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ACADEMY

QUANTITATIVE APTITUDE

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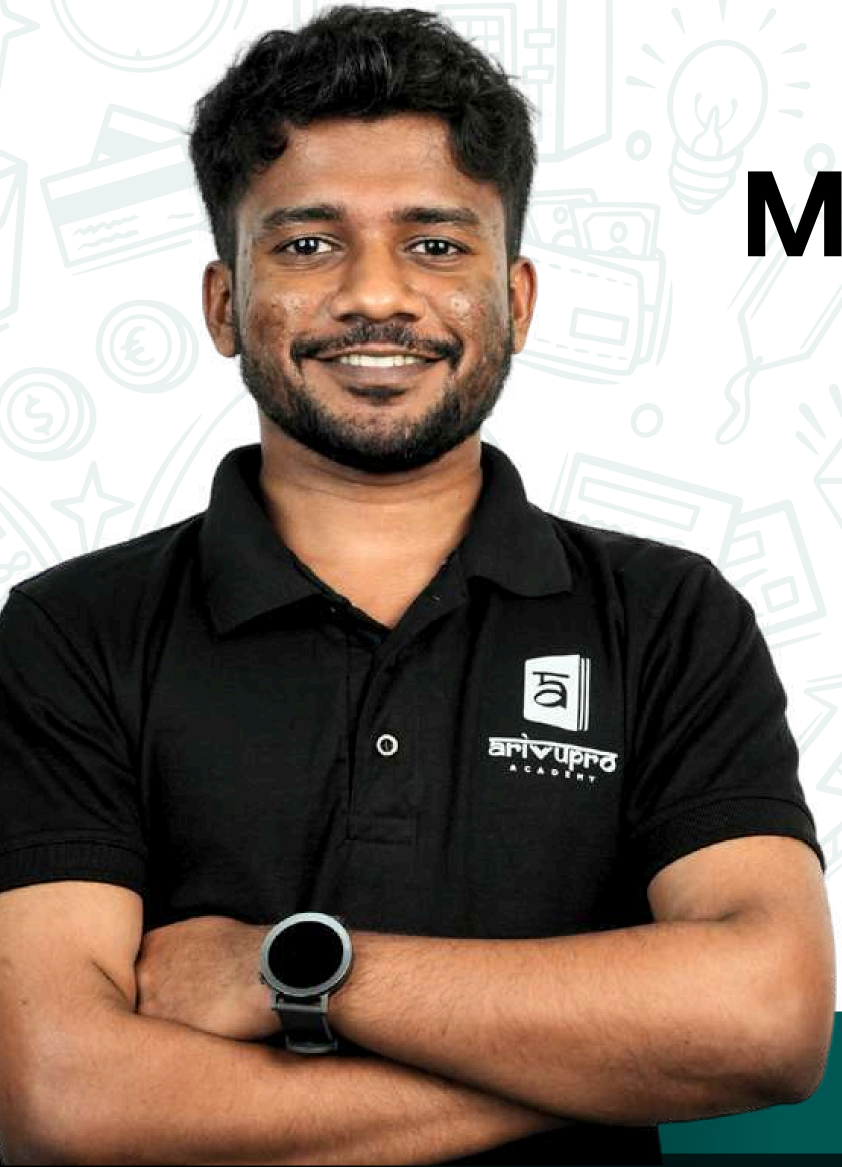
PAPER 3

MARATHON MATERIAL

Chartered Accountant



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CA FOUNDATION

Chapter 14 - Measures of Central Tendency and Dispersion

Central tendency refers to the statistical measure that identifies a single value as representative of an entire distribution. It aims to provide an accurate depiction of the entire data set.

AM
10, 20, 30, 60
10+20+30+60
÷ 4
30

Measures of Central Tendency: % 200%, 300%, 400%

Mean: The average of a data set, which is divided into:

- **Arithmetic Mean**: The sum of values divided by the number of values.
- **Geometric Mean**: The nth root of the product of the values.
- **Harmonic Mean**: The number of values divided by the sum of the reciprocals of the values. Km/s m/s Km/h Kg/Rs

Median: The middle value when data is ordered, dividing the distribution into two equal parts.

Mode: The most frequently occurring value in a data set.

Partition Values: → Median, Quartile, Decile, Percentile

Quartiles: Divide the data into four equal parts.
Deciles: Divide the data into ten equal parts.
Percentiles: Divide the data into a hundred equal parts.

M

Ideal Measure Criteria:

- Clearly defined.
- Easy to understand and compute.
- Inclusive of all data points.
- Minimally affected by outliers.
- Possesses desirable mathematical properties.

3, 5, 7, 8, 9

x	f
3	
5	
7	
8	
9	

Arithmetic Mean (AM) of ungrouped data:

Example: Find the Arithmetic Mean of the test scores for a small class of 5 students. The scores are 52, 96, 90, 88, and 79.

$\bar{x} = \frac{\sum x}{n}$

To calculate the AM, we would sum all the scores and then divide by the number of observations (which is the number of students in this case).

