

TIME AND WORK

Concepts

1. If A can do a piece of work in 10 days, then in 1 day, A will do $\frac{1}{10}$ part of the total work.
2. If A is thrice as good as B, then
 - a) In a given amount of time, A will be able to do 3 times the work B does. Ratio of work done by A and B (in the same time) = 3: 1.
 - b) For the same amount of work, B will take thrice the time as much as A takes. Ratio of time taken by A and B (same work done) = 1: 3.
- 3) Efficiency is directly proportional to the work done and inversely proportional to the time taken.

Basic Questions

Q1. A does a work in 10 days and B does the same work in 15 days. In how many days will they do the same work together?

Solution:

A does a work in 10 days

$$A \text{ 's 1 day 's work} = \frac{1}{10}$$

B does a work in 15 days

$$B \text{ 's 1 day's work} = \frac{1}{15}$$

Adding equation 1 and 2, we get,

$$A \text{ and B 's 1 day 's work} = \frac{1}{10} + \frac{1}{15} = \frac{1}{6}$$

∴ A and B will together take 6 days to do the work.

Q2. A alone can do a job in 40 days. In how many days can B alone do the job, if together they can do the job in 8 days?

Solution:

$$\Rightarrow \text{Efficiency of A: Efficiency of A + B} = (1/40) : (1/8) = 1 : 5$$

$$\Rightarrow \text{Efficiency of B / Efficiency of A} = (5 - 1)/1 = 4/1 \text{ B is}$$

4 times efficient than A,

$$\Rightarrow \text{Number of days taken by B} = 1/4 \times \text{Number of days taken by A} = 40/4 = 10 \text{ days}$$

When three persons do a work

Q3. A and B can do a work in 3 days; B & C can do it in 4 days and A & C can do it in 6 days. In how many days will A, B & C finish it, if they work together?

Solution:

⇒ Work done by A & B in 1 day = $\frac{1}{3}$... (I)

⇒ Work done by B & C in 1 day = $\frac{1}{4}$... (II)

⇒ Work done by A & C in 1 day = $\frac{1}{6}$

... (III)

Adding I), II) & III)

⇒ $2 \times (\text{work done by A, B \& C in 1 day}) = \frac{1}{3} + \frac{1}{4} + \frac{1}{6}$

∴ Work done by A, B & C in 1 day together = $\frac{1}{2} (\frac{1}{3} + \frac{1}{4} + \frac{1}{6})$

= $\frac{1}{2} ((\frac{4}{4} + \frac{3}{4} + \frac{2}{4}) / 12) = \frac{3}{8}$

∴ No. of days taken to complete the job together = $\frac{8}{3}$ days

Q4. A and B can do a piece of work in 12 days, B and C in 15 days, C and A in 20 days. How long would A take separately to do the same work?

Solution:

One day work of A and B = $\frac{1}{12}$

One day work of B and C = $\frac{1}{15}$

One day work of C and A = $\frac{1}{20}$

One day work of (A + B), (B + C) and (C + A) = $\frac{1}{12} + \frac{1}{15} + \frac{1}{20}$

= $\frac{12}{60} = \frac{1}{5}$

One day work of (A + B + C) = $(\frac{1}{2}) \times (\frac{1}{5}) = \frac{1}{10}$

One day work of A = One day work of (A + B + C) - One day work of (B + C)

= $\frac{1}{10} - \frac{1}{15} = \frac{2}{60} = \frac{1}{30}$

∴ A takes 30 days alone to do the same work.

Questions Based on Efficiency

Q5. Anil is thrice as good a workman as Arun. Together they can do a job in 12 days. In how many days will Arun finish the work alone?

Solution:

Anil is thrice as good a workman as Arun

If Anil can finish a work in x days Arun can finish the same work in $3x$ days.

$\therefore$ 1 day's work by Anil = $1/x$

1 day's work by Arun = $1/3x$

Together they can do a job in 12 days

1 day's work by (Anil + Arun) = $\frac{1}{x} + \frac{1}{3x} = \frac{4}{3x}$

Work done in 15 days = $(4/3x) \times 12 = (16/x)$

If Arun has to finish that work alone, time taken by Arun to finish the work alone

$$= \frac{\frac{16}{x}}{\frac{1}{3x}} = 48 \text{ days}$$

Q6. P is twice as good as Q and together they finish a piece of work in 36 days. The number of days taken by P alone to finish the work?

Solution:

Given,

P is twice as good as Q.

$\Rightarrow$ (P's 1 day's work) / (Q's 1 day's work) = $2 / 1$

Given,

$\Rightarrow$ (P + Q)'s 1 day's work = $1/36$

$\Rightarrow$ P's 1 day's work = $(1/36) \times (2/3) = 1/54$

$\therefore$ P alone can finish work in 54 days.

