both radios and calculators. The number of variables in solution will always equal the number of explicit constraints that are binding.
The graphic linear programming solution gives an indication of the sensitivity of the solution to changes in the constraints. If for example, additional diodes could be purchased from an outside supplier with no increase in cost, profit would be maximized by extending the iso – objective line to the next corner and producing 1,000 calculators and no radios. In this case we would have one explicit constraint (time) binding and only one decision variables (calculators) in the final solution.

Illustration 16.

The simplex calculator company makes a profit of ` 5 on each model X and ` 20 on each model Y. Each calculator requires the following time (in minutes) on the cleaning and testing machines.

	X Requirements	Y Requirements	Time Available
Cleaning	2	4	10
Testing	6	3	12

(a) State the objective function and constraints.

(b) Arrange the equations in a simplex format.

Solution:

(a) Objective function $\text{Max } Z = 5X + 20Y$

Constraints:

Cleaning $2X + 4Y \leq 10$

Testing $6X + 3Y \leq 12$

(b)

C →		5	20	0	0	
↓ Variables in solution		Decision variables				Solution Values (RHS)
		X	Y	S_1	S_2	
0	S_1	2	4	1	0	10
0	S_2	6	3	0	1	12
Z		0	0	0	0	0
C-Z		5	20	0	0	

Illustration 17.

The initial matrix of a maximization linear programming problem is as shown where the decision variables are designated A, B, etc.

C →	4	8	6	0	0	0	RHS
↓ Variables in solution							
	5	9	0	1	0	0	36
	0	8	5	0	1	0	24
	2	0	5	0	0	1	7
	0	0	0	0	0	0	0
	4	8	6	0	0	0	

(a) State the original constraint equations.

(b) How many decision variables are there?

(c) State the objective function.

Solution:

(a) $5A + 9B \leq 36$, $8B + 5C \leq 24$, and $2A + 5C \leq 7$

(b) Three

(c) Max $Z = 4A + 8B + 6C$

Comprehensive Example

Solve the following problem by Simplex Method:

Example 1

	Per Unit Resource Matrix			Profit per unit in (`)
PRODUCT/MACHINE	**M1**	**M2**	**M3**	
P1	4	3	2	20
P2	4	4	1	12
P3	4	3	1	08
Maximum capacity of the matrix	1200	900	400	

Solution:

Step 1: formulate the problem of a Generalized LPP

Let: X_1 Units be produced for product P_1.

X_2 Units be produced for product P_2.

X_3 Units be produced for product P_3.

∴ Max $Z = 20x_1 + 12x_2 + 08x_3$.

Subject to the constraints

$4x_1 + 4x_2 + 4x_3 \leq 1200.$

$3x_1 + 4x_2 + 3x_3 \leq 900.$

$2x_1 + x_2 + x_3 \leq 400.$

$x_1 \leq = 0, x_2 \leq = 0, x_3 \leq = 0$

Step 2: Convert generalized LP. To Standardized LP by introducing Slack Variables.

Matrix $z = 20x_1 + 12x_2 + 8x_3 + 0.x_4 + 0.x_5 + 0.x_6$

Subject to:

$4x_1 + 4x_2 + 4x_3 + 1.x_4 + 0.x_5 + 0.x_6 = 1200$ .... (i)

$3x_1 + 4x_2 + 3x_3 + 0.x_4 + 1.x_5 + 0.x_6 = 900$ .... (ii)

$2x_1 + x_2 + x_3 + 0.x_4 + 0.x_5 + 1.x_6 = 400$ .... (iii)

For all $x_i \geq 0$ $i = 1, 2, 3, 4, 5, 6.$

Where x_4, x_5 & x_6 are Slack variables.

Step 3: To solve the problem given in Step 2. We take the help of Simplex Method in Tabular format.

For simplicity of the problem we take the help of some useful notations.

1. C_j = Coefficient Max Z
2. B = Basis which forms the unit or identity matrix.
3. C_B = Coefficients of basis in C_j

   $C_B \times x_i$ =

   C = Coefficient of the capacity matrix

   MR = Mini Ratio

   $= \frac{\text{Element of C}}{\text{Element of Key coloumn}}$

Where Key column = Most negative of $(Z_j - C_j)$ Column

Simplex is an Iterative Algorithm. for getting optimal solution there is one stopping rule i.e., all $Z_j - C_j \leq = 0$

To generate new tables, we apply two rules i.e., Rule 1 & Rule 2.

Rule (I) (Key Row):

Dividing the key row by the key elements [key row is that row which contains Keu element & Key element isthe intercepting point of minimum MR and most negative of $Z_j - C_j$ column.

Rule 2 (Non key rows):

Old row no. – [corresponding no. in the key row × FR of that old row]

Where FR = Fixed Ratio = $\frac{\text{Element of Key coloumn}}{\text{Key element}}$

No. of columns = 3 + no of variables + 2 (The rule for counting the no. of columns in Tabular Format)

Table 1

Cj			20	12	8	0	0	0		
B	C_B	C	x_1	x_2	x_3	x_4	x_5	x_6	MR	FR
x_1	0	1200	4	4	4	1	0	0	300	2
x_2	0	900	3	4	3	0	1	0	300	3/2
x_3	0	400		1	1	0	0	1	200	1
	Z_j	0	0	0	0	0	0	0		
	$Z_j - C_j$		-20	-12	-08	0	0	0		

(2) Is the key element which occurs in 3rd row & 1st column. Therefore, x_1 will enter as a basis & x_6 excluded from the basis. For the calculation of new table. We calculated FR.

In the table 1 row 3 is the key row and row 1 & row 2 are non-key rows. Applying Rule 1 & Rule 2 , we get table 2

Table 2

Cj			20	12	8	0	0	0		
B	C_B	C	x_1	x_2	x_3	x_4	x_5	x_6	XIR	FR
x_1	0	400	0	2	2	1	0	-2	200	4/5
x_2	0	300	0	(5/2)	3/2	0	1	-3/2	120	
x_3	20	200	1	½	½	0	0	1/2	400	1/5
	Z_j	4000	20	10	10	0	0	10		
	$Z_j - C_j$		0	-2	+2	0	0	10		

(5/2) is the key elements which occurs in 2nd row & 2nd column. Hence x_2 will enter as a basis and X_5 Will be excluded from basis

In Table 2 Row 2 is the key Row & Row 1 & Row 3 are the non key rows

Cj			20	12	8	0	0	0
B	C_B	C	x_1	x_2	x_3	x_4	x_5	x_6
x_1	0	160	0	0	4/5	1	-4/5	-4/5
x_2	12	120	0	1	3/5	0	2/5	-3/5
x_3	20	140	1	0	1/5	0	-1/5	4/5
	Z_j	4240	20	12	56/5	0	4/5	44/5
	$Z_j - C_j$		0	0	16/5	0	4/5	44/5

Since all $Z_J - C_J \leq = 0$, our Solution is optimal

Therefore, product mix $x_1 = 140$ units

$x_2 = 120$ units

$x_3 = 0$ units (no production)

Therefore

Max Z = $20 \times 1 + 12 \times 2 + 8 \times 3$

$= 20 \times 140 + 12 \times 120 + 8 \times 0$

$= 4240$

4.5 Transportation

The basic transportation problem was originally developed by F.L. Hitchcock (1941) in his study entitled "the distribution of a product from several sources to numerous locations". In 1947, T.C. Koopmans independently published a study on "optimum utilization of the transportation system".

Transportation models deals with the transportation of a product manufactured at different plants or factories (supply origins) to a number of different warehouses (demand destinations). The objective is to satisfy the destination requirements within the plants capacity constraints at the minimum transportation cost. Transportation models thus typically arise in situations involving physical movement of goods from plants to warehouses, warehouses to wholesalers, wholesalers to retailers and retailers to customers. Solution of the transportation models requires the determination of how many units should be transported from each supply origin to each demands destination in order to satisfy all the destination demands while minimizing the total associated cost of transportation.

Transportation Table (Matrix)

It is convenient to represent the various data of a transportation problem in a tabular or matrix form, called the transportation table matrix(9) as follows;

O \ D	b_1	b_2	...	b_n
a_1	x_{11} c_{11}	x_{12} c_{12}	...	x_{1n} c_{1n}
a_2	x_{21} c_{21}	x_{22} c_{22}	...	x_{2n} c_{2n}
...	...	...	...	...
a_m	x_{m1} c_{m1}	x_{m2} c_{m2}	...	x_{mn} c_{mn}

The table contain mn number of cells, and each cell, and each cells contain a sub-cell, called the north- westcorner or upper left corner. The elements a1 , a2 ,......, am in the column under the source O are called the rim supplies and the elements b1 , b2 ,...,bn in row along the destination D are called the rim requirements. The notation (r,s) means the cell in the rth row and sth column.

Initial Basic Feasible Solution

Though the ultimate aim of the theory is to determine the optimal (minimal) solution, it is better to start with a basic feasible solution, and then proceed step by step to optimality. There are various methods of obtaining the initial basic feasible solution. Some of them are as follows:

(i) **Cooper charnes method of north- west corner rule, or, upper left corner rule, or, stepping stone rule;**

(ii) **Minimum row cost method;**

(iii) **Minimum column cost methods;**

(iv) **Minimum cost matrix method;**

(v) **Vogel approximation method (VAM) or unit penalty method. These methods are discussed in the following subsections. The methods are such that even though only feasible solutions are obtained by them, it can be proved that the solutions become basic also. If a tie occurs between two or more items in any methods, we can choose one of the items arbitrarily.**

North - West Corner Rule of Cooper and charnes (Stepping Stone Method)

In this rule, we prepare a transportation table as in (1), keeping the sub cells vacant. Then we fill up the subcells by x_{ij} by the following method. We take x_{11} to be the value which is minimum of a_1 and b_1, i.e., we put $x_{11} = \min(a_1, b_1)$. If $a_1 \leq b_1$, then we take $x_{11}=a_1$ and hence there will be no more materials left at the origin O_1. If $b_1 \leq a_1$ then we take $x_{11} = b_1$ and hence the capacity of the destination D_1 is exhausted, but the supply of $(a_1 - x_{11})$ is still available at O_1. Then we take $x_{12} = \min(a_1 - x_{11}, b_2)$. If $a_1 - x_{11} < b_2$, we take $x_{12} = a_1 - x_{11}$ and then the capacity of the source O_1 is exhausted. If $b_2 \leq a_1 - x_{11}$, then we take $x_{12} = b_2$, and then the capacity at D_2 is exhausted, but the supply $(a_1 - x_{11} - x_{12})$ is still available at O_1. Then we proceed to determine x_{13} in the same way. This process is continued till the supply at O_1 is exhausted, and it is bound to be exhausted as per formula (5). Some sub- cells in the first row may remain vacant. We put the zero values there.

Then we start with the source O_2 and proceed in the same way as was done for the source O_1 keeping in view whether any destination is exhausted or not. Thus, we exhaust the supply from O_2. Similarly, the other sources Om are to be exhausted. Thus, we get all values of x_{ij}.

Using these values of xij in (3), the corresponding total cost function or objective function z is evaluated. Evidently, this is a feasible solution, since all rim requirements and all rim supplies are satisfied

We not that in each allocation (i.e, determination of x_{ij}) at least one row or one column is discarded for further consideration, while the last allocation discards both a row and a column, to satisfy the rim requirements and rim supplies. Since there are m rows and n columns, the total number of positive (non-zero) allocations will be at most (m+n-1), while other allocations are all zero. Hence, there is at most (m+n-1) positive (non-zero) values of x_{ij}. Hence the solution is basic. Thus, This rule gives a basic feasible solution.
[In this method, the allocations of x_{ij} are done by the principle "first come, first served", without considering the cost coefficients c_{ij}.]

Example 2

A company has three factories O_1, O_2, O_3 whose daily production of a material are respectively 7,9,18 units. It has four warehouses D_1, D_2, D_3, D_4 whose capacities of storage are respectively 5,8,7,14 units, so that all units of productions can be stored there. The cost (in rupees) per unit materials of transporting the materials from O_1 to the four warehouses are respectively 19,30,50,10. The cost from O_2 to the warehouses are respectively 70,30,40,60. The cost from O_3 to the warehouses are respectively 40,8,70,20. Find a basic feasible solutions (allocations) and the corresponding total cost, by the method of north- west corner rule.

Solutions:

D O	b1 = 5	b2 = 8	b3 = 7	b4 = 14
a1 = 7	5	2	0	0
	19	30	50	10
a2 = 9	0	6	3	0
	70	30	40	60
a3 = 18	0	0	4	14
	40	8	70	20

Since $a1 = 7$, $b_1 = 5$, we take $x_{11} = 5$, and the remaining 2 units of a_1 as $x_{12} = 2$. It shows that $x_{13} = 0$, $x_{14} = 0$, $x_{21} = 0$, $x_{31} = 0$.

Similarly, we deal with the values of a_2, a_3. Thus, by the north-west corner rule, we get the basic feasible solution $x_{11} = 5$, $x_{12} = 2$, $x_{23} = 3$, $x_{33} = 4$, $x_{34} = 14$,
While other values of xij are zero. The corresponding total cost z is given by
$Z = 19 \times 5 + 30 \times 2 + 30 \times 6 + 40 \times 3 + 70 \times 4 + 20 \times 14 =$ ` 1015
[Here m =3, n=4 and hence (m+n-1) = 6. The table shows that there are six non-zero values of x_{ij}. Hence the feasible solution is a non - degenerate basic feasible solution. The values of x_{ij} are obtained without any reference to the cost of transport c_{ij}.]

Minimum Row cost Method

Let c1k be the smallest cost in the first row of the transportation table (1). Then we take x1k = min (a1, bk). If $a1 \le bk$, then we take x1k = a1, and hence the supply at O1 exhausted. If bk< ak, then we take x1k= bk, so that there is still $(a_1\text{-}bk)$ units available at O_1. Then we consider the next smallest cost of the first row, and proceed as above. This process is continued till the source O_1 is exhausted.

Then we consider the second row and proceed as in the first row, till the supply at O_2 is exhausted, keeping in view the capacities of the destinations. In the same way, all m rows are dealt with. This will give the values of x_{ij}. Then the total cost z is calculated by the formula (3). [In this method, the smallest costs of the rows are considered to evaluate x_{ij}, whereas in the north - west corner rule, the costs were not considered at all to get the values of x_{ij}.]

Example 3

Solve example 2 by the minimum row cost method.

Solution:

The transportation table is as follows

D / O	$b_1 = 5$		$b_2 = 8$		$b_3 = 7$		$b_4 = 14$	
$a_1 = 7$	0		0		0		7	
		19		30		50		10
$a_2 = 9$	0		8		1		0	
		70		30		40		60
$a_3 = 18$	5		0		6		7	
		40		8		70		20

The smallest cost in the table is ` 8 in the third row and second columns, and hence we take $x_{32} = 8$ so that $x_{12} = 0$, $x_{22}=0$. The next smallest cost in the table is ` 10 in the first row and fourth column and hence we take $x_{14}= 7$, so that $x_{11}=0$, $x_{13}=0$. The next smallest cost in the table is ` 19 in the first row and first column, but $x_{11} = 0$ already. The next smallest cost is ` 20 in the third row and fourth column and hence we take $x_{34} = 7$ (as $x_{14} = 7$ already and $b_4=14$), and hence $x_{24} =0$. The next smallest cost in table is ` 30 in the first row and second column, but $x_{12}=0$ already. The smallest cost ` 30 is also in the second row and second column, but $x_{22}= 0$ already. The next smallest cost of the table is ` 40 in second column, but $x_{22} = 0$ already. The next smallest cost of the table is ` 40 in second row and third column and hence we take $x_{23} = 7$, $x_{33} =0$. The smallest cost ` 40 is also in third row and first column and hence we take x31 = 3 as $x_{34} = 7$ already. Now it is seen that $x_{21} = 2$.

Thus, we finally get

$x_{14} = 7$, $x_{21} = 2$, $x_{23}=7$, $x_{31} = 3$, $x_{32}=8$, $x_{34}=7$,

and all other xij are zero. Hence

$Z = 10 \times 7 + 70 \times 2 + 40 \times 7 + 40 \times 3 + 8 \times 8 + 20 \times 7 =$ ` 814.

[This cost is lower than the cost by first and second methods, but higher than the cost of third method.]

Vogel Approximation or Unit Penalty Methods

(i) For better understanding of Vogel approximation method, we recapitulate the principles on which the previous four methods were based. In the north – west corner methods, the cost matrix is not consulted at all, but the allocations of the items to the various destinations are done on the principle of "first come, first served". In the minimum row cost method, the allocations is done first to the destinations to which the row cost is minimum, and if the source is still not exhausted, then the next allocation is done to the destinations to which the row cost is just higher than the minimum, and so on. In the minimum column cost methods, the same principals is taken with the column instead of the rows. In the minimum matrix cost method, neither the rows nor the columns are given any preference for selection, but the whole cost matrix is considered, and its minimum cost is given the first preference for allocations, and if the corresponding supply is not exhausted, the remaining materials are kept for further allocations. After the first allocations, the next higher cost in the whole matrix is given the preference, and so on.

These methods indicate that the total cost of transportation increases as the difference between the minimum cost and the next higher cost increases. Thus, the total cost depends upon the cost difference rather than their individual absolute values. This idea is utilized in Vogel method, and hence Vogel method gives a better initial basic feasible solution than the above other four methods.

(ii) In Vogel approximation method, the principle is to consider the relative minimum cost of each row and each column, computed as the difference between the minimum row (or column) cost and the next higher row (or column) cost. Then we choose the greatest of all these row and column cost difference, and the minimum cost corresponding to this greatest difference is taken as the minimum cost of the whole cost matrix (even though it may be higher than the actual minimum cost). fs The first allocation is done to the destination for which this supposed minimum cost happens. After the first allocation made in this way it will be seen that there will be at least one row or one column for which no further allocations are possible, and hence this row or column is omitted in the considerations of the next allocation. After this omission, there will be a truncken or shrunken transportation matrix. We calculate the relative row and column costs of this new matrix and make allotment as in above method. This process is continued one after another, till all allocations are completed

(iii) The explanation for selecting the highest cost difference is given as follows. Let R_1 be the difference of the minimum cost and the next higher cost in the first row. Then R_1 is the relative minimum cost of the first row. It is also called the penalty or loss in the first row. [For, if he supply a_1 in the first row is not exhausted, when the allocation is done to the destination corresponding to the minimum cost of the first row, then, the next higher cost is chosen for the purpose of the second allocation, and since R_1 is the difference between the two costs, a loss or penalty of R_1 unit of cost per unit amount of material has to be done.]

The penalties or losses for the second, third, ... mth rows are calculated in the same way as R_2, R_3, ..., R_m. The set

$$R_q = (R_1, R_2, ..., R_m)$$

is called the set of the row penalties, similarly, the set of the column penalties $C_p = (C_1, C_2, ..., C_n)$ of the first, second, ..., nth column are calculated.

The greatest of these row and column penalties is chosen for the first allocation to avoid this greatest penalty in future. It means that the destination which yields the greatest penalty or loss is to be supplied first, even if the corresponding cost is higher than the actual minimum cost. If this destination is supplied first, then the other destinations will involve less penalties. This is the reason why the greatest penalty is chosen first.

Example 6

Solve example 2, by Vogel approximation or unity penalty method.

Solutions:

The transportation table is as follows.

The transportation table is as follows.

O \ D	$b_1 = 5$	$b_2 = 8$	$b_3 = 7$	$b_4 = 14$	row penalty R_4
$a_1 = 7$	0 (19)	0 30	0 50	7 (10)	R_1 9 9 40 40
$a_2 = 9$	0 70	0 30	7 40	2 60	R_2 10 20 20 20
$a_3 = 18$	5 40	0 (8)	6 70	7 (20)	R_3 12 20 50 x
Column Penalty C_p	C_1 21 (21) X x	C_2 (22) X X X	C_3 10 10 10 10	C_4 10 10 10 (50)	

The penalties for the first, second, third rows are respectively

$$R1 = 19-10 = 9,\ R_2 = 40-30 = 10,\ R_3 = 20-8 = 12.$$

The penalties for the four columns are respectively

$C1 = 40 - 19 = 21$, $C_2 = 30 - 8 = 22$, $C_3 = 50 - 40 = 10$, $C_4 = 20 - 10 = 10$ units

The greatest of these row and column penalties is $C2 = 22$, and it corresponds to the minimum cost 8 in the third row and second column. Hence, we choose the subcell of the cell (3,2) for first supply. Thus, we get $x32=8$, and hence $x12=0$, $x22=0$. Then the second column is omitting the second column is omitted in the next consideration of allotment, as this column has been filled up.

Now, we calculate the penalties again by omitting second columns. The new row penalties are

$$R1 = 19 - 10 = 9,\ R_2 = 60 - 40 = 20,\ R_3 = 40 - 20 = 20$$

the new column penalties are

$$C1 = 40 - 19 = 21,\ (C2 = 0),\ C_3 = (50-40) = 10,\ C_4 = 20 - 10 = 10$$

The greatest of these new row and column penalties is $C_1 = 21$, and it corresponds to the minimum cost 19 in the first row and first column. Hence, we choose the subcell in the cell (1,1) for supply. Thus we get $x11 = 5$ and hence $x_{21}=0$, $x_{31}= 0$. Then the first column is omitted in the next consideration of allotment, as it has been filed up.

Now, we calculate the penalties again by omitting the first and second columns. The new row penalties is are

$$R_1 = 50-10 = 40,\ R_2 = 60-40 = 20,\ R_3 = 70 - 20 = 50,$$

and the new column penalties are

$$C_1 = 0,\ C_2 = 0,\ C_3 = 50-40 = 10,\ C_4 = 20-10 = 10.$$

The greatest of these new row and column penalties is $R_3 = 50$ and it corresponds to the minimum cost 20 in the third row and fourth column. Hence, we choose the subcell in the cell (3,4) for supply. Thus we get $x_{34} = 10$ and hence $x_{33} = 0$. Then the third row is omitted in the next consideration of allotment, as it has been filled up

Then we calculate the penalties again by omitting the first and second columnsand third row. The new row penalties are R1 = 50 – 10 = 40, R_2 = 60 – 40 = 20, R_3 = 0, and the new column penalties are C_1 = 0, C_2 = 0, C_3 = 50 – 40 = 10, C_4 = 60 – 10 = 50. The greatest of these row and column penalties is C_4 = 50 and it corresponds to the minimum cost 10 in the first row and fourth column. Hence we choose the subcell in the cell (1,4) for supply. Thus, we get x_{14} = 2 and hence x_{13} = 0, x_{24} = 2, x_{23} = 7.
Thus, all subcells have been supplied and the capacities of sources and destinations have been exhausted. The final result is
$x_{11} = 5, x_{14} = 2; x_{23} = 7, x_{24} = 2; x_{32} = 8, x_{34} = 10$,
and all other x_{ij} are zero. The corresponding total cost z is
$z = 19 \times 5 + 10 \times 2 + 40 \times 7 + 60 \times 2 + 8 \times 8 + 20 \times 10 = 779$
[The solution of example 1 by the above five methods gives the total cost z as z = (1015, 1110, 779, 814, 779). The third and fifth methods give the less total cost and hence one of them is preferable.]

Example 7

By Vogel approximation (or unit penalty) method, solve the following transportation problem.

O \ D	14	8	23
17	13	15	16
12	7	11	2
16	19	20	9

Solution:

The transport table is as follows.

The transport table is as follows.

O \ D	$b_1 = 5$	$b_2 = 8$	$b_3 = 7$	row penalty R_q
$a_1 = 17$	9 / 13	8 / 15	0 / 16	R_1 9 9 40 40
$a_2 = 12$	5 / 7	0 / 11	7 / 2	R_2 9 9 40 40
$a_3 = 16$	0 / 19	0 / 20	16 / 9	R_3 9 9 40 40
Column Penalty C_p	C_1 6 6 6	C_2 4 4 4	C_3 7 14 x	

Hence x11 = 9, x_{12} = 8, x_{21} = 5, x_{23} = 7, x_{33}= 16, and all other x_{ij} are zero. The total cost z is z = 27 + 120 + 35 + 14 + 144 = 340 units.

Example 7

By Vogel approximation (or unit penalty) method, solve the following transportation problem.

O \ D	14	8	23
17	13	15	16
12	7	11	2
16	19	20	9

Solution:

The transport table is as follows.

The transport table is as follows.

O \ D	$b_1 = 5$	$b_2 = 8$	$b_3 = 7$	row penalty R_q
$a_1 = 17$	9 / 13	8 / 15	0 / 16	R_1 9 9 40 40
$a_2 = 12$	5 / 7	0 / 11	7 / 2	R_2 9 9 40 40
$a_3 = 16$	0 / 19	0 / 20	16 / 9	R_3 9 9 40 40
Column Penalty C_p	C_1 6 6 6	C_2 4 4 4	C_3 7 14 x	

Hence $x_{11} = 9$, $x_{12} = 8$, $x_{21} = 5$, $x_{23} = 7$, $x_{33} = 16$, and all other x_{ij} are zero. The total cost z is z = 27 + 120 + 35 + 14 + 144 = 340 units.

Illustration 18

The cost conscious company requires for the next month 300, 260 and 180 tonnes of stone chips for its three constructions C1, C2 and C3 respectively. Stone chips are produced by the company at three mineral fields taken on short lease by the company. All the available boulders must be crushed into chips. Any excess chips over the demands at sites C1, C2 and C3 will be sold ex-fields. The fields are M1, M2 and M3 which will yield 250, 320 and 280 tones of stone chips respectively. Transportation costs from mineral fields to construction sites vary according to distances, which are given below in monetary unit (MU)

	To	C1	C2	C3
From	M1	8	7	6
	M2	5	4	9
	M3	7	5	5

(i)Determine the optimal economic transportation plan for the company and the overall transportation cost in MU.

(ii) What are the quantities to be sold from M1, M2 and M3 respectively?

Solution:

(i) Table: 1 Cost Matrix

To / From	C_1	C_2	C_3	Supply
M_1	8	7	6	250
M_2	5	4	9	320
M_3	7	5	5	280
Demand	300	260	180	850 / 750

From the given data we have Total Supply = 850 tonnes and total and total Demand = 740 tonnes i.e., Supply ≠ Demand.

So this is an unbalanced problem of transportation. To make it balanced we introduce a "Dummy" construction site of demand 850 - 740 = 110 tonnes and having zero cost elements for all the cells of the matrix corresponding to it.

Table: 2 Basic Feasible Solution by VAM (Optimal)

To / From	C_1	C_2	C_3	Dummy	Supply	Row Penalties 1st	2nd	3rd	4th	Row Nos. (u_i)
M_1	8	7	(140) 6	(110) 0	~~250~~ ~~140~~	6*	2	1	1	$u_1 = 0$
M_2	(300) 5	(20) 4	9	0	~~320~~ ~~20~~	4	1	5*	–	$u_2 = -2$
M_3	7	(240) 5	(40) 5	0	~~280~~ ~~40~~	5	0	0	0	$u_3 = -1$
Demand	~~300~~	~~260~~ ~~240~~	~~180~~ ~~40~~	~~110~~	850					
Column Penalties 1st	2	1	1	0						
2nd	2*	1	1	–						
3rd	–	1	1	–						
4th	–	2*	1	–						
Column Nos. (v_j)	$v_1 = 7$	$v_2 = 6$	$v_3 = 6$	$v_4 = 0$						

Row Penalty = 2nd lowest cost figure of a row – Lowest cost figure of that row. For the 1st Set of Row Penalties -

(a) For 1st row, 2nd lowest cost = 6 and lowest cost = 0

∴ Penalty = 6 – 0 = 6

(b) For 2nd Row, 2nd lowest cost = 4 and Lowest cost = 0

∴ Penalty = 4 - 0 = 4

(c) For 3rd Row, 2nd lowest cost = 5 and Lowest cost = 0,

∴ Penalty = 5 - 0 = 5

Similarly, Column Penalty = 2nd lowest cost figure of a column – Lowest cost figure of that column

For the 1st Set of Column Penalties -

(a) For 1st column, 2nd lowest cost = 7 and Lowest cost = 5, ∴ Penalty = 7 – 5 = 2

(b) For 2nd column, 2nd lowest cost = 5 and Lowest cost = 4, ∴ Penalty = 5 - 4 = 1

(c) For 3rd column, 2nd lowest cost = 6 and Lowest cost = 5, ∴ Penalty = 6 – 5 = 1

Of all these Row and Column penalties of 1st set, 6 is highest and it corresponds to 1st Row

Hence allocation should be done at that cell of 1st Row where cost is least. This corresponds to the cell (M1 - Dummy). So maximum possible unit of 110 is allocated in this cell by maintaining parity of supply and demand. With this allocation the total demand of 'Dummy' site is exhausted. But the supply of the corresponding Mineral Field (M1) is not fully exhausted. Remaining supply capacity of M1 i.e. 250 - 110 = 140 tonnes is shown as balance in the supply cell of M1. As the demand of 'Dummy' is fulfilled, the entire column for this has been shaded indicating the same. Figures of this column will no longer participate in any of the subsequent calculations of Penalty (for Rows as well as columns)

The same procedure of calculating penalty for Rows and Columns and subsequently allocating maximum possible quantity in the least cost cell corresponding to highest penalty is repeated until all the allocations are made maintaining parity of Supply and Demand.

The solution thus obtained is the Basic Feasible Solution. It is given as follows.

Table: Showing Optimum Allocation

Cell	Allocation	Cost of Transportation (`)
$M_1 - C_3$	140 tonnes	$140 \times 6 = 840$
M_1 - Dummy	110 tonnes	$110 \times 0 = 0$
$M_2 - C_1$	300 tonnes	$300 \times 5 = 1500$
$M_2 - C_2$	20 tonnes	$20 \times 4 = 80$
$M_3 - C_2$	240 tonnes	$240 \times 5 = 1200$
$M_3 - C_3$	40 tonnes	$40 \times 5 = 200$
Total	850 tonnes	` 3820

Here, m = No. of rows of the matrix = 3

n = No. of columns of the matrix = 4

$\therefore m + n - 1 = 3 + 4 - 1 = 6$

Also, no. of allocated cells = 6

As, no. of allocated cells = 6 = m + n - 1, the solution is a non degenerate one

Now the solution is tested for OPTIMALITY.

For this, Row Nos. (u_i) and column nos. (v_j) are calculated by using the equation $C_{ij} = u_i + v_j$ for all the allocated cells, where C_{ij} = Cost figure of the cell i-j

Allocated Cell	C_{ij}			$C_{ij} = u_i + v_j$	
$M_1 - C_3$	$C_{13} = 6$	C13 = u1 + v3	or, $6 = u_1 + v_3$	or, $6 = 0 + v_3$ (Assume $u_1 = 0$) or, $v_3 = 6$	(1)
M_1 - Dummy	$C_{14} = 0$	C14 = ui + v4	or, $0 = u_1 + v_4$	or, $0 = 0 + v_4$ or, $v_4 = 0$	(2)
$M_2 - C_1$	$C_{21} = 5$	C21 = u2 + v1	or, $5 = u_2 + v_1$	or, $5 = -2 + v_1$ or, $v_1 = 7$	(6)
$M_2 - C_2$	$C_{22} = 4$	C22 = u2 + v2	or, $4 = u_2 + v_2$	or, 4 = u2 + 6 or, u2 = -2	(5)

$M_3 - C_2$	$C_{32} = 5$	C32 = u3 + v2	or, $5 = u_3 + v_2$	or, $5 = -1 + v_2$ or, $v_2 = 6$	(4)
$M_3 - C_3$	$C_{33} = 5$	C33 = u3 + v3	or, $5 = u_3 + v_3$	or, 5 = u3 + 6 or, u3 = -1	(3)

Hence no. of equations = 6 and no. of unknowns = 7. So to start with a solution, it is assumed u1 = 0. thereafter all the other row nos. and column nos. are calculated. The sequence of usage of the above equations is indicated as (1), (2), (3), ... (6).

Next opportunity cost (Δ_{ij}) for all the unallocated cells are calculated using $\Delta_{ij} = C_{ij} - (u_i + v_j)$

Unallocated Cell	C_{ij}	Opportunity Cost [Δij = Cij – (ui + vj)]
$M_1 - C_1$	$C_{11} = 8$	$\Delta_{11} = C_{11} - (u_1 + v_1) = 8 - (0 + 7) = 1$
$M_1 - C_2$	$C_{12} = 7$	$\Delta_{12} = C_{12} - (u_1 + v_2) = 7 - (0 + 6) = 1$
$M_2 - C_3$	$C_{23} = 9$	$\Delta_{23} = C_{23} - (u_2 + v_3) = 9 - (-2 + 6) = 5$
M_2 - Dummy	$C_{24} = 0$	$\Delta_{24} = C_{24} - (u_2 + v_4) = 0 - (-2 + 0) = 2$
$M_3 - C_1$	$C_{31} = 7$	$\Delta_{31} = C_{31} - (u_3 + v_4) = 7 - (-1 + 7) = 1$
M_3 - Dummy	$C_{34} = 0$	$\Delta_{34} = C_{34} - (u_3 + v_4) = 0 - (-1 + 0) = 1$

As all the opportunity cost values are nonnegative, the solution is optimal.

(i) So the optimal transportation plan is as shown in Table-3 and minimum cost of transportation is ` 3820/-

(ii) Quantities to be produced by M_1, M_2 and M_3 are respectively 250,320 and 280 tonne of which 110 tonnes worth of stone chips produced by M1 will remain unused by the construction sites. So this quantity can be sold ex-field.

Illustration 19

Ladies fashion shop wishes to purchase the following quantity of summer dresses:

Dress size	I	II	III	IV
Quantity	100	200	450	150

Three manufacturers are willing to supply dresses.

The quantities given below are the maximum that they are able to supply of any given combination of orders for dresses:

Manufacturers	A	B	C
Total quantity	150	450	250

The shop expects the profit per dress to vary with the manufacturer as given below:

Size

	I	II	III	IV
A	`2.5	`4.0	`5.0	`2.0

B	`3.0	`3.5	`5.5	`1.5
C	`2.0	`4.5	`4.5	`2.5

Required:

(a) Use the transportation technique to solve the problem of how the orders should be placed with the manufacturers by the fashion shop is order to maximise profit.

(b) Explain how you know there is no further improvement possible.

Solution:

Table: 1 Profit Matrix

Manufacturer \ Dress Size	I	II	III	IV	Supply
A	2.5	4	5	2	150
B	3	3.5	5.5	1.5	450
C	2	4.5	4.5	2.5	250
Demand	100	200	450	150	850 / 900

Maximum possible supply capability of manufacturer = 850 units

Total Demand = 900 units

As Supply ≠ demand, the problem is an unbalanced one. To make it balanced, a 'Dummy' manufacturer of supply capacity = 900 – 850 = 50 units. is introduced. The profit figures for it are all zeros.

Also it is a problem of maximisation, to convert it to a problem of minimisation, a Relative Loss matrix is formed by subtracting all the profit figures given in the above matrix as well as those of Dummy from the highest profit (5.5) figure of the given matrix.

Table : 2 Relative Loss Matrix with Basic Feasible Solution

Manufacturer \ Dress Size	I	II	III	IV	Supply	Row Penalties 1st	2nd	3rd
A	(100) 3	1.5	0.5	(50) 3.5	~~150~~ 50	1	1.5	0.5*
B	2.5	2	(450) 0	4	~~450~~	2*	–	–
C	3.5	(200) 1	1	(50) 3	~~250~~ 50	0	2*	0.5
Dummy	5.5	5.5	5.5	(50) 5.5	~~50~~	0	0	0
Demand	~~100~~	~~200~~	~~450~~	~~150~~	900			
Column Penalties 1st	0.5	0.5	0.5	0.5				
Column Penalties 2nd	0.5	0.5	–	0.5				
Column Penalties 3rd	0.5	–	–	0.5				

Here, m = No. of rows of the matrix = 4 and n = No. of columns of the matrix = 4

$\therefore m + n - 1 = 4 + 4 - 1 = 7$

Also no. of allocated cells = 6 ≠ (m+ n – 1)

So the solution is a degenerate one. To resolve this, we make use of an artificial quantity 'e' and allocate this quantity at the unallocated cell which is having least cost among all the unallocated cells. It can be mentioned that the quantity 'e' is very small and for all practical purposes its value can be taken as zero.

Least cost unallocated cell is (A-III) where allocation of 'e' has to be made.

Table : 3 Showing Basic Feasible Solution (Optimal)

Manufacturer \ Dress Size	I	II	III	IV	Supply	Row Nos. (u_i)
A	3 (100)	1.5	0.5 (3)	3.5 (50)	150	$u_1 = 0$
B	2.5	2	0 (450)	4	450	$u_2 = -0.5$
C	3.5	1 (200)	1	3 (50)	250	$u_3 = -0.5$
DUMMY	5.5	5.5	5.5	5.5 (50)	50	$u_4 = 2$
DEMAND	100	200	450	150	900	
Column Nos. (v_j)	$v_1 = 3$	$v_2 = 1.5$	$v_3 = 0.5$	$v_4 = 3.5$		

To test optimality of the Basic Feasible Solution, Row Nos. (u_i) and Column Nos. (v_j) are calculated using the equation $C_{ij} = u_i + v_j$ for the allocated cells, where C_{ij} = Relative Loss figure of the cell i – j.

Allocated cell	A-I	A-III	A-IV	B-III	C-II	C-IV	Dummy-IV
C_{ij}	$C_{11} = 3$	$C_{13} = 0.5$	$C_{14} = 3.5$	$C_{23} = 0$	$C_{32} = 1$	$C_{34} = 3$	$C_{44} = 5.5$

$C_{11} = u_1 + v_1$	or, $3 = 0 + v_1$ [$u_1 = 0$, Assumed] or, $v_1 = 3$		
$C_{13} = u_1 + v_3$	or, $0.5 = 0 + v_3$ or, $v_3 = 0.5$; $C_{14} = u_1 + v_4$	or, $3.5 = 0 + v_4$	or, $v_4 = 3.5$
$C_{23} = u_2 + v_3$	or, $0 = u_2 + 0.5$ or, $u_2 = -0.5$; $C_{34} = u_3 + v_4$	or, $3 = u_3 + 3.5$	or, $u_3 = -0.5$
$C_{32} = u_3 + v_2$	or, $1 = -0.5 + v_2$ or, $v_2 = 1.5$; $C_{44} = u_4 + v_4$	or, $5.5 = u_4 + 3.5$	or, $u_4 = 2$

Opportunity Loss figures (Δif) for all the unallocated cells are calculated using the equation $\Delta_{ij} = C_{ij} - (u_i + v_j)$

A - II	$\Delta_{12} = C_{12} - (u_1 + v_2) = 1.5 - (0 + 1.5) = 0$
B - I	$\Delta_{21} = C_{21} - (u_2 + v_1) = 2.5 - (-0.5 + 3) = 0$
B - II	$\Delta_{22} = C_{22} - (u_2 + v_2) = 2 - (-0.5 + 1.5) = 1$
B - IV	$\Delta_{24} = C_{24} - (u_2 + v_4) = 4 - (-0.3 + 3.5) = 1$
C - I	$\Delta_{31} = C_{31} - (u_3 + v_1) = 3.5 - (-0.5 + 3) = 1$
C - III	$\Delta_{33} = C_{33} - (u_3 + v_3) = 1 - (-0.5 + 0.5) = 1$
Dummy - I	$\Delta_{41} = C_{41} - (u_4 + v_1) = 5.5 - (2 + 3) = 0.5$

Dummy - II $\Delta_{42} = C_{42} - (u_4 + v_2) = 5.5 - (2 + 1.5)$

$= 2$

Dummy - III $\Delta_{43} = C_{43} - (u_4 + v_3) = 5.5 - (2 + 1.5)$

$= 3$

As all the opportunity loss values are non negative, the solution is optimal.

Table Showing Optimum allocation of orders quantities

From Manufacturer	Dress Size	Allocated Quantity	Profit/unit (`)	Total (`)
(i)	(ii)	(iii)	(iv)	(v) = (iii) × (iv)
A	I	100 units	2.5	250
	IV	50 units	2	100
B	III	450 units	5.5	2475
C	II	200 units	4.5	900
	IV	50 units	2.5	125
Dummy	IV	50 units	0	0
Total	-	**900 units**	-	` 3850

Maximum Profit = ` 3850/-

Illustration 20

The products of three plants F1, F2 and F3 are to be transported to 5 warehouses W1, W2, W3, W4 and W5. The capacities of plants, demand of warehouses and the cost of transportation from one plant to various warehouses are indicated in the following table:

	W1	W2	W3	W4	W5	Plant Capacity
F1	74	56	54	62	68	400
F2	58	64	62	58	54	500
F3	66	70	52	60	60	600
Warehouse Demand	200	280	240	360	320	1500/1400

(a)Find out a distribution plan of products from plants to the warehouses at a minimum cost. What is the minimum cost?

(b)Is there any surplus capacity of the plants? If so, in which plant should we associate that surplus capacity?

(c) Is there any alternate solution for the optimum solution achieved in

Solution:

From the given data total plant capacity (1500 units) is more than the total demand of warehouses (1400 units). So the problem is unbalanced. To make it balanced, a 'Dummy' warehouse of demand

1500 - 1400 = 100 units is introduced. Cost figures corresponding to various cells of this 'Dummy' are zeros.

Table : 1 Basic Feasible Solution

Plant \ Warehouse	W_1	W_2	W_3	W_4	W_5	Dummy	Plant Capacity	Raw Penalities 1	2	3	4	5	6
F_1	74	56 (280)	54	62 (120)	68	0	~~120~~ ~~400~~	54	2	6	6	6	6
F_2	58 (200)	64	62	58	54 (200)	0 (100)	~~400~~ ~~500~~ ~~200~~	* 54	4	4	4	4	-
F_3	66	70	52 (240)	60 (240)	60 (120)	0	~~600~~ ~~360~~ ~~240~~	52	* 8	0	0	0	0
Warehouse Demand	~~200~~	~~280~~	~~240~~	~~360~~	~~120~~ ~~320~~	~~100~~	1500						
Column Penalities 1	8	8	2	2	6	0							
2	8	8	2	2	6	-							
3	8	8*	-	2	6	-							
4	8*	-	-	2	6	-							
5	-	-	-	2	6*	-							
6	-	-	-	2	8*	-							

Here, m = No. of rows = 3

n = No. of columns = 6 m + n - 1 = 3 + 6 - 1 = 8

Also no. of allocated cells = 8 = m + n - 1.

So the solution is nondegenerate.

Table : 2 Showing Basic Feasible Solution (Non Optimal)

Plant \ Warehouse	W_1	W_2	W_3	W_4	W_5	Dummy	Plant Capacity	Row Nos. (U_i)
F_1	74	(280) 56	54	(120) (-) 62	68	(+) 0	400	$u_1 = 8$
F_2	(200) 58	64	62	58	(200) (+) 54	(100) (-) 74	500	$u_2 = 0$ (left)
F_3	66	70	(240) 52	(240) (+) 60	(120) (-) 60	0	600	$u_3 = 6$
Warehouse Demand	200	280	240	360	320	100	1500	
Column Nos. (V_j)	$V_1 = 58$	$V_2 = 48$	$V_3 = 46$	$V_4 = 54$	$V_5 = 54$	$V_6 = 0$		

Calculation of Opportunity Costs for Basic Feasible Solution

Unallocated Cell	Opportunity Cost [$\Delta_{ij} = C_{ij} - (u_i + v_j)$]
$F_1 - W_1$	$\Delta_{11} = C_{11} - (u_1 + v_1) = 74 - (8 + 58) = 8$
$F_1 - W_3$	$\Delta_{13} = C_{13} - (u_1 + v_3) = 54 - (8 + 46) = 0$
$F_1 - W_5$	$\Delta_{15} = C_{15} - (u_1 + v_5) = 68 - (8 + 54) = 6$
F_1 - Dummy	$\Delta_{16} = C_{16} - (u_1 + v_6) = 0 - (8 + 0) = -8$
$F_2 - W_2$	$\Delta_{22} = C_{22} - (u_2 + v_2) = 64 - (0 + 48) = 6$
$F_2 - W_3$	$\Delta_{23} = C_{23} - (u_2 + v_3) = 62 - (0 + 46) = 16$
$F_2 - W_4$	$\Delta_{24} = C_{24} - (u_2 + v_4) = 58 - (0 + 54) = 4$

$F_3 - W_1$ $\Delta_{31} = C_{31} - (u_3 + v_1) = 66 - (6 + 58) = 2$

$F_3 - W_2$ $\Delta_{32} = C_{32} - (u_3 + v_2) = 70 - (6 + 48) = 16$

F_3 - Dummy $\Delta_{36} = C_{36} - (u_3 + v_6) = 0 - (6 + 0) = -6$

As all the Opportunity Costs are not nonnegative, the solution is non optimal i.e. further improvement is possible. For this a loop is formed starting from the cell having highest negative value which is cell (F_1 - Dummy) having a highest negative opportunity cost value of -8. The starting cell of the loop is marked with a (+) and thereafter alternately the corner cells of the loop are marked (–) and (+). Next the minimum of the allocated quantities of the cells marked (–) is subtracted from the allocated quantities of all the cells marked (–) and added to all the cells marked (+). This leads to an improved solution as shown below.

Table : 3 Showing Improved Solution (Optimal)

Plant \ Warehouse	W_1	W_2	W_3	W_4	W_5	Dummy	Plant Capacity	Row Nos. [U]
F_1	74 (8)	56 [280]	54 (0) (+)	62 [20] (–)	68 (6)	0 [100]	400	U_1 = 0 (Let)
F_2	58 [200]	64 (16)	62 (16)	58 (4)	54 [300]	0 (8)	500	U_2 = -8
F_3	66 (2)	70 (16)	52 [240] (–)	60 [340] (+)	60 [20]	0 (2)	600	U_3 = -2
Warehouse Demand	200	280	240	360	320	100	1500	
Column Nos. [V]	V_1 = 66	V_2 = 56	V_3 = 54	V_4 = 62	V_5 = 62	V_6 = 0		

Opportunity Costs (D_{ij}) for the unallocated cells are calculated same as before and shown in left bottom corner of the cells.

(a) As Dij ≥ 0, the solution is optimal.

Table -4: Showing Optimal Distribution Plan

F1	W2	280	56	15680
	W4	20	62	1240
	Dummy	100	0	0
F2	W1	200	58	11600
	W5	300	54	16200
F3	W3	240	52	12480
	W4	340	60	20400
	W5	20	60	1200
Total		1500	-	₹ 78800

Minimum Cost of Transportation is ` 78800

(b) Plant F1 is having a surplus quantity of 100 units.

(c) Presence of zero opportunity cost (in the cell F1 -W3) indicates that alternative optimum solution is possible for the problem. To get the solution, we form a loop starting from the cell F1 - W3 . The new solution is shown below-

Table-5: Showing Alternative Optimum Solution

Table-5: Showing Alternative Optimum Solution

Warehouse	W_1	W_2	W_3	W_4	W_5	Dummy	Plant Capacity
F_1	74	56 (280)	54 (20)	62	68	0 (100)	400
F_2	58 (200)	64	62	58	54 (300)	0	500
F_3	66	70	52 (220)	60 (360)	60 (20)	0	600
Warehouse Demand	200	280	240	360	320	100	1500

Table-6: Showing Alternative Optimum Distribution Plan

From Plant	To Warehouse	Quantity (Units)	Cost/Unit (`)	Total (`)
(1)	(2)	(3)	(4)	(5) = (3) × (4)
F_1	W_2	280	56	15680
	W_3	20	54	1080
	Dummy	100	0	0
F_2	W_1	200	58	11600
	W_5	300	54	16200
F_3	W_3	220	52	11440
	W_4	360	60	21600
	W_5	20	60	1200
Total		**1500**	–	**` 78800**

So the alternative solution is given above.

Illustration 21

A company has 4 factories F1 , F2 , F3 , & F4 manufacturing the same product. Production & row material cost differ from factory to factory and are given in the following table in the first two rows. The transportation cost from factories to sales departments S1 , S2 , S3 , are also given. The last two columns in the table give the sales price & the total requirement at each sales department.
The production capacity of each factory is given in the last row.

Factories / Sales Dept.	F_1	F_2	F_3	F_4	Sales price/unit	Requirement
Production cost/unit	15	18	14	13		
Row material cost/unit	10	9	12	9		
Transportation Cost/unit S_1	3	9	5	5	34	80
S_2	1	7	4	5	32	120
S_3	5	8	3	6	31	150
Availability	10	150	50	100		

Determine the most profitable production & the distribution schedule & the corresponding profit. The surplus product should be taken to yield zero profit

Solution :

Initially in this problem there are four sources (factories) and three destinations (sales Depot) .
Total Cost/unit = Production cost/unit + Raw material cost/unit + Transportation Cost/unit.
Profit/unit = Selling Price/unit – (Total Cost/unit)
Total Availability = 310 units & Total requirement = 350 units Since Total Availability not equal to total requirement so it is a unbalanced transportation problem.
Since total availability is less than total requirement, we have to introduce a dummy factory with adjustment of 40 units to make balance transportation problem.

Table showing the calculation of per unit Profit matrix

	Sales depot 1	Sales depot 2	Sales depot 3	Availability
Factory 1	34 – (15 + 10 + 3) = 6	32 – (15 + 10 + 1) = 6	1	10
Factory 2	-2	-2	-4	150
Factory 3	3	2	2	50
Factory 4	7	5	3	100
Factory 5 (Dummy)	0	0	0	40
Requirement	80	120	150	350

Table showing the Calculation of per unit cost matrix [Subtracting each element of cost from biggest element here it is '7']

	Sales depot 1	Sales depot 2	Sales depot 3	Availibility
Factory 1	1	1	6	10
Factory 2	9	9	11	150
Factory 3	4	5	5	50
Factory 4	0	2	4	100
Factory 5 (Dummy)	7	7	7	40
Requirement	80	120	150	350

Now we can apply VAM to get initial Basic feasible solutions(IBFS)

For optimality solution we will follow two steps (1) Calculation of Row Penalty (u_i) and column penalty (v_j) by trial and error method on the basis of occupied solution (IBFS) .