Formula is **AM =** $\frac{\Sigma x}{n}$, we get:

AM = $\frac{\Sigma x}{n}$ =

$= 52 + 96 + 90 + 88 + 79 \div 5$

81

45, 60, 95, 41

60.25

41, 42, 43, 79

Q) If the mean value of seven numbers 7, 9, 12, X, 4, 11, and 5 is 9, then the missing number X will be:

~~(a) 13~~

~~(b) 14~~

(c) 15

(d) 8

$\bar{x}$ = Sum of all observation / Total no: of obs

$9 = \frac{\Sigma x}{n}$

$9 = \frac{7 + 9 + 12 + x + 4 + 11 + 5}{7}$

$63 = 48 + x$

$63 \neq 61$

$63 \neq 62$

9×7 m+

Sanju

CSK RR

LHS

RHS

LHS = RHS

$9 \times 7 = 48 + x$

$x = 63 - 48 = 15$

$10 = 20 + 6 + x$

$10 - 20 - 6 = x$

Observations with equal spacing,

Example: Consider an evenly spaced set of numbers: 10, 15, 20, 25, 30. Since the spacing between each number is equal (5 in this case), the AM can be quickly found by averaging the smallest and largest numbers:

$$\text{AM} = \frac{(10 + 30)}{2} = 20$$

$\frac{\Sigma x}{n}$ $\frac{10 + 30}{2} = 20$

Theory and Generalising

For Observations with equal spacing, the arithmetic mean (AM) is simply the mean of the two extreme values.

AM of observations = AM of extreme values

This is because in a uniformly spaced series, the mean and the median coincide and both are equal to the average of the extreme values.
Consider a series of numbers: 12, 17, 22, 27, 32, 37. What is the arithmetic mean (AM) of this series?

$\frac{14+39}{2} = 26.5$

$\frac{12+37}{2} = 24.5$

24 34 44 54 64 74 = 49

Shift of origin + or −

Shift of Scale × or ÷

Transformation of the Arithmetic Mean by a Constant Factor

Generalising When a set of data is modified by applying a constant value k to each observation, the arithmetic mean (AM) is also transformed in a predictable way. The rule can be generalised as follows: 1. For Addition/Subtraction: - $New\ AM = Original\ AM \pm k$ 2. For Multiplication/Division: - $New\ AM = Original\ AM \times k$ or $New\ AM = Original\ AM \div k$ Where k is the constant value applied to each observation in the dataset.
If the AM of 10 data points is 45 and we multiply each observation by 3, what is the new AM of the data set?

$45 \times 3 = 135$

Combined Mean ($\overline{x}_c$)

Example: Two classes took the same exam. Class A, with 25 students, had an average score of 72. Class B, with 35 students, had an average score of 78. What is the combined average score for both classes?

$\Sigma x_1 = n_1\bar{x}_1$

$\Sigma x_2 = n_2\bar{x}_2$

Formula is $\overline{x}_c = \frac{n_1\overline{x}_1 + n_2\overline{x}_2 + n_3\overline{x}_3 + \ldots}{n_1 + n_2 + n_3 + \ldots}$

King Virat	Thala Dhoni	Sachin
$n_1 = 100$	$n_2 = 150$	n_3
$\bar{x}_1 = 200$	$\bar{x}_2 = 166.67$	$\bar{x}_3$

$\overline{x}_c$=Combined AM,

n_1, n_3 are Number of observations in the first and second group resp.

$\overline{x}_1, \overline{x}_2$,are AM of the first and second group

$\overline{x}_c = \frac{25\times72+35\times78}{25+35}$=?

$\bar{x}_c = 75.5$

Q) The mean of the first three terms is 14, and the mean of the next two terms is 18. The mean of all five terms is:

(a) 14.5

$n_1 = 3$, $\bar{x}_1$, $n_2 = 2$, $\bar{x}_2 = 18$

$\bar{x}_c = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} = 15.6$

$\bar{x}$

40 $n = 4$

42, and 36

wrongly taken as 48 and 29

Find correct mean

$$\frac{40 \times 4 - 48 - 29 + 42 + 36}{4}$$

40.25

(b) 15
(c) 14
(d) 15.6

Q) The mean weight of 15 students is 110 kg. The mean weight of 5 of them is 100 kg, and of another five students is 125 kg. Then the mean weight of the remaining students is:
(a) 120
(b) 105
(c) 115
(d) None of these

$\bar{x}_1$ n_1 $n_2=5$ $\bar{x}_2$

$\bar{x}_3 = ?$

$\bar{x}_c = \frac{n_1\bar{x}_1 + n_2\bar{x}_2 + n_3\bar{x}_3}{n_1+n_2+n_3} = \frac{5\times100 + 5\times125 + 5\times\bar{x}_3}{15}$

$110 = \frac{5\times100 + 5\times125 + 5\times\bar{x}_3}{15}$

110×15 M+ 5×100 M− 5×125 M− MRC ÷ 5

$\frac{110\times15 - 5\times100 - 5\times125}{5}$

Q) The average age of 15 students in a class is 9 years. Out of them, the average age of 5 students is 13 years, and that of 8 students is 5 years. What is the average of the remaining 2 students?
(a) 5 years
(b) 9 years
~~(c) 10 years~~
(d) 15 years