Work and wages

Concepts

Ratio of wages of persons doing a work is directly proportional to the ratio of efficiency of the persons

Q7. A and B can complete a piece of work in 15 days and 10 days respectively.

They got a contract to complete the work for Rs. 75000. The share of B (in Rs.) in the contracted money will be:

Ratio of number of days taken by A and B to complete the work = $15: 10 = 3: 2$

$\therefore$ Ratio of efficiency of A and B = $2: 3$

Let their share is in the ratio of $2x$ and $3x$

Now,

$$2x + 3x = 75000$$

$$\Rightarrow 5x = 75000$$

$$\therefore x = 15000$$

$$\therefore \text{Share of B} = 3x = 15000 \times 3 = \text{RS. } 45000$$

When two or more persons do a work on alternate days or hours

Q8. A and B can complete a piece of work in 10 days and 15 days respectively when working alone. Starting with A, they work on alternate days. In how many days will the work be completed?

A. Given,

$$\Rightarrow \text{A's 1 day's work} = 1/10$$

$$\Rightarrow \text{B's 1 day's work} = 1/15$$

$$(\text{A} + \text{B})\text{'s 2 days' work} = 1/10 + 1/15 = 1/6$$

$$\text{To complete work (A + B) 12 days' work} = 6 \times 1/6 = 1$$

Total time taken by both if they work alternatively = 12 days.

MHDE/W based

Concept:

More men, less days : $M \propto \frac{1}{D}$

More men, less hours : $M \propto \frac{1}{H}$

$$\boxed{MDH = \text{Constant}}$$

More efficiency, less days: $E \propto \frac{1}{D}$

More men, more work : $M \propto W$

$$\boxed{\frac{M D H E}{W} = \text{Constant}}$$

Where M = Men, D = Days, H = Hours, E = Efficiency, W = Work

Q9. 15 men can complete a task in 8 days. Three days after they started the work, 3 more men joined. In how many days will all of them together complete the remaining work?

$$\text{Total work done by 15 men in 8 days} = 15 \times 8 = 120$$

$$\text{Work done by 15 men in 3 days} = 15 \times 3 = 45$$

$$\text{Remaining work after 3 days} = 120 - 45 = 75$$

Days to complete remaining work by 18 men = $75/18 = 25/6 = 4\frac{1}{6}$ Days

Food based

This type of questions based on quantity of food required for feeding given number of persons or number of days for which given quantity of food lasts

Q10. 1200 soldiers in a fort had enough food for 28 days. After 4 days, some soldiers left the fort. Thus, food lasted for 32 more days. How many soldiers left?

Solution:

Let the number of soldiers who left the fort be x . As we know,

$$M_1 \times D_1 = M_2 \times D_2 + M_3 \times D_3$$

Given, $M_1 = 1200$ soldiers, $D_1 = 28$ days, $M_2 = 1200$ soldiers, $D_2 = 4$ days, $M_3 = (1200 - x)$ soldiers and $D_3 = 32$

$$M_1 \times D_1 = M_2 \times D_2 + M_3 \times D_3$$

$$1200 \times 28 = 1200 \times 4 + (1200 - x) \times 32$$

$$(1200 \times 28) - (1200 \times 4) = (1200 - x) \times 32$$

$$1200 (28 - 4) = (1200 - x) \times 32$$

$$(1200 \times 24) / 32 = (1200 - x)$$

$$x = 300$$

Man, Woman and Boys

When men, women and Boys together or alone do work.

Q11. It takes the same time of 66 days for 12 men to finish the same work as it takes for 24 boys. If we assign this work to 20 men and 10 boys, in how many days will they be able to finish this work?

Solution:

Let us assume the total work to be W .

Let the efficiency of each man be M units/day, and that of each boy be B units/day.

$$\Rightarrow W = 12 \times M \times 66 = 24 \times B \times 66$$

$$\therefore 12 \times M \times 66 = 24 \times B \times 66,$$

$$(M/B) = 2$$

$$\Rightarrow M = 2B$$

The task is to get 20 men and 10 boys to do the same. Let the no. of days be D .

$$\Rightarrow W = (20M + 10B) \times D$$

Replacing all the variables in terms of B, we have 24

$$\times B \times 66 = (20 \times 2B + 10B) \times D \Rightarrow 24 \times B \times 66 = 50 \times B \times$$

D

$$D = 31\frac{17}{25} \text{ Days}$$

$\therefore$ Time taken by 20 men and 10 boys to finish the same work $= 31\frac{17}{25} \text{ Days}$