Total no of initial basic feasible solutions = m+n-1 but total no of u_i and v_j are m + n so with the help of m+n-1 IBFS we can never solve m+n unknowns so any one of m+n solutions can be solved by trial and error method . Any one of the u_i or v_j will be zero on the basis of maximum number of occupied cell if no of occupied cells are same for more than one rows or one columns we can consider any of them to maintain the condition of m+n-1.
Let C_{ij} be the cost for occupied cell .Using occupied cell costs and one of the trial solution we can calculate the other row penalties and column penalties.
Where, $C_{ij} = u_i + v_j$
Let C_{ij} be the cost for occupied cell. Using occupied cell costs and one of the trial solution we can calculate the other row penalties and column penalties.
Where, $C_{ij} = u_i + v_j$
After getting all u_i and v_j then we calculate the unoccupied cell using the formula given below
$C_{ij} - (u_i + v_j)$
where, C_{ij} is the cost of unoccupied cell.

Illustration 22
Departmental store wishes to purchase the following quantities of Sprees:

Types of sprees	A	B	C	D	E
Quantity	150	100	75	250	200

Tenders are submitted by 4 different manufacturers who undertake to supply not more than the quantities mentioned below (all types of sprees combined)

Manufacturer	W	X	Y	Z
Total quantity	300	250	150	200

The store estimates that its profit/spree will vary with the manufacturer as shown in the following matrix.

			Sprees		
Manufacturers	**A**	**B**	**C**	**D**	**E**
W	275	350	425	225	150
X	300	325	450	175	100
Y	250	350	475	200	125
Z	325	275	400	250	175

How should the orders be placed?

Solution:

Profit matrix

W	275	350	425	225	150	0
X	300	325	450	175	100	0
Y	250	350	475	200	125	0
Z	325	275	400	250	175	0

Loss Matrix:

200 125 50 250 325 475 300/275/225/25

25 50 200 25

175 150 25 300 375 475 250/100/0

150 100 25/25/125/75/5

225 125 0 275 350 475 150/75/0

75 75 125* 100*

150 200 75 225 300 475

200 200/0

75/50/50/75/75/75*

150	100	75	250	200	125
0	25	0	50	0	100
	0		0		0
25	0	25	25	25	0
25	0		25	25	0
25	25		25	25	0
25			25	25	0
			25	25	0
			50	50	0

M + n - 1 allocation s are there, optimality test can be performed.

						u_i
200 / 25	125 / 25	50 / 50	250 / 50	325 / 200	475 / 25	0
175 / 150	150 / 25	25 / 25	300 / 50	375 / 50	475 / 100	0
225 / 50	125 / 75	0 / 75	275 / 25	350 / 25	475 / 0	0
150 / 0	200 / 100	75 / 100	225 / 200	300 / 0	475 / 25	-25
175	125	0	250	325	475	

As $\Delta_{ij} \geq 0$, maximum profit is as follows.

As $\Delta_{ij} \geq 0$, maximum profit is as follows.

			Qty		Maximum Profit
W	⟶	B	25 × 350	=	8750
		D	50 × 225	=	11250
		E	200 × 150	=	30000
		F	25 × 0	=	0
X	⟶	A	150 × 300	=	45000
		F	100 × 0	=	0
Y	⟶	B	75 × 350	=	26250
		C	75 × 475	=	35625
Z	⟶	D	200 × 250	=	50000
Max. Profit.			900		₹ **2,06,875**

Illustration 23

The Bombay Transport Company has trucks available at four different sites in the following numbers:

Site A	5 Trucks
Site B	10 Trucks
Site C	7 Trucks
Site D	3 Trucks

Customers – W, X and Y require trucks as shown below.

Customer W	5 Trucks
Customer X	8 Trucks
Customer Y	10 Trucks

Variable Costs of getting trucks to the Customers are given below:

From A to W	7, to X	3, to Y	6
From B to W	4, to X	6 to Y	8
From C to W	5, to X	8 to Y	4
From D to W	8 to X	4 to Y	3

Solve the above transportation problem.

Solution

	7		3		6		0	5/0	3	3*	-	-	-
5	4	5 3	6		8	2	0	10/8/3/0	4*	2	2*	2	2
	5		8	7	4		0	7/0	4	1	1	4	-
	8		4	3	3		0	3/0	3	1	1	1	1
5		8		10		2							

0	3	3	0
	0	0	
1	1	1	0
1	1	1	-
1	2	1	
-	2	1	-
-	2	5	-

	W	X	Y	Z	Ui
A	7, 6	3, 5	6, 4	0, 3	-3
B	4, 5	6, 3	8, 3	0, 2	0
C	5, 2	8, 3	4, 7	0, 1	-1
D	8, 6	4	3, 3	0, 1	-2

	W	X	Y	Z	Ui
A	7, 6	3, 5	6, 4	0, 3	-3
B	4, 5	6, 3	8, 3	0, 2	0
C	5, 2	8, 3	4, 7	0, 1	-1
D	8, 6	4	3, 3	0, 1	-2
Vj	4	6	5	0	

As Δij ≥ 0, the solution is optimum.

Allocation:

				Minimum Cost		
A	→	X	→	5 × 3	=	15
B	→	W	→	5 × 4	=	20
	→	X	→	3 × 6	=	18
	→	Z	→	2 × 0	=	0
C	→	Y	→	7 × 4	=	28
D	→	Y	→	3 × 3	=	9
				25		**₹ 90**

Illustration 24

A company has 3 plants located at different places but producing an identical product. The cost of production, distribution cost of each plant to the 3 different warehouses, the sale price at each warehouse and the individual capacities for both the plant and warehouse are given below:

Plants	F1	F2	F3		
Raw material	15	18	14		
Other expenses	10	9	12		
Distribution cost to warehouse				Sales Price in (`)	Warehouse Capacity (No)
W1	3	9	5	34	80
W2	1	7	4	32	110
W3	5	8	3	31	150
Capacity of Plant (No.)	150	100	130		

Establish a suitable table giving net profit/loss for a unit produced at different plants and distributed at different locations.

- **(a) Introduce a suitable dummy warehouse / plant so as to match the capacities of plants and warehouses.**
- **(b) Find distribution pattern so as to maximise profit / minimise loss.**
- **(c) Interpret zero value of square evaluation of an empty cell and find alternative solutions.**

Solution:

Profit matrix

6	-2	3	80
6	-2	2	110
1	-4	2	150
0	0	0	40
150	100	130	380

Loss Matrix

0 40	8 40	3	80/40/0	3/3/5
0 110	8	4	110/2	4*
5	10 20	4 130	150/20/0	1/1/6*
6	6 40	6	10/0	0/0/0

150	100	130
40	0	0
0		
0	2	1
5*	2	1
	2	1

			U
0 40	8 40	3 (1)	0
0 110	8 (0)	4 (2)	0
5 (3)	10 20	4 130	2
6 (4)	6 40	6 (6)	-2
0	8	2	

As there are m+n-1 allocations, optimality test can be performed since Δij ≥ 0,

		Quantity	Maximum Profit
F1	W1	40 × 6	240
	W2	40 × -2	-80
F2	W1	110 × 6	660
F3	W2	20 × -4	-80
	W3	130 × 2	260
F4 Dummy	W2	40 × 0	0
		380	**` 1000**

Profit ` 1,000/-

QUESTION.30
Define Job Evaluation and their steps?

ANSWER.
Job evaluation is the ranking grading, and weighing of essential work characteristics of all jobs in order to find out or rate the worth of jobs. It is a systematic approach to ascertain the labour worth of each job and is a very important concern of all employers.
Job evaluation aims at fairness and consistency so far as all wages and salaries are concerned within an organisation and when systematic and impartial, it stimulates, confidence of the employees. There are three steps for evaluations of all jobs:-
- (i) Preparation of preliminary description of each existing job.
- (ii) Analysing each job to arrive at final job descriptions and specifications.
- (iii) Analysing each job according to its approved description in order to determine its worth or value.

QUESTION.31
Explain Job Description and Specifications:

ANSWER.
The understanding of the job content or job description is the primary requirement.
Job specifications are derived from the job descriptions which have already been approved. The specification help determining the qualification required of the individual desired for the position. This in turn guides the personnel department in the selection of employees and also guides shop executives in the placement of workmen.

QUESTION.32
Explain Systems of Valuation and their Methods?

ANSWER.
Systems of Valuation: There are several systems of job evaluation.
The fundamental criteria in valuation of a job into account are to make a specific list of factors which affect job values. The many factors are:
1. Qualifications required of the worker,
2. Job difficulties,
3. Job responsibilities,
4. Working conditions.

All these factors are to be analysed in detail in order to complete the job description. The list of factors, the manner in which they are apprised and the method of finding out relative worth and money values distinguishes the various systems of valuation.
The systems of valuations which are commonly adopted are given below
1. The ranking or grading method,
2. The factor comparison method,
3. Point rating method.

Ranking or Grading Method: Under this system the titles of all jobs are written on cards and the grading is done by several competent judges. The hourly rates to be paid for different jobs are suggested by the judges without any consideration to the existing wage. The ranks or grades assigned to each job by all the judges are averaged and this average is considered the "score" for that job. Hourly rates are then fixed for jobs according to their ranking.

Factor Comparison Method: The factor comparison method analyses the job into much greater detail than the grading method. It ranks each job with respect to each factor that characterise the job and the factors are taken one at a time.

All jobs are compared and ranked first with respect to mental requirements, then skill, then physical requirements and after that responsibility and lastly working conditions. The total worth of the job is obtained by adding together money values which are assigned separately to the various levels of rank in each factor. Factor comparison method is more accurate than the simple ranking systems, since the separate factors are analysed comparatively. This method is flexible.

Point Rating Method: There are three methods of analytical evaluation of a job. They are:

1. Straight point method.
2. Weighted point method.
3. Valuation of jobs directly in money method, not specifying any maximum weight.

(i)Straight Point Method:

This method assigns equal weights for each characteristic. When evaluating a job under this system, it is assumed that all the characteristics have ranges of values between same maximum and minimum points.

(ii)Weighted Point Method:

In this method different points are assigned to the different characteristics of doing jobs.

(iii)Direct to Money Methods:

After selecting the job characteristics, ten key jobs whose rates are believed to be correct, are taken and the present wage rates of these jobs are distributed to the job characteristics by each analyst. The jobs are then ranked by the analysts for each characteristic in order of the degree to which that characteristic is present. This serves as a check to show up any errors made in the original distribution of the wages rate to the various characteristics.

QUESTIONS.33

Define Assignment and state their methods?

ANSWER.

Assignment

Assignment is a special linear programming problem. There are many situations where the assignment of people or machines etc. may be called for. Assignment of workers to machines, clerks to various check-out counters, salesmen to different sales areas are typical examples of these. The Assignment is a problem because people possess varying abilities for performing different jobs and therefore the costs of performing jobs by different people are different. Thus, in an assignment problem, the question is how the assignments should be made in order that the total cost involved is minimized.

There are four methods of solving an assignment problem and they are:

1. Complete Enumeration Method
2. Simplex Method
3. Transportation Method and
4. Hungarian Method

Hungarian Method:

The following are the steps involved in the minimization of an assignment problem under this method:

Step 1: Row Operation

Locate the smallest cost element in each row of the given cost table. Now subtract this smallest element from each element in that row. As a result, there shall be at least one zero in each row of this new table, called the reduced cost table.

Step 2: Column Operation

In the reduced cost table obtained, consider each column and locate the smallest element in it. Subtract the smallest value from every entry in the column. As a consequence of this action, there would be at least one zero in each of the rows and columns of the second reduced cost table.

Step 3: Optimality

Draw the minimum no. of horizontal and vertical lines (not the diagonal ones) that are required to cover all the zero elements. If the no. of lines drawn is equal to 'n' (the no. of rows/columns of the given Cost Matrix) the solution is optimal and proceed to step 6. If the no. of lines drawn is smaller than 'n' go to step 4.

Step 4: Improved Matrix

Select the smallest uncovered (by the lines) cost element. Subtract this element from all uncovered elements including itself and add this element to each value located at the intersection of any two lines. The cost elements through which only one-line passes remain unaltered.

Step 5: Repeat step 3 and 4 until an optimal solution is obtained

Step 6: Given the optimal solution, make the job assignments as indicated by the 'zero' elements. This is done as follows:

(a) Locate a row which contains only one zero element. Assign the job corresponding to this element to its corresponding person. Cross out the zero's if any in the column corresponding to the element, which is indicative of the fact that the particular job and person are no more available.

(b) Repeat (a) for each of such rows which contain only one zero. Similarly, perform the same operation in respect of each column containing only one 'zero' element, crossing out the zero(s), if any, in the row in which the elements lies.

(c) If there is no row or column with only a single 'zero' element left, then select a row/column arbitrarily and choose one of the jobs (or persons) and make the assignment. Thus in such a case, alternative solutions exist.

Illustration 25

Six men are available for different jobs. From past records the time in hours taken by different persons for different jobs are given below.

	JOBS						
		1	2	3	4	5	6
Men	1	2	9	2	7	9	1
	2	6	8	7	6	14	1
	3	4	6	5	3	8	1
	4	4	2	7	3	10	1
	5	5	3	9	5	12	1
	6	9	8	12	13	9	1

Find out an allocation of men to different jobs which will lead to minimum operation time.

Solution:

Job / Man	1	2	3	4	5	6
1	2	9	2	7	9	1
2	6	8	7	6	14	1
3	4	6	5	3	8	1
4	4	2	7	3	10	1
5	5	3	9	5	12	1
6	9	8	12	13	9	1

Row Operation* (Table - 1)

Job / Man	1	2	3	4	5	6
1	1	8	1	6	8	0
2	5	7	6	5	13	0
3	3	5	4	2	7	0
4	3	1	6	2	9	0
5	4	2	8	4	11	0
6	8	7	11	12	8	0

* Matrix is obtained by subtracting min. element of each row of the given Matrix from all the elements of the corresponding row.

Column Operation* (Table - 2)

Job / Man	1	2	3	4	5	6
1	0	7	0	4	1	0
2	4	6	5	3	6	0
3	2	4	3	0	0	0
4	2	0	5	0	2	0
5	3	1	7	2	4	0
6	7	6	10	10	1	0

*** Matrix is obtained by subtracting min. element of each column of Table - 1 from all the elements of the corresponding column.**

Table - 3

Job / Man	1	2	3	4	5	6
1	[0]	7	0	4	1	0
2	4	6	5	3	6	[0]
3	2	4	3	[0]	0	0
4	2	[0]	5	0	2	0
5	3	1	7	2	4	0
6	7	6	10	10	1	0

All the zeros obtained in Table - 2 are covered by minimum no. of horizontal and vertical straight lines and shown above. Here order of the given matrix = 6 and minimum no. of horizontal and vertical lines = 4.

As 4 ≠ 6, the solution is non optimal.

Table - 5

Table - 4

Job / Man	1	2	3	4	5	6
1	[0]	7	0	4	1	1
2	3	5	4	2	5	[0]
3	2	4	3	[0]	0	1
4	2	[0]	5	0	2	1
5	2	0	6	1	3	0
6	6	5	9	9	0	0

Above table is obtained by subtracting minimum uncovered element (2) of Table - 4 from all the uncovered elements and by adding the same to all the elements at the junction of the intersecting straight lines. Here minimum no. of horizontal or vertical straight lines to cover all the zeros = 6 = Order of the Matrix. So the solution is optimal.

Table - 6 Showing Optimum Solution - 1

Man \ Job	1	2	3	4	5	6
1	~~0~~	9	[0]	6	3	3
2	1	5	2	2	5	[0]
3	[0]	4	1	~~0~~	~~0~~	1
4	~~0~~	~~0~~	3	[0]	2	1
5	~~0~~	[0]	4	1	3	~~0~~
6	4	5	7	9	[0]	~~0~~

Table - 7 Showing Optimum Solution - 2

Man \ Job	1	2	3	4	5	6
1	~~0~~	9	[0]	6	3	3
2	1	5	2	2	5	[0]
3	~~0~~	4	1	[0]	~~0~~	1
4	[0]	~~0~~	3	~~0~~	2	1
5	~~0~~	[0]	4	1	3	~~0~~
6	4	5	7	9	[0]	~~0~~

Table - 8 Showing Optimum Solution - 3

Man \ Job	1	2	3	4	5	6
1	~~0~~	9	[0]	6	3	3
2	1	5	2	2	5	[0]
3	~~0~~	4	1	[0]	~~0~~	1
4	~~0~~	[0]	3	~~0~~	2	1
5	[0]	~~0~~	4	1	3	~~0~~
6	4	5	7	9	[0]	~~0~~

So the Optimal Assignments are as follows :–

	As per Table - 6		As per Table - 7			As per Table - 8		
Man	Job	Time (hrs.)	Man	Job	Time (hrs.)	Man	Job	Time (hrs.)
1	3	2	1	3	2	1	3	2
2	6	1	2	6	1	2	6	1
3	1	4	3	4	3	3	4	3
4	4	3	4	1	4	4	2	3
5	2	3	5	2	3	5	1	5

6	5	9	6	5	9	6	5	9
Total	-	22	**Total**	-	22	**Total**	-	22

Minimum total operation time = 22 hrs.

Illustration 26

A captain of a cricket team has to allot five middle batting positions to five batsmen. The average runs scored by each batsman at these positions are as follows:

		Batting Position			
	III	IV	V	VI	VII
Batsmen A	40	40	35	25	50
B	42	30	16	25	27
C	50	48	40	60	50
D	20	19	20	18	25
E	58	60	59	55	53

Make the assignment so that the expected total average runs scored by these batsmen are maximum.

Solution:

This is a problem of Maximisation. To solve it using Assignment technique it has to be converted to a Minimisation problem by forming a Relative Loss Matrix.

	Batting Position				
Batsman	**III**	**IV**	**V**	**VI**	**VII**
A	40	40	35	25	50
B	42	30	16	25	27
C	50	48	40	60	50
D	20	19	20	18	25
E	58	60	59	55	53

Relative Loss Matrix*

	Batting Position				
Batsman	**III**	**IV**	**V**	**VI**	**VII**
A	20	20	25	35	10
B	18	30	44	35	33
C	10	12	20	0	10
D	40	41	40	42	35
E	2	0	1	5	7

*** This matrix is formed by subtracting all the elements of the given matrix from the highest element (60) of it.**

Row Operation Matrix

	Batting Position				
Batsman	III	IV	V	VI	VII
A	10	10	15	25	0
B	0	12	26	17	15
C	10	12	20	0	10
D	5	6	5	7	0
E	2	0	1	5	7

Column Operation Matrix

Batting Position / Batsman	III	IV	V	VI	VII
A	10	10	14	25	[0]
B	[0]	12	25	17	15
C	10	12	19	[0]	10
D	5	[6]	4	7	0
E	2	0	0	5	7

Minimum no. of horizontal and vertical straight lines to cover all the zeros = 4 ≠ Order of the matrix(5). So the solution is non optimal.

Improved Matrix

	Batting Position				
Batsman	**III**	**IV**	**V**	**VI**	**VII**
A	10	6	10	25	[0]
B	[0]	8	23	17	15
C	10	8	15	[0]	10
D	5	2	[0]	7	~~0~~
E	6	[0]	~~0~~	9	11

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 5 = Order of the matrix. So the solution is optimal.

Batsman	Batting Position	Average runs scored
A	VII	50
B	III	42
C	VI	60
D	V	20
E	IV	60
	Total =	232

Expected maximum total runs = 232

Illustration 27

Average time taken by an operator on a specific machine is tabulated below. The management is considering replacing one of the old machines by a new one and the estimated time for operation by each operator on the new machine is also indicated.

Operator	MACHINE						
	M1	M2	M3	M4	M5	M6	New
01	2	3	2	1	4	5	6
02	4	4	6	3	2	5	1
03	6	10	8	4	7	6	1
04	8	7	6	5	3	9	4
05	7	3	4	5	4	3	12
06	5	5	6	7	8	1	6

(a)Find out an allocation of operators to the old machines to achieve a minimum operation time.

(b)Reset the problem with the new machine and find out the allocation of the operators to each machine and comment on whether it is advantageous to replace an old machine to achieve a reduction in operating time only.

(c)How will the operators be reallocated to the machines after replacement?

Solution:

Operator	Machines						
	M_1	M_2	M_3	M_4	M_5	M_6	New
01	2	3	2	1	4	5	6
02	4	4	6	3	2	5	1
03	6	10	8	4	7	6	1
04	8	7	6	5	3	9	4
05	7	3	4	5	4	3	12
06	5	5	6	7	8	1	6

Operator	Machines					
	M1	M2	M3	M4	M5	M6
01	2	3	2	1	4	5
02	4	4	6	3	2	5
03	6	10	8	4	7	6
04	8	7	6	5	3	9
05	7	3	4	5	4	3
06	5	5	6	7	8	1

Matrix after Row Operation

Operator	Machines					
	M1	M2	M3	M4	M5	M6
01	1	2	1	0	3	4
02	2	2	4	1	0	3
03	2	6	4	0	3	2
04	5	4	3	2	0	6
05	4	0	1	2	1	0
06	4	4	5	6	7	0

To find out the allocation of the Old Machines to the operators we consider the given matrix without the new machine.

Matrix after Column Operation

Operator	Machines					
	M1	M2	M3	M4	M5	M5
01	0	2	0	0	3	4
02	1	2	3	1	0	3
03	1	6	3	0	3	2
04	4	4	2	2	0	6
05	3	0	0	2	1	0
06	3	4	4	6	7	0

Minimum no. of horizontal and vertical straight lines to cover all the zeros = 5 ≠ order of the matrix (6). So the solution is non optimal.

Improved matrix

Operator	Machines					
	M1	M2	M3	M4	M5	M6
01	0	2	0	1	4	5
02	0	1	2	1	0	3
03	0	5	2	0	3	2
04	3	3	1	2	0	6
05	3	0	0	3	2	1
06	2	3	3	6	7	0

Minimum no. of horizontal and vertical straight lines to cover all the zeros = 6 = Order of the \matrix. So the solution is optimal.

Optimal Assignment

Operator					
	01	→	M3	-	2
	02	→	M1	-	4
	03	→	M4	-	4
	04	→	M5	-	3
	05	→	M2	-	3
	06	→	M6	-	1

(b) & (c)

Operator	Machines						
	M1	M2	M3	M4	M5	M6	New
01	2	3	2	1	4	5	6
02	4	4	6	3	2	5	1
03	6	10	8	4	7	6	1
04	8	7	6	5	3	9	4
05	7	3	4	5	4	3	12
06	5	5	6	7	8	1	6
Dummy	0	0	0	0	0	0	0

With the introduction of a new machine into the system, the problem becomes unbalanced one. To make it balanced, a Dummy operator is introduced and all the elements of the matrix corresponding to it are taken as zero.

(1) Matrix after Row Operation

Operator	Machines						
	M_1	M_2	M_3	M_4	M_5	M_6	New
01	1	2	1	0	3	4	5
02	3	3	5	2	1	4	0
03	5	9	7	3	6	5	0
04	5	4	3	2	0	6	1
05	4	0	1	2	1	0	9
06	4	4	5	6	7	0	5
Dummy	0	0	0	0	0	0	0

As all the columns contain zeros, the matrix after column operation will remain same. So the operation need not be done.

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 6 ≠ order of the matrix (7). So the solution is non optimal.

(2) Improved Matrix

Operator	Machines						
	M_1	M_2	M_3	M_4	M_5	M_6	New
01	0	1	0	0	3	4	5
02	2	2	4	2	1	4	0
03	4	8	6	3	6	5	0
04	4	3	2	2	0	6	1
05	4	0	1	3	2	1	10
06	3	3	4	6	7	0	5
Dummy	0	0	0	1	1	1	1

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 6 ≠ order of the matrix (7). So the solution is non optimal.

(3) Improved Matrix

Operator	Machines						
	M_1	M_2	M_3	M_4	M_5	M_6	New
01	0	2	0	0	4	5	6
02	1	2	3	1	1	4	0
03	3	8	5	2	6	5	0
04	3	3	1	1	0	6	1
05	3	0	0	2	2	1	10
06	2	3	3	5	7	0	5
Dummy	0	1	0	1	2	2	2

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 6 ≠ order of the matrix(7). So the solution is non optimal

(4) Improved Matrix Showing Optimal Solution (i)

Operator	Machines						
	M_1	M_2	M_3	M_4	M_5	M_6	New
01	0	2	0	0	5	6	7
02	0	1	2	0	1	4	0
03	2	7	4	1	6	5	0
04	2	2	0	0	0	6	1
05	3	0	0	2	3	2	11
06	1	2	2	4	7	0	5
Dummy	0	1	0	1	3	3	3

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 7 = order of the matrix. So the solution optimal.

Improved Matrix Showing Optimal Solution (ii)

Operator	Machines						
	M1	M2	M3	M4	M5	M6	New
01	0	2	0	0	5	6	7
02	0	1	2	0	1	4	0
03	2	7	4	1	6	5	0
04	2	2	0	0	0	6	1
05	3	0	0	2	3	2	11

06	1	2	2	4	7	0	5
Dummy	0	1	0	1	3	3	3

Improved Matrix Showing Optimal Solution (iii)

Operator	Machines						
	M1	M2	M3	M4	M5	M6	New
01	0	2	0	0	5	6	7
02	0	1	2	0	1	4	0
03	2	7	4	1	6	5	0
04	2	2	0	0	0	6	1
05	3	0	0	2	3	2	11
06	1	2	2	4	7	0	5
Dummy	0	1	0	1	3	3	3

Table Showing Multiple Optimum Allocations

Solution (i)			Solution (ii)			Solution (iii)		
Operators	M/C	Time (Hrs.)	Operators	M/C	Time (Hrs.)	Operators	M/C	Time (Hrs.)
01	M_1	2	01	M_3	5	01	M_4	1
02	M_4	3	02	M_4	1	02	M_1	4
03	New	1	03	New	6	03	New	1
04	M_5	3	04	M_5	0	04	M_5	3
05	M_2	3	05	M_2	3	05	M_2	3
06	M_6	1	06	M_6	7	06	M_6	1
Dummy	M_3	0	Dummy	M_1	3	Dummy	M_3	0
Total	-	13*	**Total**	-	13*	**Total**	-	13*

*** Minimum Operation Time**

From above it can be said that replacement of an old machine with the new one will result in a reduction in Total Operating Time by 17-13 = 4 Hours. So replacement decision is advantageous.

As per solutions (i) & (iii) above, Machine M3 should be replaed by a New Machine and as per Solution (iii), M1 should be replaced by a New one.

Illustration 28

Six salesmen are to be allocated to six sales regions so that the cost of allocation of the job will be minimum. Each salesman is capable of doing the job at different cost in each region. The cost matrix is given below:

		Region					
		I	II	III	IV	V	VI
Salesmen	A	15	35	0	25	10	45
	B	40	5	45	20	15	20
	C	25	60	10	65	25	10
	D	25	20	35	10	25	60
	E	30	70	40	5	40	50
	F	10	25	30	40	50	15

(Figures are in Rupees)

(a) Find the allocation to give minimum cost. What is the minimum cost?
(b) Now suppose the above table gives earning of each salesman at each region. How can you find an allocation so that the earning will be maximum? Determine the solution with optimum earning.
(c) There are restrictions for commercial reasons that A cannot be posted to region V and E cannot be posted to region II. Write down the cost matrix suitably after imposing the restrictions.

Solutions.

Salesman	Region					
	I	II	III	IV	V	VI
A	15	35	0	25	10	45
B	40	5	45	20	15	20
C	25	60	10	65	25	10
D	25	20	35	10	25	60
E	30	70	40	5	40	50
F	10	25	30	40	50	15

Matrix after Row Operation

Salesman	Region					
	I	II	III	IV	V	VI
A	15	35	0	25	10	45
B	35	0	40	15	10	15
C	15	50	0	55	15	0
D	15	10	25	0	15	50
E	25	65	35	0	35	45
F	0	15	20	30	40	5

Matrix after Column Operation

Salesman	Region					
	I	II	III	IV	V	VI
A	15	35	0	25	10	45
B	35	0	40	15	0	15
C	15	50	0	55	5	0
D	15	10	25	0	5	50
E	25	65	35	0	25	45
F	0	15	20	30	30	5

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 5 ≠ Order of the matrix (6). So the solution is non optimal.

Improved Matrix (Optimal)

Salesman	Region					
	I	II	III	IV	V	VI
A	20	35	0	30	0	45
B	40	0	40	20	0	15
C	20	50	0	60	5	0
D	15	5	20	0	0	45
E	25	60	30	0	20	40
F	0	10	15	30	30	0

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 6 = Order of the matrix. So the solution is optimal.

Table showing optimal allocation

Table showing optimal allocation

Salesman	Region	Cost (₹)
A	III	0
B	II	5
C	VI	10
D	V	25
E	IV	5
F	I	10
Total		₹ 55

Minimum Cost

(b) The given problem is a problem of Maximisation. To convert it to a problem of Minimisation, a Relative Loss Matrix is formed by subtracting all the elements of the given matrix from the highest element (70).

Relative Loss Matrix

Salesman	Region					
	I	II	III	IV	V	VI
A	55	35	70	45	60	25
B	30	65	25	50	55	50
C	45	10	60	5	45	60
D	45	50	35	60	45	10
E	40	0	30	65	30	20
F	60	45	40	30	20	55

Matrix after Row Operation

Salesman	Region					
	I	II	III	IV	V	VI
A	30	10	45	20	35	0
B	5	40	0	25	30	25
C	40	5	55	0	40	55
D	35	40	25	50	35	0
E	40	0	30	65	30	20
F	40	25	20	10	0	35

Matrix after Column Operation

Salesman	Region					
	I	II	III	IV	V	VI
A	25	10	45	20	35	0
B	0	40	0	25	30	25
C	35	5	55	0	40	55
D	30	40	25	50	35	0
E	35	0	30	65	30	20
F	35	25	20	10	0	35

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 5 ≠ Order of the matrix (6). So the solution is non optimal.

Improved Matrix

Salesman	Region					
	I	II	III	IV	V	VI
A	5	10	25	20	35	0
B	0	60	0	15	50	45
C	15	5	35	0	40	55
D	10	40	5	50	35	0
E	15	0	10	65	30	20
F	15	25	0	10	0	35

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 5 ≠ Order of the matrix (6). So the solution is optimal.

Improved Matrix (Optimal)

Salesman	Region					
	I	II	III	IV	V	VI
A	[0]	10	20	20	30	X
B	X	65	[0]	50	50	50
C	10	5	30	[0]	35	55
D	5	40	X	50	30	[0]
E	10	[0]	5	65	25	20
F	15	30	X	15	[0]	40

Table Showing Optimal Allocation

Salesman	Region	Earning (₹)
A	I	15
B	III	45
C	IV	65
D	VI	60
E	II	70
F	V	50
Total		**₹ 305**

Maximum Earning

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 6 = Order of the matrix. So the solution is optimal.

(c) The cost matrix after imposing the given restriction is

		Region					
		I	II	III	IV	V	VI
Sales man	A	15	35	0	25	α	45
	B	40	5	45	20	15	10
	C	25	60	10	65	25	10
	D	25	20	35	10	25	60
	E	30	α	40	5	40	50
	F	10	25	30	40	50	15

Cost (figures are in ₹)

(Whenever such restrictions are imposed, we have to consider the corresponding element of the given matrix as infinitely large i.e. a)

Illustration 29

Four jobs can be processed on four different machines, with one job on one machine. Resulting profits vary with assignments. They are given below:

		Machines			
		A	B	C	D
Jobs	I	42	35	28	21
	II	30	25	20	15
	III	30	25	20	15
	IV	24	20	16	12

Find the optimum assignment of jobs to machines and the corresponding profit.

Solution:

Relative Loss Matrix

Jobs \ M/cs	A	B	C	D
I	0	7	14	21
II	12	17	22	27
III	12	17	22	27
IV	18	22	26	30

As this is a problem of Maximisation, the same is converted to one of Minimisation by firming a Relative Loss Matrix where all the elements of the given matrix are subtracted from the highest element of the matrix (which is 42 in this case)

Matrix after Row Operation

Jobs \ M/cs	A	B	C	D
I	0	7	14	21
II	0	5	10	15
III	0	5	10	15
IV	0	4	8	12

Matrix after Column Operation

Jobs \ M/cs	A	B	C	D
I	0	3	6	9
II	0	1	2	3
III	0	1	2	3
IV	0	0	0	0

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 2 ≠ Order of the matrix (4)

So the solution is non optimal.

Improved Matrix (Non Optimal)

Jobs \ M/cs	A	B	C	D
I	0	2	5	8
II	0	0	1	2
III	0	0	1	2
IV	1	0	0	0

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 3 ≠ Order of the matrix (4)

So the solution is non optimal.

Further Improved Matrix [Optimal Solution (i)]

Jobs \ M/cs	A	B	C	D
I	[0]	2	4	7
II	~~0~~	[0]	~~0~~	1
III	~~0~~	~~0~~	[0]	1
IV	2	1	~~0~~	[0]

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 4 = Order of the matrix.

So the solution is optimal.

Further Improved Matrix (Optimal Solution-ii)

Jobs \ M/cs	A	B	C	D
I	[0]	2	4	7
II	~~0~~	~~0~~	[0]	1
III	~~0~~	[0]	~~0~~	1
IV	2	1	~~0~~	[0]

Assignment as per Soution (i)			Assignment as per Soution (ii)		
Jobs	M/cs	Profit (₹)	Jobs	M/cs	Profit (₹)
I	A	42	I	A	42
II	B	25	II	C	20
III	C	20	III	B	25
IV	D	12	IV	D	12
Total	–	₹ 99	Total	–	₹ 99

Maximum Profit ₹ 99

Illustration 30

A salesman has to visit five cities A, B, C, D and E. The inter-city distances are tabulated below. Note the distance between two cities need not be same both ways.

From / To	A	B	C	D	E
A	-	12	24	25	15
B	6	--	16	18	7
C	10	11	--	18	12
D	14	17	22	--	16
E	12	13	23	25	--

Note further that the distances are in km.

Required:

If the salesman starts from city A and has to come back to city A, which route would you advise him to take so that total distance travelled by him is minimised?

Solutions.

FROM \ TO	A	B	C	D	E
A	-	12	24	25	15
B	6	-	16	18	7
C	10	11	-	18	12
D	14	17	22	-	16
E	12	13	23	25	-

Row Operation*

FROM \ TO	A	B	C	D	E
A	-	0	12	13	3
B	0	-	10	12	1
C	0	1	-	8	2
D	0	3	8	-	2
E	0	1	11	13	-

*** This matrix is obtained by subtracting minimum element of each row of the given matrix from all the elements of the corresponding row**

Column Operation*

(Table - 2)

From \ To	A	B	C	D	E
A	-	0	4	5	2
B	0	–	2	4	0
C	0	1	-	0	1
D	0	3	0	–	1
E	0	1	3	5	–

* This matrix is obtained by subtracting minimum element of each column of Table-1 from all the elements of the corresponding column.

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 5 = Order of the matrix.

So the solution is optimal.

Now the solution obtained from the above table shows the travel route of the salesman as A to B, B to E, E to A which means the person is not visiting C and D at all while travelling back to A.

But this is not allowed as per the question.

So the matrix of Table-2 is examined for some of the next best solution which is depicted below

From \ To	A	B	C	D	E
A	–	[0]	4	5	2
B	~~0~~	–	2	4	[0]
C	[0]	1	–	~~0~~	1
D	~~0~~	3	[0]	–	1
E	~~0~~	1	3	[5]	–

Here the assignments have been started by encircling only zero present in the first row which means initial travel route A to B.

Then the only zero present in the last column is encircled which shows subsequent route B to E. Next the only zero of the last row is not encircled because in that case the route would have been E to A which is restricted as per the given condition. So that element of this row is considered which satisfies the restriction. It is 5 indicating the route as E to D. Next the only zero of 3rd column is encircled which means the route as D to C. Finally the only zero row present in the 3rd row is encircled which shows the route as C to A.

Hence the complete route of the Salesman is : A → B → E → D → C → A Total distance travelled = 12 + 7 + 25 + 22 + 10 = 76 Kms.

This is the optimum distance.

QUESTIONS.34
Define Scheduling?

ANSWER.
Scheduling: 'Scheduling' is the next important function of production planning and control after 'Routing'. It determines the starting and the completion timings for each of the operations with a view to engage every machine and operator of the system for the maximum possible time and; without imposing unnecessary burden over them. Scheduling is the determination of the time that should be inquired to perform each operation and also the time that should be required to perform the entire series as routed. Scheduling involves establishing the amount of work to be done and the time when each element of the work will start or the order of the work.
Scheduling technique is an important technique of determining the starting and the completion timings of each operation and that of the total manufacturing process so that the man and machines can be utilised to the maximum.
Scheduling depends upon a number of factors, e.g., routing, the method of production, quantity of production, transportation of raw materials, production capacity, the probable data of delivery specified by customers in their orders and the past records.

QUESTIONS.35
Explain Relationship between Routing and Scheduling?

ANSWER.
'Routing' and 'Scheduling' are interconnected and either of these activities cannot be undertaken independently. It is very difficult to prepare schedules without determining the routing of sequence of operations. Routing is a prerequisite for scheduling while time to be taken 'may form the basis of routing and that is fixed by scheduling. Unless route or sequence of operations, tools, equipment and plants and the persons by when operations are to be performed, are established, the time taken by each operation, the idle time of men and machine and total time for the whole process cannot be ascertained in a convincing manner.
Conversely, scheduling is equally important for routing. It is quite difficult to route an item efficiently through a plant without consulting previously-designed schedules. The main aim of routing is to pass the item through the process of manufacture by a route which is the best and the most economical. And a route or sequence of operations may be considered best which utilises the men, materials and machines to the maximum and which consumes the shortest time during the process of production. This information (time schedule of each operation) can be obtained from schedules. So, scheduling is necessary for effective routing.
Thus, we can conclude that routing and scheduling are inter-related, inter-connected and inter-dependent activities of production planning and control.

QUESTION.36

State the Principles of Scheduling:

ANSWER.

The principles of scheduling are:

(a) The principle of optimum task size: Scheduling tends to achieve its maximum efficiency when the task sizes are small and all tasks are of the same order of magnitude.

(b) The principle of the optimum Production plan: Scheduling tends to achieve its maximum efficiency when the work is planned, so that it imposes an equal/even load on all the plants/facilities.

(c) The principle of the optimum operation sequence: Scheduling tends to achieve its maximum efficiency when the work is planned so that the work centres are normally used in the same sequence.

The first principle has a tendency when applied, not only give good results but also to be self-correcting if it is ignored. For example, if in a functional batch production machine shop the loads imposed by different operations vary greatly in length then it is possible that it will be necessary to break many of the long operations into one or more small batches, in order to get the other orders completed by due date. In effect, this principle only repeats the known advantage of maintaining a high rate of stock turn over, and of single-phase ordering. The second principle merely states the obvious fact that there will be less idle time and waiting time, if all the plant is evenly loaded by the production planners, even if some of the machines are over loaded perhaps because of direct labour cost on them being lower and others are idle for part of the time due to shortage of work. The third principle says about principle of flow. Sometimes it is also true if we sequence some jobs, which need the same machine set up, at a time, this avoids machine ancillary time needed, in case, the jobs of the above type are done at different times. For example, consider drilling a 10 mm hole in five different jobs may be done at a time so that the set-up time required for five jobs can be reduced.

QUESTION.37

Different types of Forms of Schedules explain?

ANSWER.

Forms of Schedules:

Here we shall discuss the presentation of production schedules. Depending on the need and use, the Schedules can be prepared in different forms.

A Production Flow Program:

If a number of components or assemblies have to be manufactured for the final assembly line and those components are to be made concurrently, the production master flow program is prepared taking into account the sequence of operations and the time of starting and ending of each component in order to comply with the required date of completion of the product. The necessary document for this is Operation Process Chart and the Sequence of Operation.

Scheduling Systems:

Scheduling Systems may be classified into various groups as shown below:

(i) Unit scheduling system: This is used for scheduling when jobs are produced one by one and are of different types that is for job production.

(ii) Batch scheduling system: When jobs are produced to order, in batches, this is used.

(iii) Mass scheduling system: When large number of items of similar type are produced that is in mass production, this is used.

Unit Scheduling System:

Here we have two types of scheduling, one is Project scheduling and the other is Job Shop Scheduling.

Project Scheduling: Generally, a project consists of number of activities managed by different Apartments or individual supervisors. It can also be considered as a complex output made up of many interdependent activities. Examples are: Railway coach building, Shipbuilding etc. The scheduling methods used are:

(i) Project Evaluation and Review Technique (PERT),

(ii) Critical Path Method (CPM),

(iii) Graphical Evaluation and Review Technique (GERT). We can also use Bar charts, GANTT charts, Milestone chart, but these are less superior to the above.

Job Shop Scheduling:

In Job shop scheduling, we come across varieties of jobs to be processed on different types of machines. Separate records are to be maintained for each order. Only after receiving the order, one has to plan for production of the job. The routing is to be specified only after taking the order. Scheduling is done to see that the available resources are used optimally. The following are some of the factors taken into consideration for job shop scheduling. (i) Arrival pattern of the job, (ii) Processing pattern of the job, (iii) Depending on the type of machine used, (iv) Number of workers available in the shop, (v) Order of sequencing.

Arrival pattern of the job:

This is done in two ways. Firstly, as and when the order is received, it is processed on the principle of First in First Out (FIFO). Otherwise, if the orders are received from single customer at different points of time in a week/ month, then the production manager pile up all the orders and starts production depending on the delivery date and convenience (This situation is generally known as static situation)

Processing Pattern of the Job:

As the layout of Job shops is of Process type and there may be duplication of certain machines, the production planner, after receiving the order thinks of the various methods of converting the requirement of customer / order into a production plan to suit the available facilities. Depending on the process required, there may be backtracking, which is unavoidable. When facilities are busily engaged, in process inventory may be a common problem.

Machine varieties available:

Facilities available in the production shop will affect the scheduling. Here the size, capacity, precession and other factors of machines will have their influence on the scheduling

Number of Men in the production shop:

Many a time we see that the number of workers available in the job shops are very much limited, that is sometimes they are less in number than the machines in the shop (these shops are known as labour limited shop). Depending on the availability of labour, the scheduling is to be done. In case the machines available are limited and have more men (known as machine limited shops), then availability of machine dictates the scheduling.

Sequencing rules for single facility:

When we have a single facility, and the orders are in queue, then they are processed depending on the rules mentioned below:

(a) First in first served or first in first out (FIFS/FIFO): Here the jobs are processed as they come in. This commonly observes queue discipline.

(b) Shortest processing time (SPT): The jobs having shortest processing time are processed first. This is just to avoid formation of queue. For example, when you go for Xeroxing a document, and other person comes for Xeroxing a book, then the document is Xeroxed first and subsequently the book is taken up for Xeroxing.

(c) Minimum due date (MDD): Here jobs are processed in ascending order of their available time before delivery date. By doing so, we can keep up the delivery promises. To meet the delivery promises, if necessary, overtime, sub-contracting etc., may be used.

(d) Last come first served or last in first out (LCFS/LIFO): This generally happens in case of inventory stocking and using. When material piles up, the material at the top i.e., material last arrived is used first.

(e) Static slack for remaining operations (SSRO): Static slack is given by: (Time till due date - Remaining processing time/number of remaining operations). Here jobs are processed in ascending order of the operations.

(f) Dynamic slack for remaining operations (DSRO): Dynamic slack is given by: (Time till due date - expected time of remaining operations / number of remaining operations). Here the jobs are done in ascending order of the ratio dynamic slack.

Basic Scheduling Problems:

The production planner may face certain problems while preparing production plans or Schedules. Some important problems are discussed below:

(a) Flow production scheduling for fluctuating demand (known smoothening problem),

(b) Batch production scheduling, when products are manufactured consecutively,

(c) The assignment problem,

(d) Scheduling orders with random arrivals and

(e) Product sequencing

Illustration 31

The processing times (t_j) in hrs for the five jobs of a single machine scheduling is given. Find the optimal sequence which will minimise the mean flow time and find the mean flow time

Determine the sequence which will minimise the weighted mean flow time and also find the mean flow time

Job (j)	1	2	3	4	5
Processing time (t_j) hrs	30	8	10	28	16
Weight (Wj)	1	2	1	2	3

Solution:

(a) First arrange the jobs as per the shortest processing time (SPT) sequence

Job (j)	2	3	5	4	1
Processing time (t_j) hrs	8	10	16	28	30

Therefore, the job sequence that minimises the mean flow time is 2-3-5-4-1.

Computation of minimum flow time (F min)

The flow time is the amount of time the job 'j' spends in the system. It is a measure which indicates the waiting of jobs in the system. It is the difference between the completion time (C_j) and ready time (R_j) for job j.

$F_j = C_j - R_j$

Job (j)	2	3	5	4	1
Processing time (t_j) hrs	8	10	16	28	30
Completion time (C_j)	8	18	34	62	92

Since the ready time (R_j) = 0 for all j, the flow time ü$_j$ is equal to C_j for all j.

$$\text{Mean flow time} = (\overline{F}) = \frac{1}{n}\sum_{j=1}^{n} F_j = \frac{1}{5}[8+18+34+62+92] = \frac{1}{5}[214] = 42.8 \text{ hours}$$

(b) The weights are given as follows

Job (j)	1	2	3	4	5
Processing time (t_j) hrs	30	8	10	28	16
Weight (W_j)	1	2	1	2	3

The weighted processing time= $\frac{\text{Processing time (tj)}}{\text{Weight (Wj)}}$

The weighted processing time is represented as

Job (j)	1	2	3	4	5
Processing time (t_j hrs)	30	8	10	28	16
Weight (W_j)	1	2	1	2	3
Weighted Processing time (t_j/W_j)	30	4	10	14	5.31

Thus, arranging the jobs in the increasing order of tj /Wj (weighted shortest processing time WSPT) we have

Job (j)	2	5	3	4	1
Weighted Processing line (t_j/W_j)	4	5.31	10	14	30

optimal sequence that minimises the weighted mean flow time is 2-5-3-4 -1.

$$\text{Weighted Mean flow time } (\overline{F}_w) : \overline{F}_w = \frac{\sum_{j=1}^{n} W_j F_j}{\sum_{j=1}^{n} W_j}$$

Job (j)	2	5	3	4	1
Processing time (t_j) hrs	8	16	10	28	30
$F_j = (C_j - R_j)$	8	24	34	62	92
Wj	2	3	1	2	1
$F_j \times W_j$	16	72	34	124	92

The weighted mean flow time is computed as follows for optimal sequence.

Weighted mean flow time ü$_w$ is computed as

$$\bar{F}_w = \frac{(16+72+34+124+92)}{(2+3+1+2+1)} = 37.55 \text{ hrs.}$$

QUESTION.38

Explain Elements of Scheduling?

ANSWER.

(i) Demand forecasts/customer's firm orders-determine the delivery dates for finished products.

(ii) Aggregate scheduling: Tentative schedule based on demand for quarterly or monthly requirements. Enables employment of available resources in meeting the demand by adjusting the capacity. Needs rough-cut capacity planning.

(iii) Production plan: Showing output levels planned, resource requirements, and capacity limitations and inventory levels.

(iv) Master production schedule: Dates committed and desired quantity to be produced on a daily, weekly, monthly or quarterly basis.

(v) Priority planning: Master schedule is exploded into components and parts that are required to produce the product.

(vi) Capacity planning: Regulates loading of specific jobs on specific work centres or machines for specific periods of time.

(vii) Facility loading or machine loading: Loading work centres/Machines after deciding which job to be assigned to which work centre/machine i.e., actual assignment of jobs to machines taking into consideration priority sequencing and machine utilisation.

(viii) Evaluation of workload: To balance the workload on various work centres /machines when resources are scarce or limited. Excess load in one work centre or machine has to be transferred to other work centre or machine having spare capacity.

(ix) Sequencing: Priority sequencing of jobs is done to maximise workflow through work-centres or machines to minimise delay and cost of production.

Queuing models

General Structure of Queuing System The general structure of a queuing system is depicted in Figure 4.3

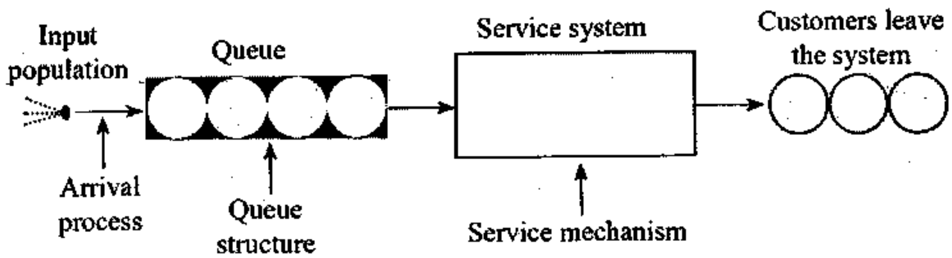


Figure 4.3: General Structure of the Queuing System

QUESTIONS.39

Write Down the different types of,The elements of a system?

ANSWER.

The elements of a system are:

1. Arrival process:

The arrivals from the input population may be classified on different basis as follows:

(a) According to source:

The source of customers for a queuing system can be infinite or finite. For example, all people of a city or state (and others) could be the potential customers at a super bazar. The number of people being very large, it can be taken to be infinite. On the other hand, there are many situations in business and industrial conditions where we cannot consider the population to be infinite – it is finite. Thus, the ten machines in a factory requiring repairs and maintenance by the maintenance crew would exemplify finite population. Removing one machine from a small, finite, population like this will have a noticeable effect on the calls expected to be made (for repairing) by the remaining machines than if there are a large number of machines, say 500.

(b) According to numbers:

The customers may arrive for service individually or in groups. Single arrivals are illustrated by customers visiting a beautician, students reaching at a library counter, and so on. On the other hand, families visiting restaurants, ships discharging cargo at a dock are examples of bulk, or batch, arrivals.

(c) According to line:

Customers may arrive in the system at known (regular or otherwise)'times, or they might arrive in a random way. The queuing models wherein customers' arrival times are known with certainty are categorized as deterministic models (insofar as this characteristic is concerned) and are easier to handle. On the other hand, a substantial majority of the queuing models are based on the premise that the customers enter the system stochastically, at random points in time.

With random arrivals, the number of customers reaching the system per unit time might be described by a probability distribution. Although the arrivals might follow any pattern, the frequently employed assumption, which adequately supports many real world situations, is that the arrivals follow Poisson distribution.

2. Service system:

There are two aspects of a service system–

(a) structure of the service system, and

(b) the speed of service.

(a) Structure of the service system:

By structure of the service system we mean how the service facilities exist. There are several possibilities. For example, there may be

(i) A single service facility A library counter is an example of this. The models that involve a single service facility are called single server models. Figure 4.2(a) illustrates such a model.