$\bar{x}_1 = 13 \quad n_1 = 5$

$\bar{x}_2 = 5 \quad n_2 = 8$

$\bar{x}_3 = ? \quad n_3 = 2$

$\bar{x}_c = 9 \quad n_1+n_2+n_3 = 15$

Q) The average salary of 50 men was ₹80, but it was found that the salary of 2 of them were ₹46 and ₹28, which was wrongly taken as ₹64 and ₹82. The revised average salary is:
(a) ₹80
(b) ₹78.56
(c) ₹85.26
(d) ₹82.92

correct average $= \frac{\text{wrong mean} \times n - \text{wrong values} + \text{correct value}}{n}$

$= \frac{80\times50 - 64 - 82 + 46 + 28}{50}$

$= 78.56$

Arithmetic Mean for Grouped Data

When calculating the arithmetic mean (AM) for grouped data, you can follow a straightforward process

Direct Method for Grouped Data:
The AM is found by dividing the sum of the product of each observation's value

3, 4, 5, 6, 7

x	f
3	1
4	1
5	1
6	1
7	1

$\Sigma f = 5$

$$\bar{x} = \frac{\Sigma f x}{\Sigma f}$$

$$= \frac{3+4+5+6+7}{5}$$

(usually the class midpoint) and its frequency by the total frequency.

Example:

Let's say we have the following data for a survey on hours spent on leisure activities per week, grouped by hours

x 2 5 8 11

Class intervals (CI) in hours	1-3	4-6	7-9	10-12
Frequency (f)	5	8	12	5

$\bar{x} = \frac{\sum fx}{\sum f}$ = MRC ÷ 30

To find the class midpoints (x), you would take the average of the upper and lower boundaries for each interval:

$$Midpoints\ of\ each\ interval\ = \frac{UCB+LCB}{2}$$

Midpoints	2	5	8	11
Frequency (f)	5	8	12	5

x	f
21	6
30	4
32	3
40	2

MRC ÷ 15 28.13

The AM is then calculated as:

$$\bar{x} = \frac{\sum f \times x}{N}$$

$$\bar{x} = \frac{(5\times 2)+(8\times 5)+(12\times 8)+(5\times 11)}{5+8+12+5} = 6.7$$

So, the AM for this set of grouped data is 6.7 hours.

Q)Calculate the average height of a class of students from the following data:

152 157 162 167 172 177

Height in cm.	150-154	155-159	160-164	165-169	170-174	175-179
No. of students	6	11	16	14	9	4

MRC ÷ 60

Σf = 163.75

Q) Find the mean of the following data:

Class Interval	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	9	13	6	4	6	2	3

(a) 23.7
(b) 35.7
(c) 39.7
(d) 43.7

15×9 M+ 25×13 M+ 35×6 M+ 45×4 M+ 55×6 M+ 65×2 M+ 75×3 M+

MRC ÷ 43 = 35.69

Q) The mean of the following data is 6. Find the value of 'P':

x	2	4	6	10	P+5
f	3	2	3	1	2

(a) 4
(b) 6
(c) 8
(d) 7

$\bar{x} = 6$ $\bar{x} = \frac{\sum fx}{\sum f}$

$6 = \frac{\sum fx}{11}$

$6\times 11 = 2\times 3 + 4\times 2 + 6\times 3 + 10\times 1 + 2P + 2\times 5$

6×11 M+ 2×3 M− 4×2 M− 6×3 M− 10×1 M− 2×5 M− MRC ÷ 2 = 7

Special Property of Arithmetic Mean:

Sum of deviations of a set of observations from their Arithmetic Mean (AM) is zero

2, 3, 4, 5 $\bar{x} = 3.5$

$2 - 3.5 = -1.5$

$3 - 3.5 = -0.5$

$$\sum(x_i - \bar{x}) = \sum[f_i(x_i - \bar{x})] = 0$$

0.5

1.5

Example:
Consider a small dataset of five values: 2, 4, 6, 8, 10.

$\sum x$

$\sum f$

$\sum x^2$

$\sum$ 150 + 160 + 170 + ...

2, 4, 5, 6

$\sum x^2 = 2^2 + 4^2 + 5^2 + 6^2$

In the given dataset of five values: 2, 4, 6, 8, 10, we'll demonstrate that the sum of the deviations from the arithmetic mean (AM) equals zero.

Step 1. Calculate the AM of the dataset:

AM = 6

Step 2. Calculate the deviation of each value from the AM (which is subtracting the AM from each value):

- Deviation for 2: 2−6=−4 +
- Deviation for 4: 4−6=−2 +
- Deviation for 6: 6−6=0 (since it's the AM) +
- Deviation for 8: 8−6=2 +
- Deviation for 10: 10−6=4 +

0

Step 3. Sum these deviations **(−4)+(−2)+0+2+4 = 0**

This is a fundamental property of the arithmetic mean which states that if you calculate the deviation of each observation from the mean (by subtracting the mean from each observation), and then sum all those deviations, the result will be zero. This holds true for both weighted and unweighted observations.

Geometric Mean (GM)

- The Geometric Mean (GM) is a measure of central tendency that is especially useful when dealing with products of variables or percentages and ratios
- Often used in the contexts of growth rates and finance.
- It is defined as the nth root of the product of n terms.
- Best measure of Central Tendency, for ascertaining the average rate of change over a period of time.