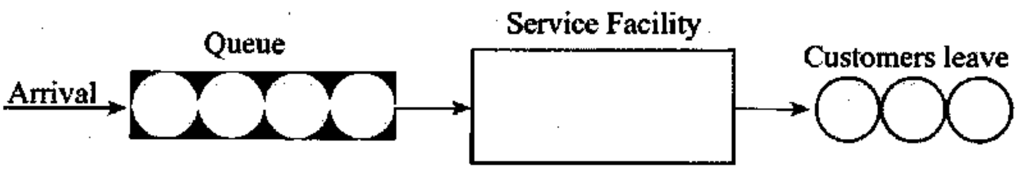


Figure 4.2 (a): Single Server, Single Queue Model

(ii) Multiple, parallel facilities with single queue:

That indicates there is more than one server. The term parallel implies that each server provides the same type of facility. Booking at a service station that has several mechanics, each handling one vehicle, illustrates this type of model. It is shown in Figure 4.2 (b).

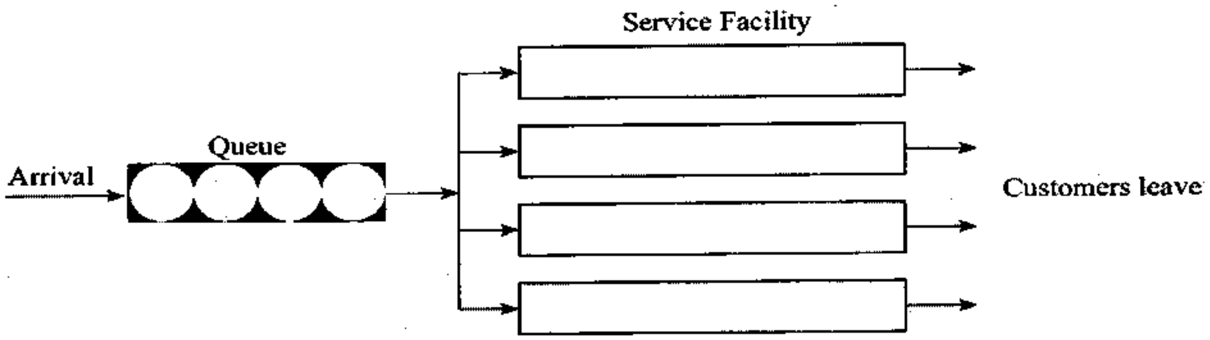


Figure 4.2 (b): Multiple, Parallel Servers, Single Queue Model

(iii) Multiple, parallel facilities with multiple queues:

This type of model is different from the earlier one only in that each of the servers has a different queue. Different cash counters in an electricity office where the customers can make payment in respect of their electricity bills is an example of this type of model. Figure 4.2 (c) portrays such a model.

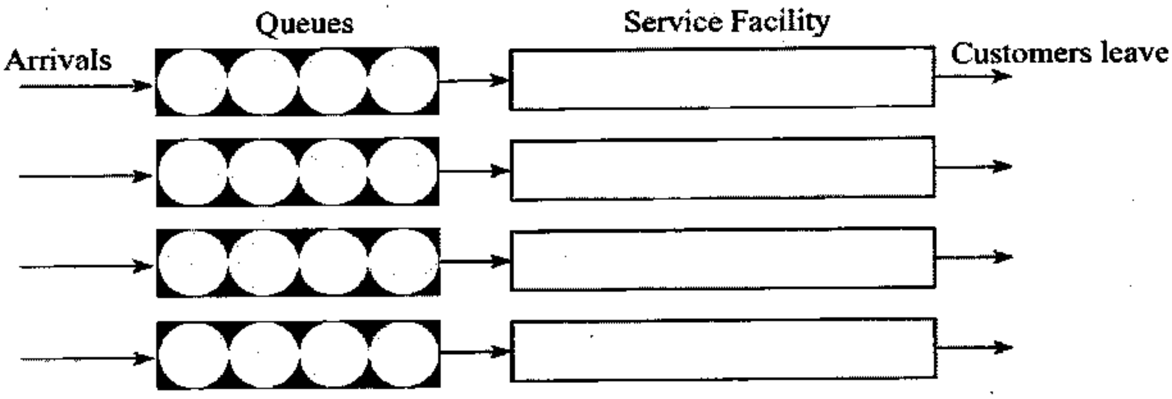


Figure 4.2 (c): Multiple, Parallel Servers, Multiple Queues Model

(iv) Service facilities in a series:

In this, a customer enters the first station and gets a portion of service and then moves on to the next station, gets some service and then again moves on to the next station . . . and so on, and finally leaves the system, having received the complete service. For example, machining of a certain steel item may consist-of cutting, turning, knurling, drilling, grinding, and packaging operations, each of which is performed by a single server in a series. Figure 4.2 (d) shows such a situation.

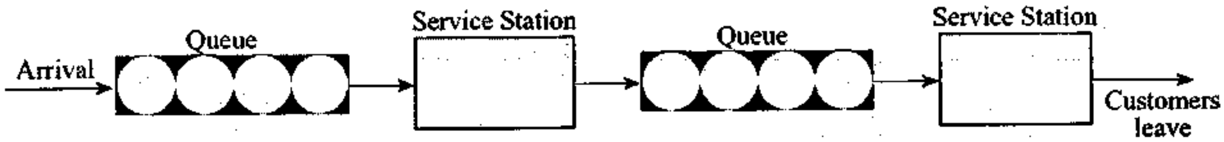


Figure 4.2 (d): Multiple Servers in Series

Besides these, there may be other possibilities as well.

(b) Speed of service:
In a queuing system, the speed with which service is provided can be expressed in either of two ways– as service rate and as service time. The service rate describes the number of customers serviced during a particular time period. The service time indicates the amount of time needed to service a customer. Service rates and times are reciprocals of each other and either of them is sufficient to indicate the capacity of the facility. Thus, if a cashier can attend, on the average, to 10 customers in an hour, the service rate would be expressed as 10 customers/hour and service time would be equal to 6 minutes/ customer, Generally, however, we consider the service time only, If these service times are known exactly, the problem can be handled easily. But, as generally happens, if these are different and not known with certainty, then we have to consider probabilities the distribution of the service times in order to analyse the queuing system. Generally, the queuing models are based on the assumption that service times are exponentially distributed about some average service time.

3. Queue structure:
Another element of a queuing system is the queue structure. In the queue structure, the important thing to know is the queue discipline which means the order by which customers are picked up from the waiting line for service. There are a number of possibilities. They are:

(a) First-come-first-served: When the order of service of customers is in the order of their arrival, the queue discipline is of the first-come-first-served type. For example, with a queue at the bus stop, the people who came first will board the bus first.

(b) Last-come-first-served: Sometimes, the customers are serviced in an order reverse of the order in which they enter so that the ones who join last are served first. For example, assume that letters to be typed, or order forms to be processed accumulate in a pile, with each new addition being put on the top of them. The typist or the clerk might process these letters or orders by taking each new task from the top of the pile. Thus, a just arriving task would be the next to be serviced provided that no fresh task arrives before it is picked up. Similarly, the people who join an elevator last are the first ones to leave it.

(c) Service-in-random-order (SIRO): Random order of service is defined as: whenever a customer is chosen for service, the selection is made in a way that every customer in the queue is equally likely to be selected. The time of arrival of the customers is, therefore, of no consequence in such a case.

(d) Priority service: The customers in a queue might be rendered service on a priority basis. Thus, customers may be called according to some identifiable characteristic (length of job, for example) for service. Treatment of VIPs in preference to other patients in a hospital is an example in point. For the queuing models that we shall consider, the assumption would be that the customers are serviced on the first-come-first-served basis.

Another thing to consider in the queuing structure is the behaviour or attitude of the customers entering the queuing system. On this basis, the customers may be classified as being (a) patient or (b) impatient. If the customers join a queue, when it exists, and wait till they enter the service station for getting service they are called patient customers. On the other hand, the queuing systems may enjoy customer behaviour in the form of defections from the queue. The customers may not select queues randomly (if there are multiple queues) and look for the shortest queue. There may be jockeying among the many queues, that is the customers may switch to other queues which are moving 'fast', and also reneging is possible—when a customer stands in the queue for some time and then leaves the system because it is working 'too slowly'. There may also be bribing or cheating by some customers for queue positions. Besides, some customers may, upon their arrival, not join the queue for some reason and decide to return for service at a later time, or may even abandon the input population altogether. In terms of the queuing theory, this is known as balking, and occurs particularly when there are limits on the time and the extent of storage capacity available to hold waiting customers. Unless otherwise specified, the storage capacity is taken to be infinite. In the queuing models that we consider, we shall assume that there is no balking or jockeying and that the customers leave the system only after receiving service, and not before. Mathematical models give way to simulation when this assumption breaks.

QUESTION.40

What are the Operating Characteristics of Queuing System ?

ANSWER.

An analysis of a given queuing system involves a study of its different operating characteristics. This is done using queuing models. Some of the more commonly considered characteristics are discussed below

1. Queue length – the average number of customers in the queue waiting to get service. Large queues may indicate poor server performance while small queues may imply too much server capacity.
2. System length – the average number of customers in the system, those waiting to be and those being serviced. Large values of this statistic imply congestion and possible customer dissatisfaction and a potential need for greater service capacity,
3. Waiting time in the queue – the average time that a customer has to wait in the queue to get service. Long waiting times are directly related to customer dissatisfaction and potential loss of future revenues, while very small waiting times may indicate too much service capacity.
4. Total time in the system – the average time that a customer spends in the system, from entry in the queue to completion of service. Large values of this statistic are indicative of the need to make adjustment in the capacity.
5. Server idle time – the relative frequency with which the service system is idle, Idle time is directly related to cost. However, reducing idle time may have adverse effects on the other characteristics mentioned above.

We now proceed to discuss some of the queuing models. It may be mentioned here that the results obtained from various models are based on the assumption that the service system is operating under equilibrium or steady state conditions. For many systems, the operating day begins in transient state with no customers in the system. It takes some initial time interval for enough customers to arrive such that a steady state balance is reached. It should be clearly understood that a steady state does not mean that the system will reach a point where the number of customers in the system never changes. Even when the system reaches equilibrium, fluctuations will occur. A steady state condition really implies that various system performance measures (the operating characteristics) would reach stable values

QUESTIONS. 41

Explain the Characteristics of Waiting Lines?

ANSWER.

There are numerous queuing models from which an analyst can choose. Naturally, much of the success of the analysis will depend on choosing an appropriate model. Model choice is affected by the characteristics of the system under investigation. The main characteristics are–

1. Population source.
2. Number of servers (channels)
3. Arrival and service patterns.
4. Queue discipline (order of service).

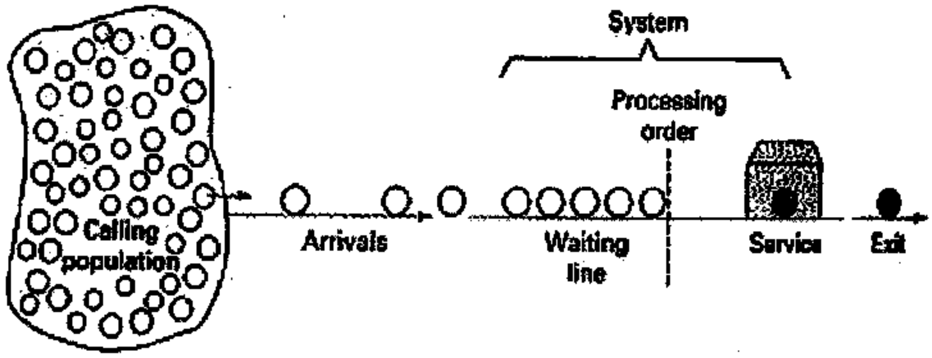


Figure 4.3: A simple queuing system

Population source;-

The approach to use in analyzing a queuing problem depends on whether the potential number of customers is limited. There are two possibilities: infinite-source and finite-source populations. In an infinite-source situation, the potential number of customers greatly exceeds system capacity. Infinite-source situations exist whenever service is unrestricted. Examples are supermarkets, drugstores, banks, restaurants, theatres, amusement centres, and toll bridges. Theoretically, large numbers of customers from the "calling population" can request service at any time. When the potential number of customers is limited, a finite-source situation exists. An example is the repairman responsible for a certain number of machines in a company. The potential number of machines that time cannot exceed the number of machines assigned to be repairer. Similarly, an operator may be responsible for loading and unloading a bank of four machines, a nurse may be responsible for answering patient calls for a 10-bed ward, a secretary may be responsible for taking dictation from three executives, and a company shop may perform repairs as needed on the firm's 20 trucks.

Number of servers (Channels); -

Channel indicates a server in a service system: The capacity of queuing systems is a function of a capacity of each server and the number of servers being used. The terms server and channel are synonymous, and it is generally assumed that each channel can handle one customer at a time. Systems can be either single-or multiple-channel. (A group of servers working together as a team, such as a surgical team, is treated as a single-channel system.) Examples of single-channel systems are small grocery stores with one checkout counter, some theatres, single-bay car washes, and drive-in banks with one teller. Multiple-channel systems (those with more than one server) are commonly found in banks, at airline ticket counters, at auto service centres, and at petrol pumps

. A related distinction is the number of steps or phases in a queuing system. For example, at theme parks, people go from one attraction to another. Each attraction constitutes a separate phase where queues can (and usually do) from. Figure 4.5 illustrates some of the most common queuing systems. Because it would not be possible to cover all of these cases in sufficient detail in the limited amount of space available here, our discussion will focus on single-phase systems

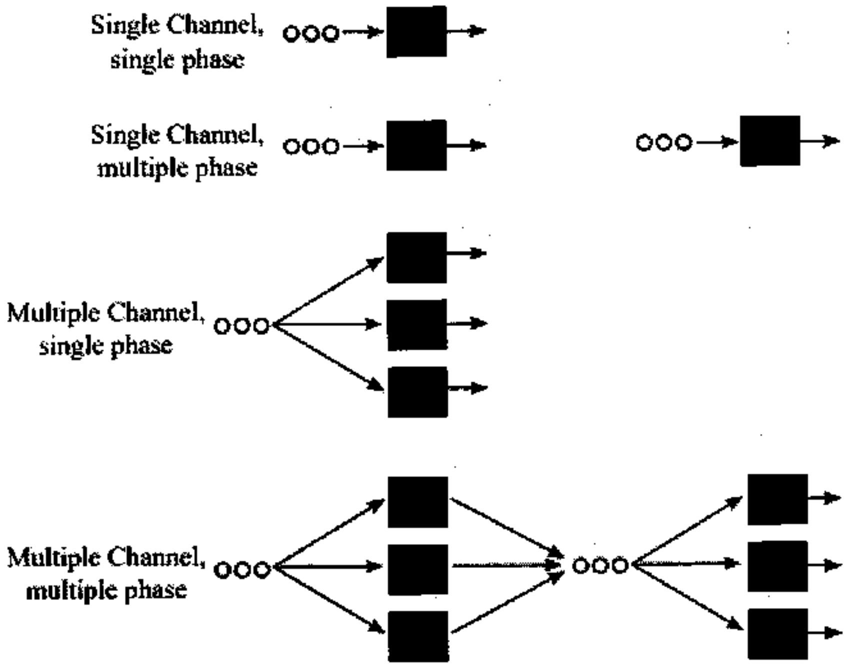


Figure 4.4: Four common variations of queuing systems

Queue Discipline;-

Queue discipline refers to the order in which customers are processed. All but one of the models to be described shortly assume that service is provided on a first-come, first-served basis. This is perhaps the most commonly encountered rule. There is first-come service at banks, stores, theatres, restaurants, four-way stop signs, registration lines, and so on. Examples of systems that do not serve on a first-come basis include hospital emergency rooms, rush orders in a factory, and main frame computer processing of jobs. In these and similar situations, customers do not all represent the same waiting costs; those with the highest costs (e.g., the most seriously ill) are processed first, even though other customers may have arrived earlier.

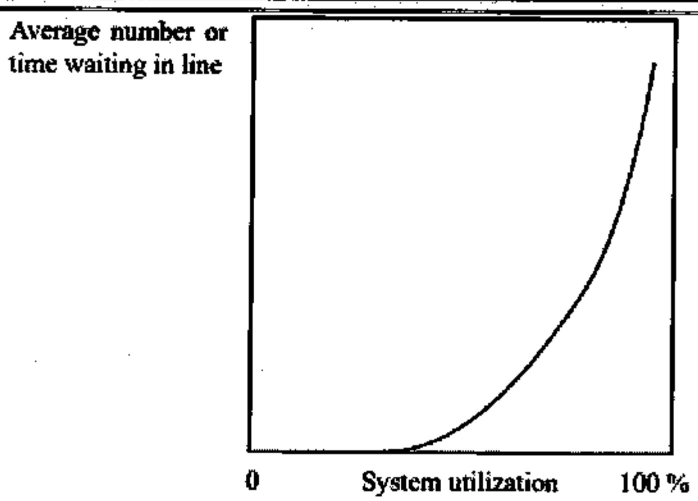


Figure 4.5: The average number waiting in line and the average time customers wait in line increase exponentially as the system utilization increases

QUESTIONS.42
What are the Measures of waiting-line performance?

ANSWER.

The operations manager typically looks at five measures when evaluating existing or proposed service systems. They relate to potential customer dissatisfaction and costs:

1. The average number of customers waiting, either in line or in the system.
2. The average time customers wait, either in line or in the system.
3. System utilization, which refers to the percentage of capacity utilized.
4. The implied cost of a given level of capacity and its related waiting line.
5. The probability that an arrival will have to wait for service.

Of these measures, system utilization bars some elaboration. It reflects the extent to which the servers are busy rather than idle. One the surface, it might seem that the operations manager would want to seek 100 percent utilization. However, as Figure 2.8.7 illustrates, increases in system utilization are achieved at the expense of increases in both the length of the waiting line and the average waiting time. In fact, these values become exceedingly large as utilization approaches 100 percent. The implication is that under normal circumstances, 100 per cent utilization is not a realistic goal. Even if it were, 100 per cent utilization of service personnel is not good; they need some slack time. Thus, instead, the operations manager should try to achieve a system that minimizes the sum of waiting costs and capacity costs.

Queuing Models: Infinite-source

Many queuing models are available for a manager or analyst to choose from. The discussion here includes four of the most basic and most widely used models. The purpose is to provide an exposure to a range of models rather than an extensive coverage of the field. All assume a Poisson arrival rate. Moreover, the models pertain to a system operating under steady-state conditions; that is, they assume the average arrival and service rates are stable. The four models described are

1. Single server, exponential service time.

2. Single server, constant service time.

3. Multiple servers, exponential service time.

4. Multiple priority service, exponential service time.

Note that the terms "server" and "channel" mean the same thing. To facilitate your use of waiting line models, Table 4.5 provides a list of the symbols used for the infinite-source models

Symbol	Represents
λ	Customer arrival rate
μ	Service rate per server
L_q	The average number of customers waiting for service
L_s	The average number of customers in the system (waiting and/or being served)
r	The average number of customers being served
ρ	The system utilization
W_q	The average time customers wait in line
W_s	The average time customer's spend in the system (waiting in line and service time)
$1/\mu$	Service time
P_o	The probability of zero units in the system
P_n	The probability of *n* units in the system
M	The number of servers (channels)
L_{max}	The maximum expected number waiting in line

Basic Relationships; -

There are certain basic relationships that hold for all infinite-source models. Knowledge of these can be very helpful in deriving desired performance measures, given a few key values. Here are the basic relationships:

Note: The arrival and service rates, represented by λ and μ , must be in the same units (e.g., customers per hour, customers per minute).

(I) System utilization: This reflects the ratio of demand (as measured by the arrival rate) to supply or capacity (as measured by the product of the number of servers, M, and the service rate, μ)

$$\rho = \frac{\lambda}{M \times \mu}$$

(II) The average number of customers being served:

$$r = \frac{\lambda}{\mu}$$

For nearly all queuing systems, there is a relationship between the average time a unit spends in the system or queue and the average number of units in the system or queue. According to Little's law, for a stable system, the average number of customers in line or in the system is equal to the average customer arrival rate multiplied by the average time in line or the system. That is,

$$L_s = \lambda W_s, \qquad \text{and} \qquad L_q = \lambda W_q$$

The implications of these are important in analysis of waiting lines. The relationships are independent of any probability distribution and require no assumptions about which customers arrive or are serviced, or the order in which they are served. It also means that knowledge of any two of the three variables can be used to obtain the third variable. For example, knowing the arrival rate and the average number in line, one can solve for the average waiting time

Figure 4.5.6: Basic Relationships

	Line	+	Service	=	System
Customers ——→	000	→	▢	→	
Average number waiting:	L_q	+		=	L_s
Average time waiting:		+		=	W_s

(III) The average number of customers (
A) Waiting in line or queue for service: L_q [Model dependent. Obtain using a table or formula.]
(B) Waiting In the system (line plus being served): $L_s = L_q + r$

(IV) The average time customers are

(A) Waiting in line or queue: $W_q = \frac{L_q}{\lambda}$

(B) Waiting in the system: $W_s = W_q + \frac{1}{\mu} \cdot \frac{L_s}{\lambda}$

All infinite-source models require that system utilization be less than 1.0; the models apply only to underloaded systems.
The average number waiting in line, L_q, is a key value because it is a determinant of some of the other measures of system performance, such as the average number in the system, the average time in line, and the average time in the system. Hence, L_q will usually be one of the first values you will want to determine in problem solving.

Illustration 32

Customers arrive at a bakery at an average rate of 16 per hour on weekday mornings. The arrival can be described by a Poisson distribution with a mean of 16. Each clerk can serve a customer in an average of three minutes; This time can be described by an exponential distribution with a mean of 3.0 minutes.

a. What are the arrival and service rates?

b. Compute the average number of customers being served at any time.

c. Suppose it has been determined that the average number of customers waiting in line is 3.2. compute the average number of customers in the system (i.e., waiting in line or being served), the average time customers wait in line, and the average time in the system.

d. Determine the system utilization for M = 1, 2, and 3 servers.

Solution:

a. The arrival rate is given in the problem: λ = 16 customers per hour. Change the service time to a comparable hourly rate by first restating the time in hours and then taking its reciprocal. Thus, (3 minutes per customer)/ (60 minutes per hour) = 1/20 = 1/ μ. Its reciprocal is μ = 20 customers per hour = Service Rate.

b. Average no. of customers being served at any time.

$r = \frac{\lambda}{\mu} = \frac{16}{20} = 0.80$ customer.

Performance Measure	Equation
Average number in line/queue	$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)}$
Probability of zero units in the system	$P_o = 1 - (\frac{\lambda}{\mu})$
Probability of *n* units in the system	$P_n = P_o(\frac{\lambda}{\mu})^n$
Performance Measure	**Equation**
Probability of less than *n* units in the system	$P_{<n} = 1 - (\frac{\lambda}{\mu})^n$

c. Given: L_q = 3.2 customers

$L_s = L_q + r = 3.2 + 0.80 = 4.0$

customers Average time customers wait in line

$= W_q + \frac{L_q}{\lambda} = \frac{3.2}{16} = 0.20$ hour, or 0.20 hour × 60 minutes/hour = 12 minutes

W_s = Average time customers wait in system = $W_q + \frac{1}{\mu}$

Waiting time in line plus service

$0.20 + \frac{1}{20}$ hour, or 15 minutes

d. System utilization is $\rho = \frac{\lambda}{M \times \mu}$.

For M = 1, $\rho = \frac{16}{1(20)} = 0.80$

For M = 2, $\rho = \frac{16}{2(20)} = 0.40$

For M = 3, $\rho = \frac{16}{3(20)} = 0.27$

Note that as the system capacity is measured by Mμ increases, the system utilization for a given arrival rate decreases.

Single server, exponential service time, M/M/1

The simplest model involves a system that has one server (or a single crew). The queue discipline is first-come, first-served, and it is assumed that the customer arrival rate can be approximate by a Poisson distribution and service time by a negative exponential distribution. There is no limit on length of queue.

Illustration 33

An airline is planning to open a satellite ticket desk in a new shopping plaza, staffed by one ticket agent. It is estimated that requests for tickets and information will average 15 per hour, and requests will have a Poisson distribution. Service time is assumed to be exponentially distributed. Previous experience with similar satellite operations suggests that mean service time should average about three minutes per request.

Determine each of the following:

a. System utilization.

b. Percentage of time the server (agent) will be idle.

c. The expected number of customers waiting to be served.

d. The average time customers will spend in the system.

The probability of zero customers in the system and the probability of four customers in the system.

Solution:

Solution:

Arrival Rate = λ = 15 customers per hour

Service Rate = $\mu = \frac{1}{\text{Service Time}} = \frac{1 \text{ customer}}{3 \text{ minutes}} \times 60$ minutes per hour = 20 customers per hour

a. System Utilisation = $\rho = \frac{\lambda}{M\mu} = \frac{\lambda}{1(20)} = 0.75$

b. Percentage of time the server will be idle = 1 − r = 1− 0.75 = 0.25, or 25 percent

c. Expected no. of customers waiting to be served = $L_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{225}{20(20-15)} = \frac{225}{(20 \times 5)} = \frac{225}{100} = 2.25$ customers

d. Average time customers will spend in the system = $W_s = \frac{L_q}{\lambda} + \frac{1}{\mu} = \frac{2.25}{15} + \frac{1}{20} = 0.20$ hours, or 12 minutes

e. Probability of zero customer in the system = $P_0 = 1 - \frac{\lambda}{\mu} = 1 - \frac{15}{20} = 0.25$ and

Probability of 4 customers in the system $P_4 = P_0 = \left(\frac{\lambda}{\mu}\right)^4 = 0.25\left(\frac{15}{20}\right)^4 = 0.079$

Single Server, Constant Service Time, M/D/1

As noted previously, waiting lines are a consequence of random, highly variable arrival and service rates. If a system can reduce or eliminate the variability of either or both, it can shorten waiting lines noticeably. A case in point is a system with constant service time. The effect of a constant service time is to cut in half the average number of customers waiting in line:

$$L_q = \frac{\lambda^2}{2\mu(\mu-\lambda)}$$

The average time customers spend waiting in line is also cut in half. Similar improvements can be realized by smoothing arrival times (e.g., by use of appointments).

Illustration 34

Wanda's Car Wash & dry is an automatic, five-minute operation with a single bay. On a typical Saturday morning, cars arrive at a mean rate of eight per hour, with arrivals tending to follow a Poisson distribution. Find

a. The average number of cars in line.

b. The average time cars spend in line and service.

Solution:

Arrive Rate = λ = 8 cars per hour

Service Rate = μ = 1 per 5 minutes, or 12 per hour

Av. no. of cars waiting in line = $L_q = \frac{\lambda^2}{2\mu(\mu-\lambda)} = \frac{8^2}{2(12)(12-8)} = 0.667$ car

Av. time cars spend in line and service = $W_S = \frac{L_q}{\lambda} + \frac{1}{\mu} = \frac{0.667}{8} + \frac{1}{12} = 0.167$ hours, or 10 minutes

Illustration 35

A departmental store has one cashier. During the rush hours, customers arrive at a rate of 20 per hour. The average number of customers that can be handled by the cashier is 24 per hour. Assume the conditions for use of the single - channel queuing model. Find out average time a customer spends in the system.

Solution.

The usual notations are given

Arrival Rate = λ = 20 customers / hour and service rate = μ = 24 customers / hour

Average no. of customers in the system = $L_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{20}{(24-20)} = \frac{20}{5} = 5$ customers

Average time a customer spends in the system = $W_s = \frac{L_s}{\lambda} = \frac{5}{20} = \frac{1}{4} = 0.25$ hour = 15 mins.

Illustration 36

As a tool service centre the arrival rate is two per hour and the service potential is three per hour. Simple queue conditions exist

The hourly wage paid to the attendant at the service centre is 1.50 per hour and the hourly cost of a machinist away from his work is 4.

Calculate:

(i) The average number of machinists being served or waiting to be served at any given time.

(ii) The average time a machinist spends waiting for service.

(iii) The total cost of operating the system for an eight - hour day.

(iv) The cost of the system if there were two attendants working together as a team, each paid 1.50 per hour and each able to service on average 2 per hour.

Solution:

Arrival rate = λ = 2 per hour

Service rate = μ = 3 per hour

(i) Average number of machinists being served or waiting to be served at any given time:

$$L_s = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{2}{(3-2)} = 2$$

(ii) Average Time a machinist spends waiting for the services:

$$W_q = \frac{\lambda}{\mu} \times \frac{1}{(\mu-\lambda)} = \frac{2}{3} \times \frac{1}{(3-2)} = 0.667 \text{ hours}$$

It means a machinist spends 40 minutes (ie., 60 × 0.667) in the queue.

(iii) Average time in the system

$$W_s = \frac{1}{(\mu-\lambda)} = \frac{1}{(3-2)} = 1 \text{ hour}$$

Average number of machinists in the system = 2	[As per (i) above]
Cost of two machinists being away from work = 4 × 2 =	`8.00 per hour
Attendant cost=	<u>1.50 per hour</u>
	9.50 per hour

Cost of 8- hour day = 8 hrs × 9.50 = 76.00

(iv) It is assumed that there is still a single service point, but the average service rate with 2 attendants now is 4 per hour

∴ Now λ = 2 per hour

m = 4 per hour

∴ Average number of machinists in the system = $L_S = \frac{\lambda}{\mu-\lambda} = \frac{2}{(4-2)} = 1$

Illustration 37

Workers come to tool store room to enquire about special tools (required by them) for accomplishing a particular project assigned to them. The average time between two arrivals is 60 seconds and the arrivals are assumed to be in Poisson distribution. The average service time (of the tool room attendant) is 40 seconds.

Determine:

(i) average queue length,

(ii) average length of non-empty queues,

(iii) average number of workers in system including the worker being attended

(iv) mean waiting time of an arrival,

(v) average waiting time of an arrival who waits

Solution:

Here, Arrival Rate = $\lambda = \frac{60}{60}$ per second = 1 per minute

Service Rate = $\mu = \frac{60}{60}$ per second = 1.5 per minute

(i)Average queue length:

$$Lq = \frac{\lambda}{\mu} \text{ x } \frac{\lambda}{(\mu-\lambda)} = \frac{1}{1.5} \text{ x } \frac{1}{(1.5-1)} = \frac{1}{0.75} = \frac{3}{4}$$

(ii) Average length of non-empty queues

$$L_n = \frac{\mu}{\mu-\lambda} = \frac{1}{(1.5-1)} = 3 \text{ workers}$$

(iii) Average number of workers in the system:

$$L_s = \frac{\lambda}{\mu-\lambda} = \frac{1}{(1.5-1)} = 2 \text{ workers}$$

(iv) Mean waiting time of an arrival

$$W_q = \frac{\lambda}{\mu} \times \frac{\lambda}{(\mu-\lambda)} = \frac{1}{1.5} \times \frac{1}{(1.5-1)} = \frac{3}{4} \text{ minutes}$$

(v) Average waiting time of an arrival who waits

$$W_n = \frac{1}{\mu-\lambda} = \frac{1}{(1.5-1)} = 2 \text{ minutes}$$

Question.43
Define Simulation.

ANSWER.

The techniques of LPP, Transportation, and Assignment are used for optimization of various types of problem faced in business situations. However, all the business situations can not be solved with the above techniques only. There may be some complex situations, where numbers of assumptions are also necessary. It may be quite often possible to simulate the given system and study the behaviour.

To simulate means to imitate. In general, simulation involves developing a model of real phenomenon and then performing experiments on the model evolved. It is to be noted that it is a descriptive and not optimizing technique. In simulation, a given system is copied and the variables and constants associated with it are manipulated in that artificial environment to examine the behaviour of the system. For ex: aerodynamic testing, scaled down models of airplanes and placing them in work tunnels etc.

Thus, also in a complex situation of business a given system is taken and simulated for obtaining the required results.

It consists of four phases:

1. Definition of the problem and statement of objectives.
2. Construction of an appropriate model
3. Experimentation with the model constructed.
4. Evaluation of the results of simulation.

Monte Carlo Simulation:

Although simulation can be of many types, our discussion will focus on the probabilistic simulation using the Monte Carlo method. Also called computer simulation, it can be described as a numerical technique that involves modelling a stochastic system with objective of predicting the system's behaviours. The chance element is a very significant feature of Monte Carlo simulation and this approach can be used when the given process has a random, or chance, component

In using the Monte Carlo method, a given problem is solved by simulating the original data with random number generators. Basically, its use requires two things, first, as mentioned earlier, we must have a model, that represents an image of the reality of the situation. Here the model refers to the probability distribution of the variable in question. What is significant here is that the variable may not be known to explicitly follow any of the theoretical distribution like Poisson, Normal and so on. The distribution may be obtained by direct observation or from past records. To illustrate, suppose, that a bakery keeps a record of the sale of the number of cakes of a certain type and the information relating to sales of 200 days is as below –

Demand (No of cakes):	5	6	7	8	9	10	11	12	**Total**
(No of days) :	4	10	16	50	62	38	12	8	**200**

Illustration 38

State the major two reasons for using simulation to solve a problem

A confectioner sells confectionery items. Past data of demand per week in hundred kilograms with frequency is given below:

Demand/Week	0	5	10	15	20	25
Frequency	2	11	8	21	5	3

Using the following sequence of random numbers, generate the demand for the next 10 weeks. Also find out the average demand per week.

Random numbers	35	52	13	90	23	73	34	57
	35	83	94	56	67	66	60	

Solution:

Random No. Range Table for demand				
Demand per week	Frequency (f)	Probability ($p = f \div \Sigma f$)	Cumulative Probability	Range† of Random Nos.
0	2	.04	.04	00-03
5	11	.22	.26	04-25
10	8	.16	.42	26-41
15	21	.42	.84	42-83
20	5	.10	.94	84-93
25	3	.06	1.00	94-99
	$\Sigma f = 50$	1.00		

†As the given Random Nos. are of 2 digits, the ranges of Random Nos. has also been considered to have 2 digits only. Also the range of Random Nos. corresponds to cumulative probability values which lies between 0 & 1 and can be correlated as nos. between 00 and 99.

Simulated Values for next 10 weeks		
Weeks	Random Nos.	Demand
1	35*	10*
2	52	15
3	13	5
4	90	20
5	23	5
6	73	15
7	34	10
8	57	15
9	35	10
10	83	15
	-	120

*From Table (I), Random No. 35 appears in the range of 26-41. Also the demand for this range is 10.
Average weekly demand = 120 /10 = 12

Illustration 39

The manager of a book store has to decide the number of copies of a particular tax law book to order. A book costs ` 60 and is sold for ` 80. Since some of the tax laws change year after year, any copies unsold while the edition is not current must be sold for ` 30. From past records, the distribution of demand for this book has been obtained as follows:

Demand (No of copies)	15	16	17	18	19	20	21	22
Proportion	0.05	0.08	0.20	0.45	0.10	0.07	0.03	0.02

Using the following sequence of random numbers, generate the demand for 20 time periods(years). Calculate the average profit obtainable under each of the courses of action open to the manager. What is the optimal policy?

14	02	93	99	18	71	37	30	12	10
88	13	00	57	69	32	18	08	92	73

Solution:

	Random No. Range Table		
Demand	**Probability**	**Cumulative Probability**	**Random No. Range**
15	.05	.05	00-04
16	.08	.13	5-12
17	.20	.33	13-32
18	.45	.78	33-77
19	.10	.88	78-87
20	.07	.95	88-94
21	.03	.98	95-97
22	.02	1.00	98-99
Total	**1.00**	–	–

Calculation of demand and profit for next 20 years					
Year	Random Numbers	Expected demand	No. of books unsold if stock is		
			16*	17*	18*
1	14	17	-	-	1
2	02	15	1	2	3
3	93	20	-	-	-
4	99	22	-	-	-
5	18	17	-	-	1
6	71	18	-	-	-
7	37	18	-	-	-
8	30	17	-	-	1
9	12	16	-	1	2
10	10	16	-	1	2
11	88	20	-	-	-
12	13	17	-	-	1
13	00	15	1	2	3
14	57	18	-	-	-
15	69	18	-	-	-
16	32	17	-	-	1
17	18	17	-	-	1
18	08	16	-	1	2
19	92	20	-	-	-
20	73	18	-	-	-
	Total		2	7	18

*Looking at the simulated demand pattern, these stock figures have been chosen to find out optimal course of action. Stock figures of 20 or more have not been considered because it is quite obvious that such figures will not give optimal course of action due to high losses for the unsold books.

Statement Showing Computation of Profit			
No. of Books order (n)	No. of Books sold in 20 years (n × 20 – Books unsold)	*Net Profit (`)	Average Profit/Year (Profit ÷ 20)
15	15 × 20 = 300	6000	300
16	16 × 20 – 2 = 318	6300 (318 × 20) – 2 × 30	315
17	(17 × 20) – 7 = 333	6450 (333 × 20) –7 × 30	322.5
18	(18 × 20) – 18	6300 (342 × 20) – 18 × 30	315

* Net Profit = No. of books sold × 20# – No. of books unsold × 30**
Selling price/book = 80, Cost/book = 60
Profit /book = 80 – 60 = 20
Selling price of any unsold book = 30
**Loss incurred/unsold book = ` 60 – 30 = 30
Since profit is maximum for 17 books order, the optimal policy is to order 17 books per year.

Illustration 40

A Small retailer has studied the weekly receipts and payments over the past 200 weeks and has developed the following set of information:

Weekly Receipts (₹)	Probability	Weekly Payments (₹)	Probability
3000	0.20	4000	0.30
5000	0.30	6000	0.40
7000	0.40	8000	0.20
12000	0.10	10000	0.10

Using the following set of random numbers, simulate the weekly pattern of receipts and payments for the 12 weeks of the next quarter, assuming further that the beginning bank balance is ` 8000. What is the estimated balance at the end of the 12-weekly period? What is the highest weekly balance during the quarter? What is the average weekly balance for the quarter?

Random Numbers

For Receipts	03	91	38	55	17	46	32	43	69	72	24	22
For payments	61	96	30	32	03	88	48	28	88	18	71	99

According to the given information, the random number interval is assigned to both the receipts and the payments.

Solution:

Range of random numbers							
Receipt (`)	Probability	Cumulative probability	Range	Payments (`)	Probability	Cumulative probability	Range
3000	0.20	0.20	00-19	4000	0.30	0.30	00-29
5000	0.30	0.50	20-49	6000	0.40	0.70	30-69
7000	0.40	0.90	50-89	8000	0.20	0.90	70-89
12000	0.10	1.00	90-99	10000	0.10	1.00	90-99

Simulation of Data for a period of 12 weeks					
Week	Random No. for receipt	Expected Receipt (`)	Random No. for payment	Expected Payment (`)	Week end Balance (`)
Opening Balance					8000
1	03	3000	61	6000	5000 (8000 + 3000 – 6000)
2	91	12000	96	10000	7000
3	38	5000	30	6000	6000
4	55	7000	32	6000	7000
5	17	3000	03	4000	6000
6	46	5000	88	8000	3000

Simulation of Data for a period of 12 weeks					
Week	Random No. for receipt	Expected Receipt (`)	Random No. for payment	Expected Payment (`)	Week end Balance (`)
7	32	5000	48	6000	2000
8	43	5000	28	4000	3000
9	69	7000	88	8000	2000
10	72	7000	18	4000	5000
11	24	5000	71	8000	2000
12	22	5000	99	10000	(3000)

Estimated balance at the end of 12th week = (3,000)

Highest balance	= 7,000
Average balance during the quarter	= 45,000/12 = 3,750

Illustration 41

An automobile production line turns out about 100 cars a day, but deviations occur owing to many causes. The production is more accurately described by the probability distribution given below:

Production/Day	Prob.	Production/Day	Prob.
95	0.03	101	0.15
96	0.05	102	0.10
97	0.07	103	0.07
98	0.10	104	0.05
99	0.15	105	0.03
100	0.20		
		Total	1.00

Finished cars are transported across the bay, at the end of each day, by ferry. If the ferry has space for only 101 cars, what will be the average number of cars waiting to be shipped, and what will be the average number of empty spaces on the boat? Use following Random Numbers to simulate the data provided above - 20, 63, 46, 16, 45, 41, 44, 66, 87, 26, 78, 40, 29, 92, 21.

Solution:

	Simulation of data of an Automobile Production line		
Production/day	Probability	Cumulative Probability	Random No. Range
95	0.03	0.03	00–02
96	0.05	0.08	03-07
97	0.07	0.15	08-14
98	0.10	0.25	15-24
99	0.15	0.40	25-39
100	0.20	0.60	40-59
101	0.15	0.75	60-74
102	0.10	0.85	75-84
103	0.07	0.92	85-91
104	0.05	0.97	92-96
105	0.03	1.00	97-99
	1.00		

Simulated data				
Day	**Random No.**	**Production**	**No.of cars waiting to be shipped**	**No. of empty space on the boat**
1	20	98	-	3
2	63	101	-	-
3	46	100	-	1
4	16	98	-	3
5	45	100	-	1
6	41	100	-	1
7	44	100	-	1
8	66	101	-	-
9	87	103	2	-
10	26	99	-	2
11	78	102	1	-
12	40	100	-	1
13	29	99	-	2
14	92	104	3	-
15	21	98	-	3
	Total		6	18

Average no. of cars waiting to be shipped = 6/15 = 0.40 per day

Average no. of empty space on the boat = 18/15 = 1.2 per day

Illustration 42

A book store wishes to carry 'Ramayana' in stock. Demand is probabilistic and replenishment of stock takes 2 days (i.e. if an order is placed on March 1, it will be delivered at the end of the day on March 3). The probabilities of demand are given below:

Demand (daily)	0	1	2	3	4
Probability	0.05	0.10	0.30	0.45	0.10

Each time an order is placed, the store incurs an ordering cost of ` 10 per order. The store also incurs a carrying cost of ` 0.50 per book per day. The inventory carrying cost in calculated on the basis of stock at the end of each day

The manager of the bookstore wishes to compare two options for his inventory decision.

A. Order 5 books when the inventory at the beginning of the day plus order outstanding is less than 8 books.

B. Order 8 books when the inventory at the beginning of the day plus order outstanding is less than 8 Currently (beginning 1st day) the store has a stock of 8 books plus 6 books ordered two days ago and expected to arrive next day.

Using Monte-Carlo Simulation for 10 cycles, recommend, which option the manager, should choose. The two-digit random numbers are given below:

89	34	70	63	61	81	39	16	13	73

Solution:

Demand	Probability	Cumulative Probability	Random No. Range
0	0.05	0.05	00-04
1	0.10	0.15	05-14
2	0.30	0.45	15-44
3	0.45	0.90	45-89
4	0.10	1.00	90-99

Option - A

Day	Random No.	Demand	Opening Stock	Ordered Quantity receipt	Closing Stock	Quanity for which Order Placed
1	89	3	8	-	5	-
2	34	2	5	6	9	-
3	70	3	9	-	6	0
4	63	3	6	-	3	5
5	61	3	3	0	0	-
6	81	3	0	5	2	5
7	39	2	2	-	0	5
8	16	2	0	5	3	-
9	13	1	3	5	7	-
10	73	3	7	-	4	5
					39	

Ordering cost 4 × 10	` 40
Carrying cost 0.5 × 39	` 19.50
Total Cost	` 59.50

Option B

Day	R No.	Demand	Opening Stock	Ordered Quantity receipt	Closing Stock	Quantity for which Order placed
1	89	3	8	-	5	-
2	34	2	5	6	9	-
3	70	3	9	-	6	-
4	63	3	6	-	3	8
5	61	3	3	-	0	-
6	81	3	0	8	5	-
7	39	2	5	-	3	8
8	16	2	3	-	1	-
9	13	1	1	8	8	-
10	73	3	8	-	5	-
					45	

Ordering cost 2×10	20.0
Carrying cost 0.5×45	22.50
Total Cost	42.50

Illustration 43

After observing heavy congestion of customers over a period of time in a petrol station, Mr. Petro has decided to set up a petrol pump facility on his own in a nearby site. He has compiled statistics relating to the potential customer arrival pattern and service pattern as given below. He has also decided to evaluate the operations by using the simulation technique.

Arrivals		Services	
Inter-arrival time (minutes)	Probability	Service time (minutes)	Probability
2	0.22	4	0.28
4	0.30	6	0.40
6	0.24	8	0.22
8	0.14	10	0.10
10	0.10		

Assume:
(i) The clock starts at 8:00 hours
(ii) Only one pump is set up.
(iii) The following12 Random Numbers are to be used to depict the customer arrival pattern: 78, 26, 94, 08, 46, 63, 18, 35, 59, 12, 97 and 82.
(iv) The following 12 Random Numbers are to be used to depict the service pattern: 44, 21, 73, 96, 63, 35, 57, 31, 84, 24, 05, 37
You are required to find out the
(i) probability of the pump being idle, and (ii) Average time spent by a customer waiting in queue.

Solution:

Inter-arrival time				Service time			
Minutes	Probability	Cumulative probability	Range of Random No.	Minutes	Probability	Cumulative probability	Range
2	0.22	0.22	00-21	4	0.28	0.28	00-27
4	0.30	0.52	22-51	6	0.40	0.68	28-67
6	0.24	0.76	52-75	8	0.22	0.90	68-89
8	0.14	0.90	76-89	10	0.10	1.00	90-99
10	0.10	1.00	90 - 99	–	–	–	–

Sl. No.	Random No. for inter arrival time	Inter arrival time (Mins.)	Entry time in queue as per clock	Service start time as per clock	Random no for service time	Service time (Mins.)	Service end time as per clock	Waiting time of customer (Mins.)	Idle time (Mins.)
1	78	8	8.08	8.08	44	6	8.14	-	8
2	26	4	8.12	8.14	21	4	8.18	2	-
3	94	10	8.22	8.22	73	8	8.30	-	4
4	08	2	8.24	8.30	96	10	8.40	6	-
5	46	4	8.28	8.40	63	6	8.46	12	-
6	63	6	8.34	8.46	35	6	8.52	12	-
7	18	2	8.36	8.52	57	6	8.58	16	-
8	35	4	8.40	8.58	31	6	9.04	18	-
9	59	6	8.46	9.04	84	8	9.12	18	-
10	12	2	8.48	9.12	24	4	9.16	34	-
11	97	10	8.58	9.16	05	4	9.20	18	-

12	82	8	9.06	9.20	37	6	9.26	14	-
Total Time								140	12

Average time spent by the customer waiting in the queue = 140/12 = 11.67 minutes
Probability of idle time of petrol station = Total idle time /Total Operating = 12/86 = 0.1395 time of the Service Channel*
*Service End Time - 9.26 Hrs. Service Channel opened at 8.00 hrs. i.e. Total Time of the Service Channel = 1 hr. 26 Mins = 86 Mins.

Illustration. 44
A retailer deals in a perishable commodity. The daily demand and supply are variables. The data for the past 500 days show the following demand and supply:

Availability (Kg.)	Supply (No. of days)	Demand (Kg.)	Demand (No. of days)
10	40	10	50
20	50	20	110
30	190	30	200
40	150	40	100
50	70	50	40

The retailer buys the commodity at 20 per kg. and sells at 30 per kg. Any commodity remains at the end of the day, has no sales value. Moreover the loss on unsatisfied demand is ` 8 per Kg. Given the following pair of random numbers, simulate 6 days sales, demand and profit: (31, 18) (63, 84) (15, 79) (07, 32) (43, 75) (81, 27). The first random number in the pair is that of supply and the second random number is for demand.

Solution:

Table-1: Probability Distribution (Supply)

Supply	Probability	Cum. Prob.	Range	Range of Random Nos. for simulation
10	40/500 = 0.08	0.08	0 - 0.08	00 - 07
20	50/500 = 0.10	0.18	0.08 - 0.18	08 - 17
30	190/500 = 0.38	0.56	0.18 - 0.56	18 - 55
40	150/500 = 0.30	0.86	0.56 - 0.86	56 - 85
50	70/500 = 0.14	1.00	0.86 - 1.00	86 - 99

Table-2: Probability distribution (Demand)

Demand	Probability	Cum. Prob.	Range	Range of Random Nos. for simulation
10	50/500 = 0.10	0.10	0 - 0.10	00 - 09
20	110/500 = 0.22	0.32	0.10 - 0.32	10 - 31
30	200/500 = 0.40	0.72	0.32 - 0.72	32 - 71
40	100/500 = 0.20	0.92	0.72 - 0.92	72 - 91
50	40/500 = 0.08	1.00	0.92 - 1.00	92 - 99

Table-3: Showing simulated data

SIMULATED DATA FOR SUPPLY			Simulated data for demand		
Day	Random No.	Supply (Kg.)	Day	Random No.	Demand (Kg.)
1	31	30	1	18	20
2	63	40	2	84	40
3	15	20	3	79	40
4	07	10	4	32	30
5	43	30	5	75	40
6	81	40	6	27	20

Table-4: Statement Showing Supply, Demand and Profit

Day	Supply	Demand	*Sales Revenue	Cost (II)	Loss due to unsatisfied demand (III)	Profit (`)
(a)	(b)	(c)	(d)	(e) = (b) × 20/kg	(f) = [(c)-(b)]×8/kg	(g) = (d)-(c)-(f)
1	30	20	600	600	-	Nil
2	40	40	1,200	800	-	400
3	20	40	600	400	160	40
4	10	30	300	200	160	-60**
5	30	40	900	600	80	220
6	40	20	600	800	-	-200**

* (1) Sales revenue = Demand × Selling price, when Demand < Supply
(2) Sales revenue = Supply × Selling price, when Demand > Supply
** Negative figures indicate loss

Illustration 45.

Using empirical data A process planner is working on plans for producing a new detergent. She wishes to simulate a raw material demand in order to plan for adequate materials - handing and storage facilities. On the basis of usage for a similar product introduced previously, she has developed a frequency distribution of demand in tons per day for a 2-month period. Use this data (shown below) to simulate the raw material usage requirements for 7 periods (days).

Demands, X (tons/ day)	10	11	12	13	14	15	
Frequency (days)	6	18	15	12	6	3	Total = 60

SOLUTION:

(1) Data are given in frequencies.

(2) To formulate a probability distribution, divide each frequency by the total (60), for example, 6 60 = .10 and 18 60 = .30. Then formulate a cumulative probability distribution by successively summing the probability values.

Demand (tons/day)	Frequency (days)	Probability P(X)	Cumulative probability
10	6	0.10	0.10
11	18	0.30	0.40
12	15	0.25	0.65
13	12	0.20	0.85
14	6	0.10	0.95
15	3	0.05	
1.00	60	1.00	

(3) Next, assign random - number intervals so that the number of values available to each class corresponds with the probability. Using 100 two - digit numbers (00-99), we assign 10 percent (00-09) to the first class, 30 percent (10-39) to the second class, and so on.

Demand (tons/day)	Probability P(X)	Corresponding Random Numbers
10	.10	00-09
11 ←	← .30 ←	10-39 ←
12	.25	40-64
13	.20	64-84
14	.10	85-94
15	.05	95-99
	1.00	

RN = 27

(4) We obtained random numbers (RN) from column 1 of Appendix I (for convenience), so the first seven numbers are:

27 13 80 10 54 60 49

The first RN, 27, falls into the second class of the distribution and corresponds to a demand of 11 tons per day.

Random Number	27	13	80	10	54	60	49
Simulated Demand	11	11	13	11	12	12	12

(5) This extremely small simulation yields a mean of X = 11.7 tons and a standard deviation of s = .76 tons. The expected value from the empirical probability distribution is E(X) = [XP(X)] = 12.05 tons, suggestion that the small sample size of only 7 periods has resulted in some error. A much larger sample should be simulated before the simulation results are used for making decisions.

Note that the width of the random number "target" in each class corresponds exactly to the relative frequency of the class. This helps to ensure that the simulated results have the same type of distribution as the original data. This is more apparent in the graphic method where the vertical distances on the graph correspond to the relative frequencies of the respective classes.

Illustration 46.

Empirical data collected on the time required to weld a transformer bracket were recorded to the nearest ¼ minute, as shown in the accompanying table.

Weld Time (min)	Numbers of Observation
< .25	0
.25 < .75	24
.75 < 1.25	42

1.25 < 1.75	72
1.75 < 2.25	38
2.25 < 2.75	14
2.75 < 3.25	10

A. Formulate a cumulative distribution in percentage terms.

B. Graphs the frequency and cumulative distributions.

C. A simulation is to be conducted using random numbers. What simulated weld times (to the nearest .25 minute) would result from the random numbers 25, 90, and 59?

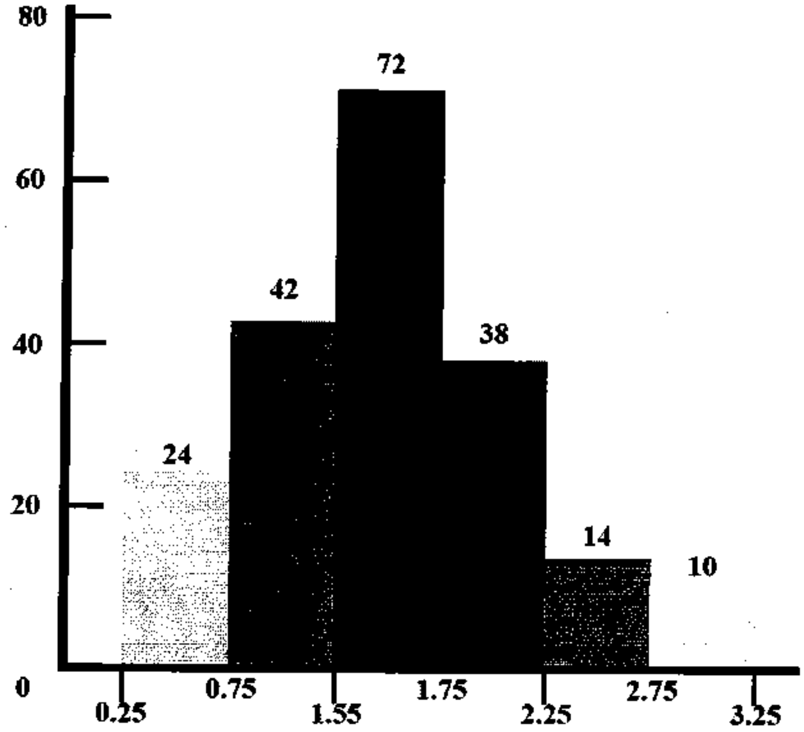
D. What proportion of the times exceed 2.0 minutes?

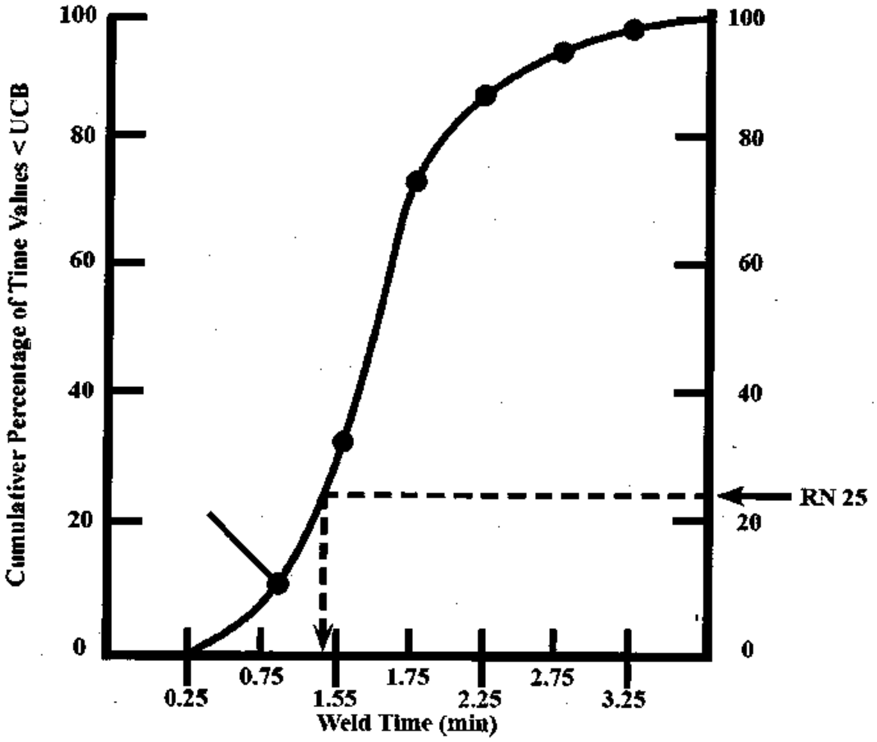
SOLUTION:

A. Cumulative distributions are usually formulated on a scale where the cumulative percentage is "more than" or "less than" a corresponding X axis amount. We shall use a "less than" percentage and so will need to identify the upper- class boundaries (UCB) as the Y coordinates for the cumulative distribution.

Weld Time (Min)	Frequency In Numbers	Upper - Class Boundary (UCB)	Cumulative Number Of Times < UCB	Cumulative Percentage Of Time < UCB
< .25	0	.25	0	0
.25 < .75	24	.75	24	12
.75 < 1.25	42	1.25	66	33
1.25 < 1.75	72	1.75	138	69
1.75 < 2.25	38	2.25	176	88
2.25 < 2.75	14	2.75	190	95
2.75 < 3.25	10	3.25	200	100

B. The frequency distribution is constructed by extending vertical lines from the class boundaries to the appropriate frequency level for the class. For the cumulative distribution, values of the cumulative percentage of time < UCB are plotted at weld times corresponding to the UCB. For example, the frequency (12 percent) is plotted at UCB = .75 (as illustrated below).





C. The simulated time for random number (RN) 25 is determined by entering the cumulative graph at 25 (as shown by the arrow) and proceeding horizontally to the curve and then down to the weld time. The resultant is a reading of 1.0 minute (rounded to the nearest. 25 minutes). Times for random number 90 and 59 are 2.5 and 1.5 minutes, respectively. (A larger graph would lend more accuracy.)

D. From the cumulative distribution, about 12 percent of the times exceed 2.0 minutes.

Illustration 47

How simulated times can be used to gain a knowledge of the interface of two assembly activitiesIn an aircraft assembly operation, activities A precedes activity B, and inventory may accumulate between the two activities. With the use of random numbers, a simulated sample of performance times yielded the values shown (minutes) in the accompanying table.

A → B

Activity A		Activity B	
Random Number	Time (min)	Random Number	Time
07	.3	63	.5
90	.8	44	.4
02	.2	30	.4
50	.5	98	.9
76	.6	30	.4
47	.5	72	.6
13	.3	58	.5
06	.3	96	.9
79	.7	37	.4

A. Simulated the assembly of six parts, showing idle time in activity B, waiting time of each part, and number of parts waiting. Note: omit the first random number of A so that activity B begins at time zero.

B. What was the average length of the waiting line ahead of B (in number of units)?

C. What was the average output per hour of the assembly line?

SOLUTION:

A. Our interest lies in activity b, so we can set up a table (below) to show when parts arrive at B, how long it takes B, how long it takes B to work on them, and the resultant idle and waiting times:

Part Number	Part Available for Activity B at Time	Activity B Beginning Time	Activity B Ending Time	Activity B Idle Time	Waiting Time of Part	Number parts Waiting at B End time
1	-	0	.5	0	0	0
2	.8	.8	1.2	.3	0	1
3	1.0	1.2	1.6	0	.2	1
Part Number	Part Available for Activity B at Time	Activity B Beginning Time	Activity B Ending Time	Activity B Idle Time	Waiting Time of Part	Number parts Waiting at B End time
4	1.5	1.6	2.5	0	.1	1
5	2.1	2.5	2.9	0	.4	2
6	2.6	2.9	3.5*	0	.3	2
7	2.9				1.0 **	2
8	3.2					

* Total run time.

**Total waiting time.

Activity B begins at 0, and it takes .5 minute to complete the first part. B is then idle for .3 minute until part 2 arrives from A at .8 minutes. Part 2 takes .4 minute, so the ending time is .8 + .4 = 1.2 minutes. By this time part 3 has been waiting. 2 minutes because it became available at .8 + .2 = 1.0 minute; but work could not be begun on it until 1.2 minutes. However, before activity B is finished on part 3 at 1.6 minutes, part 4 has arrived (at 1.0 + .5 = 1.5 minutes) and so one part is waiting. We continue systematically in this manner through part 6, noting that when it is finished at time were 3.5 minutes, there are two parts waiting, for their availability times were 2.9 minutes and 3.2 minutes, respectively.

B. The average length of the waiting line (that is, average inventory) ahead of B can be expressed in equation form as follows:

Average inventory= Total waiting time/ Total run time

= 1.0 assembly minute/3.5 minutes

= 0.29 assembly

C. Average output per hour:

Units/hr = $\frac{6 \text{ unit}}{3.5 \text{ min}}\left(\frac{60 \text{ min}}{\text{hr}}\right)$ = 102.9 units/hr

= = 102.9 units/hr.

Illustration 48

The Tit-Fit Scientific Laboratories is engaged in producing different types of high class equipment for use in science laboratories. The company has two different assembly lines to produce its most popular product 'Pressure'. The processing time for each of the assembly lines is regarded as a random variable and is described by the following distributions.

Process Time (minutes)	Assembly A1	Assembly A2
10	0.10	0.20
11	0.15	0.40
12	0.40	0.20
13	0.25	0.15
14	0.10	0.05

Using the following random numbers, generate data on the process times for 15 units of the item and compute the expected process time for the product. For the purpose, read the numbers vertically taking the first two digits for the processing time on assembly A1 and the last two digits for processing time on assembly A2.

4134	8343	3602	7505	7428
7476	1183	9445	0089	3424
4943	1915	5415	0880	9309

In the first stage, we assign random number intervals to the processing times on each of the assemblies.

SOLUTION:

Computation of Random Interval for Processing Time

	A1			A2		
Process time Minutes	Pi	ΣPi	Range	Pi	ΣPi	Range
10	0.10	0.10	0-9	0.20	0.20	0-19
11	0.15	0.25	10-24	0.10	0.60	20-59
12	0.40	0.65	25-64	0.20	0.80	60-79
13	0.25	0.90	65-89	0.15	.095	80-94
14	0.10	1.00	90-99	0.05	1.00	95-99

Simulated date for 15 units

	Random No.	Process Time	Random No.	Process Time	Total
1	41	12	34	11	23
2	74	13	76	12	25
3	49	12	43	11	23
4	83	13	43	11	24
5	11	11	83	13	24
6	11	11	83	13	24
7	36	12	02	10	22
8	94	14	45	11	25
9	54	12	15	10	22
10	75	13	05	10	23
11	00	10	89	13	23
12	08	10	80	13	23
13	74	13	28	11	24
14	34	12	24	11	23
15	93	14	09	10	24
		182		167	349

Average Process time for

A1 = 182/15 = 12.13 Minutes A2 = 167/15 = 11.13 Minutes

For product = 349/15 = 23.27 Minutes

Expected process time for the product = 23.27 minutes (12 .13 + 11.13)

Illustration 49

A businessman is considering taking over a certain new business. Based on past information and his own knowledge of the business, he works out the probability distribution of the monthly costs and sales revenues, as given here:

Cost (in ₹)	Probability	Sales Revenue (₹)	Probability
17000	0.10	19000	0.10
18000	0.10	20000	0.10
19000	0.40	21000	0.20

20000	0.20	22000	0.40
21000	0.20	23000	0.15
		24000	0.05

Use the following sequences of random numbers to be used for estimating costs and revenues. Obtain the probability distribution of the monthly net revenue.

	82	84	28	82	36	92	73	91	63	29
Sequence 1	27	26	92	63	83	02	10	39	10	10
	39	72	38	29	71	83	19	72	92	59
Sequence 2	49	39	72	94	04	92	72	18	09	00

b. Repeat the analysis in (a) by using the following random number streams:

	20	63	46	16	45	41	44	66	87	26
Sequence 1	78	40	29	92	21	36	57	03	28	08
	23	57	99	84	51	29	41	11	66	30
Sequence 2	41	80	62	74	64	26	41	40	97	15

SOLUTION:

Cost (₹)	**Probability**	**Cumulative Probability**	**Random Range**	**Cost (₹)**	**Probability**	**Cumulative Probability**	**Random Range**
17000	0.1	0.1	00-09	19000	0.1	0.1	00.09
18000	0.1	0.2	10-19	20000	0.1	0.2	10-19
19000	0.4	0.6	20-59	21000	0.2	0.4	20-39
20000	0.2	0.8	60-79	22000	0.4	0.8	40-79
21000	0.2	1.0	80-99	23000	0.15	0.95	80-94
				24000	0.05	1.00	95-99

Month	**Random No. for Sales**	**Cost (₹)**	**Random No. for Sales**	**Cost (₹)**	**Monthly net Revenue (₹)**
1	82	21000	39	21000	-
2	84	21000	72	22000	1000
3	28	19000	38	21000	2000
4	82	21000	29	21000	-
5	36	19000	71	22000	3000

6	92	21000	83	23000	2000
7	73	20000	19	20000	-
8	91	21000	72	22000	1000
9	63	20000	92	23000	3000
Month	**Random No. for Sales**	**Cost (₹)**	**Random No. for Sales**	**Cost (₹)**	**Monthly net Revenue (₹)**
10	29	19000	59	22000	3000
11	27	19000	49	22000	3000
12	26	19000	39	21000	2000
13	92	21000	72	22000	1000
14	63	20000	94	23000	3000
15	83	21000	04	19000	(2000)
16	02	17000	92	23000	6000
17	10	18000	72	22000	4000
18	39	19000	18	20000	1000
19	10	18000	09	19000	1000
20	10	18000	00	19000	1000
					35000

Average = 35000/20 = ₹ 1750.

Line Balancing

Line balancing is arranging a production line so that there is an even flow of production from one work station to the next, i.e. so that there are no delays at any work station that will leave the next work station with idle time.

Line balancing is also defined as "the apportionment of sequential work activities into work stations in order to gain a high utilization of labour and equipment and therefore minimize idle time." Balancing may be achieved by rearrangement of the work stations or by adding machines and / or workers at some of the stations so that all operations take about the same amount of time.

Line Balancing Procedure in Assembly Layouts

Step 1: Determine what tasks must be performed to complete one unit of a finished product and the sequence in which the tasks must be performed. Draw the precedence diagram.

Step 2: Estimate the task time (amount of time it takes a worker or a worker/machine combination to perform each task).

Step 3: Determine the cycle time (the amount of time that would elapse between products coming off the end of the assembly line if the desired hourly production rate is met.)

Step 4: Assign each task to a worker and balance the assembly line. This process results in determining the scope of each worker's job or which tasks that he or she will perform.

Steps Involved in Combining of the Tasks into Worker's Jobs

1. Starting at the beginning of the precedence diagram, combine tasks into a work station in the order of the sequence of tasks so that the combined task times approach but do not exceed the cycle time or multiples of the cycle time.
2. When tasks are combined into a workstation, the number of multiples of the cycle time is the number of workers required at the work station, all performing the same job.

QUESTION 44.
How to Analysis of Line Balancing Problems

ANSWER:
The procedure involves the following steps

1. Determine the no. of work stations and time available at each work station.
2. Group the individual tasks into amounts of work at each work station.
3. Evaluate the efficiency of grouping

When the available work time at any station exceeds that which can be done by one worker, additional workers must be added at that work station.

The key to efficient line balancing is to group activities or tasks in such a way that the work times at the work station are at or slightly less than the cycle time or a multiple of cycle time if more than one worker is required in any workstation.

Determination of cycle time (CT) : When the amount of output units required per period (period may be hour, shift, day or week etc.) is specified and the available time per period is given (i.e., the number of working hours per shift, number of shifts per day, number of working days per week etc.) then,

$$\text{Cycle time (CT)} = \frac{\text{Available time per period}}{\text{Output units required per period}}$$

Cycle time is the time interval at which completed products leave the production line.

Determination of the Ideal or Theoretical Minimum Number of Workers Required in the Line

$$\text{Ideal or theoretical minimum no. of workers required in the assy. line / production line} = \frac{\text{Total operation or task time} \times \text{Output units required per period}}{\text{Available time per period per worker}}$$

$$N = \Sigma t \times \left(\frac{1}{CT}\right) = \frac{\Sigma t}{CT}$$

QUESTION 45.

Define Balancing Efficiency and write Terminology Used in Line Balancing

ANSWER:

Balancing Efficiency: An efficient line balancing will minimize the amount of idle time. The balance efficiency can be calculated as:

(i) $E_{ffB} = \frac{\text{Output of task time}}{\text{Input by workstation times}} = \frac{\Sigma T}{CT \times N}$

Where, Σt = Sum of the actual worker times or task times to complete one unit

CT = Cycle time;

N = No. workers or work stations

(ii) $E_{ffB} = \frac{\text{Theoretical minimum number of worker}}{\text{Actual Number of worker}}$

The grouping of tasks is done with the aid of a precedence diagram. The precedence diagram is divided into work

zones or stations and the appropriate activities are granted under each workstation until the cycle time is as fully utilized as possible.

Terminology Used in Line Balancing:

(i) **Tasks:** Element of work or activity

(ii) **Task precedence:** Indicates the sequence in which tasks must be performed. Except the beginning task, all other tasks have preceding tasks.

(iii) **Task times:** The amount of time required for an automatic machine or a well trained worker to perform a task.

(iv) **Cycle time:** The interval of time between two successive products coming off the end of a production line or assembly line.

(v) **Productive time per hour:** The duration (in minutes) a work station or machine is working in each-hour. The productive time per hour is lesser than the actual available time due to lunch break, breakdown, personal time for the worker, start-ups and shutdowns.

(vi) **Work station:** Physical location where a particular set of tasks is performed. Workstation could be either a machine or equipment operated by a worker or an automatic machine or a machine operated by a robot.

(vii) Work centre: A physical location where two or more identical workstations are located in order to provide the needed production capacity.

(viii) Theoretical minimum number of workstations: The least number of work stations that can provide the required production calculated by:

N_t = Sum of all task time(Σt)

Cycle Time (CT

Cycle time = $\frac{\text{Available time}}{\text{Output required}}$

(ix) Actual number of workstations: The total number of workstations required on the entire production line, calculated as the next higher integer value of the number of workstations working.

(x) Utilisation: The percentage of time that a production line is working. This is calculated as

Utilisation or Balance efficiency

$= \frac{\text{Minimum number of workstations}}{\text{Actual number of workstations}} \times 100 = \frac{\Sigma T}{CT \times N}$

Question 46

Define Line Balancing Procedure.

ANSWER.

Steps:

(1) Calculate the cycle time and determine the theoretical minimum number of workstations

$N_t = \frac{\text{Sum of all task time}(\Sigma t)}{\text{Cycle Time}(CT)}$

Cycle time = $\frac{\text{Available time}}{\text{Output required}}$

(2) Compute the actual number of workstation (N) required by rounding up the theoretical number of workstations to the next higher integer value.

(3) Assign the tasks to the workstations beginning with station 1. Tasks are assigned to work stations moving from left to right through the precedence diagram.

(4) Before assigning each task to a workstation, use the following criteria to determine which tasks are eligible to be assigned to a workstation.

a) All preceding tasks in the sequence have been assigned already.

b) The task time does not exceed the time remaining at the workstation.

If no tasks are eligible to be assigned to a particular workstation, move to the next workstation.

(5) After each task assignment, determine the time remaining at the current work station by subtracting the sum of times for tasks already assigned to the work station from the cycle time.

(6) When there is a tie between two tasks (parallel tasks) to be assigned, use one- of these rules:

a) Assign the, task with the longest task time

b) Assign the task with greatest number of followers.

(7) Continue assignment of tasks until all tasks have been assigned to workstations.

(8) Calculate the idle time (or balance delay), percent idle time and efficiency of balancing the line.

Illustration 50

Table shows the time remaining (number of days until due date) and the work remaining (number of day's still required to finish the work) for 5 jobs which were assigned the letters A to E as they arrived to the shop. Sequence these jobs by priority rules viz., (a) FCFS, (b) EDD, (c) LS, (d) SPT and (e) LPT.

Job	Number days until due date	Number of days of work remaining
A	8	7
B	3	4
C	7	5
D	9	2
E	6	6

SOLUTION:

a) FCFS (First come first served): Since the jobs are assigned letters A to E as they arrived to the shop, the sequence according to FCFS priority rule is A B C D E

b) EDD (Early due date job first) rule: Taking into account the number of days until due date, the sequence of jobs as per EDD rules is

Job	B	E	C	A	D
No. of days units/due date	3	6	7	8	9

Here the job having earliest due date is sequenced first and the others are sequenced in ascending order of due date.

c) L.S. (Least slack) rule also called as Minimum slack rule.

Calculation of slack:

Slack = (Number of days until due date) - (Number of days of work remaining)

Job	No. of days until/due date	No. of days of work remaining	Slack (Days)
A	8	7	8 - 7 = 1
B	3	4	3 - 4 = -1
C	7	5	7 - 5 = 2
D	9	2	9 - 2 = 7
E	6	6	6 - 6 = 0

Sequence:

Job	B	E	A	C	D
Slack	-1	0	1	2	7

Here the jobs are sequenced in ascending order of magnitude of their respective slacks.

d) SPT (Shortest Processing Time job first) also referred as SOT (Shortest Operation time job First) rule or MINPRT (Minimum Processing time job first) rule. As per this rule, jobs are sequenced in ascending order of magnitude of their respective processing time.

Sequence:

Job	D	B	C	E	A
Processing Time (Days)	2	4	5	6	7

e) LPT (Longest Processing time job first) also referred to as LOT (Longest operation time job first) rule. As per this rule jobs are sequenced in descending order of magnitude of their respective processing times.

Sequence:

Job	A	E	C	B	D
Processing Time (Days)	7	6	5	4	2

Illustration 51

The following jobs have to be shipped a week from now (week has 5 working days)

Job	A	B	C	D	E	F
Number of days of work remaining	2	4	7	6	5	3

Sequence the jobs according to priority established by (a) least slack rule (b) critical ratio rule.

SOLUTION:

a) Calculation of slack:

Number of days until due date is 1 week i.e. 5 days for all jobs

Job (1)	No. of days until/due date (2)	No. of day of work remaining (3)	Slack (Days) (4) = (2) - (3)
A	5	2	3
B	5	4	1
C	5	7	- 2
D	5	6	- 1
E	5	5	0
F	5	3	2

Sequence:

Job	C	D	E	B	F	A
Slack (Days)	-2	-1	0	1	2	3

Jobs are sequenced in ascending order of magnitude of respecive slack values.

b) Calculation of Critical ratio :

Critical Ratio = Due Date - Date Now / Lead Time Remaining = DD - DN / LTR = Available time till due date / Operation time still needed to complete the job

Critical ratio for job A = 5/2 = 2.5

Critical ratio for job B = 5/4 = 1.25

Critical ratio for Job C = 5/7 = 0.71

Critical ratio for job D = 5/6 = 0.83

Critical ratio for job E = 5/5 = 1.0

Critical ratio for job F = 5/3 = 1.67

Job having least critical ratio is given the first priority and so on.

Sequence :	C	D	E	B	F	A
Critical Ratio :	0.71	0.83	1.0	1.25	1.67	2.5

Illustration 52

In a factory, there are six jobs to perform, each of which should go through two machines A and B, in the order AB. The processing timings (in hours) for the jobs are given here. You are required to determine the sequence for performing the jobs that would minimize the total elapsed time, T. What is the value of T?

Job	Machine A	Machine B
1	7	3
2	4	8
3	2	6
4	5	6
5	9	4
6	8	1

SOLUTION:

a) The least of all the times given in the table is for job 6 on machine B. So, perform job 6 in the end. It is last in the sequence. Now delete this job from the given data.

b) Of all timings now, the minimum is for job 3 on machine A. So, do the job 3 first.

c) After deleting job 3 also, the smallest time of 3 hours is for job 1 on machine B. Thus, perform job 1 in the end (before job 6).

d) Having assigned job 1, we observe that the smallest value of 4 hours is shared by job 2 on machine A and job 5 on machine B. So, perform job 2 first and job 5 in the end.

e) Now, the only job remaining is job 4, it shall be assigned the only place left in the sequence. The resultant sequence of jobs is, therefore, as follows:

3		2		4		5		1		6

This sequence is the optimal one. The total elapsed time, T, is obtained in Table 2.8.16 as equal to 36 hours

Table: Calculation of Total Elapsed Time (T)

Job	Machine A		Machine B	
	In	Out	In	Out
3	0	2	2	8
2	2	6	8	16
4	6	11	16	22
5	11	20	22	26
1	20	27	27	30
6	27	35	35	36

As shown in this table, the first job, job 3, starts at lime 0 on the machine A and is over by time 2, when it passes to machine B to be worked on till time 8. The job 2 starts on the machine A at time 2 as the machine is free at that lime. It is completed at time 6 and has to wait for 2 hours before it is processed on machine B, starting at time 8 when this machine is free, Similarly, the various jobs are assigned to the two machines and the in and out times are obtained.

4.9 Lean Operations

Lean operation has its roots in the Toyota Automobile Co., of Japan, where waste was to be avoided at all costs:

- the waste in time caused by having to repair faulty products
- the waste of investment in keeping high inventories and
- the waste of having idle workers.

QUESTION 47. What are the element of Lean Production
ANSWER. The elements of lean production are: • To consider the organization in terms of supply chain of value streams that extends from suppliers of raw materials, through transformation to the final customer. • To organize workers in teams and to have everyone in the organization conscious of his or her work. • To produce products of perfect quality and to have continuous quality improvement as a goal. • To organize the operation by product or cellular manufacturing, rather than using a functional or process lay-out. • To operate the facility in a just-in-time mode.

4.10 Just-In-Time (JIT)

Objectives of JIT manufacturing: The specific goal of JIT manufacturing is to provide the right quality level at the right place. Customer demand always determines what is right. JIT tries to build only what internal and external customers want and when they want it. The more focused objectives of JIT are:

1. Produce only the products (goods or services) that customers want.
2. Produce products only as quickly as customers want to use them.
3. Produce products with perfect quality.
4. Produce in the minimum possible lead times.
5. Produce products with features that customers want and no others.
6. Produce with no waste of labour, materials or equipment, designate a purpose for every movement to leave zero idle inventory.
7. Produce with methods that reinforce the occupational development of workers.

Overview of JIT manufacturing

JIT manufacturing includes many activities:

(i) **Inventory reduction:** JIT is a system for reducing inventory levels at all stages of production viz. raw materials, work-in-progress and finished goods.

(ii) **Quality improvement:** JIT provides a procedure for improving quality both within the firm and outside the firm.

(iii) **Lead time reduction:** With JIT, lead time components such as set-up and move times are significantly reduced.

(iv) **Vendor control/Performance improvement:** JIT gives the buying organisation greater power in buyer- supplier relationship. The firm moves from a situation where multiple suppliers are used to a situation where only one or two suppliers are used for supplying most of the parts. With fewer suppliers, the buying organisation has more power because it is making larger purchases from each vendor. Also, the buying organisation can now impose higher requirements on each supplier in terms of delivery and quality.

(v) **Continuous Improvement:** In the JIT system, existing problems are corrected and new problems identified in a never-ending approach to operations management.

(vi) **Total Preventive Maintenance:** JIT emphasizes preventive maintenance to reduce the risk of equipment break-downs which may cause production hold ups and increase in manufacturing cycle time due to delays.

(vii) **Strategic Gain:** JIT provides the firm's management with a means of developing, implementing and maintaining a sustainable competitive advantage in the market place

MEASUREMENT TECHNIQUES OF PRODUCTIVITY INDEX

5.1 Introduction

Productivity implies development of an attitude of mind and a constant urge to find better, cheaper, easier and safe ways of doing a job, which could be either manufacturing an article or providing a service. Since the beginning of the industrial era, the manufacturers or producers have been facing the problem of how to use the available resources and factors of production to the best of their ability and capacity so as to get the maximum output with the minimum cost of production. Industrial revolution, social, technological and scientific developments, changes in economic systems are the various efforts made in this direction and the process of development and changes is still on. New machines, methods and technology are being invented and used in the industrial field to minimise the wastage of men, materials and machines. It is all to increase the productivity

Productivity is the quality or state of being productive. It is some relationship of outputs to inputs. It is a concept that guides the management of a production system, and measures its success. It is the quality that indicates how well labour, capital, materials and energy are utilised. Productivity improvement is sought everywhere because it supports a higher standard of living, helps control inflation, and contributes towards a stronger national economy.

Productivity is an indicator reflecting the changes in the performance of the enterprise and having some sort of input-output comparison relating to various activities of an organisation. It also facilitates the management to control and plan the future operations of the enterprise.

Productivity is the talk of the day and it is generally regarded as efficiency in industrial production to be measured by some relationship between outputs and inputs. The increase in productivity is looked upon as the key to prosperity at all levels. In its modern sense, it refers to the relationship between the result and the means employed or to be more specific between the product and the factors used for obtaining it. It is the quantitative relationship between what we produce and the resources which we use to obtain it. It can also be termed as the ratio of what is produced to what is required to produce it. The higher is the ratio, the greater is the productivity. Thus, it seeks to measure the economic soundness of the use of the means of production. It means productivity can be considered higher if the same amount of production is obtained with lesser inputs or it will be lower if the same quantity of production is obtained with larger quantity of inputs. It is higher when there is maximum production with the least usage of resources.

A productivity index is a device of numerically expressing the ratio between outputs and the inputs of the resources. These indices are prepared by comparing the volume of output of goods with the labour employed on that job or the profits of the firm with the capital employed. If the comparison shows an upward trend in indices, it is a sign of improved or better productivity and vice-versa.

The productivity is a measure of how much input is required to achieve a given output.

Symbolically:

$P = \frac{O}{I}$ where P = Productivity;

O = Output,

I = Input.

The output may be measured in terms of the units of goods produced or the value of goods and services produced. The input, on the other hand, can be referred to as the combination of different factors, i.e., raw materials, machinery, worker's time, power, efforts and imagination of entrepreneur and the managers. A unit of input, therefore, can be expressed as one worker, or one hour of labour time or one tonne of raw materials, or one kw of electricity and so on. Thus, it is very clear from the above description that the productivity can be calculated or measured for each one of the factors comprising of the input or of all the factors together. The productivity of labour, for example, can be found out by ascertaining the ratio between the quantity of goods produced and the number of workers or man-hours employed on the production of such output.

QUESTION 1.
What are the importance of the concept of productivity can be viewed from the following points?

ANSWER.
1. To beat the competition: It is an age of cut-throat competition. There may be other commodities which can serve as the substitute of a particulars 'product' and can attract the consumers' for purchasing. The firm whose productivity is higher can only beat the competition and can exist in the market for long.
2. Guide to Management: The productivity indices are very useful for the management and can be used for different purposes. These indices can serve as a valuable guide to the management for improving the performance of its enterprise. The productivity measures can be used for the following purposes:
(a) Strategic : With the help of productivity indices, the efficiency of different firms can be measured, analysed and compared. The necessary steps can be taken to improve the productiveness of the firm taking in view the productiveness of the other competitive firms.
(b) Tactical: Different units or the sectors of the firm can also be compared as regards to their productivity and the productivity of the less productive units or sectors can be improved.
(c) Planning: A firm uses different inputs in producing the goods. A comparison of relative benefits accruing from the use of different inputs can be done and the most beneficial input can be used in production. It helps the management to plan for the future.
(d) Administration: Productivity indices indicate the progress of the firm over a period of years. The productivity of different inputs, including labour, can be measured individually. The individual productivity indices help the management in bargaining with the labour leaders, trade unions and the Government in case of labour disputes regarding welfare activities. Thus administration can be improved with the help of productivity indices.
3. An Indicator of Progress: In economically backward countries, productivity improvement is the basic aspect of progress. It implies the development of an attitude of mind and a constant urge to go for better, cheaper, quicker and safer ways of doing a job which could be either manufacturing a product or providing a service. In an urge to improve the productivity, new inventions take place. Thus productivity is an aspect of basic progress.
4. Maximum utilisation of Scarce Resources: In order to provide the articles or commodities to the consumers at the lowest possible cost, the productivity urges to utilise the available resources to the maximum possible extent so that there is full satisfaction of customers. The productivity processes

and techniques are designed to facilitate more efficient work involving less fatigue to the workers by improvement in the layout of the plant, better working environment and simplification of works

5. Key to National Prosperity: The productivity, in fact, has become synonymous to the progress. Higher productivity is an index of more production with the same inputs at lower cost. It enables industry to offer goods to the general public at cheaper rates and results in expansion of markets. The working conditions and wages of workers will improve and industrialists too will get larger profits. Thus higher productivity is the key to national prosperity. The secrets of Japan, China, South Korea and Western countries' prosperity lie in increased productivity.

6. Prosperity to Work force: The higher productivity is a boon to workers also. It brings improved working conditions, better wages and salaries, better welfare activities for labourers, etc. Thus their standard of living is going to be improved.

7. Other Uses:

(i) Higher productivity increases the profits and reserve funds of the industry that can be used for expansion and modernisation.

(ii) It increases the goodwill of the firm due to cheaper goods to the public, well-off staff and more profits and better financial position.

(iii) It improves the competitive strength of the company in export markets through reduction in cost of production and quality products

QUESTION 2.

Explain Measurement of Productivity?

ANSWER.

Measurement of Productivity: The productivity or the performance of various input and output factors can be measured in many ways. These measures are mainly based on the following two criteria:

(i) **Change in output per unit of input** indicates the change in the performance of corresponding input during the given period, e.g., change in output per worker or per man-hour will signify the change in performance of labour.

(ii) **Change in input per unit of output** during a given period signifies the change in the performance of the corresponding input factor, e.g., change in man-hour or workers per unit of output will also indicate the change in the performance of the labour input.

Productivity measurement implies the use of standards set for each input factor in terms of output. In circumstances where standards are not in use, productivity can be measured only when the output is converted into 'units or work' which is defined as the amount of work that can be performed by one unit of input. Thus, productivity can be measured by dividing the output by the performance of each input factor taken together.

Some of the well-known indices of productivity are given below:

(A) **Man-hour output:** The most widely used index of productivity is to work out the output per man hour it can be put as -

Productivity = Units of output / Total man-hours

(B) **Productivity Ratio:** The rate of return on capital employed is a valuable and widely used guide to many types of business decisions. This ratio of profit to capital employed is a valuable means of measuring the performance of divisions, sections, plants, products and other components of a business, and can be calculated as–

Productivity = Net Profit / Capital employed

(C) **Use of Financial Ratios:** There are many situations when time standards cannot be set and therefore, it is very difficult in such cases to measure the productivity by a direct method. In these cases, financial ratios can be used to measure the productivity by using its sales turn-over. But

'added value' is a more useful approach for measuring productivity. 'Added value' means output – inputs.
The most common financial ratio of productivity is–
Productivity = Added Value / Labour Costs
Productivity = Added Value / Conversion Costs
The first ratio gives the financial productivity of labour force and the second ratio gives the financial productivity of all the resources of the company put together.
(D) Other Useful Measures: There are many other useful productivity ratios to measure the productivity of various input factors. These are:

(i) Manpower Productivity = $\frac{Value\ of\ output\ of\ goods\ or\ services}{No.of\ workers\ or\ man\ hours\ used}$

ii) Materials Productivity = $\frac{Value\ of\ output\ of\ goods\ or\ services}{Units\ (or\ cost)\ of\ materials\ used}$

(iii) Capital Productivity = $\frac{Value\ of\ output\ of\ goods\ or\ service}{Capital\ assets\ employed}$

(iv) Energy Productivity = $\frac{\text{Value of output of goods or services}}{\text{Units (or cost) of energy used}}$

A combined measure of productivity can be taken as

Productivity = $\frac{\text{Value of output of goods or services}}{\text{Values of (labour + capital + materials + others inputs)}}$

There may be other input factors such as insurance, taxes, advertising etc. and their productivity can be measured likewise.
Each measure requires different kinds of data and only rarely such information is available for all commodities in an industry on continuous basis.

QUESTION 3.
What are the Tools of productivity or how to increase productivity?

ANSWER:
The productivity of an enterprise can be improved by improving the performance of various inputs and other factors affecting productivity. For this purpose, use of following tools can be recommended.
1. Human Aspects: Under this, cooperation of workers is sought in the following ways:
(i) More workers participation in management or in decision making through joint consultation.
(ii) Improving communication services.
(iii) Improving mutual trust and cooperation through improved job procedures, better training of employees, more workers incentives by implementing various incentive schemes, and labour welfare programmes. (iv) Better planning of work, more effective management, more democracy in administration, improved human relations and selection and training of personnel at various levels of management are some human efforts from the side of management in order to improve the productivity.
2. Supply of Inputs:
(i) Improvement in the nature and quality of raw materials and their supplies to the work.
(ii) Proper provision of plant, equipment and their maintenance.
(iii) Introduction of more and more machines and equipment in place of physical work.
(iv) Fuller utilisation of manpower and efficiency or capacity of plant and equipment employed.
3. Technological Aspects:
Certain methodological and technological developments are also necessary to improve the productivity of the concern. These are;

(i) Work, time and motion studies to determine better ways and means of doing a job.
(ii) Implementing various simplification, specialisation and standardisation programmes.
(iii) Applying control techniques comprising of production, planning and control, cost control and quality control techniques.
(iv) Improving layout of plants, shops and machine tools, and material handling and internal transportation system.
(v) Improving inspection techniques so as to minimise the wastage and defective work.

QUESTION 4.
Which Factors affecting industrial productivity?

ANSWER.
Factors affecting industrial productivity:
Productivity is defined to be some ratio between output and input. Thus all factors which affect output and inputs will also affect the measure of productivity.
The following factors affect the productivity.
1. Technological Development: Technical factors including the degree of mechanisation, technical know-how, raw materials, layout and the methods and techniques of work determine the level of technological development in any industry. The principal factors in technological development affecting productivity are:
(a) The Size of the Plant: The size of the plant and the capacity utilisation has direct bearing on productivity. Production below or above the optimum level will be uneconomical and will tend towards lower level of productivity.
(b) Research and Development: Investment in research and development may yield better method of work and better design and quality of products.
(c) Plant and Job Layout: The arrangement of machines and positions in the plant and the set-up of the work-bench of an individual worker will determine, how economically and efficiently production will be carried out.
(d) Machine and Equipment Design: Whether the design of machinery and equipment is modern and in keeping with the limitations and capacities of the workers, will also determine the production efficiency and level of productivity.
(e) Production Processes: Advanced production processes involving the use of modern integrated and automatic machinery and semi-processed materials have been known to help in raising levels of productivity.
(f) Power, Raw Materials etc. Improved quality of raw materials and increased use of power have a favourable effect on productivity.
(g) Scientific Management Techniques: Scientific management techniques such as better planning of work, simplification of methods, time and motion study, emphasis for reduced wastage and spoilage have positive effects on productivity.
It will be realised that technological development requires a great amount of funds and general economic and technical environment in the country. Thus capital plays an important role in increasing the productivity through implementing technological development. It should also be recognised that such developments influence the job performance of employees. With better machines, tools and processes, it should be considered that both ability and willingness to work should be increased.
2. Individual Factors: Individual factors such as knowledge, skill and attitude also affect the productivity of industry. Knowledge is acquired through training, education and interest on the part of learner. Skill is affected by aptitude (one's capacity to learn a particular kind of work), personality (emotional maturity, balance of mind etc.) as also by education, experience, training etc.

Increased knowledge, skill and aptitude certainly increase the productivity and a person deficient in these personal attributes is less productive than an average man.

The attitude (willingness of employee to work for the organisation) of employees towards the work and the organisation, affect their productivity to a great extent. Knowledge and skill without willingness are futile. The urge to work is a complex phenomenon governed by several factors such as formal and informal organisation, leadership, need, satisfaction, influence of trade unions etc. These factors motivate the workers to work better and with enthusiasm

3. Organisation Factors: Organisation factors include various steps taken by the organisation towards maintaining better industrial relations such as delegation and decentralisation of authority, participative management (workers' participation in management), organisational efficiency, proper, personnel policies relating to selection, placement, promotion, wage, salary levels, incentives, merit rating, job evaluation, training and provision for two-way communication, supervision, etc. These factors also influence motivation. Likewise the existence of groups with higher productivity as their goal is likely to contribute to the organisational objectives. These facts were brought out by Hawthorne experiments in U.S.A. A properlymotivated worker will certainly contribute to the industrial productivity.

4. Work Environment: The importance of proper work environment and physical conditions on the job has been emphasised by industrial psychologists and human engineers. Better work environment ensures the greatest ease at work through better ventilation and lighting arrangement, improved safety devices, reduction in noise, introduction of suitable rest-pause etc.

5. Other factors: There are several other factors that affect productivity. These are:

(a) Natural Factors: Physical, geographical and climatic conditions influence the productivity to a large extent. Abundance of natural resources affects the productivity and similarly climate affects the efficiency of workers to a great extent.

(b) Managerial Factors: The industrial productivity is influenced very much through managerial ability and leadership. The managerial ability of utilising the available resources to the maximum, organising capacity, foresightedness, decision-making ability and entrepreneurship are certain factors that contribute to productivity.

(c) Government Policy: Government policies towards industry also contribute to the industrial productivity. Taxation policy, financial and administrative policy, tariff policy and protection policy affect the productivity to a large extent.

Thus, the above factors are responsible for the increased productivity

Production and Productivity:

Production and productivity are not synonymous. Production refers to the volume, value or quantity of goods and services produced during a given period by a worker, plant, firm or economy. It is the sum total of results achieved by the various factors used together. Productivity, on the other hand, is not concerned with the volume of production. It is the ratio of output and input factors of an enterprise. It shows the efficiency of production or the efficiency level of input factors. In other words, productivity is relative to the resources used in turning out a certain amount of physical output, while production is used, more or less, in absolute sense. The distinction between these two terms becomes more clear when we find that increase in production does not necessarily mean the increase in productivity. If increase in production is attributed to the increase in the inputs of production in the same proportion, the production will have increased but productivity may have declined or may remain constant because the ratio of output and inputs has shown a decline or has not shown any improvement

Illustration 1

In a particular plant there are 10 workers manufacturing a single product and the output per month consisting of 25 days of that particular product is 200. How much is the monthly productivity ?

Solution:

Monthly productivity per worker = 200/ 10 = 20 units

Illustration 2

There are two industries A and B manufacturing hose couplings. The standard time per piece is 15 minutes. The output of two small scale industries is 30 and 20 respectively per shift of 8 hours. Find the productivity of each per shift of 8 hours. What is the expected production of each per week consisting of 6 days?

Solution:

Productivity $= \frac{\text{Actual production}}{\text{Standard production}}$

Standard production of hose complings per shift $= \frac{8 \times 60}{15} = 32$ pcs

Productivity of industry A $= \frac{30}{32} = \frac{15}{16}$ *and productivity of industry B* $= \frac{20}{32} = \frac{5}{8}$

If the productivity is expressed in percentage, the same for A is 15 /16 x 100 = 93.75%

and productivity of industry B is 5 /8 x 100 = 62.5%

Production per week of industry A = 30 × 6 = 180 nos. (Assuming the industry to work for one shift per day)

Production per week of industry B = 20 × 6= 120 nos. (Assuming the industry to work for one shift per day)

Illustration 3

The following data is available for a machine in a manufacturing unit:

Hours worked per day	8
Working days per month	25
Number of operators	1
Standard minutes per unit of production	
Machine time	22
Operator time	8
Total time per unit	30

(i) If plant is operated at 75% efficiency, and the operator is working at 100% efficiency, what is the output per month?

(ii) If machine productivity is increased by 10% over the existing level, what will be the output per month?

(iii) If operator efficiency is reduced by 20% over the existing level, what will be the output per month?

Solution:

(a) Hours worked per day = 8

Working days per month = 25

Hours worked per month = 25 × 8 = 200 hrs.

Machine time = 22 minutes Operator time = 8 minutes

Total time per unit = 30 minutes = ½ hr

No. of units that can be produced/month/operator = 200/1/2= 400
As the no. of operator is 1, possible monthly production = 400 units. As the plant operates at 75% efficiency. Monthly production = 400 × 75/ 100 = 300 units.
(b) If machine productivity is increased by 10% i.e. Machine time = 22 × 100/ (100 + 10) = 20 minutes.
Then, total time = 20 + 8 = 28 minutes
Monthly production = 400 x 30/ 28 x 75 / 100 = 321 units
(c) If operator efficiency reduced by 20% i.e.
Operator time = 8 × (100 + 20)/ 100 = 8 × 1.2 = 9.6 minutes.
Total time = 22 + 9.6 = 31.6 minutes.
Monthly production = 400 x 30/ 31.6 x 75 /100 = 284 units.
(Efficiency reduced by 20%. Instead of 100%, now 80% job is completed in 8 minutes. That means, operators time is increased to 10 minutes)

Illustration 4
The following data is available for a manufacturing unit :

No. of operators	15
Daily working hours	8
No. of days per month	25
Std. production per month	300 units
Std. Labour hours per unit	8

The following information was obtained for November 2015:

Man days lost due to absentism	30
Unit produced	240
Idle Time	276 man hours

Find the following:-
(a) Percent absentism
(b) Efficiency of utilisation of labour
(c) Productive efficiency of labour
(d) Overall productivity of labour in terms of units produced per man per month

Solution:
No. of days per month = 25
Daily working hrs. = 8
No. of operators = 15
No. of Man days per month = 15 × 25 = 375
Man days. Total working hrs. per month = 375 × 8 = 3,000
Hours lost in absentism in a month = 30 × 8 = 240
(a) Percent absentism =240 x 100/3000 = 8%
(b) Efficiency of utilisation of labour = $\frac{\text{Standard labour hour to produce 240 units}}{\text{Total labour hour}} \times 100$

$$\frac{240\text{hrs} \times 100}{3000 \text{ hrs}} = 8\%$$

(c) Standard time required to produce 240units = 240 × 8 = 1920 labour-hours.
In November, man hours lost = 30 × 8 = 240
idle time (in hours) = 276
Total loss of time = 516 hours.
Productive hours available in November = 3000 Less,
Total loss of time = (516) Actual labour-hours = 2484 hours

Efficiency of labour = $\frac{\text{Std. Labour hrs}}{\text{Actual Labour hrs}} = \frac{1920 \times 100}{2484} = 77.3\%$

(d) 15 men produces 300 units,

Std. labour productivity = 300/15 = 20 units.

In November, overall productivity = 240/15 = 16 units. (Ans.)

i.e. productivity falls by 25%.

Illustration. 5

An incentive scheme allows proportionate production bonus beyond 100% performance level. Calculate the amount of

(i) Incentive bonus and

(ii) Total payment received by an operator on a particular day during which the following particulars apply:

Operation	Assembling pocket transistor radio set
Work Content	30 Standard minutes per assembled set
Attended Time	8 Hours
Time spent on unmeasured work	2 Hours
Numbers of sets assembled during the day	15
Wage rate	₹ 4 per hour

(iii) What is the net labour productivity achieved by the operator during the day?

Solution:

Total standard minutes worked during the day = 30 × 15 = 450, working time = 8 - 2 = 6 hours = 360 minutes.

Performance = (450 × 100) /360 = 125% i.e incentive is payable on 25% which is above 100%

(i) Incentive bonus = 0.25 × 6 × 4 = ` 6 for six hours on measured work

(ii) Guaranteed wage for 8 hours = 8 × 4 = 32; Total earnings for the days

= ` (6 + 32) = 38

(iii) Net labour productivity = Output in units / Net man hours = 15 / 6 = 2.5 sets per hour

5.2 Five Key Aspects of Productivity

Productivity:

Productivity is commonly defined as a ratio between the output volume and the volume of inputs. In other words, it measures how efficiently production inputs are being used in an economy to produce a given level of output. Productivity is considered a key source of economic growth and competitiveness and, as such, is basic statistical information for many international comparisons and country performance assessments. For example, productivity data are used to investigate the impact of product and labour market regulations on economic performance. Productivity growth constitutes an important element for modelling the productive capacity of economies. It also allows analysts to determine capacity utilisation, which in turn allows one to gauge the position of economies in the business cycle and to forecast economic growth.