$GM=(x_1 \times x_2 \times \ldots)^{1/n}$

GM For Ungrouped data

3, 6, 12

Example: Find the GM of 3, 6 & 12

~~a) 7~~ b) 6 c) 5.5 d) 7.5

Geometric Mean, $G = \left(x_1 \times x_2 \times x_3 \times \ldots\ldots x_n\right)^{\frac{1}{n}}$

n *is the number of observation*

$G=(x_1 \times x_2 \times x \ldots)^{1/n}$

$GM=(3 \times 6 \times 12)^{1/3}$

Finding n^{th} *root other than square root is not possible in simple calculators. So students are advised to use* ***option hitting method*** *to solve the questions on GM.*

216

$\surd$ 12 times

$\times 1 \div 3$

+1 $x = (12 \text{ times})$

$(option)^n = \ Product\ of\ observations$

Here $n = 3$

Try option a)

Then $(option)^n$= $(7)^3 = 343$

$(GM)^n = x_1 \times x_2 \times \dots$

$Product\ of\ observations$= 3x6x12=216

$(option)^n \neq \ Product\ of\ observations$

$(GM)^3 = 216$

So Option a) is wrong

Try option b)

Then

$(option)^n$= $(6)^3 = 216$

$343 \neq 216$

$Product\ of\ observations$= 3x6x12=216

So $(option)^n = \ Product\ of\ observations$

So G=6

$n=4$

$10 \times 15 \times 20 \times 25$ ⌐ 12times −1 ×1÷4 +1

Q)Find GM of 10,15,20,25

a) 15.5487 b) 16.5487 c) 14.5487 d) 17.5487

16.5487^4

Q) The geometric mean of 3, 7, 11, 15, 24, 28, 30, 0 is: $x=, x= \dots$ 12 times

200%, 250%, 300%

200×250×300 ⌐ 12times −1 ×1÷3 +1 $(x = (12 \text{ times})$

(a) 6

(b) 0

(c) 9

(d) 12

GM for grouped data

Example: Suppose you have a dataset of test scores with the following scores and frequencies:

scores (x)	5	7	10
f	2	3	1

a)6.64 b) 7.4 c) 8.76 d) 5.67

Geometric Mean (GM) for discrete grouped data

$$G = \left(x_1^{f_1} \times x_2^{f_2} \times x_3^{f_3} \times \dots\dots\dots x_n^{f_n}\right)^{\frac{1}{N}}$$

$G = \left(x_1^{f_1} \times x_2^{f_2} \times x_3^{f_3} \times \dots\right)^{1/N}$

$$N = \Sigma f = 6$$

*Finding n^{th} root other than square root is not possible in simple calculators. So students are advised to use **option hitting method** to solve the questions on GM.*

$(Option)^N = x_1^{f_1} \times x_2^{f_2} \times x_3^{f_3} \times \ldots\ldots\ldots x_n^{f_n}$

Try option a)

Then

$(option)^N = (6.64)^6 = 85705.46$

$x_1^{f_1} \times x_2^{f_2} \times x_3^{f_3} = 5^2 \times 7^3 \times 10^1 = 85750$

Both are approximately equal

So Option A is the correct answer

$N = \Sigma f$

Suppose you have a dataset of test scores with the following scores and frequencies:

scores (x)	4	5	6	7
f	2	3	1	5

a) 4.7 b)5.7 c)6.7 d)7.7

$GM = (4^2 \times 5^3 \times 6^1 \times 7^5)^{1/11}$

Combined Geometric mean

If x and y are two variables, the GM of their product xy is equal to the product of their individual geometric means

$$GM(xy) = GM(x) \times GM(y)$$

This means that if you multiply two sets of numbers together and then calculate the GM of the resulting set, you would get the same result as if you calculated the GMs of the two original sets separately and then multiplied them.

- Similarly, the GM of the quotient $\frac{x}{y}$ is equal to the quotient of their individual geometric means:

$$GM\left(\frac{x}{y}\right) = \frac{GM(x)}{GM(y)}$$

$G(ab) = GM(a) \times GM(b)$

Harmonic mean(HM)

- The Harmonic Mean (HM) is particularly useful when dealing with rates or ratios.
- It is defined as the reciprocal of the arithmetic mean (AM) of the reciprocals of a set of numbers.

Harmonic mean(HM) For Ungrouped data

$$HM = \frac{n}{\Sigma\frac{1}{x}}$$

$= \frac{4}{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}}$

2, 3, 4, 5

Reciprocals

$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$

$\frac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}}{4}$

$\frac{4}{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}}$

$\frac{2ab}{a+b}$ = avg speed

a, b

$\frac{2}{\frac{1}{a}+\frac{1}{b}}$ $\frac{2}{\frac{a+b}{ab}}$ $\frac{2ab}{a+b}$

Q) The harmonic mean of 1, 1/2, 1/3, ..., 1/n is:

(a) 1/(n + 1)
(b) 2/(n + 1)
(c) (n + 1)/2
(d) 1/(n − 1)

$\frac{n}{\Sigma\frac{1}{x}} = \frac{n}{1+2+3+\ldots\ldots+n}$

$= \frac{\cancel{n}}{\frac{\cancel{n}(n+1)}{2}} = \frac{2}{n+1}$

$\frac{1}{\frac{1}{2}}$ $1\div\frac{1}{2}$ 1×2

1, 2, 3, 4....n

$\frac{n+1}{2} = \frac{2}{n+1}$

Q) A man travels from Agra to Gwalior at an average speed of 30 km per hour and back at an average speed of 60 km per hour. What is his average speed?