It is quite easy to say that productivity is output divided by input. But the term 'output' is ambiguous since there is no simple way of totalling the products and services. Again, when we consider input, we come across diverse factors. In order to produce anything, we need people, capital, land, facilities, machine tools, mineral deposits, energy resources, ingenuity, activity, climate, electrical power, organisation, rational price and a host of other factors. So, productivity can be more correctly stated as the relationship between achieving a result and the time it takes to accomplish it. Hence productivity = results/time. If a carpenter can complete a job in 5 hours and another completes the same job in 10 hours, the former's productivity is double the latter

$$\text{Productivity} = \frac{\text{Output in a Period of Time}}{\text{Input in the Same Period of Time}}$$

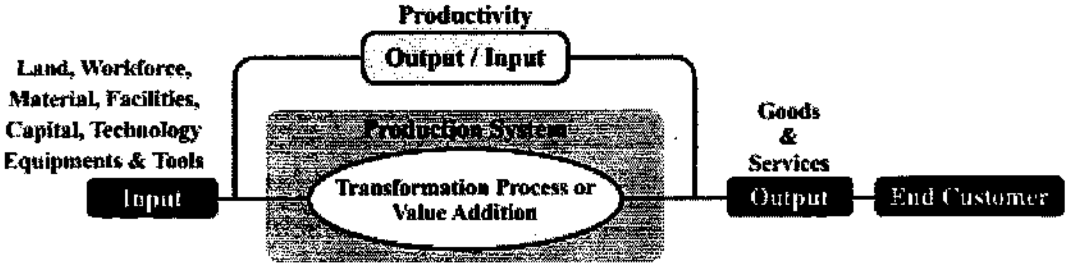


Figure 5.1: System Concept of productivity

- Employee productivity can be defined as the amount of work (or output) produced by an employee in a specific period of time. As a manager, it's important to understand how long it takes your teammates to complete specific tasks, and if there are any roadblocks or distractions along the way that you could help them overcome. 'We often assume that productivity means getting more things done each day. Wrong. Productivity is getting important things done consistently," says James Clear. He further added "And no matter what you are working on, there are only a few things that are truly important. Being productive is bout maintaining a steady, average speed on a few things, not maximum speed on everything. "Productivity in the workplace will often translate into good customer service, healthier work relationships, and motivated employees. Employee productivity can be measured in three ways: Measure goals, Measure quality of work, and measure the amount of work completed; and some of the ways to increase employee productivity in the workplace are - Improve Workplace conditions, Allow flexible schedules, Set clear deadlines and expectations, Encourage self-care and time off, Optimize meetings, Coach employees on their priorities, Learn time management skills, and Boost morale by celebrating accomplishments

- Material productivity in manufacturing units is a vital factor in ensuring a high level of effectiveness and efficiency. Materials is one of the basic inputs which constitute 50 to 70 percent of the total value of the output of selected companies. Therefore, to improve the performance of the selected organisations, material productivity will have to be improved. For calculating the material productivity ratio, material output (Net sales) is divided by the material input; the ratio reveals the output received in constant prices per rupees of material input. Suppose the base year material productivity ratio is 100,material index below 100 will mean low productivity and above 100 will mean improvement in productivity in comparison with the productivity of the base year.
- Land, labour and capital are the three basic inputs of the production process. But they do not make contribution to total output separately or independently. They produce goods and services only when brought together in the presence of an organizing authority or catalyst. This catalyst is, of course, management, and the three factors of production are the resources or inputs at the disposal of management. The competence and attitudes of managers have an important bearing on productivity. In many organisations, productivity is low despite latest technology and trained manpower. This is due to inefficient and indifferent management. Competent and dedicated managers can obtain extraordinary results from ordinary people. Job performance of employees depends on their ability and willingness to work. Management is the catalyst to create both. Advanced technology requires knowledgeable workers who in turn work productively under professionally qualified managers. No ideology can win a greater output with less effort. It is only through sound management that optimum utilization of human and technical resources can be secured. The Manager is the dynamic, life-giving element in every business. Without his leadership, the resources of production remain resources and never become production. In competitive economy, above all, the quality and performance of the managers determine the success of a business, indeed they determine its survival' [P. F. Drucker: The Practice of Management.
- Apart from these, some other factors also have a great impact on productivity. This includes:

QUESTION.5

Types of Technological Factors

ANSWER.

- Product design
- Plant layout
- Size and capacity of the plant
- Location of plant
- Timely supply of raw material
- Repairs and maintenance
- Material handling system
- Research and development
- Inventory control

QUESTION.6

Define;

1.Natural Factors

2. Social Factors

3. Political Factors

4. Economic Factors

ANSWER.

Natural Factors

It is quite obvious that there are some factors that are not under the control of anyone. These are natural factors. The physical, geographical, geological, and climatic conditions fall in this category. These factors highly influence industries that carry out extraction activity

Social Factors

We live in a society, and we have to follow its culture, traditions, customs, rules, and norms. Also, it poses a significant influence on productivity. However, the social factors, differ from place to place. This means what is considered wrong in India, might not be considered wrong in other countries like Japan or USA and vice versa.

Political Factors

To increase productivity, law and order, peace, and stability of the government are a must. Industrial policy, tariff policy, and taxation also have an influence on the firm's productivity.

Economic Factors

There are certain factors that also have an impact on productivity such as:

Market size, Banking and credit facilities and Transport and communication system

- Every organisation strives to create and sustain a climate of perpetual interest and passion among its personnel in order to improve results through the efficient use of facilities and resources. Productivity improvement techniques assure larger revenues, which can be used to instal cutting-edge technology and improve the overall performance of the business. Individual productivity development solutions include methods for managing time and increasing job efficiency. An organisation may undertake a number of key steps toward improving productivity.

(a) Develop productivity measures for all operations; measurement is the first step in managing and controlling an organisation.

(b) Look at the system as a whole in deciding which operations are most critical; it is over-all productivity that is important.

(c) Develop methods for achieving productivity improvement, such as soliciting ideas from workers (perhaps organizing teams of workers, engineers, and managers), studying how other firms have increased productivity, and re-examining the way work is done.

(d) Establish reasonable goals for improvement.

(e) Make it clear that management supports and encourages productivity improvement. Consider incentives to reward workers for contributions.

(f) Measure improvements and publicize them.

Some of the key techniques for managers to improve productivity at workplaces are as follows:

Time Your Time, Initiate Regular Breaks, Self-Imposed Deadlines, Quit the Habit of Multitasking, Encourage the "2-Minute Rule", Avoid Unnecessary Meetings, Use the Unexpected 'Bonus' Time, Be Proactive and Not Reactive.

5.3 TQM Basic Tools and Certification

Total Quality Management:

A philosophy that involves everyone in an organisation in a continual effort to improve quality and achieve customer satisfaction.

QUESTION. 7
What are the Basic Concepts in TQM?

ANSWER.
1. Top management commitment and support.
2. Focus on both internal and external customers.
3. Employee involvement and empowerment.
4. Continuous improvement (KAIZEN)
5. Partnership with suppliers
6. Establishing performance measures for processes.

QUESTION. 8
What are the Essentials of TQM Focus?

ANSWER.
1. Customer satisfaction
2. Leadership
3. Quality policy
4. Organisation structure
5. Employee involvement
6. Quality costs
7. Supplier selection and development
8. Recognition and reward.

QUESTION. 9
Which are the Underlying Principles in TQM?

ANSWER.
1. Strive for quality in all things (Total Quality)
2. The customer is the creation of quality
3. Improve the process or systems by which products are produced
4. Quality improvement is continuous, never ending activity (continuous improvement-Kaizen)
5. Worker involvement is essential
6. Ground decisions and actions on knowledge
7. Encourage team work and cooperation.

QUESTION. 10

Explain the Scope of TQM

ANSWER.

1. An integrated organisational infrastructure
2. A set of management practices
3. A wide variety of tools and techniques

TQM is Japanese approach to quality. The term TQM refers to a quest-for quality in an organisation. TQM is a process that underlines three philosophies. One is never-ending push to improve, which is referred to as continuous improvement; the second is the involvement of every employee in the organisation and the third is the goal for customer satisfaction, which means meeting or exceeding customer expectations. It often focuses on benchmarking world-class standards, product and service design and purchasing. In addition, TQM involves a number of other elements such as:

- Team approach,
- Employee empowerment
- Decisions based on facts rather than opinions,
- Knowledge of quality tools [flow charts, check sheets, histograms, pareto analysis, scatter diagrams etc.]
- Quality at the source and
- Inclusion of suppliers as a part of quality improvement programme

TQM is a process of continuous improvement at every level of the organisation-the centre of the entire process-is-customer satisfaction. TQM implies that the organisation is doing everything it can to achieve quality at all stages of the process, from customer demands, to product design, to engineering

TQM seeks to breakdown communication barriers among employees and also between the organisation and its external stakeholders, in order to increase cross-functional integration and provide new avenues for co-operation to improve quality. It would be incorrect to think of TQM merely a collection of techniques. Rather, TQM reflects a whole new attitude toward quality. It is about the culture of an organisation. To truly reap the benefits of TQM, the culture of an organisations must change. In other words, TQM organisation strives to develop co-operative relationships with its suppliers and distributors so that continuous improvement of quality becomes their goals too. Ford, Motorola, and GM have taken steps to develop long-term relationships with their suppliers and distributors.

5.4 ISO Standard Basics

Quality Certification

Many international businesses recognize the importance of quality certification. The EU, in 1987, established ISO [International Organisation for Standardisation] 9000 certification. Two of the most well known of these are ISO 9000 and ISO 14,000. ISO 9000 pertains to quality management. It concerns what an organisation does to ensure that its products or services are suitable to customers expectations. ISO 14,000 concerns minimization of harmful effects to the environment caused by its operations. Both ISO 9000 and ISO 14000 are related to an organisations processes rather than its products and services and they stress continual improvement

ISO 9000 is composed of the national standard bodies of 91 countries. About 90 countries have adopted ISO 9000 as national standards. This certification is intended to promote the idea of quality at every level in the organisation.

ISO certification is an elaborate and expensive process. Any firm seeking this certification needs to document how its workers perform every function that affects quality and install mechanisms to ensure that, they follow on expected lines. ISO 9000 certification entails a complex analysis of management systems and procedures. Rather than judging the quality of a particular product, ISO 9000 evaluates the management of the entire manufacturing process, from purchasing, to design, to training. A firm that seeks this certification must fill out a report and then be certified by a team of independent auditors. With certification comes registration in an ISO directory, that firms seeking suppliers can refer to, for a list of certified companies. They are generally given preference over unregistered companies.

There are essentially five standards associated with the ISO 9000 series. The series, if we place them on a continuum, would range from design and development through procurement, production, installation and servicing. Whereas, ISO 9004 only establishes guidelines for operation, ISO 9001, 9002 and 9003 are well-defined standards.

Quality System

9001 Model for Quality Assurance in Design, Production, Installation and Servicing. (To be used when conformance to specified requirements is to be assured by the supplier during several stages that may include design/development, production, installation and servicing)

9002 Model for Quality Assurance in Production and Installation. (To be used when conformance to specified requirements is to be assured by the supplier during production and installation).

9003 Model for Quality Assurance in Final Inspection Test. (To be used when conformance to specified requirements is to be assured by the supplier solely at final inspection and test).

Guidelines for Use

9000 Quality Management and Quality Assurance Standards - Guidelines for Selection and Use.

9004 Quality Management and Quality System Elements - Guidelines.

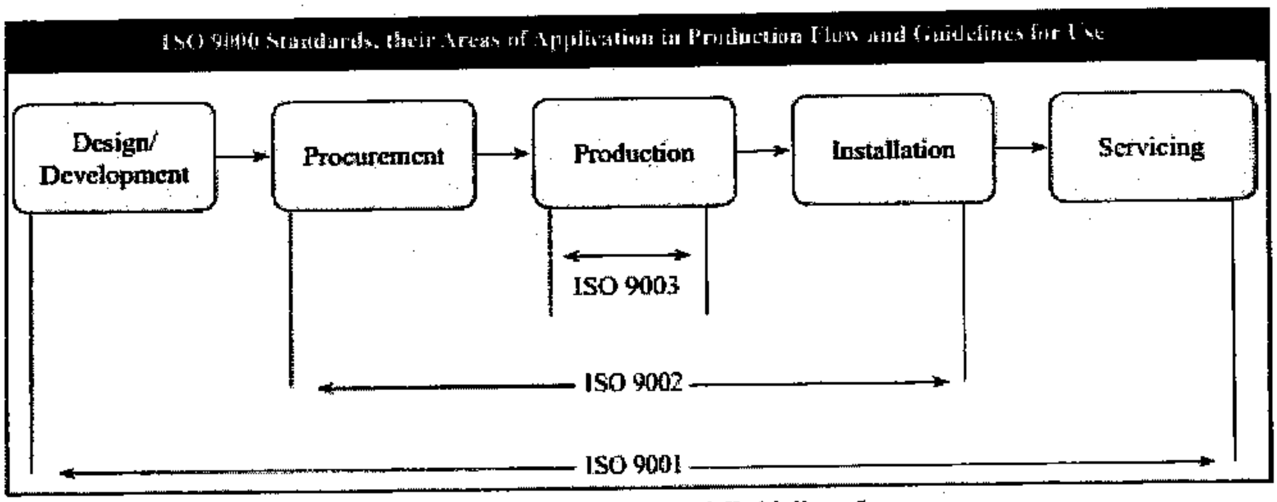


Figure 5.2: Production flow and Guidelines for use

ISO certification is a must for doing business with any member of the EU. In addition to the benefits of accessing the EU, ISO 9000 certification and registration is particularly helpful for companies that do not currently have a quality management system, as it provides guidelines for establishing the system and making it effective. The latest version of ISO 9000 forms the basis of eight quality management principles

1. A system approach to management
2. Continual improvement
3. Factual approach to decision making
4. Mutually beneficial supplier relationships
5. Customer focus
6. Leadership
7. People involvement 8. Process approach.
8. Process approach.

ISO standards are reviewed every five years and revised if needed. This helps ensure they remain useful tools for market place. The challenges faced by business and organisations today are very different from few decades ago and ISO 9001 has been updated to take this new environment into account

The last version was ISO 9001: 2008 and has been replaced by further updated version ISO 9001:2015 on and from September 2018.

Illustration 6

Compute the productivity per machine hour with the following data. Also draw your interpretation.

Month	No. of machines employed	Working hours	Production Units
January	400	220	99,000
February	550	180	1,00,000
March	580	220	1,25,000

Solutions

Month	No. of machines employed	Working hours	Machine hours	Production Units
January	400	220	88,000	99,000
February	550	180	99,000	1,00,000
March	580	220	1,27,600	1,25,000

P = Productivity per machine hour

= Number of units produced /Machine hours

For January P = 99,000/88,000 = 1.125

February P = 100,000 / 99,000 = 1.010

March P = 125,000 /127,600 = 0.980

Interpretation: Though the total production in number of units is increasing, the productivity is declining

Illustration 7

Calculate the standard production per shift of 8 hours duration, with the following data: Observed time per unit = 5 minutes, Rating Factor -120%, Total allowances = 30% of normal time.

Solution:

Normal time per unit = Observed time / unit × Rating factor = 5 × (120/100) = 6minutes

Allowances = 30% of normal time = (30 × 6)/100 = 1.8 minutes

Standard time/unit = Normal time/unit + Allowances = 6 + 1.8 = 7.8 minutes / unit Standard production in shift of 8 hours = (8 × 60)/7.8 = 61.54 units.

Illustration 8

Study in the Packaging Department of a Soft drinks Manufacturing unit revealed the following facts for a worker Basant Rao Patil.

Activity Element \ Cycle No.	1	2	3	4	Performance Rating
(A) Get empty car toon	0.15min	0.25min	—	0.17min	90%
(B) Place 30 bottles in the cartoon	1.56min	*	1.80min	1.75min	105%
(C) Close the car toon & set aside	0.20min	†	0.10min	0.15min	95%
(D) Smoking	—	0.50min	—	—	—

* Bottles slipped out of hands and broke

† Empty cartoon not set aside and used for packaging in the next cycle.

Calculate the standard production by Basant Rao in a shift of 8 hours when the units standard rules allow 10% as Allowance Factor.

Solution:

Average time for Activity Element A = $\frac{0.15+0.25+0.17}{3}$ = 0.19 min.

Average time for Activity Element B = $\frac{1.56+1.80+1.75}{3}$ = 1.703 min. Average

Average time for Activity Element C = $\frac{0.20+0.10+0.15}{3}$ = *0.15 min.*

Computation of Normal Time

Activity Element	Average time (Mins)	Performance Rating (%)	Normal Time (Mins)
(1)	(2)	(3)	(4) = (2) × (3) ÷ 100
A	0.19	90	0.171
B	1.703	105	1.788
C	0.15	95	0.142
Total	–	–	2.101

So Normal Time for the job of packaging = 2.101 Mins

Standard Time = $\frac{\text{Normal Time}}{1-(\text{Allowance Factor } 100)} = \frac{2.101}{1-\frac{10}{100}}$ =2.334 Mins.

Standard Production in a shift of 8 hours = $\frac{8 \times 60}{2.334}$ =205.66 cartoons.

Illustration 9

A department works on 8 hours shift, 288 days a year and has the usage data of a machine, as given below:

Product	Annual Demand (units)	Processing time (Standard time in hours)
A	325	5.0
B	450	4.0
C	550	6.0

Calculate (a) Processing time needed in hours to produce products A, B and C, (b) Annual production capacity of one machine in standard hours, and (c) Number of machines required

Solution:

(a)The processing time needed in hours to produce products A, B and C in the quantities demanded using the standard time data:

Product	Annual Demand (units)	Processing time (standard time in hours)	Processing time needed to produce demand quantity (hrs.)
A	325	5.0	325 × 5 = 1,625
B	450	4.0	450 × 4 = 1,800
C	550	6.0	550 × 6 = 3,300
			Total = 6,725 hrs.

(b)Annual production capacity of one machine in standard hours = 8 × 288 = 2,304 hours per year.

(c)Number of machines required = Work load per year / Production capacity per Machine = 6,725 / 2,304 =2.90 machines = 3 machines

Illustration 10

Following results are recorded in a study of work sampling carried for 100 hours in a Machine Shop.

1. Total no. of observations recorded – 2500
2. No. of observations in which no working activity is noticed – 400
3. Ratio of Mannual to Machine elements – 2 : 1
4. Average Rating Factor – 115%
5. No. of articles produced during the study period – 6000

As per the policy of the company, rest and personal allowances are taken as 12% of Normal Time.
Calculate Standard Time to produce an article.
Given that the shop produces 42000 articles per month of 25 working days by 5 workers working for a shift of 8 hours per day. Consider absentism to be 7%.
Compute Efficiency of utilisation of Labour and Productive Efficiency of Labour.

Solution:

Percentage of working time $= \frac{2500-400}{2500} x100 = 84\%$

Actual working time in a study of 100 hours = 84 hours = 84 × 60 = 5040 mins. Production – 6000 articles

Time required to produce an article = 5040/ 6000 = 0.84 mins

Of this Manual time = 0. 84 x 2/ 3 (Ratio of Manual to Machine activity elements = 2:1)
= 0.56 mins

Machine time = 0.84 × 1/ 3 = 0.28 min.

Normal Time of man = Time of man as per study × Rating Factor/ 100
= 0.56 115/ 100 . = 0.644 min.

Normal Time of machine = 0.28 min

Allowances for man = 12% of Normal time of Man = 0.12 × 0.644 = 0.077 min

Standard Time for Man to produce an article = Normal Time of Man + Allowances
= 0.644 + 0.077 = 0.721 min.

Standard Time for machine = 0.28 min.

Standard Time to produce an article = 0.28 + 0.721 = 1.001 mins.

Standard time required to produce 42000 articles = 42000 × 1.001 = 42042 mins. = 700.7 hours.

No. of days/month - 25, Daily working hours - 8, No. of workers - 5

Total available working hours/month = 5 × 25 × 8 = 1000

Actual working hours/month = 1000 × 0.93 [Since Absentism = 7%]

= 930

Efficiency of utilisation of Labour = $\frac{\text{Standard time to produce 42000 article}}{\text{Total available hours}}$ x 100

= 700.7/ 1000 x 100 70.07 %

Illustration 11

A cement factory in Madhya Pradesh works 7 days a week in 3 shifts per days having maintenance in the first shift of around 2 hours. It has roughly 100 workers which produces only pozzolanic properties cement better known as PPC. The output per month is around 2500 tonnes of PPC. Find the productivity per worker?

Solution:

Productivity per worker = 2500/100 = 25 tonnes.

Illustration 12

Compare the productivity of two plant of tobacco company situated in two different state Y and Z in an 8-hour shift. The standard time in manufacture a tobacco packet is 10 min. The output is 40 and 55 of two different plants in a shift. Find also the expected productivity of both plants in a week.

Solution:

Productivity =Actual production/Standard production

Standard production of tobacco plant is = 8 × 60 / 10 = 48 packets.

Productivity of plant located in state Y = 40/48 = 0.833 (83.33%)

Productivity of plant located in state Z = 55/48 = 1.146 (114.6%)

Now, expected productivity of plant in Y = 40 × 7 = 280 Packets (if it works for 7 days with one shift)

And, expected productivity of plant in Z = 55 × 7 = 385 Packets (if it works for 7 days with one shift)

Illustration 13

For the given data of manufacturing unit which produces spare parts of HEMM the operators time, machine time and total time are 10, 28 and 38 minutes respectively. If there are one operator and working hour per day is 8 hr and considering 22 working days in a month. Find

(a) If plant is working at 65% efficiency, what is the expected output per month?

(b) If plant productivity is increased by 20% over the existing level, what will be the output per month?

(c) If operator efficiency is reduced by 30% due to injury over the existing level, what will be the output per month?

Solution:

Working hours per month = 22 × 8 = 176 hrs.

No. of units that can be produced/month by the operator = 176 × 60/38= 277.89 approx 278

a. Now if the plant efficiency = 65% and since there is only one operator and its efficiency is 100% then expected production of spare parts = 277.89 × 0.65 = 180.62 Units.

b. If the plant efficiency increases by 20% new output will be New machine time is 28 × 100/120 = 23.33 minutes, and then the total time = 10 + 23.33 = 33.33 min New monthly production = 277.89 × (38/33.33) × 0.65 = 205.93 Units

c. If the operator's efficiency is reduced by 30% then new production will be New Operator's time = 10 × (130/100) = 13 min So new total time = 13 + 28 = 41 minutes. Now new monthly production = 277.89 × (38/41) × 0.65 = 167.41 units

Illustration 14

Following are the data related to call centre Firm which gives tech and non tech support to large IT companies. It has 20 executives to address the queries which has 8 hr a shift having on an average 24 working days in month. On an average the company is able to address around 290 calls in a month.

Additional data for the current month is obtained

(a) No of call logged for the month = 250

(b) Idle time = 275-man hours

(c) Man days lost (absenteeism) = 28.

Find

1. Efficiency of utilisation of manpower
2. Absenteeism (%)
3. Overall productivity of manpower.

Solution;

No of man days per month = 20 × 24 = 480 Man days

Total working hr per month = 480 × 8 = 3840.

Hr lost in absenteeism in a month = 28 × 8 = 224

1. Efficiency of utilisation of manpower = (250 × 8 / 3840) × 100 = 52.08%
2. Absenteeism = (224 hr/3840hr) ×100 = 5.833%.
3. 20 men logs 290 calls in a month

So the St. manpower productivity = 290/20 = 14.5 calls.

In the current month overall productivity = 250/20 = 12.5 calls

So, the productivity has fallen from 14.5 to 12.5 i.e. 13.8%

Illustration 15

Find the productivity of IT firm in terms of business achieved for the following data and comment

Quarter	No of Employees	Working Hours	Business Achieved (`)
Q1	1600	800	1000000
Q2	1500	750	1024000
Q3	1700	775	1300000
Q4	2000	900	1200000

Solution:

Quarter	No of Employees	Working Hours	Man Hours	Business Achieved (`)	Productivity
Q1	1600	800	1280000	1000000	0.78125
Q2	1500	750	1125000	1024000	0.910222
Q3	1700	775	1317500	1300000	0.986717
Q4	2000	900	1800000	1200000	0.666667

Man hour of Q1 = No of Employee × Working Hours = 1600 × 800 = 1280000
Productivity in Q1 = 1000000/1280000 = 0.78125
From the above table we can say that the productivity of Q3 is best then follows Q2 then Q1 and the least is Q4.

Illustration 16
Find the standard production for 8 hr shift. If allowance = 25% of normal time, Observer time per unit is 7 min and the rating factor is 110%.

Solution
Normal time per unit = Observed time / unit × Rating factor = 7 × (110/100) = 7.7 minutes Now,
Allowances = 25% of normal time = (25 × 7.7)/100 = 1.925 minutes
So, Standard time/unit = Normal time/unit + Allowances = 7.7 + 1.925 = 9.625 minutes / unit
Hence, Standard production in shift of 8 hours = (8×60)/9.625 = 49.89 units (50 Units approx.)

Illustration 17
A captive plant works for one shift in a day i.e. 8 hr a shift for 200 days in a year to cater for large automobile company. It produces three product having annual demand as 425, 429 and 546 units respectively. The processing time (standard time in hr) are 4, 5 and 5.5 hours respectively. Calculate

(a)Processing time required to produce all three products.
(b)Annual production
(c)And number of machine required

Solution:
(a)Product 1 → Processing time needed to produce demand quantity (hrs.) = Annual Demand × Processing time (Standard time in hr) = 425 × 4 = 1700 hrs
Product 2 → Processing time needed to produce demand quantity (hrs.) = 429 × 5 = 2145 hrs
Product 3 → Processing time needed to produce demand quantity (hrs.) = 546 × 5.5 = 3003 hrs.
Hence total time needed= 6848hrs
(b)Annual production capacity for a single machine = 8 × 200 =1600 hrs for a year.
(c)Minimum number of machines required = 6848/1600 = 4.29 (5 machine Approx.)

Illustration 18
Below data are collected related to work study for 150 hrs on a floor shop employing 7 labours having a shift of 8 hrs in a day.

(a)Number of observations documented in total = 3000
(b)Number of observations in which no working activity is observed = 500
(c)Manual to machine ratio = 3:2
(d)Average Rating factor = 120%
(e)Number of product produced during the period of study = 7000

Company has its own policy regarding personal allowance which is pegged at 11% of normal standard time to produce a product.
The floor shop produces 49000 products per month for 24 working days, it has an absenteeism of around 6%.
Calculate efficiency of utilisation of Labour and Productive Efficiency of Labour.

Solution:
Percentage of working time = ((3000-500)/3000) × 100 = 83.33%

Actual working time in a study of 150 hrs = 150 × 0.8333 × 60 = 7500 min.
Production = 7000 units.
Time required for one unite to produce = 7500/7000 = 1.0714 min
So, manual time on this is 1.0714 × (3/5) = 0.643 mins and machine time is 1.0714 × (2/5) = 0.43 mins.
Normal time of labour = time of labour as per study × Rating Factor/100 = 0.643 × 120/100= 0.772 min.
And normal time of machine = 0.43 min.
Now, if allowance is considered which is 11% of normal time which bring the standard time for the labour to produce = 0.772 × 1.11 = 0.857 min.
Hence Standard time required to produce a product = 0.857 + 0.43 = 1.286 min.
And. Standard time required to produce 49000 units = 49000 × 1.286 = 63,038 min = 1050 hrs.
Now, total available working hrs for 24 working day of 8 hrs shift of 7 labours = 1344 hr in a month
Taking absenteeism in consideration actual working hour left = 1344 × 0.94 = 1263.4 hrs.
Efficiency of utilisation of Labour= 1050/1344 = 0.7813 (78.13%)
And
Productive efficiency of Labour = 1050/1263.4 = 0.8312 (83.12%)

6.1 Project Planning

Introduction

Planning g begins with well-defined objectives. The project team may be drawn from several organizational departments, e.g., engineering, production, marketing, and accounting. Project definition involves identifying the controllable and uncontrollable variables involved, and establishing project boundaries. Performance criteria should relate to the project objectives, which are often evaluated in terms of time, cost, and resource utilisation.

Project planning is part of project management, which relates to the use of schedules such as Gantt charts to plan and subsequently report progress within the project environment. Project management is the discipline of organizing and managing resources (e.g. people) in such a way that the project is completed within defined scope, quality, time and cost constraints. A project is a temporary and one-time endeavour undertaken to create a unique product or service, which brings about beneficial change or added value. This property of being a temporary and one-time undertaking contrasts with processes, or operations, which are permanent or semi-permanent ongoing functional work to create the same product or service over and over again. The management of these two systems is often very different and requires varying technical skills and philosophy, hence requiring the development of project managements.

The first challenge of project management is to make sure that a project is delivered within defined constraints. The second, more ambitious challenge is the optimized allocation and integration of inputs needed to meet predefined objectives. A project is a carefully defined set of activities that use resources (money, people, materials, energy, space, provisions, communication, etc.) to meet the predefined objectives.

Initially, the project scope is defined and the appropriate methods for completing the project are determined. Following this step, the durations for the various tasks necessary to complete the work are listed and grouped into a work breakdown structure. The logical dependencies between tasks are defined using an activity network diagram that enables identification of the critical path. Float or slack time in the schedule can be calculated using project management software. Then the necessary resources can be estimated and costs for each activity can be allocated to each resource, giving the total project cost. At this stage, the project plan may be optimized to achieve the appropriate balance between resource usage and project duration to comply with the project objectives. Once established and agreed, the plan becomes what is known as the baseline. Progress will be measured against the baseline throughout the life of the project. Analysing progress compared to the baseline is known as earned value management.

QUESTION 1.
What is Gantt Chart

ANSWER.
Gantt Chart is a principal tool used in scheduling and also in some methods of loading. This chart was originated by the American engineer Henry L. Gantt and consists of a simple rectangular grid, divided by series of parallel horizontal and vertical lines. The vertical lines always divide the horizontal scale units of time. The time units can be in years, months, weeks, days, hours, minutes or even seconds according to the work for which it is prepared. In this chart, the time which an activity takes in completing the task is represented by the horizontal line. The length of the line is drawn in proportion to the duration of time. Generally, the time in the chart should flow from left to right and activities be listed from top to bottom. The progress of the work may be show by a bar or a line within the uprights of the activity symbol and its length should represent the amount of work completed. Horizontal lines divide the chart into sections which can represent various work tasks (work schedule) or work centres (load schedule). When it shows only work tasks-products, orders, or operations to be completed, it is known as Work Schedule. When it shows the same task opposite the work centres at which they are produced factories, departments, workshops, machine tools or men it is known as Load Chart.
The units scheduled or loaded on these charts are always the same because these work tasks are known as having a known standard time. The work tasks can be represented on the chart by numbers or symbols. The symbols used on the chart may vary from company to company.

QUESTION 2.
Define Network Analysis and its important characteristics

ANSWER.
Network Analysis: Routing is the first step in production planning. In small projects, routing is very simple. Sequence of operations is almost decided and the operations can be performed one after the other in a given sequence. But in large project, this is rather a difficult problem. There may be more than one route to complete a job. The function of production manager is to find out the path which takes the least time in completing the project.
In a big project, many activities are performed simultaneously. There are many activities which can be started only at the completion of other activities. In such cases, a thorough study is required to collect the complete details about the project and then to find out a new, better and quicker way to get the work done in a decent way. In such cases, the first step is to draw some suitable diagram showing various activities and their positions in the project. It should also explain the time to be taken in completing the route from one operation to the other. It also defines the way in which the delay in any activity can affect the entire project in terms of both money and time. Such a diagram is called network diagram. A network is a picture of a project, a map of requirements tracing the work from a departure point to the final completion objective. It can be a collection of all the minute details involved or only a gross outline of general functions.
Important characteristics in a Network Analysis: The following are some important points to remember in a network analysis:

(i) The objective is to finish within the specified time otherwise there is a penalty.
(ii) Various activities are to be completed in an order; however, a number of activities are performed simultaneously while there are many other activities, which can be started only when some other activities are completed.
(iii) The cost of any activity is proportional to its time of completion.
(iv) There can be hurdles in the process and the resources to be allocated may be limited. A network graph consists of a number of points or nodes, each of which is connected to one or more

of the other nodes by routes or edges. It is a set of operations and activities describing the time orientation of a composite project.

QUESTION 3.
Explain Concept of Network drawing

ANSWER.

Important Concept of Network drawing:

A Network can be considered as a means of graphically depicting all the operations involved in a Project. When a Network is constructed then it is essential to maintain the relationship between various Activities of the Project.

Some of the key concepts of Network drawing can start with defining some of the key terminology of Network. These are –

Activity: – All projects may be viewed as a number of operations which when completed will cause the completion of the project. Each of these operations is termed as an **Activity** of the project which require expenditure of time and resources for accomplishment.

In a Network diagram, an Activity is depicted by a single arrow (→). This is not scaled and as such its length has no bearing on the time the Activity takes for its completion. In other words, the length of Activity arrow is drawn conveniently so that the clarification of relationship of activities is proper. It does not depict the importance of time. The head of the arrow shows the direction of flow of the Activity. An Activity cannot begin until the preceding one/ones is/are not completed.

Predecessor Activity means the Activity that must be completed prior to the start of an Activity.

Successor Activity cannot be started until are or more of the other activities are completed but immediately succeed them.

Concurrent Activities means the Activities which can occur simultaneously.\

Event: – An **Event** represents a specific accomplishment in the project and takes place at a particular instant of time and does not, therefore consume time or resources. It can be considered as a time-oriented reference point that signifies the end of an activity and start of another. Events are represented by circles (◯) in a Network diagram, Events are also known as Nodes.

All Activity arrows must begin and end with Event nodes as shown below :-

Start Event is also called tail event & Finish Event is called Head Event of the activity.

Merge Event is that event where more than one Activity ends. In the diagram below 3 and 4 are the Merge Events.

Burst Event is that Event from where more than one Activity starts. In the diagram below, 2 and 1 are the Burst Events.

Merge and Brust Events are those Events where more than one Activity ends and from where more than one Activity starts. In other words, these are the combination of both Merge and Brust Events.

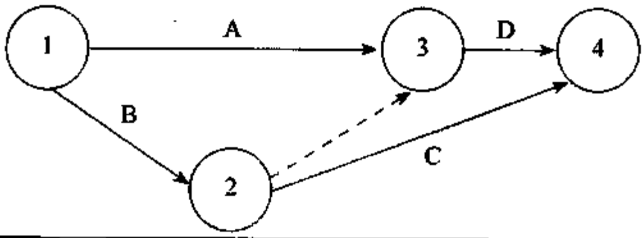
Dummy Activity: – Activities occurring simultaneously, is a very common feature in a project. Also it can so happen that two Activities are having same Start and End Events. To resolve such situations, Dummy Activities are introduced. Hence as a rule there is only one Activity between two Events. With the use of Dummy Activity, other activities can be identified by unique end events.

Dummy Activities consume no time or resource. In Network diagrams these are represented by dashed arrows (- - - ➔) and is inserted in the Network to clarify activity pattern in the following situations

(a) to make activities with common start and end Events distinguishable

(b) to identify and maintain the proper precedence relationship between activities that are not connected by events.

For the situation where A & B are concurrent activities, C is dependent on B and D is dependent on both A & B we have no other option but to introduce a Dummy Activity (Shown in the diagram) to clearly represent the precedence relationship of the Activities.



QUESTION 4.

What is the Procedure for drawing a network diagram

ANSWER.

Procedure for drawing a network diagram: The procedure for drawing a network diagram may be explained below.

There are three basic questions and the network depends on them.

These questions are:

- Which operation must be completed before each given operation can be started?
- Which activities can be carried out in parallel?
- Which operation immediately succeeds other given activities?

The common practice is simply to work backward through the list of operations, generating the immediate predecessors for each operation.

QUESTION 5.

Define Slack and float, and use of Float Information in Decision Making

ANSWER.

Slack - Slack signifies the freedom for rescheduling or to start the job. It can be calculated by the difference between EFT and LFT for any job. A job for which the slack time is zero is known as critical job. The critical path can be located by all those activities or events for which slack time is either zero or float time is the least. The abbreviations EFT and LFT given in the above line have the following explanation.

EFT (Earliest Finish Time) - this is the sum of the earliest start time plus the time of duration for any event.

LFT (Latest Finish Time) - It is calculated from the LFT of the head event. For its calculation total project time is required. The total project time is the shortest possible time required in completing the project.

Floats - Floats in the network analysis represent the difference between the maximum time available to finish the activity and the time required to complete it. There are so many activities where the maximum time available to finish the activity is more than the total time required to complete it. This difference is known as floats.

Floats may be total, free, and independent:

Total Float: Total float is the maximum amount by which duration time of an activity can be increased without increasing the total duration time of the project. Total float can be calculated as follows:

(i) First, the difference between Earliest Start Time (EST) of tail event and Latest Finish Time (LFT) of head event for the activity shall be calculated.

(ii) Then, subtract the duration time of the activity from the value obtained in (i) above to get the required float for the activity.

The total float can be helpful in drawing the following conclusions:

(a) If total float value is negative, it denotes that the resources for completing the activity are not adequate and the activity, therefore, cannot finish in time. So, extra resources or say critical path needs crashing in order to reduce the negative float.

(b) If the total float value is zero, it means the resources are just sufficient to complete the activity without any delay.

(c) If the total float value is positive, it points out that total resources are in excess of the amount required or the resources should be reallocated to avoid the delay otherwise the activity will be delayed by so much time.

Free Float: It is that fraction from total float of an activity which can be used for rescheduling the activity without affecting the succeeding activity. If both tail and head events are given their earliest times, i.e., EST and EFT the Free Float can be calculated by deducting head slack from total float, i.e.,

Free Float = Total float - Slack time of the head event.

Independent Float: It is the time by which the start of an activity can be rescheduled without affecting the earliest start time of any immediately following activities assuming that the preceding activity has finished at its latest finish time. It may be calculated as

Independent Float = Free Float - Tail Slack

or, Independent Float = Free Float -Slack Time of tail event. The basic difference between slack and float time is that a slack is used with reference to events whereas float is used with reference to activity

Use of Float Information in Decision Making: The float information can be used in decision-making in the following ways:

(i) Total float can affect both the previous and the subsequent activities.

(ii) Total float can be used without affecting the subsequent activities.

(iii) Independent float can be used in allocating the resources elsewhere and increasing the time of some noncritical activities.

(iv) Negative float signifies reduction in target time to finish the work in time

QUESTION 6.

Define Critical Path Method (CPM).

ANSWER.

Critical Path Method (CPM): The critical path analysis is an important tool in production planning and scheduling. Gnatt charts are also one of the tools of scheduling but they have one disadvantage for which they are found to be unsuitable. The problem with Gnatt Chart is that the sequence of operations of a project or the earliest possible date for the completion of the project as a whole cannot be ascertained. This problem is overcome by this method of Critical Path Analysis.

CPM is used for scheduling special projects where the relationship between the different parts of projects is more complicated than that of a simple chain of task to be completed one after the other.

This method (CPM) can be used at one extreme for the very simple job and at other extreme for the most complicated tasks.
A CPM is a route between two or more operations which minimises (or maximises) some measures of performance. This can also be defined as the sequence of activities which will require greatest normal time to accomplish. It means that the sequence of activities which require longest duration are singled out. It is called at critical path because any delay in performing the activities on this path may cause delay in the whole project. So, such critical activities should be taken up first.
One of the purposes of critical path analysis is to find the sequence of activities with the largest sum of duration times, and thus find the minimum time necessary to complete the project. The path of the Network with the critical series of activities is known as the **'Critical Path'**.
Under CPM, the project is analysed into different operations or activities and their relationship are determined and shown on the network diagram. So, first of all a network diagram is drawn. After this the required time or some other measure of performance is posted above and to the left of each operation circle. These times are then combined to develop a schedule which minimises or maximises the measure of performance for each operation. Thus CPM marks critical activities in a project and concentrates on them
Thus CPM technique is a very useful analysis in production planning of a very large project.

QUESTION 7.
Explain PERT (Programme Evaluation and Review Technique) and its Major Features of PERT

ANSWER.
The following are the main features of PERT:
(i) All individual tasks should be shown in a network. Events are shown by circles. Each circle represents an event–a subsidiary plan whose completion can be measured at a given time.
(ii) Each arrow represents an activity –the time - consuming elements of a programme, the effort that must be made between events.
(iii) Activity time is the elapsed time required to accomplish an event. In the original PERT, three-time values are used as follows:
- t_1 (Optimistic time): It is the best estimate of time if everything goes exceptionally well.
- t_2 (Most likely time): It is an estimated time what the project engineer believes necessary to do the job or it is the time which most often is required if the activity is repeated a number of times.
- t_3 (Pessimistic time): It is also an estimate of time of an activity under adverse conditions. It is the longest time and rather is more difficult to ascertain.

The experiences have shown that the best estimator of time out of several estimates made by the project engineer is:

$t = \frac{t_1+4t_2+t_3}{6}$ and the variance of t is given by: $V(t) = \left(\frac{t_3 - t_1}{6}\right)^2$

(iv) The next step is to compute the critical path and the slack time.
A critical path or critical sequence of activities is one which takes the longest time to accomplish the work and the least slack time

QUESTION 8.
Difference in PERT and CPM.

ANSWER.
Although these techniques (PERT and CPM) use the same principles and are based on network analysis yet they are different in the following respects from each other:

(i) PERT is appropriate where time estimates are uncertain in the duration of activities as measured by optimistic time, most likely time, and pessimistic time, whereas CPM (Critical Path Method) is good when time estimates are found with certainty. CPM assumes that the duration of every activity is constant and therefore does not deal with uncertainty in time.
(ii) PERT is concerned with events which are the beginning or ending points of operation while CPM is concerned with activities.
(iii) PERT is suitable for non-repetitive projects while CPM is designed for repetitive projects.
(iv) PERT can be analysed statistically whereas CPM not.
(v) PERT is not concerned with the relationship between time and cost, whereas CPM establishes a relationship between time and cost and cost is proportionate to time.

6.2 Project Life Cycle

QUESTION 9.
What is project Management?

ANSWER.
A project is defined as a sequence of activities undertaken for getting a set of tasks done to achieve the desired business goals successfully. Project Management centres on planning and managing everything involved in delivering a Project.

QUESTION 10.
What is project?

ANSWER.
A project is defined as a one-time activity with a series of tasks that produces a specific outcome to achieve organizational goals.
Projects are a set of interdependent tasks that have a common goal. No matter what the project is, each project is broken down into objectives and what needs to be done to achieve them, ensuring that the project stays on track and is completed as per plan.
The primary constraints of a project are:
- Time - the schedule for the project to reach completion
- Cost - the budget allocated for the project to meet its objectives and complete it on time
- Scope - the specific deliverables of the project
- Quality - the standard of the outcome of the project

QUESTION 11.
Explain Phases of Project Management Life Cycle

ANSWER.
1. Project Initiation.
It provides an overview of the project, along with the strategies required to attain desired results. It is the phase where the feasibility and business value of the project are determined
The Project Charter is considered to be the most important document of any project as it comprises:
- Business vision and mission
- Project goals and benefits
- List of stakeholders
- List of stakeholders

- Project deliverables
- Risks associated with the project
- Project budget and resources

2. Planning phase:

- Create a Statement of Work document to flesh out the details of project deliverables
- Develop a Work Breakdown Structure
- Create a project plan, assign team members (and other resources) to the various tasks and build a detailed project timeline
- Identify the Project Team roles and other resources for the project. At this stage, the Project Manager - working with a project staffing function - will most likely identify specific people for some of the key roles needed for the success of the project.
- Create a risk mitigation plan to identify potential risks and develop a strategy to minimize them
- Incorporate an effective change management plan for necessary changes in the project and to avoid bottlenecks
- Create a communication plan to schedule interactions with relevant stakeholders

3. Project Quality Management:

- The main principle of project quality management is to ensure the project will meet or exceed stakeholder's needs and expectations.
- Project Quality management consists of four main processes:
- Quality Definition
- Quality Assurance
- Quality Control
- Quality Improvements
- Quality Definition:

  Quality management implies the ability to anticipate situations and prepare actions that will help bring the desired outcomes. The goal is the prevention of defects through the creation of actions that will ensure that the project team understands what is defined as quality
- Quality Assurance:

  Quality Assurance is a process to provide confirmation based on evidence to ensure to the donor, beneficiaries, organization management and other stakeholders that product meet needs, expectations, and other requirements. It assures the existence and effectiveness of process and procedures tools, and safeguards are in place to make sure that the expected levels of quality will be reached to produce quality outputs.
- Quality Control:

  Quality control is the use of techniques and activities that compare actual quality performance with goals and define appropriate action in response to a shortfall.
- Quality Improvements:

  Quality improvement refers to the application of methods and tools to close the gap between current and expected levels of quality by understanding and addressing system deficiencies and strengths to improve, or in some cases, re-design project processes.

4. Execution Phase

This stage is where the bulk of the project happens. Deliverables are built to make sure the project is meeting requirements. This is where most of the time, money, and people are pulled into the project. This phase happens in tandem with the execution phase. As the project moves forward, the project manager must make sure all moving parts are seamlessly headed in the right direction. If adjustments to the project plan need to be made due to unforeseen circumstances or a change in direction, they may happen here.

During the controlling and monitoring phase, project managers may have to do any of the following:

- Manage resources
- Monitor project performance
- Risk management
- Perform status meetings and reports
- Update project schedule
- Modify project plans.

5. Project Closure:

The closing phase is a critical step in the project management life cycle. It signals the official end of the project and provides a period for reflection, wrap-up, and organization of materials.

Project managers can:

- Take inventory of all deliverables
- Tie up any loose ends
- Hand the project off to the client or the team that will be managing the project's day-to-day operations
- Perform a post-mortem to discuss and document any learnings from the project
- Organize all project documents in a centralized location
- Communicate the success of the project to stakeholders and executives
- Celebrate project completion and acknowledge team members

6.3 Gantt Charts

Introduction

Gantt chart is a graphical representation of a series of activities drawn to a time scale. Horizontal axis (X-axis) represents time and vertical axis (Y-axis) shows the activities to be performed. The Gantt chart shows activities to specific jobs at individual/work centers by horizontal bars. Also known as a 'bar chart' because of its graphic presentation of the information, the position and the length of the horizontal bar indicate the start and completion date of the activity.

QUESTION 12.
Define Strength and Limitations of Gantt Charts

ANSWER.

Strengths of Gantt Charts.

Gantt charts are preferred for various reasons, which are as follows:

- Very simple to understand by everyone e.g. foreman, engineers, managers, and top management.
- Provide useful information in a format that is simple to develop and interpret
- It is a good tool for planning as well as monitoring the progress of the work. It helps schedulers to evaluate the progress of a project at various levels.
- Helps in loading the work center in relation to the available capacity.
- It provides the user with a quick, visual indication of the actual status of each order and its anticipated or planned status.
- The scheduler could easily incorporate changes in timing, machine loads, and current status.
- Some common changes make Gantt charts fairly flexible to apply. It indicates the need for reassessing the resources incase the load at one work station becomes too much. Workforce could be temporarily adjusted to meet the high demand of the heavily loaded workstation by shifting the manpower from a relatively less loaded work center. Even multi-purpose equipments are shifted from less loaded work centers to heavily loaded work centers.

- Gantt charts suit the requirements of a wide range of media from ruled paper to mechanical devices and computer systems.

Limitations

- It does not convey the variability of the task duration, equipment performance (including breakdowns), and human potential, any one of which could influence the accuracy of loading the work centers
- It does not clearly indicate the details regarding progress of activities.
- It does not give a clear indication of the interrelationship between separate activities.
- The chart is static and has to be updated periodically to account for new job arrivals and revised time estimates for existing jobs.

Illustration 1

A project consists of seven activities. Activities P, Q, R run simultaneously. The relationships among the various activities is as follows:

Activity	Immediate Successor
P	S
Q	T
R	U

Activity "V is the last operation of the project and it is also immediate successor to S, T and U. Draw the network of the project.

SOLUTION:

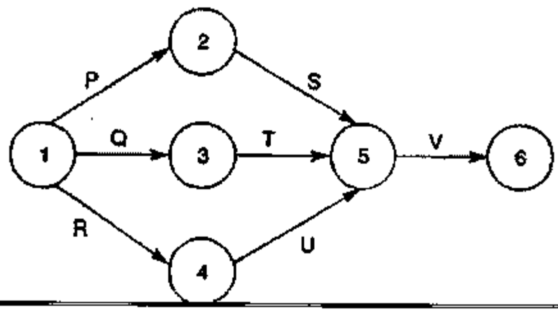


Illustration 2

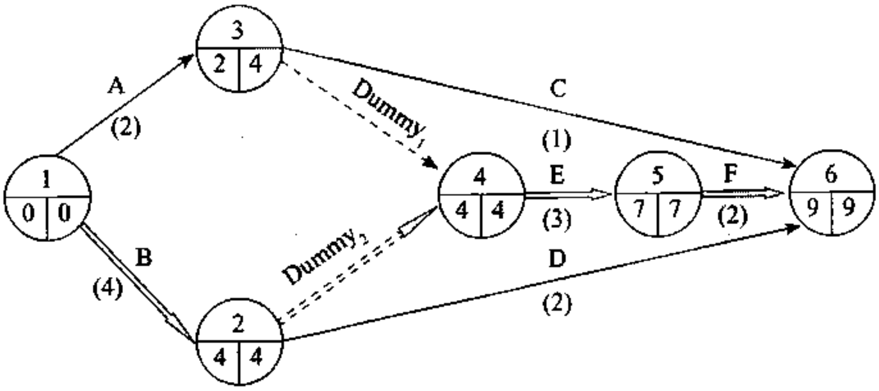
Project with the following data is to be implemented. Draw the network and find the critical path.

Activity	Predecessor	Duration (days)	Cost (₹ Day)
A	-	2	50
B	-	4	50
C	A	1	40
D	B	2	100
E	A,B	3	100
F	E	2	60

1. What is the minimum duration of the project?
2. Draw a Gantt chart for early start schedule.
3. Determine the peak requirement of money and the day on which it occurs in the above schedule

SOLUTION:

(1)



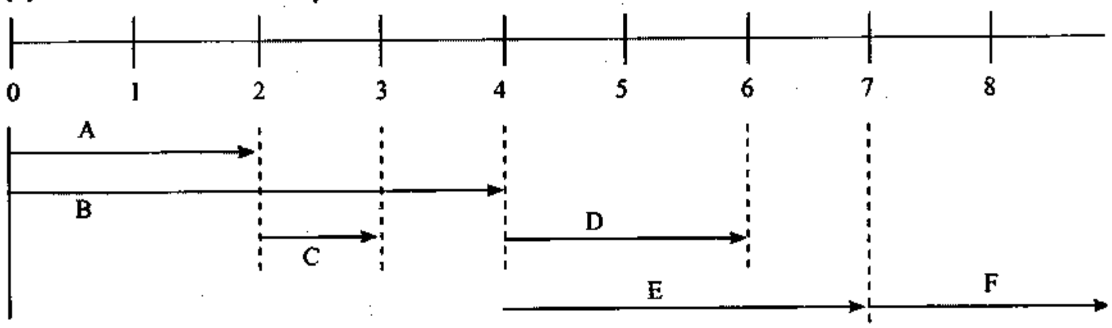
Critical Path B – Dummy₂ – E – F

Minimum duration of the project = 9 days

Table: Activity Relationship

Activity	t	ES (EF- t)	EF	LS (LF- t)	LF	Event Slack (LS-ES) (LF-EF)	On Critical Path
A	2	0	2	2	4	2	No
B	4	0	4	0	4	0	Yes
C	1	4	5	8	9	4	No
D	2	4	6	7	9	3	No
E	3	4	7	4	7	0	Yes
F	2	7	9	7	9	0	Yes

(2) Gantt Chart for Early Start Schedule



(3) Peak requirement of money will occur during simultaneous occurrence of Activities.

From the Network diagram above, it can be said that the following Activities need to occur simultaneously

(i) A & B – Either during the days 1 & 2 or during the days 3 & 4 of Project Duration, which will require (₹50 for A + ₹ 50 for B) per day i.e. ₹ 100 per day

(ii) B & C – Either on day 3 or on day 4 of the project and it will require (₹50 for B + ₹40 for C) per day i.e. ₹90 per day

(iii) C, D & E – During day no. 5 or day no. 6 and cost is ₹ (40 + 100 + 100) = ₹240 per day

(iv) C, D & F – During day no. 8 or day no. 9 and cost is ₹ (40 + 100 + 60) = ₹200 per day

(v) D & E – During day nos. 5 & 6 or 6 & 7. Cost is ₹ (100 + 100) = ₹200 per day

(vi) D & F – During day nos. 8 & 9. Cost = ₹ (100 + 60) = ₹160 per day

(vii) C & E – Either on day no. 5 or 6 or 7. Cost to be incurred = ₹ (40 + 100) = ₹140 per day

From above we can say that C can occur by using either of the options (ii), (iii), (iv) & (vii). As cost fo option (ii) is least one should decide fo it at a cost of ₹90 per day.
Similarly, D can occur by either of the option (iii), (iv), (v) & (vi) above. As (vi) is the least cost option of all these, one should go for it at a cost of ₹160 per day.
Hence the Project Activities should follow the sequence given below:-
(a) A & B to start at their Earliest Time (i.e. 0) and occur simultaneously till day 2 @ ₹100 per day
(b) C can start either at its Earliest Time (i.e. 2) or on day 3 and occur simultaneously with B either on day 3 or 4 @ ₹90 per day
(c) E being Critical Activities must have to start at it's earliest time (i.e. 4) and occur @ ₹100 per day
(d) F being Critical Activity has to start on Earliest Time (i.e. 7) and will occur concurrently with D is the during days 8 & 9 @ ₹160 per day.
Hence peak requirement of money is ₹ 160 per day and it will occur at days 8 and 9.

Illustration 3

A project has the following time schedule

Activity	1-2	1-3	1-4	2-5	3-6	3-7	4-6	5-8	6-9	7-8	8-9
Time (months)	2	2	1	4	8	5	3	1	5	4	3

Construct a PERT network and compute
- ➢ Construct a PERT network and compute
- ➢ Total float for each activity

Also, find the minimum number of cranes the project must have for its activities 2-5, 3-7, 5-8 and 8-9 without delaying the project given that one crane is sufficient to carry out the work involved in each activity if taken care of individually.

SOLUTION:

Steps:
1. Moving forward, find EF times (choosing the Maximum at activity intersection)
2. Maximum EF = LF = Critical Path Time.
3. Return path find LF (Choosing the Minimum at activity intersection)
4. Note LF, EF from network (except activity intersections)

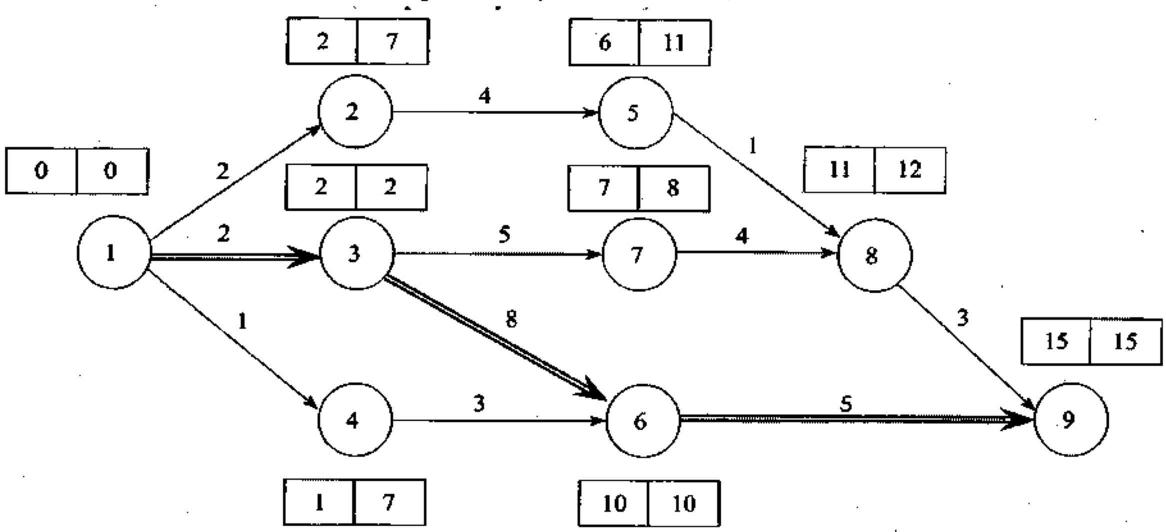


Table: Activity Relationship

Activity	Duration Months (t_{ij})	Earliest Start (ES_{ij})	Earliest Finish ($EF_{ij} = ES_{ij} + t_{ij}$)	Latest Start ($LS_{ij} = LF_{ij} - t_{ij}$)	Latest Finish (LF_{ij})	Total Float ($TF_{ij} = LS_{ij} + ES_{ij} = LE_{ij} - EF_{ij}$)
1 -2	2	0	2	5	7	5
1 -3	2	0	2	0	2	0
1 -4	1	0	1	6	7	6
2-5	4	2	6	7	11	5
3-6	8	2	10	2	10	0
3-7	5	2	7	3	8	1
4-6	3	1	4	7	10	6
5-8	1	6	7	11	12	5
6-9	5	10	15	10	15	0
7-8	4	7	11	8	12	1
8-9	3	11	14	12	15	1

Critical path is 1-3-6-9 with duration 15 months

Minimum number of cranes

- Finish 3 – 7 at its earliest finish time 7 with one crane
- Finish 2 – 5 at its latest finish time 7 + 4 =11 with the same crane by starting the activity at its latest start time 7
- Finish 5 – 8 at its latest finish time 11 + 1 = 12 with the same crane by starting the activity at its latest start time 11
- Finish 8 – 9 at its latest finish time 12+ 3=15 with the same crane by starting the activity at its latest start time 12

Therefore, one crane will be sufficient if start time of the following activities are:

- Activities 2-5 – 7
- Activities 5-8 – 11
- Activities 8-9 – 12

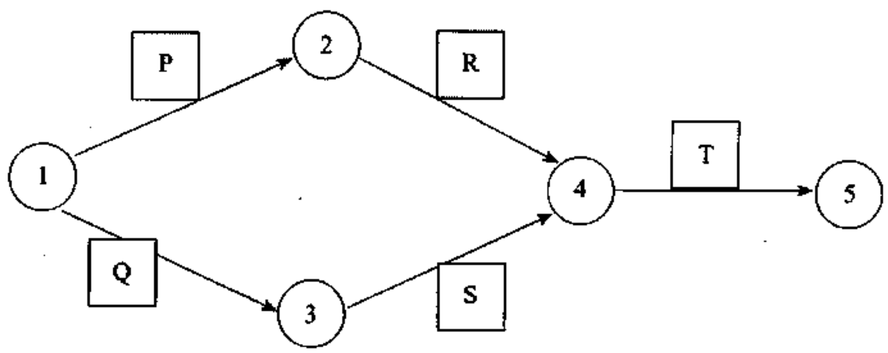
Illustration 4

A project consists of five activities. Activities P and Q run simultaneously. The relationship among the various activities is as follows:

Activity	Immediate Successor
P	R
Q	S

Activity T is the last operation of the project and it is also immediate successor to R and S. Draw the network of the Project.

SOLUTION:



6.4 PERT and CPM

QUESTION 13.
Define Network Analysis and its Applications

ANSWER.

Network analysis is the general name given to certain specific techniques which can be used for planning, management and control of project. It often acts as a network management tool for breaking down projects into components or individual activities and recording the result on a flow chart or network diagram. These results generally reveal information that is used to determine duration, resource limitations and cost estimates associated with the project.

It offers insight into what is occurring at each critical point of the network. Project management and efficient resource allocation are two critical aspects of the production and operations managers' responsibilities. Since a project is non-repetitive and temporal in nature, the mode of management differs from the usual job shop or other related types of scheduling.

Network analysis enables us to take a systematic quantitative structural approach to the problem of managing a project through to successful completion. Also, since it has a graphical representation, it can be easily understood and used by those with a less technical background.

Network is a graphical representation of all the Activities and Events arranged in a logical and sequential order. Network analysis plays an important role in project management. A project is a combination of interrelated activities all of which must be executed in a certain order for its completion. Activity is the actual performance of the job. This consumes resources (Time, human resources, money, and material. An event refers to start or completion of a job. This does not consume any resources.

Applications:

- Construction of a Residential complex,
- Commercial complex,
- Petro-chemical complex
- Ship building, Aircraft Manufacturing
- Satellite mission development
- Installation of a pipe line project etc

QUESTION 14.
Explain The procedure of drawing a network?

ANSWER.
The procedure of drawing a network is:

1. **Specify the Individual Activities:** From the work breakdown structure, a listing can be made of all the activities in the project. This listing can be used as the basis for adding sequence and duration information in later steps.

2. **Determine the Sequence of the Activities:** Some activities are dependent on the completion of others. A listing of the immediate predecessors of each activity is useful for constructing the CPM network diagram.

3. **Draw the Network Diagram:** Once the activities and their sequencing have been defined, the CPM diagram can be drawn. CPM originally was developed as an activity on node (AON) network, but some project planners prefer to specify the activities on the arcs.

4. **Estimate Activity Completion Time:** The time required to complete each activity can be estimated using past experience or the estimates of knowledgeable persons. CPM is a deterministic model that does not take into account variation in the completion time, so only one number is used for an activity's time estimate.

5. **Identify the Critical Path:** The critical path is the longest-duration path through the network. The significance of the critical path is that the activities that lie on it cannot be delayed without delaying the project. Because of its impact on the entire project, critical path analysis is an important aspect of project planning.

The critical path can be identified by determining the four parameters for each activity. The four parameters are Earliest Start, Earliest Finish, Latest Finish and Latest Start.

QUESTION 15.
Define the Rules for drawing the network diagrams.

ANSWER.
In a network diagram, arrows represent the activities and circles represent the events.

- The tail of an arrow represents the start of an activity and the head represent the completion of the activity.
- The event numbered 1 denotes the start of the project and is called initial event.
- Event carrying the highest number in the network denotes the completion of the project and is called terminalevent.
- Each defined activity is represented by one and only arrow in the network.
- Determine which operation must be completed immediately before other can start.
- Determine which other operation must follow the other given operation.
- The network should be developed on the basis of logical, analytical and technical dependencies betweenvarious activities of the project.

QUESTION 16.
Define basic network construction.

ANSWER:
The basic network construction - Terminology used.
Network representation: There are two types of systems -

AOA system (Activity on Arrow system)	**AON system (Activity on Node system)**
In this activities are represented by an arrows.	In this method activities are represented in the circles.

A project consists of tasks with definite starting and ultimate ending points and hence a project manager is saddled with the responsibilities of getting job done on schedule within allowable cost and time constraint specified by the management. Typically, all projects can be broken into:

Separate activities - where each activity has an associated completion time (time from the start of the activity to its finish).

Precedence relationships - which govern order in which we may perform the activities.

The main problem is to bring all these activities together in a coherent fashion to complete the project at a required time.

Apart from the traditional method of adding activity durations, these exist two different techniques for network analysis namely the PERT - Program Evaluation and Review Technique and CPM – Critical Path Management. PERT has the ability to cope with uncertainty in activity completion times while CPM emphasized on the tradeoff between cost of the project and its overall completion time.

The CPM has the advantage of decreasing completion times by probably spending more money.

PERT-CPM

- CPM - Critical Path Method
- PERT - Program Evaluation Review Techniques

Project

- **Scientific**
- **Commercial**
- **Education**

- Any project is composed of related activities
- Activities are composed of related events.
- Each activity is divided into three parts:
  - I. Independent Activity
  - II. Dependent Activity
  - III. Dummy Activity
- For completion of each activity except dummy, time is required.
- Time taken for completion of the activity divided into two parts:
  - I. Deterministic time
  - II. Probabilistic time
- Probabilistic time is divided into three parts:
  - I. Optimistic time(t_o)
  - II. Most likely time(t_m)
  - III. Pessimistic time(t_p)

  where t_e = expected time = $(t_o + 4 \times t_m + t_p)/6$

  Variance = $((t_p - t_o)/6)^2$
- For calculation of expected time & Variance we apply Beta Distribution.
- Time calculated based on <u>activities</u> as well as <u>events</u>.

Activities → Expected Time → t_o, t_m, t_p

Events → EST | EFT | LST | LFT

- EST - Earliest Start Time
- EFT - Earliest Finish Time
- LST - Latest Start Time
- LFT - Latest Finish Time

- EST and EFT are forward process.
  - First, we calculate EST then EFT.
- Where EFT = EST + d_{ij} ; d_{ij} = duration (expected time) for i-j activity
- LST and LFT are backward process.
  - First we calculate LFT then LST
    Where LST = LFT - d_{ij}

QUESTION 17.
What are the steps to be followed for solving PERT/CPM problem

ANSWER.

- Step 1: Draw the network diagram.

Diagrammatic presentation of the project which composed of:

- Dependent activity
- Independent activity
- Dummy activity

- Step 2: Calculate te(expected time/duration) for each activity from the problem given.
- Step 3: Calculate EST, EFT, LFT & LST for the problem.
- Step 4: Calculate Total Float (TF)
  Where, TF = LFT – EFT, for checking the Critical Path.
  - Critical Path is the path which contains maximum activity with maximum duration.
- Step 5: After calculating Critical Path, calculate Critical Path Duration (CPD) and Variance for each activities.
- Step 6: At last we calculate total time (approx.) taken for the completion of the project using Normal Distribution.

Activities:

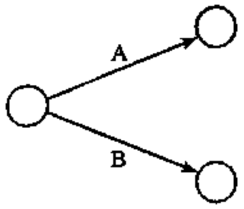
Let,

A – Independent Activity
B – Independent activity
C – Dependent activity (Dependent on A and B)
D – Dependent Activity (dependent on C)

- For independent activity starting point must be same.
- The dependent activity C will start after the completion of A and B.
- The dependent activity D will start after the completion of C.

Let's take a look at the Network Diagram of the above processes.

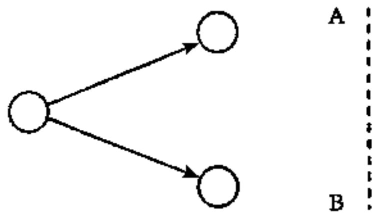
Network Diagram:



For Network diagram:

- ○ - Starting or Finishing point
- → - Activity along with its direction
- ⇢ - Dummy activity

- As A and B are independent activities of the process so their starting point is also same but since they aredifferent activities their directions are different.
- Now from where the activity C will be starting – from the end of A or from the end of B? This remains a question to complete the Network Diagram. Here we need to add a Dummy Activity after the end of A & B.



- Now the Dummy Activity is added but the direction is still not clear so that we can start C.
- The duration of the activity or the Time taken by the independent activities will decide the direction the Dummy activity to maintain the sequence of the project and go further in the process.

Let the time taken for each activity be:

Activity	Time Taken(max)
A – Independent Activity	3
B – Independent activity	4
C – Dependent activity (Dependent on A and B)	5
D – Dependent Activity (dependent on C)	6

For A – 3 days is needed at max.
B – 4 days is needed at max.

- Then the direction of the dummy activity (in this case) will be from A to B since A will finish before B anyhow.

For a Dummy Activity:

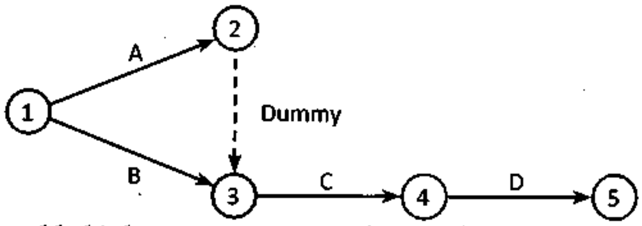
- The starting point of the dummy activity will be the end point of the activity which ends first.
- If both the independent activities finish at the same point then the direction can be in any way.

- Dummy Activity is required to maintain the sequence of the project. Therefore, it has no time to complete.

i.e. For the dummy activity the time taken is 0 units always.

- Now we can complete the Network Diagram as below.

- For understanding the Network Diagram we need some numbering techniques:
  - Assigning numbers to starting and finishing points in a particular order.



- After the numbers are added it becomes easier to denote the activities according the path they follow.
  - A (1-2) - A starts at 1 and finishes at 2
  - B (1-3) - B starts at 1 and finishes at 3
  - Dummy (2-3) - Dummy starts at 2 and finishes at 3
  - C (3-4) - C starts at 3 and finishes at 4
  - D (4-5) - D starts at 4 and finishes at 5
- There is a Source and a Destination for every project. In our case Source is 1 and Destination is 5.
- Every project aims at starting from the Source and reach the Destination through a certain path.
- In our case, we have the Network Diagram above with the path defined from 1 to 5 as follows:
- Path I: 1 - 2 - 3 - 4 - 5
- Path II: 1 - 3 - 4 - 5
- Calculating the path duration of each path:
- Path I: 3 days (1 - 2) + 0 days (2 - 3) + 5 days (3 - 4) + 6 days (4 - 5) = 14 days
- Path II: 4 days (1 - 3) + 5 days (3 - 4) + 6 days (4 - 5) = 15 days
- Path II takes maximum time to reach the destination from source or to complete the project. Therefore, it is the Critical Path.
- Critical Path (CP) -> 1 - 3 - 4 - 5
- Critical Path Duration (CPD) = 15 days
- Critical Path Activities (CPA) -> B(1 - 3), C(3 - 4), D(4 - 5)
- Critical Path contains the major activities of the project thus the activities are called Critical Path Activities.
  - Table showing the calculation of EST, EFT, LST, LFT and TF:

Activities	Time(d_{ij})	EST	EFT (= EST + dij)	LST (= LFT - dij)	LFT	TF (= LFT - EFT)
A (1-2)	3	0	3	1	4	1
B (1-3)	4	0	4	0	4	0*
Dummy (2-3)	0	3	3	4	4	1
C (3-4)	5	4	9	4	9	0*
D (4-5)	6	9	15	9	15	0*

- To calculate the above values, here are the few things to be remembered:
  - In case of EST, if the starting point is same then EST will be same.
  - In case of LFT, if the finishing point is same then the LFT will be same.
  - For independent activities, EST = 0.
  - For the last activity LFT = EFT.
- Calculation behind the above table created:

- EST and EFT are Forward Processes i.e. we start from the first activity and end at the last activity. And we go for EST first then EFT is calculated.
- Since A and B are independent activities, their EST = 0.
- The Dummy activity can only start after the completion of A. So the EST of the Dummy activity will be the Time taken to complete A i.e. 3
- C can start after the completion of B. So the EST of C will the time taken to complete B i.e. 4
- D can start only after the completion of C. So the EST of D will the time taken to complete C plus the time taken to start C i.e. 4(to start C) + 5(to complete C) = 9.
- EST is filled. Now we get the EFT by adding the time/duration (dij) of each activity to EST. EFT completed.

- LST and LFT are Backward Processes i.e. we start from the last activity and move towards the first activity. And we go for LFT first and then LST is calculated.
- For the last activity LFT = EFT i.e. for D, LFT = 15
- For C, we don't need to consider the time for D but we need to consider every other activity. Thus, LFT of C will be the LFT of D minus the Time taken to complete D. i.e. 15(LFT of D) - 6(Time taken for D) = 9
- For Dummy activity, we don't need to consider the time for C & D but we need to consider every other activity. Thus, LFT of Dummy will be the LFT of D minus the Time taken to complete C & D. i.e. 15(LFT of D) - 6(Time taken for D) - 5(Time taken for C) = 4
- For B, it needs to finish before the start of C. So it's LFT = EST of C i.e. 4.
  - Also, for B, we don't need to consider the time for Dummy, C & D but we need to consider A. Thus, LFT of B will be the LFT of D minus the Time taken to complete Dummy, C & D. i.e. 15(LFT of D) - 0(Time for Dummy) - 6(Time taken for D) - 5(Time taken for C) = 4.
  - If we think it otherwise, the LFT of Dummy is 4. And B needs to finish either on 4th day or before (from the start of the project), it can't take more time than that. But it itself needs 4 days to complete the activity. Thus, LFT will be 4 only.
- For A, It is an independent activity but it needs to complete before start of C. B doesn't count in case of A because both follows separate path. From the last activity till Dummy the LFT stays at 4 which means A can complete the activity at 4th day as well even if its time required is 3 days, and also Dummy doesn't have any stipulated time to complete. A should only be bothered about the start of C.
  - Also for A, we don't need to consider the time for Dummy, C & D from the critical path duration. Thus LFT of A will be the LFT of D minus the Time taken to complete Dummy, C & D. i.e. 15(LFT of D) - 0(Time for Dummy) - 6(Time taken for D) - 5(Time taken for C) = 4.
- LFT is filled now. Then calculate the LST by subtracting each value by the time/duration (dij) of each activity. LST is also filled.
  - TF is calculated as TF = LFT - EFT. The table is complete now.
  - The zeroes in the TF column are assigned a * to indicate the Critical Path Activities
- If the TF of an activity is zero, then they are called the Critical Path Activities. This is the other way of finding the CPAs where the results stays same as before when we found it through the network diagram.

Illustration 5.

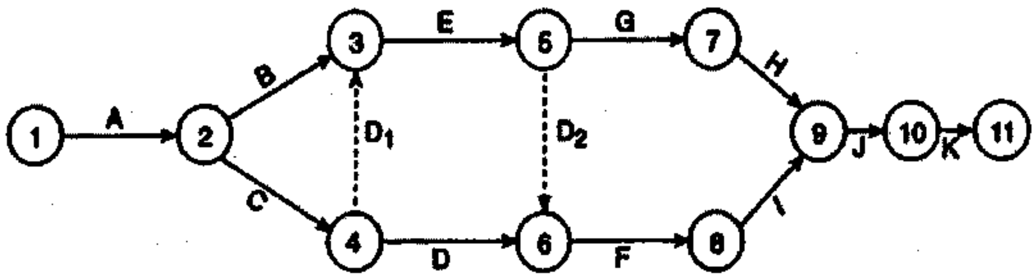
XYZ Auto-manufacturing company has to prepare a design of its latest model of motorcycle. The various activities to be performed to prepare design are as follows:

Activity	Description of activity	Preceding activity
A	Prepare drawing	–
B	Carry out cost analysis	A
C	Carry out financial analysis	A
D	Manufacture tools	C
E	Prepare bill of material	B, C
F	Receive material	D,E
G	Order sub-accessories	E
H	Receive sub-accessories	G
I	Manufacture components	F
J	Final assembly	I,H
K	Testing and shipment	J

Prepare an appropriate network diagram.

SOLUTION:

The network diagram will be as follows:



Where D1 and D2 are dummy activities.

Illustration 6.

The following table gives data on normal time & cost and crash time & cost for a project.

Activity	Normal		Crash	
	Time (days)	Cost (₹)	Time (days)	Cost (₹)
1–2	6	600	4	1,000
1–3	4	600	2	2,000
2–4	5	500	3	1,500
2–5	3	450	1	650
3–4	6	900	4	2,000
4–6	8	800	4	3,000
5–6	4	400	2	1,000
6–7	3	450	2	800

The indirect cost per day is ₹100.

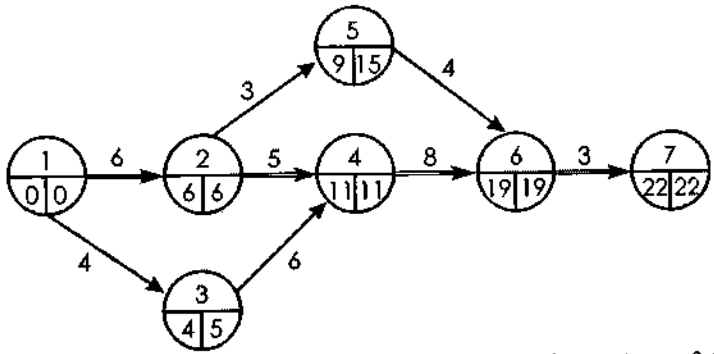
(i) **Draw the network and identify the critical path.**

(ii) **What are the normal project duration and associated cost?**

(iii) **Crash the relevant activities systematically and determine the optimum project completion time and cost.**

SOLUTION:

i. **The network for normal activity times indicates a project time of 22 days with the critical path 1-2-4-6-7.**



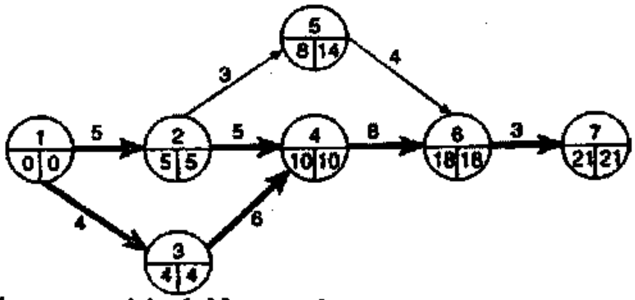
ii. **Normal project duration is 22 days and the associated cost is as follows: Total cost = Direct normal cost + Indirect cost for 22 days.**
= 4,700 + 100 × 22 = ₹ 6,900.

iii. **For critical activities, cost - slope is given below:**

Critical activity	Cost-slope* (`/day)
1-2	$\frac{1000 - 600}{6 - 4} = 200$
2-4	$\frac{1500 - 500}{5 - 3} = 500$
4-6	$\frac{3000 - 800}{8 - 4} = 550$
6-7	$\frac{800 - 450}{3 - 2} = 350$

$$\text{*Cost slope} = \frac{\text{Crash Cost} - \text{Normal Cost}}{\text{Normal Time} - \text{Crash Time}}$$

Of the activities lying on the critical path, activity 1–2 has lowest cost slope Therefore, we shall first crash this activity by just one day.
Duration = 21 days, and cost = 4700 + 1 × 200 + 100 × 21 = ₹ 7000.



Other activities too have become critical. Now we have 2 critical paths:
1→2→4→6→7 and 1→3→4→6→7.

To reduce duration of the activity further, we shall have to reduce duration of both the paths. We have following alternatives:

Crash activity 6 – 7 by 1 day at a cost of ₹350.

Crash activity 4 – 6 by 4 days at the cost of ₹550 per day.

Crash activities 1–2 and 1 – 3 by 1 day each at a cost of ₹ (200 + 700) = ₹900.

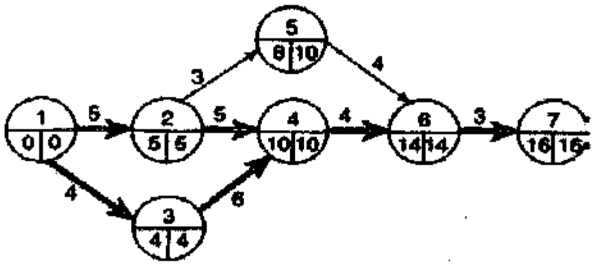
Crash activities 2 – 4 and 3 – 4 by 2 days each at a cost of ₹ (500 + 550) = ₹1050/day.

Thus, we shall first crash activities 6 – 7 by 1 day and then activity 4 – 6 by 4 days.

On crashing activity 6 – 7 by 1 day, cost = 4900 + 350 × 1 + 100 × 20 = ₹7250, and duration = 20 days. Next

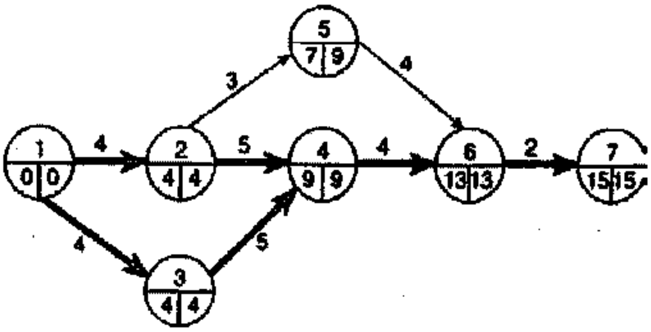
we crash 4–6 by 4 days.

Cost = 5250 + 550 × 4 + 100 × 16 = ₹9050. Duration = 16 days.



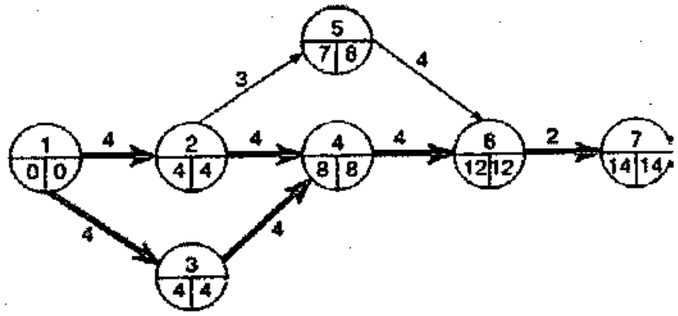
Next, we crash activities 1–2 and 3–4 by 1 day each.

Cost = 7450 + 200 × 1 + 550 × 1 + 100 × 15 = ₹9700.



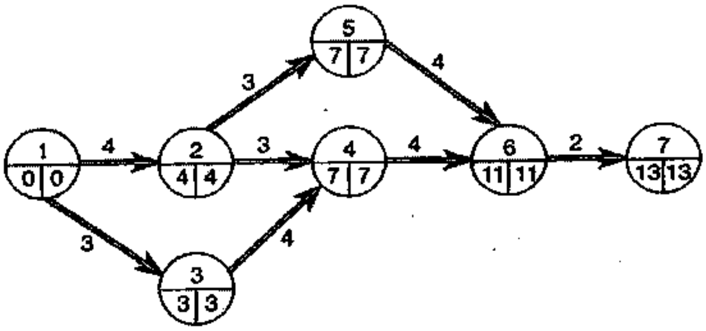
we crash activities 2 ⟶ 4 and 3 ⟶ 4 by 1 day each.

Cost = 8200 + 500 × 1 + 550 × 1 + 100 × 14 = ₹10,650. Duration = 14 days.



We crash activities 1–3 and 2–4 by 1 day each.

Cost = 9250 + 700 × 1 + 500 × 1 + 100 × 13 = ₹11,750 Duration = 13 days.



Now there are three critical paths:

1–2–5–6–7, 1–2–4–6–7, 1–3–4–6–7

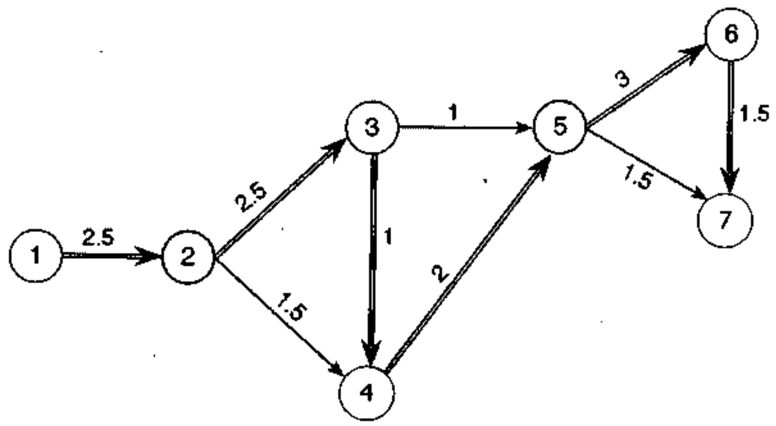
Also, no further crashing is possible. Hence minimum duration of the project =13 days with cost ₹11,750.

Illustration 7.

Draw the network for the following activities and find critical path and total duration of project.

Activity	Duration (months)	Activity	Duration (months)
1-2	2.5	4-5	2.0
2-3	2.5	5-6	3.0
2-4	1.5	6-7	1.5
3-4	1.0	5-7	1.5
3-5	1.0		

SOLUTION:



Paths	Duration
1-2-3-5-6-7	2.5+2.5+1+3+1.5 = 10.5
1-2-3-5-7	2.5+2.5+1+1.5 = 7.50
1-2-3-4-5-6-7	2.5+2.5+1+2+3+1.5 = 12.5 (Critical path)
1-2-3-4-5-7	2.5+2.5+1+2+1.5 = 9.5
1-2-4-5-7	2.5+1.5+2+1.5 = 7.5
1-2-4-5-6-7	2.5+1.5+2+3+1.5 = 10.5

Illustration 8.

The following activities must be accomplished in order to complete a construction project:

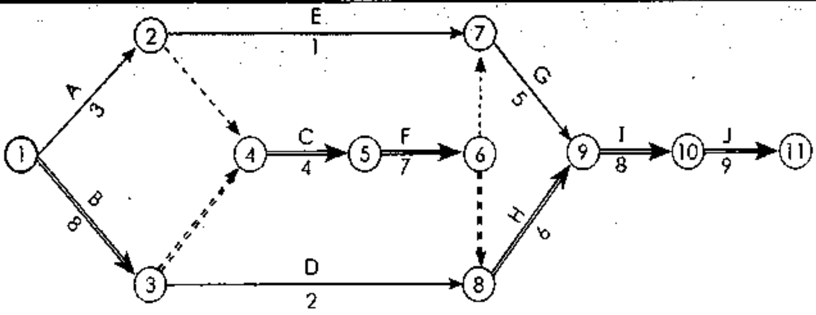
Activity	A	B	C	D	E	F	G	H	I	J
Time	3	8	4	2	1	7	5	6	8	9
Predecessors	–	–	AB	B	A	C	EF	DF	GH	I

- Construct a network diagram for this project. Find the CP and the duration of the project.
- Assume that you are project manager of the project mentioned above. The project has progressed for 10 weeks and the status is follows:

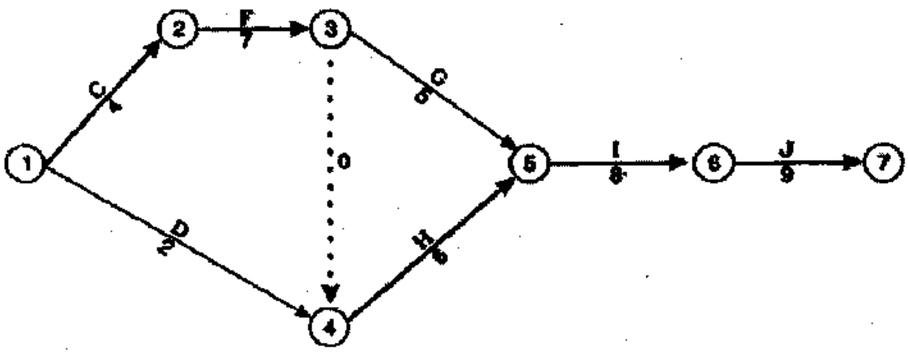
Activities completed: A, B, E. Other activities have not started as yet.

- If no managerial action is taken at all when will the project get completed?
- What action might you take to get the project back to a schedule that can be completed by the end of week 42?

SOLUTION:



Paths	Duration (weeks)	Paths	Duration (weeks)
1-2-7-9-10-11	26	1-3-4-5-6-7-9-10-11	41
1-2-4-5-6-7-9-10-11	36	1-3-4-5-6-8-9-10-11	42
1-2-4-5-6-8-9-10-11	37	1-3-8-9-10-11	33
Critical Path: BCFHIJ. Duration 42 weeks.			



Paths	1-2-3-5-6-7	1-2-3-4-5-6-7	1-4-5-6-7
Duration (weeks)	33	34 Critical Path: CFHIJ	25

For completing the project as per original schedule, the project activities on the critical path should be reduced by 2 weeks. For example, we may reduce any one of the activities CFHIJ by 2 weeks or any two activities by one week each.

Illustration 9.

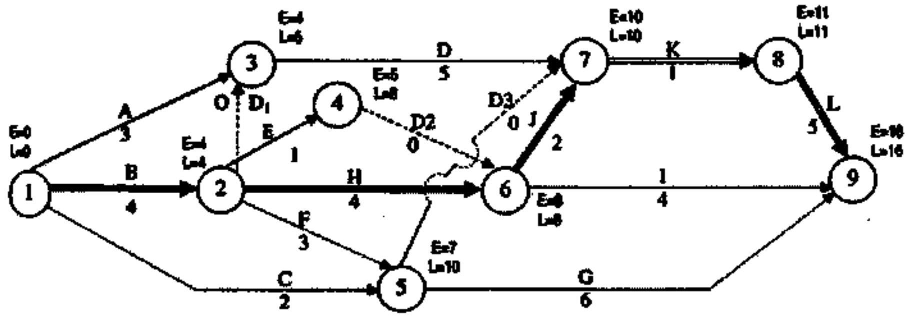
Given is the following information regarding a project:

Activity	A	B	C	D	E	F	G	H	I	J	K	L
Dependence	-	-	-	AB	B	B	FC	B	EH	EH	CDFJ	K
Duration	3	4	2	5	1	3	6	4	4	2	1	5

Draw the Network Diagram and identify the Critical Path and Project Duration.

SOLUTION:

Network Diagram:



Network Table:

Activity	Duration	EST	LST	EFT	LFT	Total Float	Free Float	Independent Float
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
A	3	0	2	3	5	2	2 - 1 = 1	1 - 0 = 1
B	4	0	0	4	4	0	0	0
C	2	0	8	2	10	8	8 - 3 = 5	5 - 0 = 5
D1	0	4	5	4	5	1	1 - 1 = 0	0
D	5	4	5	9	10	1	1 - 0 = 1	1 - 1 = 0
E	1	4	7	5	8	3	3 - 3 = 0	0
F	3	4	7	7	10	3	3 - 3 = 0	0
G	6	7	10	13	16	3	3 - 0 = 3	3 - 3 = 0
D2	0	5	8	5	8	3	3 - 0 = 3	3 - 3 = 0
H	4	4	4	8	8	0	0	0
I	4	8	12	12	16	4	4 - 0 = 4	4 - 0 = 4
J	2	8	8	10	10	0	0	0
D3	0	7	10	7	10	3	3 - 0 = 3	3 - 3 = 0
K	1	10	10	11	11	0	0	0
L	5	11	16	16	16	0	0	0

Critical path is B – H – J – K – L. Expected Duration = 16 days

The columns are updated in the following order as under:

1. Activity (including Dummies) are listed from the Question and network Diagram
2. Duration (including Dummies) are listed from the Question and Network Diagram
3. EST = E value of LHS/ Tail Event from Diagram.
4. LFT = L value of RHS/ Head Event from Diagram.
5. EFT = EST + Duration as per Column (2). Hence Column (5) = Column (3) + Column (2)

6. LST = LFT – Duration as per Column (2). Hence column (4) = Column (6) – Column (2)
7. Total Float = [LET - EFT] or [LST - EST] = [Col.(6) – Col.(5)] or [Col.(4) – Col.(3)]
8. Free Float = Total Float – Head Event Slack i.e. [Col.(7) – difference between L and E of RHS Event].
   Note: If Total Float is Zero, Free Float is also equal to Zero If a negative value is derived, it is restricted to zero.
9. Independent Float = Free Float – Tail Event Slack i.e. [Col (8) – Difference between L and E of LHS Event].
   Note: If Free Float is Zero, Independent Float is also equal to Zero. If a negative value is derived, it is restricted to zero.

Note:
- The activities whose Total Float is Zero are Critical Activities. These Total Floats are circled and the respective activities are indicated by double in the network diagram.
- Dummy Activities may or may not lie on the critical path. However, in this question, the dummy activities do not fall on the Critical Path.

Illustration 10.

A project with normal duration and cost along with crash duration and cost for each activity is given below:

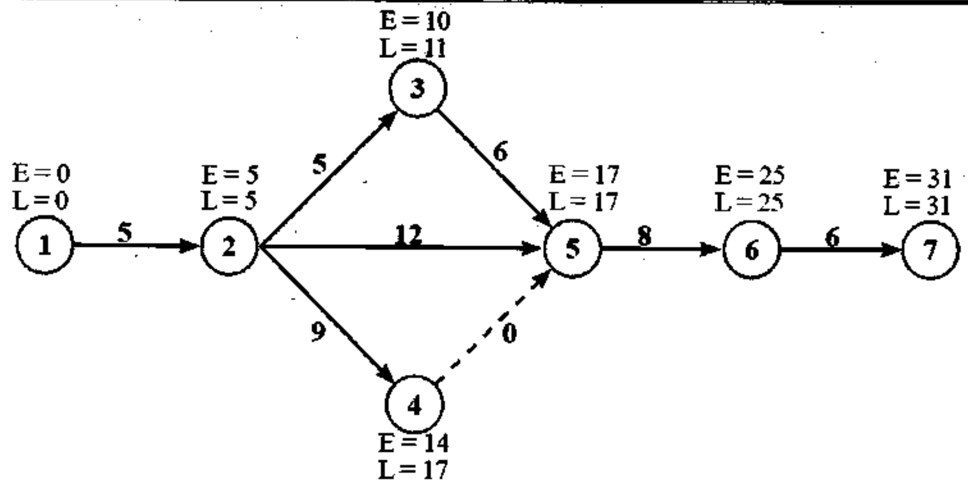
Activity	Normal time (Hrs.)	Normal cost (₹)	Crash time (Hrs.)	Crash cost (₹)
1-2	5	200	4	300
2-3	5	30	5	30
2-4	9	320	7	480
2-5	12	620	10	710
3-5	6	150	5	200
4-5	0	0	0	0
5-6	8	220	6	310
6-7	6	300	5	370

Overhead cost is ₹50 per hour.

Required:

Draw network diagram and identify the critical path.

SOLUTION:



Paths →	1-2-5-6-7 (Let's denote this by A)	1-2-3-5-6-7 (Let's denote this by B)	1-2-4-5-6-7 (Let's denote this by C)
Duration	31 hours	30 hours	28 hours
The critical path is A. Its duration is 31 hours			

Illustration 11.

What are the difference between CPM and PERT.

SOLUTION:

CPM originated from construction project while PERT evolved from R & D projects. Both CPM and PERT share the same approach for constructing the project network and for determining the critical path of the network.

There is some basic differences between PERT and CPM

PERT	CPM
1. Time estimate is probabilistic with uncertainty intime duration. Three-time estimates.	1. Time estimate is deterministic with known timedurations. Single time estimate
2. Event oriented	2. Activity oriented
3. Focused on time	3. Focused on time-cost trade off
4. More suitable for new projects	4. More suited for repetitive projects

Illustration 12.

Construct a network diagram satisfying the following conditions.

A<D,C; B<E; D<G,F; E,F<H; G,H,C<I

[Hint: X<Y,Z means both Y and Z cannot start until X is complete.]

SOLUTION:

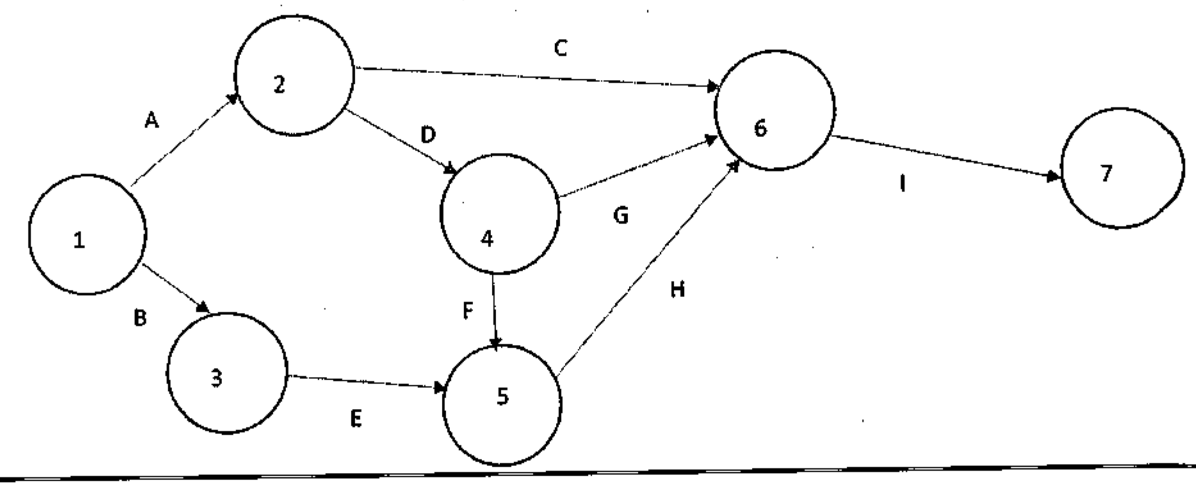


Illustration 13.

Construct the network diagram from the data given below and find

(a) total duration of the project

(b) Critical Path

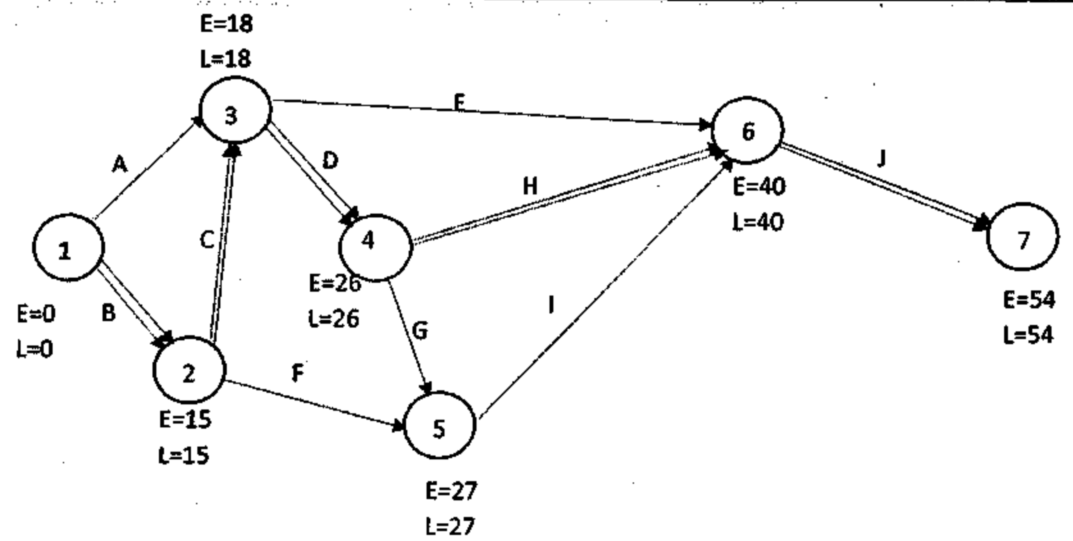
(c) EST, EFT, LST, LFT.

(d) Total float of each activity

Activity	A	B	C	D	E	F	G	H	I	J
Duration	15	15	3	5	8	12	1	14	3	14
Predecessor Activity	-	-	B	A,C	A	B	D	D	F,G	E,H,I

SOLUTION:

Activity (i-j)	Time (tij)	Earliest Start (ESTij)	Earliest Finish (EFTij = ESTij + tij)	Latest Start (LSTij = LFTij- tij)	Latest Finish (LFTij)	Total Float (TFij = LSTij + ES- Tij = LETij - EFTij)
A(1-3)	15	0	3	15	18	3
B (1-2)	15	0	0	15	15	0
C(2-3)	3	15	15	18	18	0
D (3-4)	8	18	18	26	26	0
E (3-6)	12	18	28	30	40	10
F(2-5)	5	15	32	20	37	17
G (4-5)	1	26	36	27	37	10
H (4-6)	14	26	26	40	40	0
I (5-6)	3	27	37	30	40	10
J (6-7)	14	40	40	54	54	0



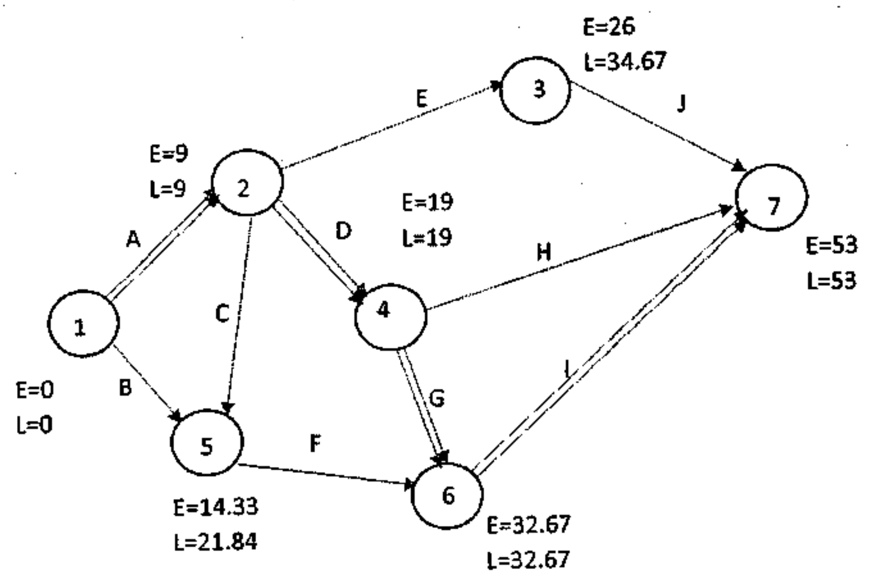
Critical Path 1-2-3-4-6-7
Critical Activity: B -C -D -H-J.

Illustration 14.

For the given data find the expected duration of the project and variance of the project.

Activity	Optimistic time (to)	Most likely Time (tm)	Pessimistic time(tp)
1-2	6	9	12
1-5	4	7	8
2-3	14	17	20
2-4	7	10	13
2-5	3	5	9
3-7	13	18	25
4-6	10	14	16
4-7	12	15	18
5-6	9	11	12
6-7	17	20	25

SOLUTION:



Activity	Optimistic time (to)	Most likely Time (tm)	Pessimistic time (tp)	σ^2= (tp-to/6)^2	te = to+4tm+tp/6
1-2	6	9	12	1.00	9.0
1-5	4	7	8	0.44	6.7
2-3	14	17	20	1	17.0
2-4	7	10	13	1	10.0
2-5	3	5	9	1	5.33
3-7	13	18	25	4	18.33
4-6	10	14	16	1	13.67
4-7	12	15	18	1	15.00
5-6	9	11	12	0.25	10.83
6-7	17	20	25	1.78	20.33

The critical path is 1 - 2 - 4 - 6 - 7

Variance of the critical path = 1 + 1 + 1.78 = 3.78

SD of the critical path = SD of the network diagram = $\sqrt{(3.78)}$ = 1.944

Illustration 15.