(a) 38 km per hour
(b) 40 km per hour
(c) 45 km per hour
(d) 35 km per hour

$\frac{2\times 30\times 60}{90}$

Q) A lady travels at a speed of 20 km/h and returns at a quicker speed. If her average speed of the whole journey is 24 km/h, find the speed of the return journey (in km/h):

a, b

(a) 25
(b) 30
(c) 35
(d) 38

HM = 24

$HM = \frac{2ab}{a+b}$

$24 = \frac{2\times 20\times b}{20+b}$

$480 + 24b = 40b$

$\frac{480}{16} = b$ $b = 30$

Harmonic mean(HM) For Grouped data

$$HM = \frac{N}{\Sigma\frac{f}{x}}$$

$HM = \frac{N}{\Sigma\frac{f}{x}}$

Q: Imagine you have a dataset representing the speeds of a car at different times, and you want to calculate the HM of the speeds:

Speeds (km/h)	40	50	60
frequencies	1	2	3

$$HM = \frac{N}{\sum \frac{f}{x}} = \frac{6}{\frac{1}{40} + \frac{2}{50} + \frac{3}{60}}$$

1 ÷ 40 M+
2 ÷ 50 M+
3 ÷ 60 M+
MRC
÷ =
× 6
= 52.17

$\frac{6}{4+6}$

$\frac{6}{10}$

Combined HM

$$Combined\ HM = \frac{n_1 + n_2 + n_3 + n_4 \ldots}{\frac{n_1}{H_1} + \frac{n_2}{H_2} + \frac{n_3}{H_3} + \ldots}$$

If there are two groups with 75 and 65 as harmonic means and containing 15 and 13 observations then the combined HM is given by
a) 65. b) 70.36 c) 70 d) 71

$$HM_c = \frac{m_1 + m_2}{\frac{m_1}{H_1} + \frac{m_2}{H_2}} = \frac{15 + 13}{\frac{15}{75} + \frac{13}{65}} = 70$$

$\frac{54 \times 5}{36 \times 3 + 5 \times 4}$

36×3 M+
5×4 M+ MRC
÷ =
× 54 × 5

Relationship between AM, GM and HM

- If the observations **are positive and equal,** then all three means are equal, i.e.,
  AM = GM = HM.

2, 2, 2

$AM = \frac{2+2+2}{3} = 2$ | $GM = (2 \times 2 \times 2)^{1/3} = (2^3)^{1/3} = 2$

A, B, C P

A > G > H

- If the observations **are positive and distinct**
  AM > GM > HM
- Additionally, for any two observations a and b, the GM can be found as the square root of the product of AM and HM, which is:

$$GM = \sqrt{AM \times HM}$$

$G = \sqrt{AH}$

This relationship is useful in various statistical and financial analyses, particularly when dealing with rates of return or growth rates over time.

distinct

Q) If A be the A.M. of two positive unequal quantities X and Y, and G be their G.M., then:
(a) A < G
(b) A > G
(c) A ≤ G
(d) A ≥ G

$\boxed{A > G} > H$

Q) The harmonic mean H of two numbers is 4, and their arithmetic mean A and the geometric mean G satisfy the equation $2A + G^2 = 27$. Then the numbers are:
~~(a) (1, 3)~~
~~(b) (9, 5)~~
(c) (6, 3)
(d) (12, 7)

$HM = \frac{2ab}{a+b}$

$4 = \frac{2ab}{a+b}$

$\frac{2 \times 6 \times 3}{9} = 4$

LHS = RHS

Q) A person purchases 5 rupees worth of eggs from 10 different markets. You are to find the average number of eggs per rupee purchased from all the markets taken together. The suitable average in this case is:
(a) A.M.
(b) G.M.
(c) H.M
(d) None of the above.

Q) If the A.M. and G.M. of two numbers are 30 and 24 respectively, find the numbers:
~~(a) 12 and 24~~
(b) 48 and 12
~~(c) 30 and 30~~
~~(d) 40 and 20~~

$\frac{a+b}{2} = 30$

$GM = \sqrt{a \times b}$
$= \sqrt{48 \times 12}$
$= 24$

Q) The A.M. and H.M. of two numbers are 5 and 3.2 respectively. Then G.M. will be:
(a) 4.4
~~(b) 4.2~~
(c) 4.0
(d) 3.8