A marketing organization is planning a questionnaire survey on behalf of their client to assess market potential of instant foods. The following activities are involved in this project:

Task			Duration(days)	
	Precedence	Optimistic	Most(likely)	Pessimistic
A. Design		2	3	4
questionnaire				
B. Sample design		6	10	20

Task			Duration(days)	
	Precedence	Optimistic	Most(likely)	Pessimistic
C.Testing of		2	4	6
questionnaire				
and refinements				
D.Recruiting				
interviewers	B	2	3	10
E. Training of	D, A	1	1	1
interviewers				
F. Allocation of	B	4	5	6
interviewers				
to territories				
G. Conducting	C,E,F	5	12	25
interviews				
H. Evaluation of results	G	6	10	20

(a) Find the expected duration and variance of each task.
(b) Draw an arrow diagram (network) of the project.
(c) Calculate EST, EFT, LST, LFT & TF
(d) Identify the critical path.
(e) Find the critical path duration of the project.
(f) What percentage of the project will be complete in 44 days?

Activity	A	M	B	Te	Variance	EST	EFT	LST	LFT	TF
A	2	3	4	3	1/9	0	3	12	15	12

(g) Find the no of day by which approximately 100% of the project will be completed

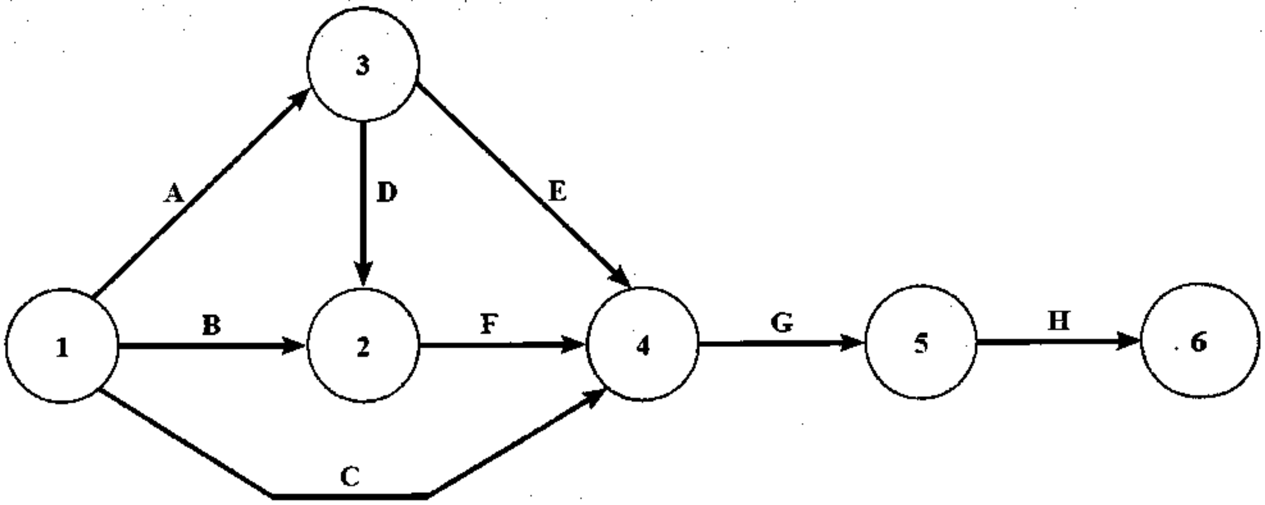
SOLUTION:

te = Expected time
A = Optimistic time;
M = Most likely time;
B = Pessimistic time

Activity	A	M	B	Te	Variance
A	2	3	4	3	1/9
B	6	10	20	11	49/9
C	2	4	6	4	4/9
D	2	3	10	4	16/9
E	1	1	1	1	0
F	4	5	6	5	1/9
G	5	12	25	13	100/9
H	6	10	20	11	49/9

$t_e = (A+4M+B)/6$

$\text{Variance}(t) = [(B-A)/6]^2$



Activity	A	M	B	Te	Variance	EST	EFT	LST	LFT	TF
A	2	3	4	3	1/9	0	3	12	15	12
B	6	10	20	11	49/9	0	11	0	11	0
C	2	4	6	4	4/9	0	4	12	16	12
D	2	3	10	4	16/9	11	15	11	15	0
E	1	1	1	1	0	15	16	15	16	0
F	4	5	6	5	1/9	11	16	11	16	0
G	5	12	25	13	100/9	16	29	16	29	0
H	6	10	20	11	49/9	29	40	29	40	0

There are two Critical Path

(i) 1---2---3---4---5---6 = (11 + 4 + 1 + 13 + 11) = 40

(ii) 1---2---4---5---6 = (11 + 5 + 13 + 11) = 40

As both the critical path suggest for both cases 40 days required to complete the project, so we calculate the standard deviation of critical path

$CSD_1 = \sqrt{\frac{49}{9} + \frac{16}{9} + 0 + \frac{100}{9} + \frac{49}{9}} = 14.9/3 = 4.966$ (Approx.)

$CSD_2 = \sqrt{\frac{49}{9} + \frac{1}{9} + \frac{100}{9} + \frac{49}{9}} = 14.1/3 = 4.7$ (Approx.)

Here, CSD_2 performing better than CSD_1, so we select the 2nd Critical Path.

Then here, $\mu = 40$, $\sigma = 4.7$

Since in 44 days taken then the percentage of work done

$P(T = 44) = P((T-\mu)/\sigma <= ((44-40)/4.7))$

$P(Z <= 4/4.7)$

$P(Z <= 0.8) = 0.78814$

Nearly 79% of the project will be completed during 44 days.

For the completion of 100% of the project we can take the 3 sigma limit

$P(T <= n)$

$P((T-\mu)/\sigma <= (n-\mu)/\sigma) = P(Z <= 3)$

$P(Z <= (n-40)/4.7) = P(Z <= 3)$

$n = 4.7 \times 3 + 40$

n = 54 day (Approx)

Illustration 16.

A management institute plans to organize a conference on use of "Operation Research for decision making". In order to co-ordinate the project, it has decided to use a PERT network. The major activities and time estimate for activity has been compiled as follows:

Sl. No.	Activity description	Time estimate (a-m-b)	Activity that must precede
A	Design conference meeting theme	1-2-3	None
B	Design front cover of conference proceedings	1-2-3	A
C	Design brochure	1-2-3	A
D	Compile list of distinguished speakers	2-4-6	A
E	Finalize brochure and print it	2-5-14	C and D
F	Make travel arrangements for distinguished speakers.	1-2-3	D
G	Send brochures	1-3-5	E
H	Receive papers for conference	10-12-20	G
I	Edit papers	3-5-7	H
J	Print proceedings	5-10-15	B and I

(a) Draw the network.

(b) Calculate expected time for each activity and variance for each activity.

(c) Calculates EST, EFT, LST, LFT, TF

(d) Identify critical path.

(e) Find the no of day by which approximately 90% of the project will be completed

SOLUTION:

te = Expected time

A = Optimistic time;

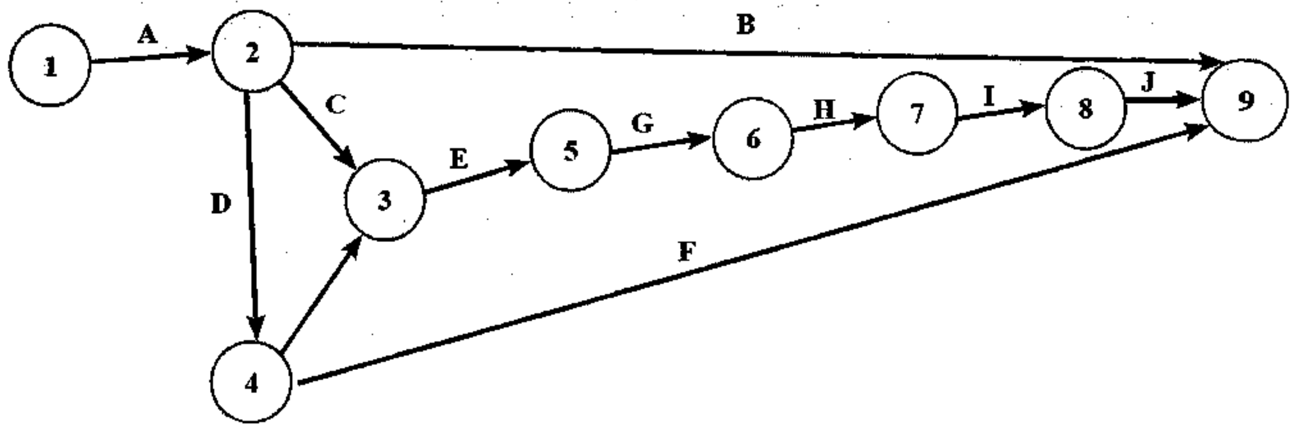
M = Most likely time;

B = Pessimistic time

Activity	A	M	B	Te	Variance
A	1	2	3	2	1/9
B	1	2	3	2	1/9
C	1	2	3	2	1/9
D	2	4	6	4	4/9
E	2	5	14	6	4
F	1	2	3	2	1/9
G	1	3	5	3	4/9
H	10	12	20	13	25/9

$t_e = (A + 4M + B)/\ 6$

$\text{Variance}(t) = (B - A)/\ 6^2$



Activity	A	M	B	t_e	Variance (t)	EST	EFT	LST	LFT	TF
A	1	2	3	2	1/9	0	2	0	2	0
B	1	2	3	2	1/9	2	4	41	43	39
C	1	2	3	2	1/9	2	4	4	6	2
D	2	4	6	4	4/9	2	6	2	6	0
E	2	5	14	6	4	6	12	6	12	0
F	1	2	3	2	1/9	6	8	41	43	35
G	1	3	5	3	4/9	12	15	12	15	0
H	10	12	20	13	25/9	15	28	15	28	0
I	3	5	7	5	4/9	28	33	28	33	0
J	5	10	15	10	25/9	33	43	10	43	0
Dummy						6	6	6	6	6

Critical Path = 1 - 2 - 3 - 5 - 6 - 7 - 8 - 9

= (2 + 4 + 6 + 3 + 13 + 5 + 10) = 43

$$\text{CSD} = \sqrt{\frac{1}{9} + \frac{4}{9} + 4 + \frac{4}{9} + \frac{25}{9} + \frac{4}{9} + \frac{25}{9}} = 3.31$$

Let n be the no. of days by which 90% of the project will be completed.

P [T<= n] = 0.90

P[(T-43)/3.31<=(n-43)/3.31} = 0.90 = 1.28

(n-43)/3.31 = 1.28

n = (1.28 × 3.31) + 43

n = 47.23 = 48 days

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997

6.5 BASICS OF MS PROJECT

Microsoft Project:

Microsoft Project is a project management software program developed and sold by Microsoft, designed to assist a project manager in developing a schedule, assigning resources to tasks, tracking progress, managing the budget, and analyzing workloads.

Project creates budgets based on assignment work and resource rates. As resources are assigned to tasks and assignment work estimated, the program calculates the cost, equal to the work times the rate, which rolls up to the task level and then to any summary task, and finally to the project level.

Each resource can have its own calendar, which defines what days and shifts a resource is available. Microsoft Project is not suitable for solving problems of available materials (resources) constrained production. Additional software is necessary to manage a complex facility that produces physical goods.

QUESTION 18.
Define Project Management and what is Scheduling?

ANSWER:
MS Project is feature rich, but project management techniques are required to drive a project effectively. A lot of project managers get confused between a schedule and a plan. MS Project can help you in creating a Schedule for the project even with the provided constraints. It cannot Plan for you. As a project manager you should be able to answer the following specific questions as part of the planning process to develop a schedule. MS Project cannot answer these for you.

- What tasks need to be performed to create the deliverables of the project and in what order? This relates to the scope of the project.
- What are the time constraints and deadlines if any, for different tasks and for the project as a whole? This relates to the schedule of the project.
- What kind of resources (man/machine/material) are needed to perform each task?
- How much will each task cost to accomplish? This would relate to the cost of the project.
- What kind of risk do we have associated with a particular schedule for the project? This might affect the scope, cost and time constraints of your project.

From the perspective of Project Management Methodology, a Plan and Schedule are not the same. A **plan** is a detailed action-oriented, experience and knowledge-based exercise which considers all elements of strategy, scope, cost, time, resources, quality and risk for the project.

Scheduling is the science of using mathematical calculations and logic to generate time effective sequence of task considering any resource and cost constraints. Schedule is part of the Plan. In Project Management Methodology, schedule would only mean listing of a project›s milestones, tasks/activities, and deliverables, with start and finish dates. The schedule is linked with resources, budgets and dependencies.
However, in this tutorial for MS Project (and in all available help for MS Project) the word 'Plan' is used as a 'Schedule' being created in MS Project. This is because of two reasons.
One, MS Project does more than just create a schedule it can establish dependencies among tasks, it can create constraints, it can resolve resource conflicts, and it can also help in reviewing cost and schedule performance over the duration of the project. So it does help in more than just creating a Schedule. Thus, it makes sense for Microsoft to market MS Project as a Plan Creator rather than over-simplifying it as just a Schedule Creator.

A project manager should also be able to answer other project-related questions as well. For example –

- Why this project needs to be run by the organization?
- What's the best way to communicate project details to the stakeholders?
- What is the risk management plan?
- How the vendors are going to be managed?
- How the project is tracked and monitored?
- How the quality is measured and qualified?

MS Project can help you –

- Visualize your project plan in standard defined formats.
- Schedule tasks and resources consistently and effectively.
- Track information about the work, duration, and resource requirements for your project.
- Generate reports to share in progress meetings

A blank project file can be daunting, especially if you're new to project management. But with a few clicks, you can tap the power of Project to convert your to-do list into a full-fledged project for you to manage and share with your team and stakeholders.

Here are a few starting points:

- Add tasks
- Outline tasks
- Link tasks
- Change your view
- Print your project

Add tasks:

1. Click View > Gantt Chart.
2. Type a name in the first empty Task Name field at the bottom of the task list, and press Enter.

Want more? If adding tasks one at a time starts to take too long, you can also:

- Add multiple tasks at once.
- Cut and paste a list from another program.
- Import a tasks list from a SharePoint site.

Outline tasks

Indent and outdent tasks to show hierarchy – that is, to turn your task list into an outline of your project. An indented task becomes a subtask of the task above it, which becomes a summary task.

1. Click View > Gantt Chart.
2. In the Task Name column, click the task you want to indent.
3. Click Task > Indent Task. The task becomes a subtask.
4. Click Outdent Task to move the task back to the level of the task above it. It's no longer a subtask.

Want more? Use subtasks and summary tasks to show phases, easily navigate through a large project, and more.

Link tasks

You can link any two tasks in a project to show their relationship (also called a task dependency). Dependencies drive the project schedule – once you link the tasks, every change you make to one affects the other, which affects the next one, and so on.

1. Click **View > Gantt Chart**.
2. Hold down Ctrl and click the two tasks you want to link (in the **Task Name** column).
3. Click **Task > Link the Selected Tasks**.

Want more? Project supports four kinds of task links to show different relationships. Want to change the link type or remove the link completely?

Change your view

Project starts you off with the tried-and-true Gantt Chart, but you have dozens of other options for viewing your tasks and resources and how they're all connected. You can change any view to meet your specific needs.

1. Click the View tab.
2. In the Task Views group or Resource Views group, click the view that you want to use.
3. To see all the available views, click Gantt Chart > More Views, and then choose from the options in the More Views dialog box.

Want more? There's a lot more to learn here! Need some help choosing the right view of your project?

Print your project

Printing a view or report in Project is similar to printing in other Office programs:

Click File > Print > Print.

Want more? Getting *only* the specific project information you want to share with your stakeholders into your printout can involve some prep work before you hit the print button:

- Prepare a view for printing
- Prepare a report for printing

Show the critical path of your project in MS Project

Every task is important, but only some of them are critical. The critical path is a chain of linked tasks that directly affects the project finish date. If any task on the critical path is late, the whole project is late.

The critical path is a series of tasks (or sometimes only a single task) that controls the calculated start or finish date of the project. The tasks that make up the critical path are typically interrelated by task dependencies. There are likely to be many such networks of tasks throughout your project plan. When the last task in the critical path is complete, the project is also complete.

Show the critical path in the Gantt Chart view

The Gantt Chart view will likely be your most used view for showing the critical path.

1. Choose View > Gantt Chart.
2. Choose Format, and then select the Critical Tasks check box.

Tasks on the critical path now have red Gantt bars.

Show the critical path in other task views

You can see the critical path in any task view by highlighting it.

1. On the **View** tab, pick a view from the **Task Views** group.
2. Staying on the **View** tab, select **Critical** from the **Highlight** list. The critical path shows up in yellow.
3. To see *only* the tasks on the critical path, choose the **Filter** arrow, then pick **Critical**.

View the critical path in a master project

When you're managing a master project, whole subprojects can be on the critical path. You can see if this is true by telling Project to treat the subprojects like they are summary tasks.

1. Choose **File** > **Options.**
2. Choose **Schedule**, and then scroll down to the **Calculation options for this project** area.
3. Make sure the **Inserted projects are calculated like summary tasks** box is selected

Change what tasks show up on the critical path

Typically, critical tasks have no slack. But you can tell Project to include tasks with one or more days of slack on the critical path so you can see potential problems coming from farther away.

1. Choose **File** > **Options.**
2. Choose **Advanced**, and then scroll down to the **Calculation options for this project** area.
3. Add a number to the **Tasks are critical if slack is less than or equal to** box.

Show multiple critical paths

You can set up your project schedule to display as many critical paths as you need to keep tabs on your project.

1. Choose **File** > **Options.**
2. Choose **Advanced**, scroll down to the bottom, and then select **Calculate multiple critical paths.**
3. Choose **View** > **Gantt Chart.**
4. Choose **Format**, and then select **Critical tasks.**

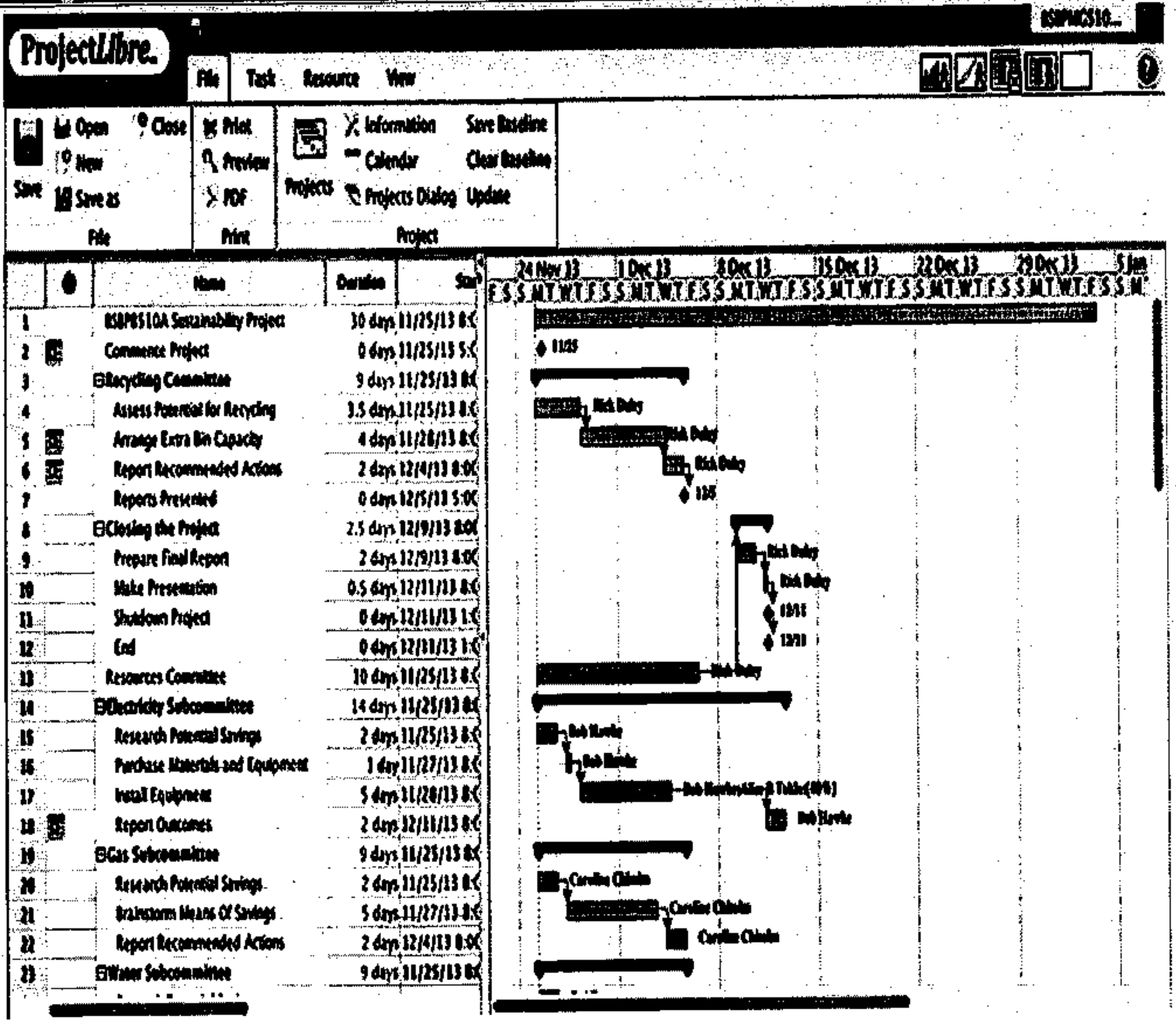


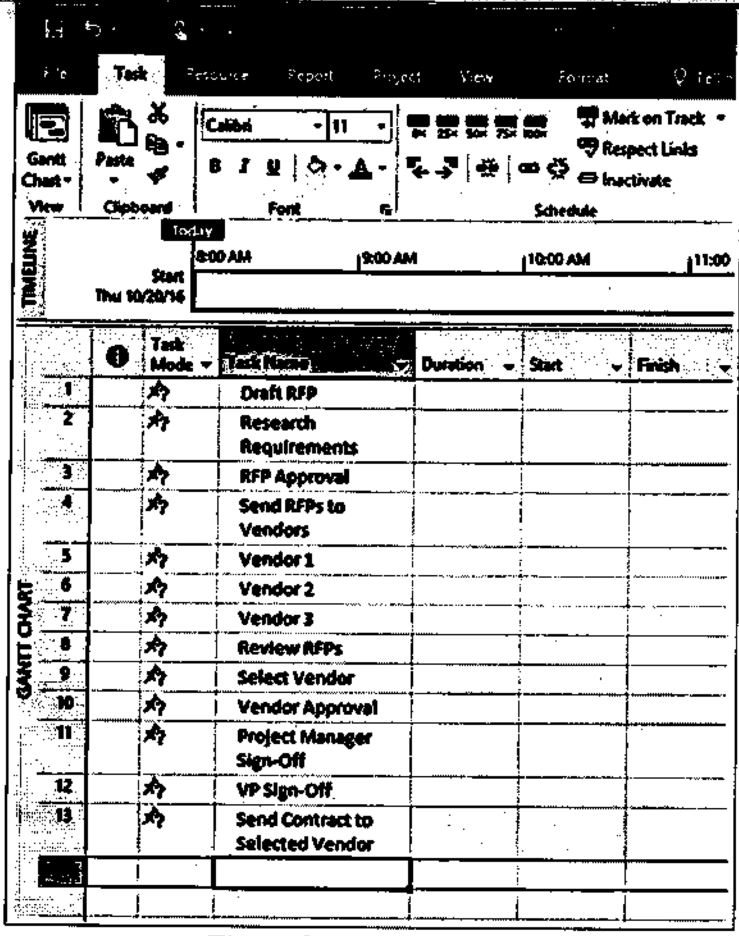
Figure 6.1: MS Project - 1

QUESTION 19.

How to Create a Timeline in Microsoft Project Tutorial:

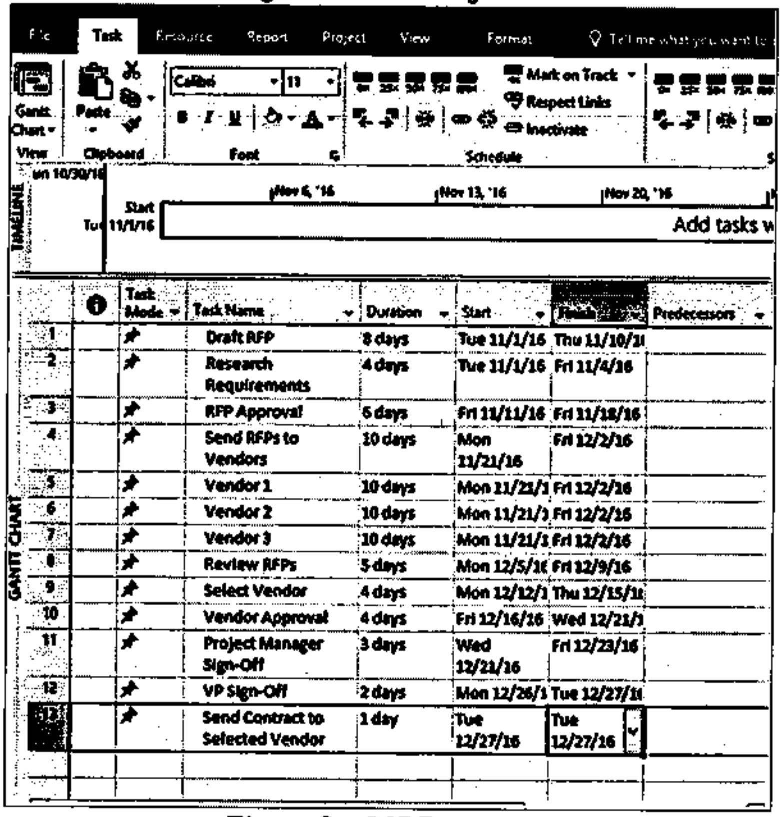
ANSWER:

1. **Create a Task List** You'll need to build a list of required tasks. To get started, open Microsoft Project, click Blank Project, and type each task into a cell under Task Name.
2. **Add Start and Finish Dates to Each Task** To enter start and end dates, click the Start cell that corresponds to the first task and enter a date (if you click the down arrow in the cell, a calendar will appear and you can use that to select a date). Then tab over to the Finish row and enter an end date. Microsoft will automatically enter the amount of time it will take to complete the task in the Duration row. You'll notice that as you add the dates, bar charts will be added to the timeline in the right-hand pane.
3. **Add Tasks to the Timeline** To add tasks to the Timeline, click the View tab and click the Timeline bar that appears above the task list. Then right-click on a Task cell and choose Add to Timeline from the list and click it to add the task to the timeline.



	Task Mode	Task Name	Duration	Start	Finish
1		Draft RFP			
2		Research Requirements			
3		RFP Approval			
4		Send RFPs to Vendors			
5		Vendor 1			
6		Vendor 2			
7		Vendor 3			
8		Review RFPs			
9		Select Vendor			
10		Vendor Approval			
11		Project Manager Sign-Off			
12		VP Sign-Off			
13		Send Contract to Selected Vendor			

Figure 6.2: MS Project - 2



	Task Mode	Task Name	Duration	Start	Finish	Predecessors
1		Draft RFP	8 days	Tue 11/1/16	Thu 11/10/1	
2		Research Requirements	4 days	Tue 11/1/16	Fri 11/4/16	
3		RFP Approval	6 days	Fri 11/11/16	Fri 11/18/16	
4		Send RFPs to Vendors	10 days	Mon 11/21/16	Fri 12/2/16	
5		Vendor 1	10 days	Mon 11/21/1	Fri 12/2/16	
6		Vendor 2	10 days	Mon 11/21/1	Fri 12/2/16	
7		Vendor 3	10 days	Mon 11/21/1	Fri 12/2/16	
8		Review RFPs	5 days	Mon 12/5/16	Fri 12/9/16	
9		Select Vendor	4 days	Mon 12/12/1	Thu 12/15/1	
10		Vendor Approval	4 days	Fri 12/16/16	Wed 12/21/1	
11		Project Manager Sign-Off	3 days	Wed 12/21/16	Fri 12/23/16	
12		VP Sign-Off	2 days	Mon 12/26/1	Tue 12/27/1	
13		Send Contract to Selected Vendor	1 day	Tue 12/27/16	Tue 12/27/16	

Figure 6.3: MS Project - 3

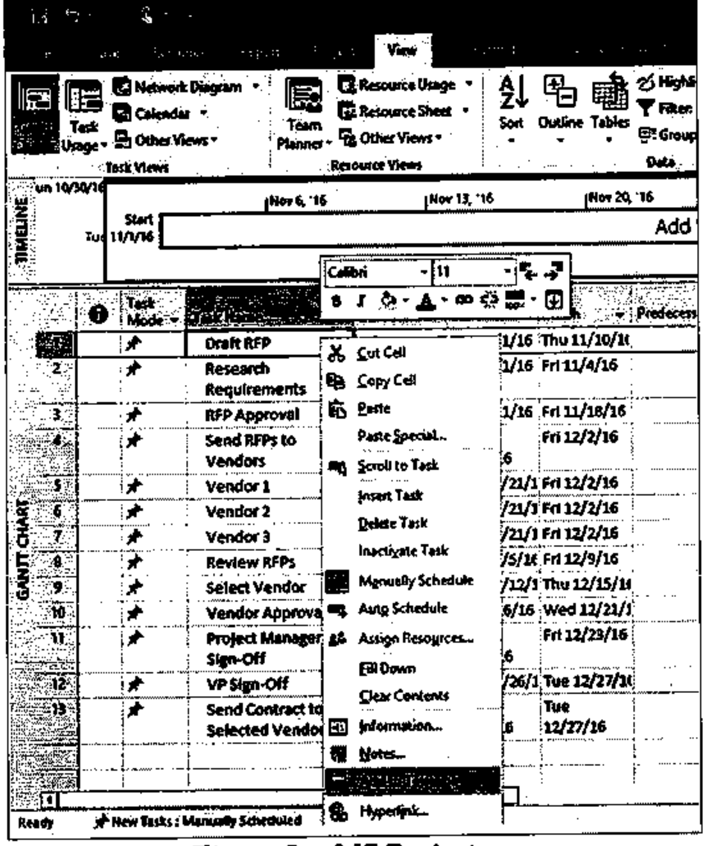


Figure 6.4: MS Project - 4

QUESTION 20.

How to Set Up Resources in Microsoft Project

ANSWER:

The term "resources" typically refers to people, but can also mean documentation or a certain type of work that will be needed to complete the project.

1. Open the Resource Sheet

From the *View* tab, click *Resource Sheet*. The Resource Sheet will appear on the screen.

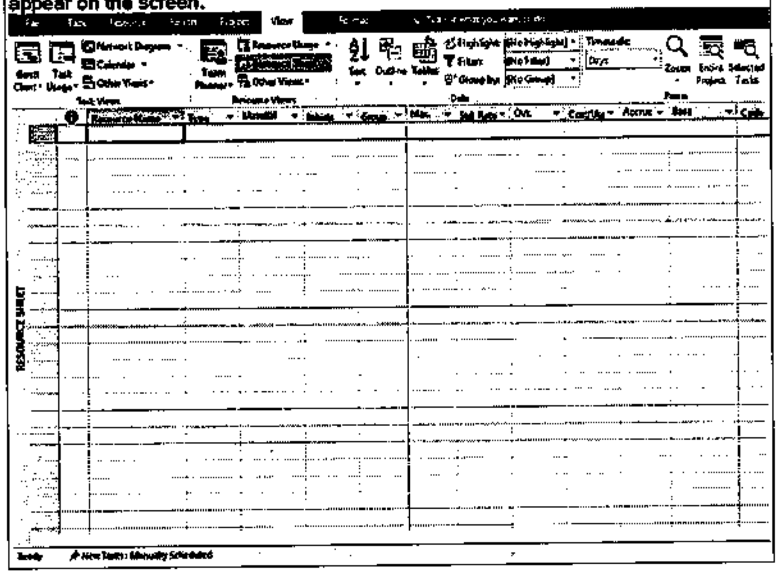


Figure 6.5: MS Project - 5

2. Add Resources Type the name of the resource needed in the Resource Name field and complete the remainder of the information: Type, Material (if it's a material), Initials, Max (max amount of time), Standard Rate, Overtime, Cost/Use, Accrue, Base, and Code.

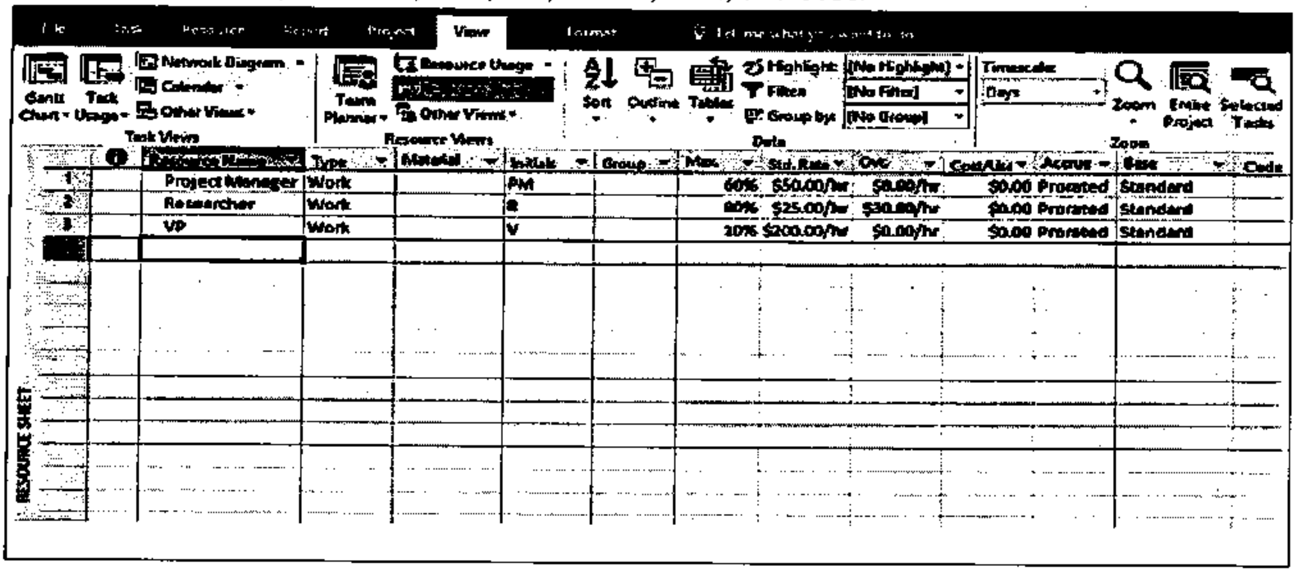


Figure 6.6: MS Project - 6

Once your resources are added to the project, you can easily view who is available to take on the task based on their workload, and manage how much time each team member will spend on tasks in the **Resource Management** view.

QUESTION 21.

How to Assign Tasks in MS Project

ANSWER.

Once you have a list of resources for your project, you'll want to assign tasks. This will help you better manage the project and get work done in a specific time period. One of the benefits of MS Project is that it can calculate how long it will take a person to complete the task based on their availability. If it's a particularly important part of the project that needs to be done quickly, you can assign multiple people to it and Microsoft Project will decrease the time it takes to complete the task based on how many resources are assigned. This also lets the people assigned to the project know how much time is required of them.

1. **Switch to the Gantt chart:** To assign tasks, you'll need to switch to the Gantt chart. Click the Gantt chart icon in top left corner of the window
2. **Open the Task Form** You should still be in the View tab. Click the Details box in the ribbon. The Task Form should appear on the lower half of the screen. If it doesn't appear, click the down arrow in the Details box and select Task Form.

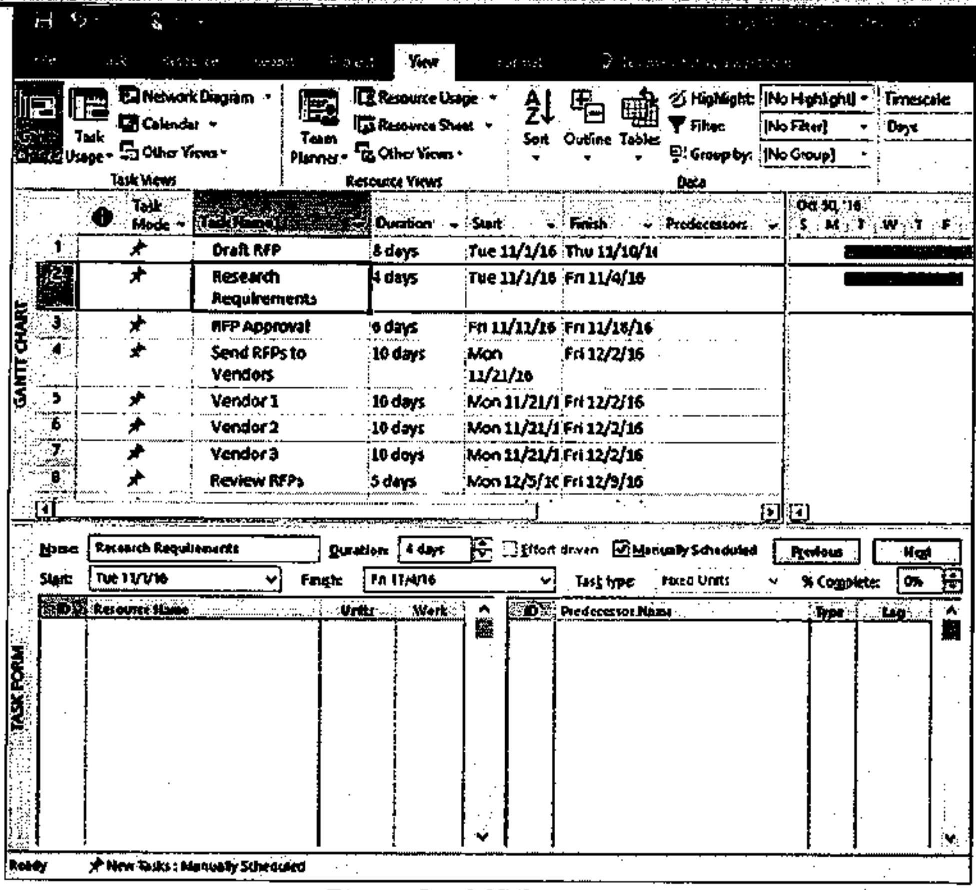


Figure 6.7: MS Project - 7

3. **Select a Task to Assign** Click a task in the Gantt chart view and it will appear in the Name section of the Task Form. Click the box under Resource Name and choose a resource from the drop-down menu. Then click OK.

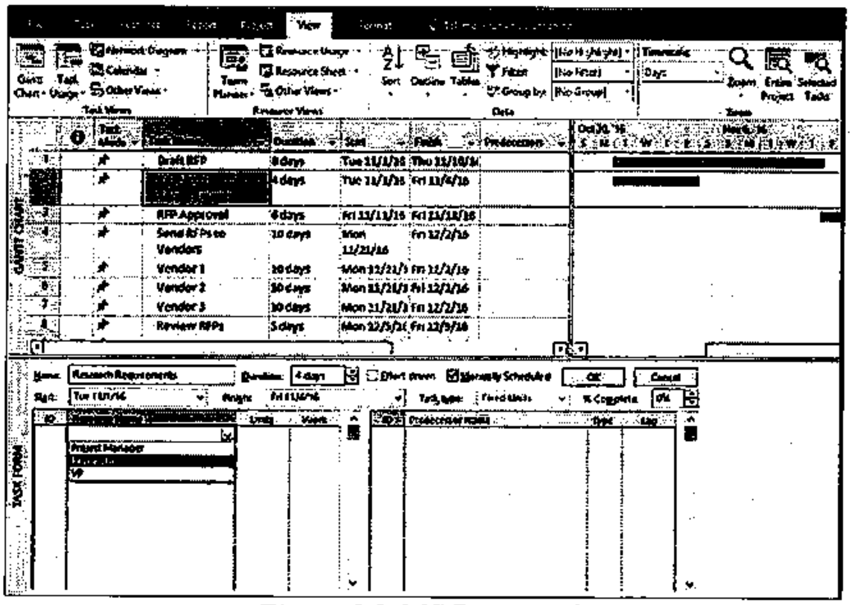


Figure 6.8: MS Project - 8

You can add another person to the same task by clicking the area under *Resource Name* and choosing the name you want. Click *OK*. As you assign tasks, the amount of time will be added to the Gantt chart.

QUESTION 22.

How to Schedule Tasks Automatically or Manually

ANSWER.

With Microsoft Project 2016 you can schedule tasks manually or automatically. When you opt to manually schedule tasks it's up to you schedule all new tasks and track them to ensure they are being completed on time. If you choose Automatic scheduling, Project will schedule tasks based on dependencies, calendars, and constraints among other things. The default option when creating tasks is to schedule them manually, here it is mentioned how to change the setting to automatic.

1. Access Microsoft Project Settings

Click *File* on the menu bar and choose *Options* (it's the last choice in the left column). Click *Edit project settings*.

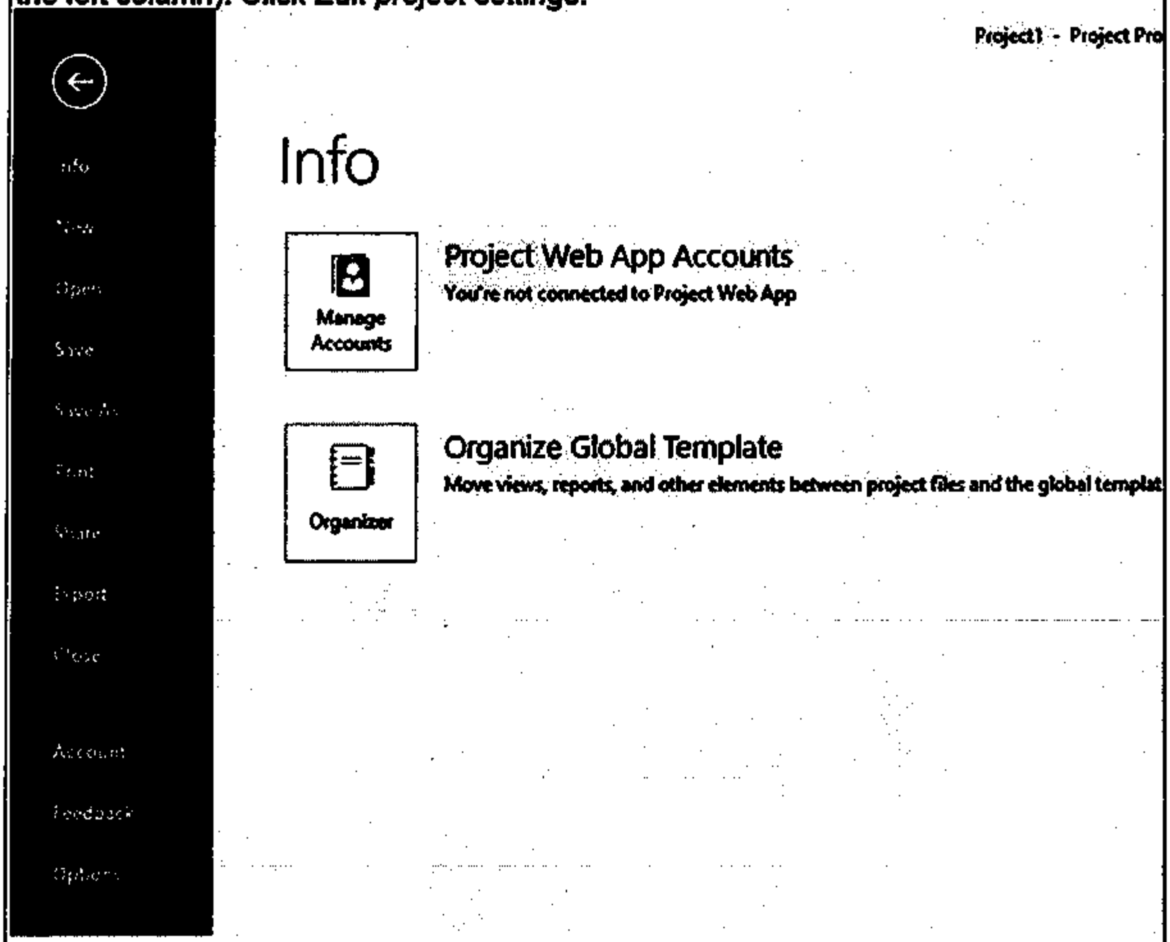


Figure 6.10: MS Project - 10

2. Change Schedule Options When the Project Options form appears on the screen, click *Schedule* in the left column.

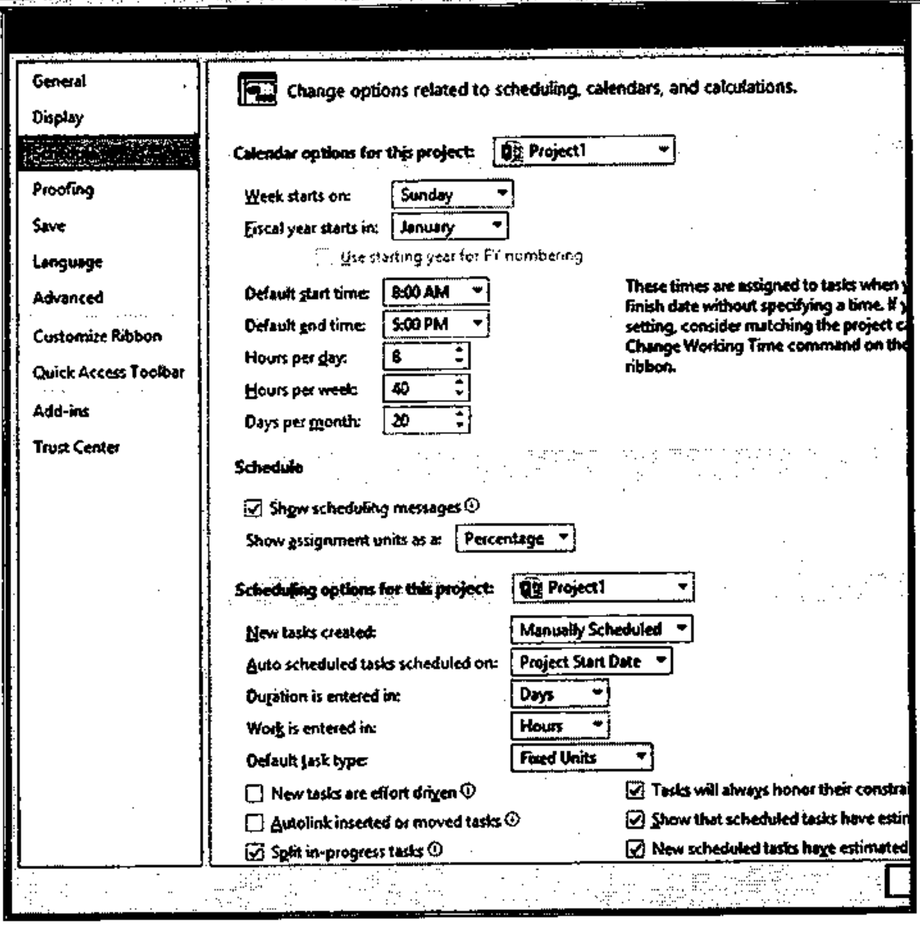


Figure 6.11: MS Project - 11

Next, under Scheduling Options for this Project section, click the drop-down menu for New Tasks Created. The default is set to Manually Scheduled. Select and click Auto Scheduled and click the OK button.

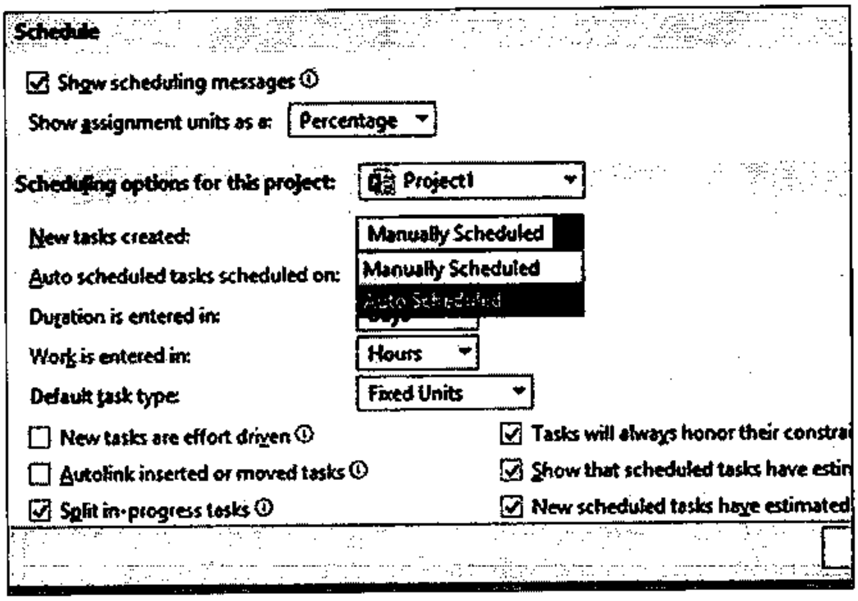


Figure 6.12: MS Project - 12

QUESTION 23.

How to Create Task Dependencies

ANSWER:

Dependencies occur when one task can't move on to the next phase until a particular task is completed before it. Creating dependencies involves linking tasks in the Gantt chart view. In Microsoft Project, you can link any two tasks. Once tasks are linked, every change made to the predecessor affects the successor.

1. **Switch to Gantt Chart View** You should still be in the Gantt chart view. If you're not, click the Gantt chart icon in top left corner of the window.
2. **Select Tasks to Link** Click the *Task* tab in the menu bar. Identify the two tasks in the list that you want to link. Click the first task and press and hold the Ctrl key and select the second task. Click the chain icon in the ribbon to link the tasks. You'll see an arrow appear on the Gantt chart that connects the items.

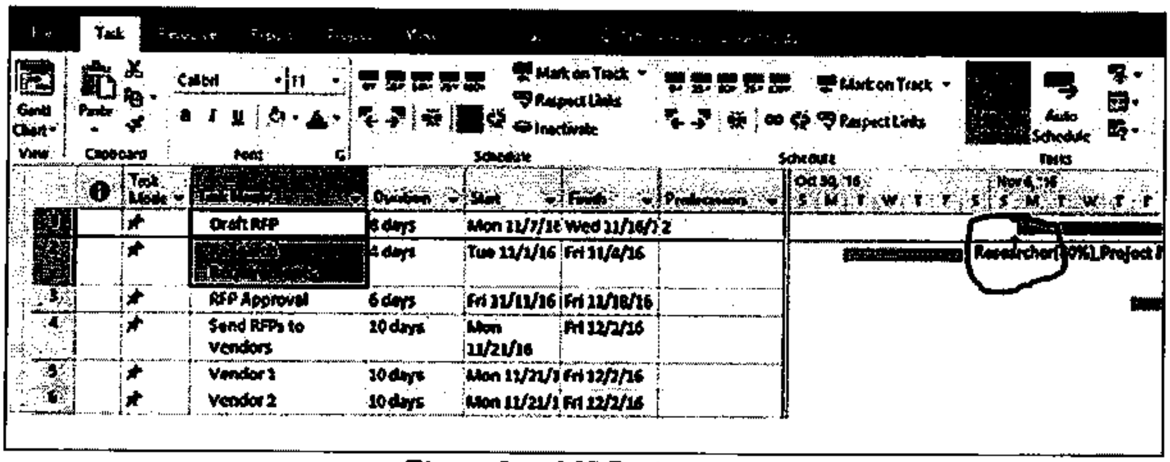


Figure 6.13: MS Project - 13

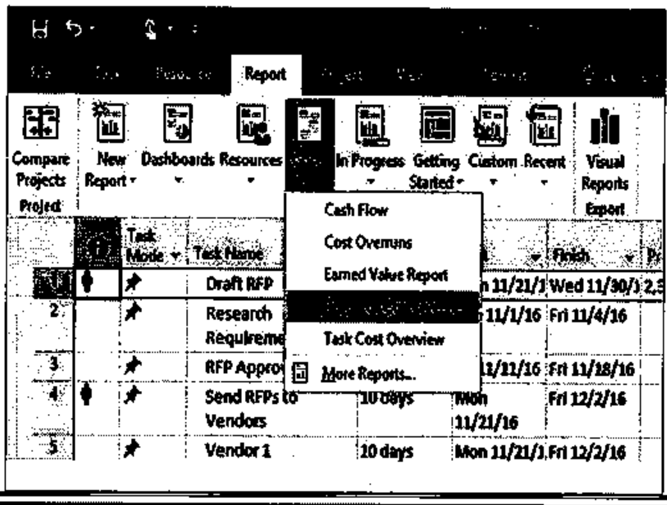
QUESTION 24.

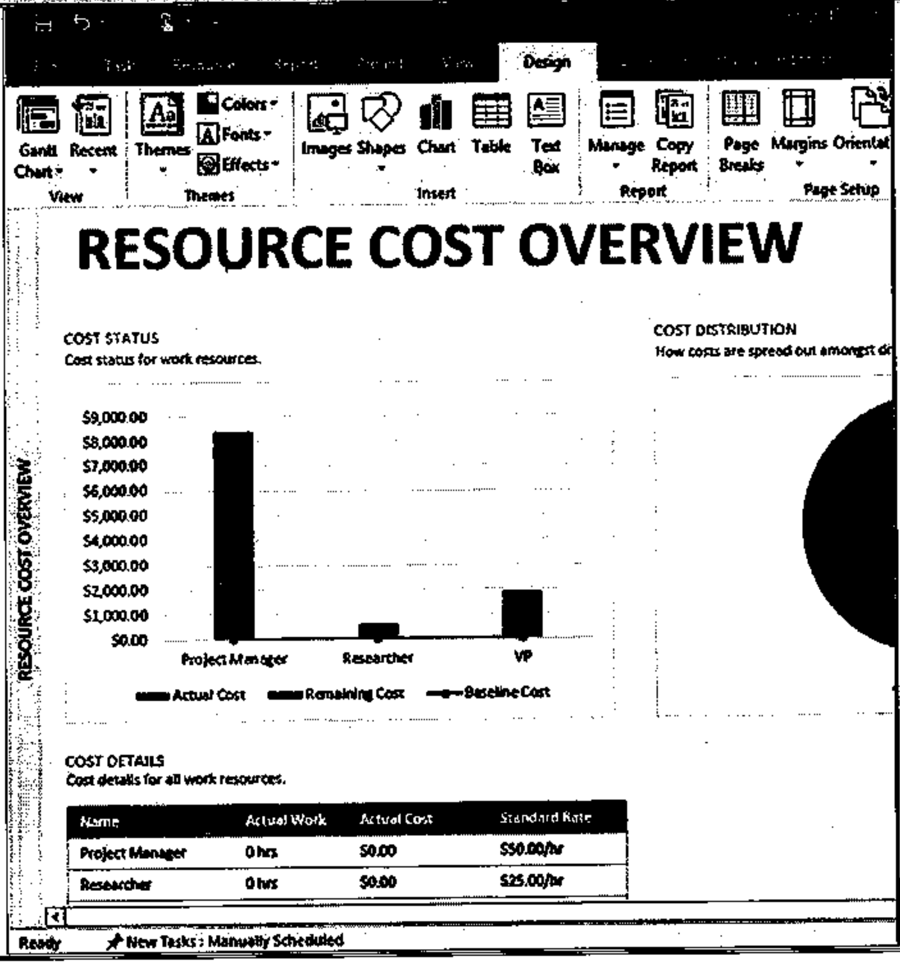
How to Generate a Cost Report in Microsoft Project 2016

ANSWER:

Once you've entered time and resources information to the best of your ability, you can use Project to run a Cost Overview report. Here's how to create a Resource Cost Overview report:

1. **Select the Report Tab** Click the *Report* tab to get a quick overview of the reports you can run.
2. **Choose a Cost Report to Run** Click the arrow below *Costs* in the ribbon and click Resource Cost Overview.



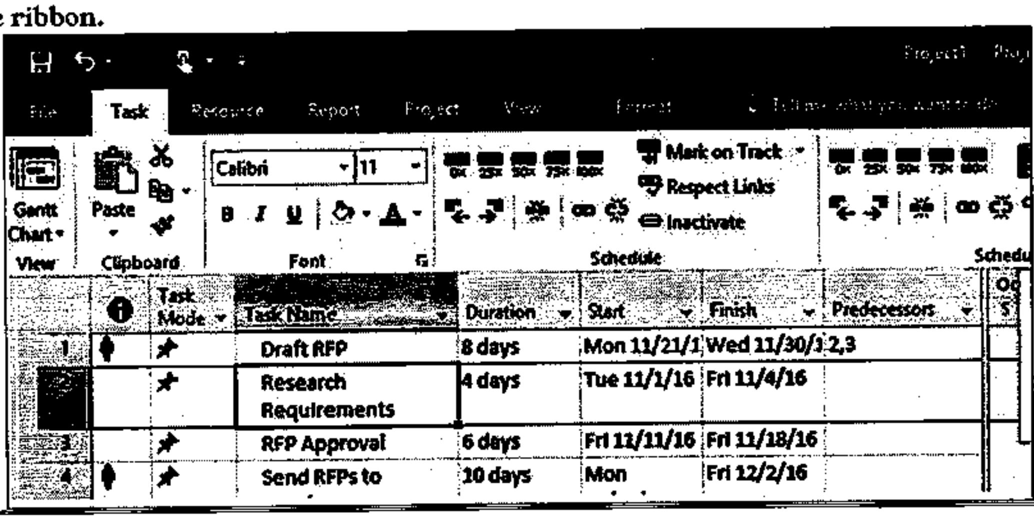


QUESTION 25.
How to Track the progress of your MS Project

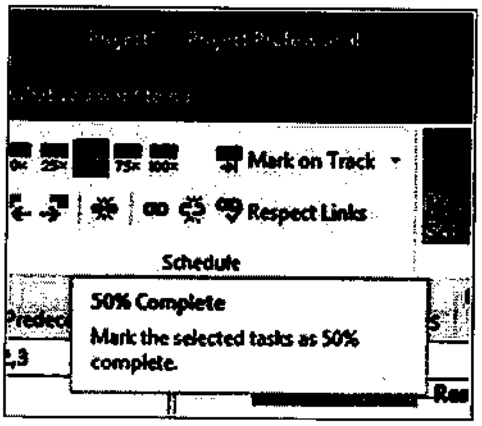
ANSWER:

With Microsoft Project, you can keep an eye on tasks to see if things are running on time or behind schedule. This will be easy to view as long as you keep the status of tasks updated during the length of your project.

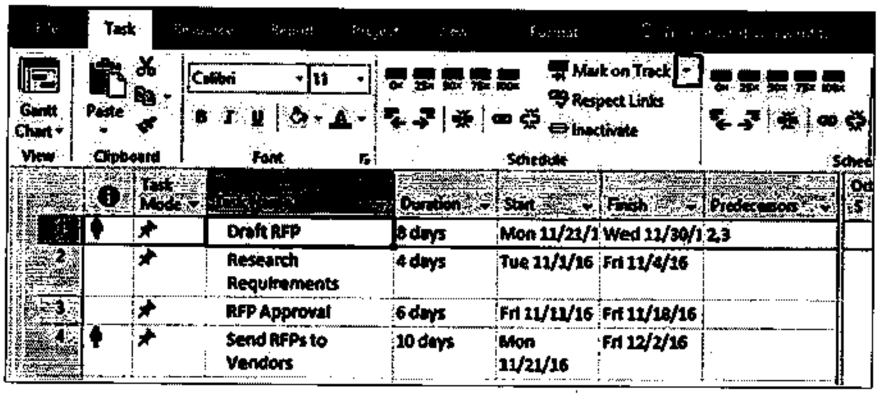
1. **Mark Tasks That Are on Track** Click the Task tab in the menu bar to see all the task options. Click a task that you want to update. If the task is on track, click the Mark on Track button in the ribbon.



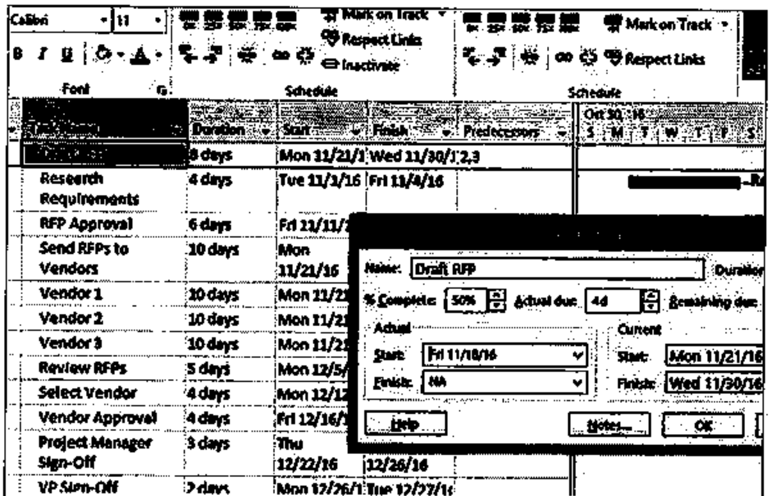
2. **Use Predetermined Percentages to Track Tasks** To the left of the Mark on Track option, there are percentages that you can use to denote the progress of a task. Click a task to update and click 0%,25%, 50%, 75%, or 100%. You'll see a line drawn through the corresponding bar on the Gantt chart that signifies how much of the task is complete.



3. **Update Tasks**
Sometimes tasks fall behind or get accomplished ahead of schedule. You can use the Update Task option to update the status. Click the down arrow next to Mark on Track and click Update Tasks

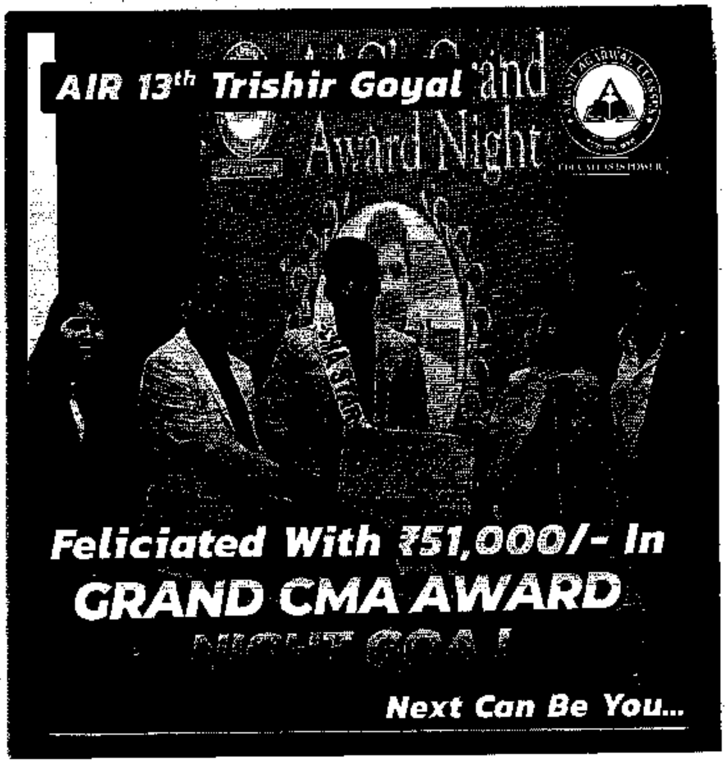


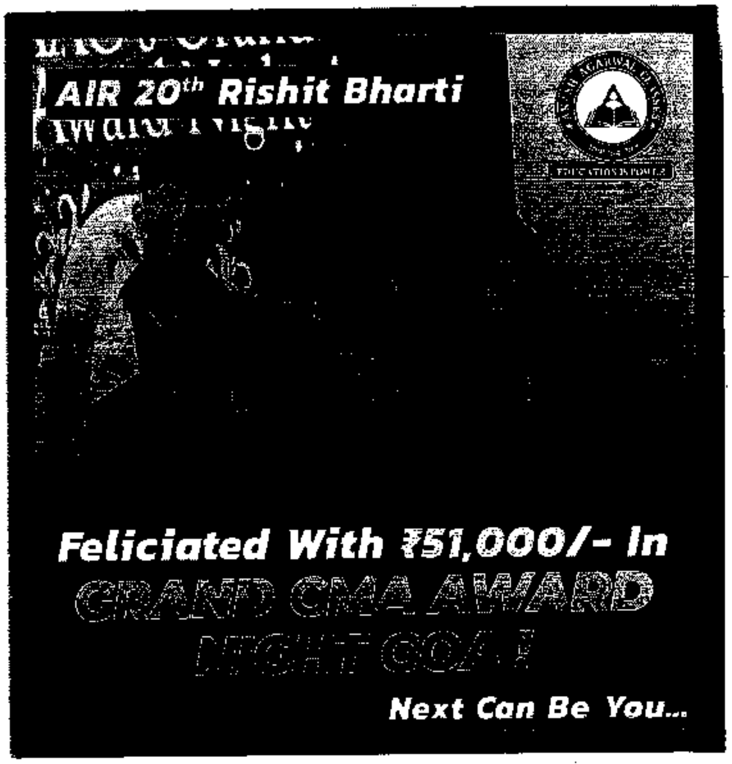
A dialogue box will appear where you can update status and change start and end dates. Make any changes and click OK.



These are all the steps you need to get started and create a project, assign and manage tasks, and run reports in Microsoft Project 2016.

CMA STUDENTS AND TEAM AAC CELEBRATING SUCCESS OF CMA INTER CLEARED STUDENT IN GOA





TOKEN OF MOTIVATION TO ACHIEVERS

.....NEXT CAN BE YOU.

7.1 Breakdown Maintenance

Introduction

Here the production facility is run without much routine maintenance until it is breakdown. Once the machine breakdown it is taken for repair and inspected to find out the defects. After identifying the defect, the required repair is planned and the spares are procured to repair the machine. As the breakdowns are random in nature and the machine cannot be used during the repair period, production hours are lost hence the productivity is reduced. Repair maintenance is not a recommended practice, in general, but many a time many organizations prefer this, because they do not want to keep the machine idle for maintenance. But they ignore the fact that the break down repair costs more than the regular maintenance practice. It is however, an economical way of maintaining certain non-critical items whose repair and down time costs are less this way than with any other system of maintenance.

7.2 Preventive Maintenance

Introduction

A system of scheduled, planned or preventive maintenance tries to minimize the problems of breakdown maintenance. It locates weak parts in all equipments, provides them regular inspection and minor repairs thereby reducing the danger of unanticipated breakdowns. The underlying principle of preventive maintenance is that prevention is better than cure. It involves periodic inspection of equipment and machinery to uncover conditions that lead to production breakdown and harmful depreciation. The system of preventive maintenance varies from plant to plant depending on the requirement of the plant. Any company, adopting the preventive maintenance should keep the record of failure of various components and equipment, which help the maintenance department to statistically analyze the failure pattern and replace the item before it fails, so that the breakdown can be eliminated. This reduces the unanticipated breakdowns, increases the availability of the equipment for production purpose, maintain optimum productive efficiency of equipment and machinery reduces the work content of maintenance job, increases productivity and safety of life of worker.

Production department or maintenance department depending on the size of the plant generally takes up preventive maintenance work. As the preventive maintenance is a costly affair, it is better to maintain records of cost (both labour, materials used and spares used) and a valuation of the work done by the department will show us what benefits are derived from preventive maintenance. The analytical approach to evaluate the work done by preventive maintenance is

(i) (Inspections incomplete) / (Inspections scheduled) × 100 should be less than 10%
(ii) (Hours worked for maintenance) / (Scheduled hours) × 100 = Performance of the department.

(iii) Down time to be given as a ratio of the available hours and to be compared against a standard to be worked out for each company or against a figure of the past. The ratio is given as:
= Down time in hours/ Available hours (where Available Hours = working days × hours per day × number of machines). Here down time is the total time of stoppage of the machine for scheduled and unscheduled maintenance work.

(iv) Frequency of break downs = (Number of break downs) / (Available machine hours)

(v) Effectiveness of planning = (Labour hours on scheduled maintenance) / (Total labour hours spent on maintenance).

OR

(Down time due to scheduled maintenance)/(Down time due to total maintenance work)

QUESTION 1. What are the Advantages of preventive maintenance
ANSWER. 1. Reduced breakdowns and downtime, 2. Greater safety to workers, 3. Fewer large scale repairs, 4. Less standby or reserve equipment or spares, 5. Lower unit cost of the product manufactured, 6. Better product quality, 7. Increased equipments life and 8. Better industrial relations.

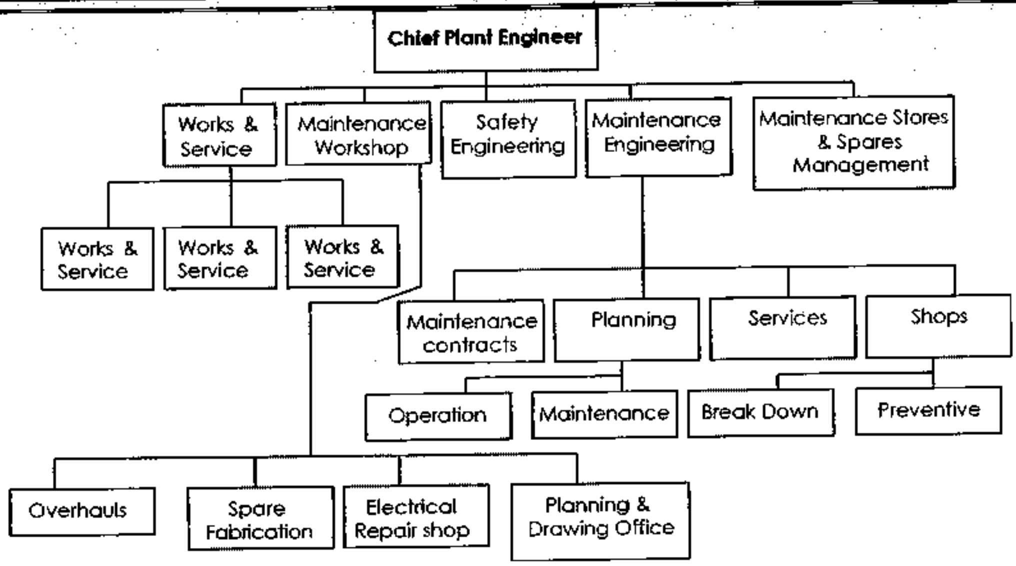
7.3 Routine Maintenance

Introduction

It includes lubrication, cleaning, periodic overhaul; etc. This is done while the equipment is running or during pre- planned shut-downs. Running maintenance is the work which can be carried out while the facility is in service.

QUESTION 2.
Define Maintenance Techniques
ANSWER. It can be discussed as under: In some cases, the loss and inconvenience due to breakdown of equipment is so high that standby equipment is kept. As soon as the original equipment fails, the standby facility is employed to avoid interruption and downtime. Standby machines are often kept to reduce the loss due to the breakdown of a key machine. Breakdown maintenance also requires use of standby machines. The main question here is how many standby machines to keep and for how long. In order to decide this, a cost benefit analysis of standby machines should be made. There are various costs involved in standby machines. First, there is interest cost on capital investment. Secondly, space is needed to keep standby machines. Thirdly, there is depreciation in the value of standby machines. Fourthly, periodic checking and servicing is necessary to keep the standby machines in good condition. The benefits of standby machines consist of protection against a complete shutdown or shut down of operations. It avoids loss of production and, therefore, it is necessary to estimate loss of future failures a table of expected costs and benefits can be prepared. Shifting production during breakdown. Under this method spare capacity is maintained not in the form of standby machines but by allowing rest to running machines at intervals and by rotation. If one machine in a production line requires shutdown, the output is maintained by shifting to underutilized machines in other lines. For such application, the capacities of different machines must be properly matched.

QUESTION 3.
Define Maintenance Organization And there rules.
ANSWER. At least 50 to 60 percent of investment of any organization is spent on Building and Production facilities. Hence, it is worthwhile to give due consideration for effective maintenance of these items. The maintenance department will look after the upkeep of equipments, buildings and other. For effective contribution of its work, the maintenance department must have proper place in the organization and it must also have a good organizational structure. While organizing a maintenance department one must remember that there should be clear division of authority with little or no overlap. Vertical lines of authority and responsibility must be kept as short as possible. Keep the span of control to an extent of 3 to 6 for a manager. The organizational structure should be flexible. The structure should be designed to suit the types of maintenance work involved. Depending on the need, the maintenance activity may be centralized or decentralized.



Outline of a maintenance department of a large organization

Organizing Maintenance Work

In order to facilitate proper control of maintenance work; we must enforce three rules as below.

1. Maintenance Request:
   This must be made in writing to a central point in the organization. No work should be carried out without the knowledge and approval of maintenance supervisor - if this discipline is not followed by the organization, it leads to wastage of skilled manpower and inability of the maintenance personnel to schedule essential maintenance work.
2. Maintenance Stores:
   Non-availability of vital spare parts when required to meet an emergency like breakdown, may lead to excessive shutdown of the plant and equipment. A large number of items or materials are required to be stored and it involves investing valuable funds from the working capital. A proper stores management is essential as a backup service of good maintenance.
3. Records of Maintenance Work Done
   Paper work for maintenance is crucial for establishing a good maintenance organization and is often neglected. The records of maintenance work carried out from time to time have to be kept equipment wise. History cards or logbooks of all the plants and equipment must be compiled meticulously giving details of materials used, components replaced and time spent by the workforce.
   Creation and maintaining this database is essential for proper planning and control, which alone will lead to effective and efficient maintenance.
   To get the full benefits of effective maintenance the following requirement is to be fulfilled:
   a. Good Supervision and administration of maintenance department,
   b. Good and clear instructions to be given to maintenance crew regarding the repair,
   c. Proper control of work in coordination with production department,
   d. Good training should be given to the maintenance personnel,
   e. Good scheduled maintenance program should be chalked out,
   f. Proper maintenance record keeping is a must,
   g. There should be adequate stock of spare parts, particularly insurance spares.

QUESTION 4.
Define Maintenance Problem

ANSWER.

The main problem in maintenance analysis is to minimize the overall cost of maintenance without sacrificing the objectives. There are two alternatives before management. One is to repair a machine or equipment only when it breaks down. This will save expense of inspection and replacement of a part before its lifetime ends. The other alternative is to replace the equipment before the expiry of its working life. This will involve cost of periodic shutdown for checkup and repairs. However, it will avoid the loss due to sudden failure or breakdown.

The two types of cost - cost of premature replacement and cost of breakdown - need to be balanced. The objective is to minimize total maintenance cost and downtime. Economic analysis is helpful in finding a judicious combination of two types of maintenance. The relationship between preventive maintenance time and repair time is also significant. Preventive maintenance policy is justified only when the average downtime and its cost is less than the average time taken to carry out breakdown repairs. If the machine happens to be part of production line, the breakdown of a machine would throw the entire production line out of gear while a preventive maintenance schedule might enable the repair to be performed during a scheduled idle time of the line.

7.4 Replacement of Machine

Introduction

Wear and obsolescence are the two main causes for replacement of machinery in every aspect of life. The reduction of wear is therefore a primary concern while designing appliances. Wear and tear due to passage of time and/or normal usage of plant and machinery is an accepted fact. Technological obsolescence is a major danger which business firms face in modern era. With the development of new and better techniques or equipment of performing a particular function, existing equipment and machines become uneconomical. Whenever a firm decides to switch over to new machines or improved product designs, existing machine designs are said to be obsolete. Hence, obsolescence is a major issue in the procurement and installation of machinery and equipment. A machine is technically obsolete when another machine can do the same job more efficiently, with reduced time and also at a lower cost. Technological obsolescence arises due to continuous improvements in the methods and techniques of production and sometime the rate of improvement is so fast that it becomes economical to replace the machinery before its expected life. A machine may be replaced to reduce the running costs of the concerned machine and the new machines productivity will be more. In replacement decisions, the basic problem is to decide whether to replace a machine or equipment at present or at a future date. It is, therefore, necessary to determine whether obsolescence or deterioration has reached the point where the reduction in operating costs expected from replacement justifies the net capital expenditure involved in installing the new machine and disposing of the old one.

Any function aimed at bringing back or restoring an item to its original or acceptable condition or to keep it and retain its health as well as workability is known as Maintenance.

QUESTION 5.
What are the Objectives of Maintenance

ANSWER.

(i) To keep all the production facilities and other allied facilities such as building and premises, power supply system, etc. in an optimum working condition,
(ii) To ensure specified accuracy to products and time schedule of delivery to customers,
(iii) To keep the down time of the machine at minimum, so that the production program is not disturbed,
(iv) To keep the production cycle with in the stipulated range,
(v) To modify the machine tools to meet the augmented need for production,
(vi) To improve productivity of existing machine tools and to avoid sinking of additional capital,
(vii) To keep the maintenance cost at a minimum as far as possible, there by keeping the factory Overheads at minimum,
(viii) To extend the useful life of plant and machinery, without sacrificing the level of performance.

Illustration 1.

M/s Nirmala Toolkit Pvt. Ltd. has a workshop comprising of 20 tool machines of similar type. To improve the preventive maintenance plan, the workshop manager collects the data of failure history of the machines as under

Elapsed time after Maintenance attention (in month)	Probability of failure
1	0.20
2	0.15
3	0.15
4	0.15
5	0.15
6	0.20

It costs ₹150 to attend a failed machine and rectify the same. Compute the yearly cost of servicing the broken-down machines.

SOLUTION:

Expected time before failure.

= 0.20×1 + 0.15×2 + 0.15×3 + 0.15×4 + 0.15×5 + 0.20×6 = 3.5 months

Therefore, number or repair/machine/annum = 12/3.5

Considering 20 machines and ₹150 to attend a failed machine the yearly cost of servicing

= 12/3.5 × 20 × 150 = ₹10286.

Illustration 2.

A Public transport system is experiencing the following number of breakdowns for months over the past 2 years in their new fleet of vehicles:

Number of breakdowns	0	1	2	3	4
Number of months this occurred	2	8	10	3	1

Each break down costs the firm an average of ` 2,800. For a cost of ` 1,500 per month, preventive maintenance can be carried out to limit the breakdowns to an average of one per month. Which policy is suitable for the firm?

SOLUTION:

Converting the frequencies to a probability distribution and determining the expected cost/month of breakdowns we get:

No. of breakdowns (x)	Frequency in months(f)	Probability (p = f/Σf)	Expected no. of breakdowns (px)
0	2	0.083	0.000
1	8	0.333	0.333
2	10	0.417	0.834
3	3	0.125	0.375
4	1	0.042	0.168

	Σf = 24	Σp = 1	Total 1.710 = Σpx

Expected Breakdown cost per month; Expected no. of breakdowns per month × cost of each breakdown = 1.710 × ₹2800 = ₹4788.

Preventive maintenance cost per month: -

Average cost of one breakdown/month = ₹2, 800

Maintenance contract cost/month = ₹1,500

Total = ₹4,300

Thus, preventive maintenance policy is suitable for the firm.

Illustration 3.

Indian Electronics, manufactures TV sets and carries out the picture tube testing for 2000 hours. A sample of 100 tubes was put through this quality test during which two tubes failed. If the average usage of TV by the customer is 4 hours/day and if 10,000 TV sets were sold, then in one year how many tubes were expected to fail and what is the mean time between failures for these tubes?

SOLUTION:

The total test time = (100 tubes) × 2000 hours = 200,000 tube-hours.

There are two tubes which have failed and hence the total time is to be adjusted for the number of hours lost due to the failures during the testing.

The lost hours are computed as = $2 \times \frac{2000}{2}$ = 2000 hours.

The assumption is made here is that each of the failed tubes have lasted an average of half of the test period. Therefore, the test shows that there are two failures during (2,00,000 - 2000) = 1,98,000 tube hours of testing. During 365 days a year (four hours a day) for 10,000 tubes the number of expected failures

$\frac{1,98,000}{2} \times 10,000 \times 365 \times 4$ = 147.47 =148 tubes approximately.

Mean time between failures = $\frac{1,98,000 \text{ tubes hrs.of testing}}{2 \text{ failure}}$

=99,000 tubes hours per failure = $\frac{99,000}{4 \times 365}$ = 67.8 tubes year per failure

Illustration 4.

M/s XYZ Pvt. Ltd has 50 identical machines in its facilities. The company has the recorded figure for cost of preventive maintenance (Cp) and cost of breakdown maintenance (Cb) as ₹20 and ₹100 respectively. The company wants to reduce the breakdown occurrence while minimizing Cp. Given is the data on breakdown occurrence.

Probabilities of machine breakdown, by month:

Months after servicing that breakdown occurs (i)	Probability that breakdown will occur (Pi)	$i.P_i$
1	0.10	0.10
2	0.05	0.10
3	0.05	0.15
4	0.10	0.40

5	0.15	0.75
6	0.15	0.90
7	0.20	1.40
8	0.20	1.60
Total	1.00	5.40

SOLUTION:

The mean time before failure is 5.4 months and the expected cost with no preventive maintenance would be $100 \times \frac{50}{5.4}$ = ₹925.93 per month. The following calculations show Bj, the expected number of breakdowns between preventive maintenance intervals, for the possible intervals, that may be considered.

$B_1 = MP_1 = 50\ (0.10) = 5$

$B2 = m\ (P1 + P2) + B1P1 = 50(0.10+0.05) + 5(0.10) = 8$

$B_3 = 50\ (0.10 + 0.05 + 0.05) + 8\ (0.10) + 5\ (0.05) = 11.05$

Accordingly, B4 = 16.75, B5 = 25.63, B6 = 35.5, B7 = 48.72, B8 = 63.46.

The costs of various preventive maintenance intervals are summarized in the table below:

Cost of alternative preventive maintenance intervals –

Number of months between preventive services (j)	B_j Expected Number of Breakdown in jmonths	Expected cost/ month to Repair Breakdown $C_R \times B_j/j$	Cost per monthfor preventive service every j month $C_R(M)/j$	Total expected cost per month of preventive maintenance andrepair
(1)	(2)	(3)	(4)	(5)
1	5.00	500.00	1000.00	1500.00
2	8.00	400.00	500.00	900.00
3	11.05	368.33	333.33	701.66
4	16.75	418.75	250.00	668.75
5	25.63	512.60	200.00	712.60
6	35.50	591.67	166.67	758.34
7	48.72	696.00	142.86	838.86
8	63.46	793.25	125.00	918.25

A policy of performing preventive maintenance every 4 months results in the lowest average cost, about ₹669. This amount is ₹257 per month less than the ₹926 expected cost without preventive maintenance. This policy would reduce the costs by (257 ÷ 926) × 100 = 27.75% below the cost of repairing the machines only when they breakdown.

7.5 Spare Parts Management

QUESTION 6.
Define Spare Parts Management and Spars type.

ANSWER.
Spare Parts Management (SPM) is a complex activity in a large manufacturing plant as it is one of the critical enablers for providing better services to the customers during several stages of the product life cycle. Usually, SPM deals with two types of parts such as one which is required very frequently ("fast moving") and the one which is used quite rarely ("slow moving or non-moving"). While the managers keep stock for the former category, the later posits a greater challenge to the works manager. Further, given the fact of changing consumer preferences and needs coupled with technological progress, organizations are compelled to bring new products and/or differentiate the existing products every now and then, SPM invokes a methodical and record-based careful approach to provide services to the customers those who have bought the products before obsolescence or differentiation. Further, for preventive maintenance also spare parts are required often. Hence, it is necessary to maintain a comprehensive database management for SPM that helps to-

(a) Keep records for specific and general purpose spare parts used in preventive and breakdown maintenance
(b) Track the movement of the spare parts and consumption pattern
(c) Plan for future maintenance activities on short and mid-term basis
(d) Demand forecasting for spare parts requirement on short, medium and long-term basis.
(e) Plan for collaboration with spare parts vendors and managing them
(f) Optimize the spare parts inventory

Preventive maintenance is very important but failure cannot be able to eliminated. To avoid failures spare parts, play a vital role. Failure statistics are useful in calculating spare parts for preventive maintainance and breakdown maintenance also. Spares can be classified as per service level/understocking cost.

- **Regular spares**
  The spare parts required regularly and in substantial number. Both reliability and per unit cost of these items are less.
  Service level = Ku/Ku + Ko, Ku = Opportunity cost of understock of one unit Ko = Opportunity cost of overstock of one unit.
- **Insurance spares**
  An insurance spare is a spare part that you hold in your spare parts inventory, that you would not expect to use in the, normal life of the plant and equipment but if not available when needed it would result in significant losses.
- **Capital spares**
  Capital spares are spare parts which, although acknowledged to have a long life or a small chance of failure, would cause a long shutdown of equipment because it would take along time to get a replacement for them.
  $\Sigma_{i=0}^{N-1} P_i \leq C3 - C / C3 \leq \Sigma Pi$
- **Rotable spares**
  Rotable items are generally thought of as items of plant or assets that periodically are changed out for repair or overall.
  The management of rotable items and repairable spare parts is different to the management of other inventory items and proper control requires greater cooperation between maintenance and stores/inventory management. The successful management of these items

is far more active than other spare parts as maintenance and store/ inventory personnel must work together to ensure that there is visibility of the status of items.

Illustration 5.

Compute the requirement of spares for breakdown maintenance for an item that exhibits a Poissonian behavior for failure rates with a mean breakdown rate of five items per month. If the lead time for procuring these spares is one month and a service level of 90 per cent is to be used, what buffer stock of these items should be maintained? (A fixed re-order quantity system of inventory is being used).

SOLUTION:

Buffer stock is required to cover the lead time only, i.e. to cover one month's period.

Mean consumption rate = 5 per month

Referring to the Poisson distribution table for a = 5, we have for

x = 7. Cumulative probability = 0.867

x = 8 Cumulative probability = 0.932

Thus, with seven items only 86.7 per cent service level is attained; with eight items 93.2 per cent service level is obtained. Since one would err on the higher side of the service level, the value of x = 8 is chosen.

This means, the amount of spares stock that has to be kept must correspond to a maximum demand rate Dmax of eight during the lead time. In other words we should keep a Buffer Stock = Dmax - Daverage

during a lead time = 8 - 5 = 3 items.

Thus, buffer stock desired is three numbers of the given spare part.

Illustration 6.

The main shaft of an equipment has a very high reliability of 0.990. The equipment comes from Russia and has a high downtime cost associated with the failure of this shaft. This is estimated at ₹2 crore as the costs of sales lost and other relevant costs. However, this spare is quoted at ₹10 lakh at present. Should the shaft spare be procured along with the equipment and kept or not?

SOLUTION: The expected cost of down-time

= (Probability of failure) × (Cost when break-down occurs)

= (1 - 0.990) × (₹2 crore) = ₹2 lakh

However, the cost of procuring the spare now is ₹10 lakh. Therefore, expected cost of downtime is less than the cost of spare; hence the spare need not be bought along with the equipment.

Illustration. 7.

PQR company has kept records of breakdowns of its machines for 300 days work year as shown below:

No. of breakdown	Frequency in days
0	40
1	150
2	70
3	30
4	10
	300

The firm estimates that each breakdown costs ₹650 and is considering adopting a preventive maintenance program which would cost ₹200 per day and limit the number of breakdowns to an average of one per day. What is the expected annual savings from preventive maintenance program?

SOLUTION:

Step 1: To determine the expected number of breakdowns per year:

No. of breakdowns (x)	Frequency of breakdowns in days i.e.,f(x)	Probability distributionof breakdowns P(x)	Expected value of breakdowns X P(x)
0	40	40/300 = 0.133	Nil
1	150	150/300 = 0.500	0.500
2	70	70/300 = 0.233	0.466
3	30	30/300 = 0.100	0.300
4	10	10/300= 0.033	0.132
Total	300	1.000	1.400

Step 2:

Total no. of breakdowns per day = 1.40

Cost of breakdown per day = 1.40 × 650 = ₹910

Cost of preventive maintenance programme per day = ₹200+ ₹ 650 = ₹ 850

Expected annual savings from the preventive maintenance programme = (910 - 850) × 300 days = 60×300 = ₹ 18,000

Illustration 8

A firm is using a machine whose purchase price is ₹15,000. The installation charges amount to ₹3,500 and the machine has a scrap value of only ₹1,500 because the firm has a monopoly of this type of work. The maintenance cost in various years is given in the following table:

Year	1	2	3	4	5	6	7	8	9
Maintenance Cost (₹)	260	760	1100	1600	2200	3000	4100	4900	6100

The firm wants to determine after how many years should the machine be replaced on economic considerations, assuming that the machine replacement can be done only at the year end.

SOLUTION:

Cost of machine, C = ₹15,000 + ₹ 3,500 = ₹18,500

Scrap value, S = ₹1,500.

Year	Maintenance Cost, M_i (₹)	Cumulative Maintenance Cost, ΣM_i (₹)	Cost of Machine - Scrap Value(₹)	Total Cost $T_{(n)}$ (₹)	Annual Cost $A_{(n)}$ (₹)
(i)	(ii)	(iii)	(iv)	(v)= (iii)+(iv)	(vi)=(v)/n
1	260	260	17,000	17,260	17,260
2	760	1,020	17,000	18,020	9,010
3	1,100	2,120	17,000	19,120	6,373
4	1,600	3,720	17,000	20,720	5,180
5	2,200	5,920	17,000	22,920	4,584
6	3,000	8,920	17,000	25,920	4,320
7	4,100	13,020	17,000	30,020	4,288*
8	4,900	17,920	17,000	34,920	4,365
9	6,100	24,020	17,000	41,020	4,557

Lowest average cost is ₹4,288 approx., which corresponds to n = 7 in above table. Thus machine needs to be replaced every 7th year.

Illustration 9.

A large computer installation contains 2,000 components of identical nature which are subject to failure as per probability distribution that follows:

Month End:	1	2	3	4	5
% Failure to date:	10	25	50	80	100

Components which fail have to be replaced for efficient functioning of the system. If they are replaced as and when failures occur, the cost of replacement per unit is ₹3. Alternatively, if all components are replaced in one lot at periodical intervals and individually replace only such failures as occur between group replacement, the cost of component replaced is ₹1.

(a) Assess which policy of replacement would be economical.
(b) If group replacement is economical at current costs, then assess at what cost of individual replacement would group replacement be uneconomical.
(c) How high can the cost per unit in-group replacement be to make a preference for individual replacement policy?

SOLUTION:

(a) **Computation of failures & Mean life**

Month (X)	Probability of Failure (P)	P X
1	0.10	0.10
2	0.15	0.30
3	0.25	0.75
4	0.30	1.20
5	0.20	1.00
		$\Sigma p_i x_i$ = 3.35 month

Average Life of a component = 3.35 Months

Average No. of Replacements = 2000/3.35 = 597 per month

Cost of Individual Replacement = 597 × ₹ 3 = ₹ 1791 per

Computation of expected No. of Replacements:

Month	Expected number of components to be replaced by the month end	
1	$N_1 = N_0P_1 = 2000 \times 0.1$	200
2	$N_2 = N_0P_2 + N_1P_1 = 2000 \times 0.15 + 200 \times 0.1$	320
3	$N_3 = N_0P_3 + N_1P_2 + N_2P_1 = 2000 \times 0.25 + 200 \times 0.15 + 320 \times 0.1$	562
4	$N_4 = N_0P_4 + N_1P_3 + N_2P_2 + N_3P_1 = 2000 \times 0.3 + 200 \times 0.25 + 320 \times 0.15 + 562 \times 0.1$	754.2
5	$N_5 = N_0P_5 + N_1P_4 + N_2P_3 + N_3P_2 + N_4P_1 = 2000 \times 0.2 + 200 \times 0.3 + 320 \times 0.25 + 562 \times 0.15 + 754.2 \times 0.1$	699.72

Computation of Average cost

Month (x)	Cumulative number of component Replace individually by month end	Cost		Total Cost (Tc)	Average Cost = Tc/n
		Individual	Group		
		₹	₹	₹	₹per month
1	200	600	2000	2600	2600
2	520	1560	2000	3560	1780
3	1082	3246	2000	5246	1748.67*
4	1836.2	5508.6	2000	7508.6	1877.15
5	2535.92	7607.76	2000	9607.76	1921.55

Since the average cost is lowest in 3rd month, the optimal interval i.e. replacement is 3 months. Also the average cost is less than ₹1791 of individual replacement, **the group replacement policy is better.**

(b) **Let 'K' be the cost of Individual Replacement**

Month	Average Cost of Group Replacement	Average cost of Individual Replacement	'K' Value* (₹)
1	(2000 + 200 K)/1	597 K	5.04
2	(2000 + 520 K)/2	597 K	2.97
3	(2000 + 1082 K)/3	597 K	2.82
4	(2000 + 1836.2 K)/4	597 K	3.62
5	(2000 + 2535.92 K)/5	597 K	4.45

* To obtai the value of K use the equation Average cost of Individual Replacement = Average

If group replacement is anything smaller than 2.82, then Group Replacement would be uneconomical.

(c) Let 'a' be the unit cost of Group Replacement Policy

Month	Average Cost of Group Replacement	Average of Individual Replacement	'a' Value (₹)
1	(2000 a + 600)/1	1791	0.60
2	(2000 a + 1560)/2	1791	1.01
3	(2000 a + 3246)/3	1791	1.06
4	(2000 a + 5508.6)/4	1791	0.83
5	(2000 a + 7607.76)/5	1791	0.67

When unit cost is more than ₹1.06 then Individual Replacement policy would be better.

Illustration 10

An electric company which generates and distributes electricity conducted a study on the life of poles. The repatriate life data are given in the following table:

Life data of electric poles

Year after installation:	1	2	3	4	5	6	7	8	9	10
Percentage poles failing:	1	2	3	5	7	12	20	30	16	4

- If the company now installs 5,000 poles and follows a policy of replacing poles only when they fail, how many poles are expected to be replaced each year during the next ten years? To simplify the computation, assume that failures occur and replacements are made only at the end of a year.
- If the cost of replacing individually is ₹160 per pole and if we have a common group replacement policy it costs ₹80 per pole, find out the optimal period for group replacement.

SOLUTION:

Chart showing Optimal Replacement Period

Average life of the pole - $1 \times 0.01 + 2 \times 0.02 + 3 \times 0.03 + 4 \times 0.05 + 5 \times 0.07 + 6 \times 0.12 + 7 \times 0.20 + 8 \times 0.3 + 9 \times 0.16 + 10 \times 0.04 = 7.05$ years.

No. of poles to be replaced every year $= \frac{5000}{7.05} = 709$

Average yearly cost on individual replacement = 709 × ₹160 = ₹1,13,440.

Group Replacement: Initial Cost = 5,000 × ₹80 = ₹4,00,000.

Year	No. of poles to be replaced	Yearly cost of individual replacement @ ₹ 160/pole (₹)	Cumulative cost of individual replacement (₹)	Total cost of individual replacement as well as group replacement (₹)	Average Annual Cost = TotalCost / Year (₹)
1	5,000 × 0.01 = 50	8,000	8,000	4,08,000	4,08,000
2	5,000 × 0.02 + 50 × .01 = 101	16,160	24,160	4,24,160	2,12,080
3	5,000 × 0.03 + 50 × 0.02 + 101 × 0.01 = 152	24,320	48,480	4,48,480	1,49,493

4	$5,000 \times 0.05 + 50 \times 0.03 + 101 \times 0.02 + 152 \times 0.01 = 256$	40,960	89,440	4,89,440	1,22,360
5	$5,000 \times 0.07 + 50 \times 0.05 + 101 \times 0.03 + 152 \times 0.02 + 256 \times 0.01 = 362$	57,920	1,47,360	5,47,360	1,09,472
6	$5,000 \times 1.2 + 50 \times 0.07 + 101 \times 0.05 + 152 \times 0.03 + 256 \times 0.02 + 362 \times 0.01 = 6023$	9,63,680	11,11,040	15,11,040	2,51,840

Optimal replacement at the end of the 5th year.

Illustration 11

Product A has a Mean Time Between Failures (MTBF) of 30 hours and has a Mean Time To Repairs (MTTR) of 5 hours. Product B has a MTBF of 40 hours and has a MTTR of 2 hours.

i Which product has the higher reliability?

ii Which product has greater maintainability?

iii Which product has greater availability?

SOLUTION:

i. Product B, with higher MTBF (i.e. 40 hours) than Product A (i.e. 30 hours), is more reliable since it has lesser chance of failure during servicing.

ii. By MTTR we mean the time taken to repair a machine and put it into operation. Thus Product B, with lesser MTTR (i.e., 2 hours) than Product A (i.e., 5 hours), has greater maintainability.

iii. Availability of a machine/product = $\frac{MTBF}{MTBF + MTTR}$
Therefore, Availability of Product A = 30/ (30+5) = 30/35 = 85.714% Availability of Product B = 40/(40+2) = 40/42 = 95.238%

Hence, Product B has more availability.

Illustration 12

Maharashtra Trucking Company (MTC) has a fleet of 50 trucks. The past data on the breakdown of the trucks show the following probability distribution (for a new truck as well as for one which has been repaired after a breakdown).

Months after Maintenance	Probability of Breakdown
1	0.10
2	0.20
3	0.30
4	0.40

Each breakdown costs ₹3,000 on an average; which includes cost of time lost and cost of materials and manpower.

The manager of MTC knows the importance of preventive maintenance. He estimates the costs of the preventive maintenance to be ₹500 per such preventive action. What should be the appropriate maintenance policy in terms of the mix of preventive and breakdown maintenance

SOLUTION:

First, let us compute the cost of a totally breakdown maintenance policy.

The expected number of months between failures

$= 0.1 (1) + 0.2 (2) + 0.3 (3) + 0.4 (4) = 3.0$

Cost per month of totally breakdown maintenance policy

$$= \frac{\text{(No.of trucks)(Cost per breakdown)}}{\text{(Expected number of months between failure)}}$$

$$= \frac{(50)(₹3000)}{(3.0)} = ₹5000$$

Now let us compute the costs of different periodicities of preventive maintenance.

(i) Preventive maintenance (PM) period one month

No. of breakdowns within the period of one month:

$B_1 = (50) \times (0.1) = 5$

Cost of breakdown = 5 × ₹3000 = ₹15,000

Cost of preventive maintenance = ₹500 × 50 = ₹ 25,000

Total Cost during the PM period = ₹ 40,000

Therefore, cost per month for this policy is

= 40,000 ÷ 1 = ₹40,000

(ii) Preventive maintenance (PM) period two months

No. of breakdowns within 2 months:

$B_2 = (50) \times (0.1 + 0.2) + (50) \times (0.1) \times (0.1) = 15.5$

Cost of breakdown = (15.5) × ₹3000 = ₹46,500

Cost of prev. maintenance = ₹500 × 50 = ₹25,000

Total cost during the PM period = ₹71,500

Therefore, cost per month for this policy:

₹71,500 ÷ 2 months = ₹35,750

(iii) Preventive maintenance period 3 months

No. of breakdowns within 3 months:

$B_3 = (50) \times (0.1 + 0.2 + 0.3) + (50 \times 0.1)(0.1 + 0.2) + (50 \times 0.1 \times 0.1)(0.1)$

$= 30 + 1.5 + 0.05 = 31.55$

Cost of breakdown = 31.55 × ₹3000 = ₹94,650

Cost of preventive maintenance = 50 × ₹500 = ₹25,000

Total = ₹1,19,650

Therefore, cost per month for this policy

= ₹1,19,650 ÷ 3 months = ₹39,883.33

(iv) Preventive maintenance period 4 months

No. of breakdowns within 4 months

$B_4 = [(50) \times (1.0)] + [(50) \times (0.1) \times (0.1 + 0.2 + 0.3) + (50 \times 0.1 \times 0.1) \times (0.1 + 0.2) + (50 \times 0.1 \times 0.1 \times 0.1) \times (0.1) + (50 \times 0.1 \times 0.2) \times (0.1)] + [(50 \times 0.2) \times (0.1 + 0.2) + (50 \times 0.2 \times 0.1) \times (0.1)] + [(50 \times 0.3 \times (0.1)]$

$= 57.855$

Cost of breakdown = (57.855) × (₹3,000) = ₹1,73,565

Cost of preventive maintenance = 50 × ₹500 = ₹ 25,000

Total = ₹1,98,565

Therefore, cost per month for this policy is ₹1,98,565 ÷ 4 months = ₹49,641.25

Comparing the costs per month of different policies, we see that the policy of preventive maintenance every two months is the most economic policy.