$GM = \sqrt{AH}$
$= \sqrt{5 \times 3.2}$
$= \sqrt{16} = 4$

Median (Positional average) **Definition**: The median is the value that lies in the middle of a dataset when it is ordered in either ascending or descending sequence. **Significance**: It is considered a positional average because its value is determined by the position within the ordered list, not by the actual magnitude of the values. **Usage**: Median is particularly useful in datasets with outliers or non-symmetric distributions, as it accurately reflects the center of the dataset without being affected by extreme values. Like the arithmetic mean (AM), the median is also rigidly defined, which means it has a specific calculation method and is not influenced by the actual values of the data, just their position.
Partitional Values **Definition**: Partitional values are specific data points that divide the dataset into equal parts. Q_1, Q_2, Q_3 $Q_2 \rightarrow M$ **Types:** **Quartiles**: Divide the data into four equal parts. The second quartile is the median of the data. $D_1, D_2, D_3, D_4 \ldots D_9$ **Deciles**: Divide the data into ten equal parts. $P_1, P_2, P_3 \ldots P_{99}$ **Percentiles**: Divide the data into one hundred equal parts, with each percentile representing 1% of the dataset. **Usage**: These are used for understanding the distribution of data, such as determining the spread or variability, and for comparing different sets of data. They are also vital in the construction of box plots in exploratory data analysis.

Ungrouped Data

1. Increasing order

$M = \left[\frac{1}{2}(n+1)\right]^{th} Value$ *n is number of observations*	2, 3, 4 3, 2, 4 2, 3, 4 n = 3 $\left(\frac{1}{2}(n+1)\right)^{th}$ value $= \frac{4}{2}^{th}$ value $= 2^{nd}$ value
Quartiles	1, 2, 3, 4 5, 6, 7, 8, 9, 10

2, 3, 5, 9, 8, 6, 4, 3, 2, 1
1, 2, 2, 3, 3, 4, 5, 6, 8, 9
$\left(\frac{1}{2}(n+1)\right)^{th}$ value
$\left(\frac{1}{2}(11)\right)^{th}$ value
5.5^{th} value
5^{th} value + 0.5 × gap
3 + 0.5 × 1

$= 3.5$

$Q_3 = \left(\frac{3}{4}(n+1)\right)^{th}$ value

	$Q_i = \left[\frac{i}{4}(n+1)\right]^{th}$ *Value* $\quad i = 1, 2, 3$ *n is number of observations* Q_1, *Lower Quartile or First Quartile* , $i = 1$ Q_3, *Upper Quartile, third Quartile*, $i = 3$ Q_2, *Median or second Quartile*, $i = 2$ *Median* $M = Q_2 = \frac{Q_1 + Q_3}{2}$
Deciles	$D_i = \left[\frac{i}{10}(n+1)\right]^{th}$ *Value* $\quad i = 1, 2, 3, 4.... 9$ *n is number of observations* $D_7 = \left(\frac{7}{10}(n+1)\right)^{th}$ value
Percentiles	$P_i = \left[\frac{i}{100}(n+1)\right]^{th}$ *Value* $\quad i = 1, 2, 3, 4..... 98, 99$ *n is number of observations* $P_{72} = \left(\frac{72}{100}(n+1)\right)^{th}$ value

Q) Determine the median, Q1, D4, P76 of the following set of numbers:
12, 37, 24, 45, 29, 11, 33.

$n = 7$

11, 12, 24, 29, 33, 37, 45

$Q_1 = \left(\frac{1}{4}(n+1)\right)^{th}$ value

$= \left(\frac{8}{4}\right)^{th}$ value $= 2^{nd}$ value

$= 12$

$D_4 = \left(\frac{4}{10}(8)\right)^{th}$ value $= 3.2^{th}$ value

$= 3^{rd}$ value $+ 0.2 \times$ gap

$= 24 + 0.2 \times 5$

$= 25$

$P_{76} = \left(\frac{76}{100} \times 8\right)^{th}$ value

$= (6.08)^{th}$ value

$= 6^{th}$ value $+ 0.08 \times$ gap

$= 37 + 0.08 \times 8$

$= 37.64$

Q) In a class of 11 students, 3 students failed a test. 8 students who passed secured 10, 11, 20, 15, 12, 14, 26, and 24 marks respectively. What will be the median marks of the students?
(a) 12
(b) 15
(c) 13
(d) 13.5

$F_1, F_2, F_3, 10, 11, 12, 14, 15, 20, 24, 26$

$\left(\frac{1}{2}(n+1)\right)^{th}$ value $= 6^{th}$ value

Q) The median of the following numbers, which are given in ascending order, is 25. Find the value of X:

11, 13, 15, 19, $\boxed{(X + 2), (X + 4)}$, 30, 35, 39, 46

(a) 22
(b) 20
(c) 15
(d) 30

$\frac{x+2+x+4}{2} = 25$

Q) 50th Percentile is equal to
(a) Median
(b) Mode
(c) Mean
(d) None

$P_{50} = \left(\frac{50}{100}(n+1)\right)^{th}$ value

$= \left(\frac{1}{2}(n+1)\right)^{th}$ value

$= M$

9, 10, 11, 12, 15, 18, 20, 25

Q) The 3rd decile for the numbers 15, 10, 20, 25, 18, 11, 9, 12 is
(a) 13
(b) 10.70
(c) 11.00
(d) 11.50

$D_3 = \left(\frac{3}{10}(n+1)\right)^{th}$ value

$= \left(\frac{3}{10}(9)\right)^{th}$ value

$= 2.7^{th}$ value

$= 2^{nd}$ value $+ 0.7 \times$ gap

$= 10 + 0.7 \times 1 = 10.7$

Discrete Grouped	
Quartiles	$Q_i = \left[\frac{i}{4}(N+1)\right]^{th}$ Value $\quad i = 1, 2, 3$ $N = \Sigma f$, f is the corresponding frequency
Deciles	$D_i = \left[\frac{i}{10}(N+1)\right]^{th}$ Value $\quad i = 1, 2, 3, 4.... 9$ $N = \Sigma f$, f is the corresponding frequency
Percentiles	$P_i = \left[\frac{i}{100}(N+1)\right]^{th}$ Value $\quad i = 1, 2, 3, 4..... 98, 99$ $N = \Sigma f$, f is the corresponding frequency

Find P_{45}

Q_1 P_{45}

x	3	4	5	9	10
f	3	5	11	5	2
CF	3	8	19	24	26 N

$Q_1 = \frac{1}{4}(27)$

$P_{45} = \left(\frac{45}{100} \times 27\right)^{th}$ value

$= 12.15$

$D_6 = \frac{6}{10} \times 27$

Continuous grouped data

Median	$$M = l_1 + \left(\frac{\frac{N}{2} - cf}{f}\right) \times c$$ $M = LCB + \left(\frac{\frac{N}{2} - cf_{pre}}{f_{median}}\right) \times c$
Quartiles	$Q_i = LCB + \left(\frac{\frac{iN}{4} - cf_{pre}}{f_{position}}\right) c$

	Percentile class	Decile class		Q_3 class
CI	20-30	30-40	40-50	50-60	60-70
F	2	4	5	3	10
CF	2	6	11	14	24

Find D_4, P_{21}, Q_3

$$\frac{iN}{10} = \frac{4 \times 24}{10} = 9.6$$

$$D_4 = LCB + \left(\frac{\frac{4N}{10} - C_{f\,pre}}{f_D}\right) c$$

$$= 40 + \left(\frac{9.6 - 6}{5}\right) \times 10$$

$$= 47.2$$

$$\frac{21 \times 24}{100} = 5.04$$

29.5 – 39.5

	0 – 9	10 – 19	20 – 29	30 – 39	40 – 49
f	4	5	6	7	5
	4	9	15	22	27

Q_3

$$\frac{iN}{4} = \frac{3 \times 27}{4} = 20.25$$

$$Q_3 = LCB + \left(\frac{\frac{3N}{4} - Cf_{pre}}{f_Q} \right) c$$

$$= 29.5 + \left(\frac{20.25 - 15}{7} \right) \times 10$$

$$= 37$$

Deciles	
Percentiles	$LCB + \left(\frac{\frac{iN}{100} - Cf_{pre}}{f_{per}}\right) c$

Q) Find the median of the following:

median class

C.I.	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
f	2	3	4	5	6
CF	2	5	9	14	20

(a) 35
(b) 32
(c) 36
(d) 37.5

1. Find $\frac{N}{2}$ $\qquad \frac{20}{2} = 10$

2. Find the median class

3. $M = LCB + \left(\frac{\frac{N}{2} - Cf_{pre}}{f_{med}}\right) c = 30 + \left(\frac{10-9}{5}\right) \times 10$

$= \underline{\underline{32}}$

Mode (Z)

Mode (Z) Fashionable Average

Narration with an example
Imagine a fashion retailer tracking the number of each size of jeans sold over a month. The sales data is as follows:

- Size 28: 10 sales
- Size 30: 25 sales
- Size 32: 40 sales
- Size 34: 15 sales
- Size 36: 5 sales

In this dataset, size 32 jeans have the **highest frequency** of sales, with 40 pairs sold. Therefore, the mode of the jean sizes sold is size 32.
This indicates that the most popular, or fashionable, size among customers is size 32

— hence the term "**Fashionable Average.**"

The concept of "Mode" refers to the value that appears most frequently in a dataset. To put it another way, it's the number or category that occurs the most in your data collection, which can indicate the most common outcome or preference among the measured items.
It's also possible for a dataset to have more than one mode. If another size had also been sold 40 times, then the dataset would be **bimodal**, indicating two modes. If there are more than two modes, it is referred to as **multimodal**.

Mode for Discrete Distribution

Question 1	Find mode for the observations 1,3,3,4,6,7 3
Question 2	Find mode for the observations 1,3,3,4,4,6 3,4
Question 3	Find mode for the observations 1,3,4,5,6,7 No mode

Mode for discrete grouped Distribution

Question 1
Imagine a fashion retailer tracking the number of each size of jeans sold over a month. The sales data is as follows:

Size	S	M	L	XL	XXL
No of sales	5	8	12	6	3

Find **mode**

L

Mode for continuous grouped Distribution

Mode for continuous grouped Distribution

Narration with an example

Example: Find the mode of the distribution

Weight(kg)	**60-62**	**63-65**	**66-68**	**69-71**	**72-74**
f	**14**	**117**	**132**	**123**	**19**

Step 1:
If the class intervals are **inclusive then we shall convert them into exclusive classes** by setting up new boundaries

Weight(kg)	**59.5-62.5**	**62.5-65.5**	**65.5-68.5**	**68.5-71.5**	**71.5-74.5**
f	**14**	**117**	**132**	**123**	**19**

Step 2: Finding the **modal class;** Class with the highest frequency

Here Modal Class is 65.5-68.5

$Z = LCB + \left(\frac{f_m - f_{pre}}{2f_m - f_p - f_{next}}\right) C$

Step 3: Apply the below formula

$$Z = l_1 + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times C$$

$= 65.5 + \left(\frac{132 - 117}{2 \times 132 - 117 - 123}\right) 3$

$= 67.375$

- l_1 is the lower class boundary (LCB) of the modal class,
- f_1 is the frequency of the modal class (the class with the highest frequency),
- f_0 is the frequency of the class preceding the modal class (pre modal class),
- f_2 is the frequency of the class following the modal class (post modal class).
- C is the length of the class interval (modal class length),

Here

$l_1 = 65.5 \qquad f_1 = 132, \qquad f_0 = 117$

$f_2 = 19 \quad C = 68.5 - 65.5 = 3$

$Z = 65.5 + \left(\frac{132-117}{264-117-19}\right) \times 3 =$

Note: The Final Answer should fall within the Modal Class.

Q) Find the mode of the following data: modal class

Class Interval	3 – 6	6 – 9	9 – 12	12 – 15	15 – 18	18 – 21
Frequency	2	5	10	23	21	12

(a) 25
(b) 4.6
(c) 14.6
(d) 13.5

$Z = LCB + \left(\frac{f_m - f_{pre}}{2f_m - f_p - f_n}\right) c = 12 + \left(\frac{13}{2\times23-10-21}\right)3 = 14.5$

Relationship between mean, Median & mode

The relationship between the mean, median, and mode for different types of distributions can be understood with examples:

Symmetrical Distribution: **Mean= Median= Mode**
For Positively Skewed (Skewed Right) Distribution: **mean > median > mode.**
For Negatively Skewed (Skewed Left) Distribution: **mode > median> mean**
For Moderately Skewed Distribution: The empirical rule known as **Karl Pearson's coefficient of skewness** states that for a moderately skewed distribution, the difference between the mean and mode can be related to the mean and median through the formula: **Mean-mode = 3(mean-Median) Or ~~Mode = 3median - 2Mean~~**

Q) If the difference between the mean and mode is 63, then the difference between the mean and median will be ________.
(a) 63
(b) 31.5
(c) 21
(d) None of the above

63 = 3(Mean − Median)

21.

Q) If the mode of a data is 18 and the mean is 24 then the median is:
(a) 18
(b) 24
(c) 22
(d) 21

Mean - Mode = 3 (Mean - Median)
24 - 18 = 3 (24 - Median)
$\frac{6}{3}$ = 24 - Median
Medi = 22

Q) The pair of averages whose value can be determined graphically?
(a) Mode, Median
(b) Mean, Mode
(c) Mean, Median
(d) None of the above

Mode → Histogram
Median → Ogive

Q) The ordering of a particular design of a cloth showroom, a _______ size, would be more appropriate.
(a) Median
(b) Mean
(c) Mode
(d) All of these

Q) In a moderately skewed distribution, the values of mean & median are 12 & 8 respectively. The value of mode is:
(a) 0
(b) 12
(c) 15
(d) 30

Mean - Mode = 3 (mean - Median)
12 - Mode = 3 (4)
Mode = 0

Q) Which of the following is a positional average?
(a) Median
(b) G.M.
(c) H.M.
(d) A.M.

Q) Along a road, there are 5 buildings of apartments, marked as 1, 2, 3, 4, 5. The number of people residing in each building is available. A bus stop is to be set up near one of the buildings so that the total distance walked by the residents to the bus stop from their buildings must be kept to a minimum. One must consider involving _______ to

1 2 3 4 5

find the position of the bus stop.

(a) Mean
(b) Median
(c) Mode
(d) Weighted mean

Q) Which one of these is least affected by extreme values? 2, 2, 3, 3, 4, 5, 7
(a) Mean
(b) Median
(c) Mode
(d) None

Effect of Extreme Values on Mean, Median, and Mode

- **Arithmetic Mean (AM)**:
  - **Affected** by extreme values and sampling fluctuations.
  - The sum of deviations from the mean is always **zero**: $\sum(x-\bar{x})=0$
- **Median**:
  - **Not affected** by extreme values or sample fluctuations.
  - The sum of absolute deviations is **minimum** when taken from the median: $\sum|x-M|$ is minimum
- **Mode**:
  - **May or ~~may not be affected~~** by extreme values.
  - Mode represents the **most frequently occurring value** in a dataset.
- The **mean** is useful when data is symmetrically distributed, but it is sensitive to outliers.
- The **median** is a better measure of central tendency for skewed distributions.
- The **mode** is useful for categorical data and identifying the most common value in a dataset.

Changes in Origin & Scale

Changes in Origin & Scale
Narration with an example
If x & y are 2 variables related as ax + by + c = 0 & a Central tendency of x is given. Then how will you find the central tendency of y.

Example: If x & y are related as $3x - 4y - 10 = 0$ & $\bar{x}$ is 15. Find AM of y

Step 1 *Solve y*

$$3x - 4y - 10 = 0$$
$$3x - 4y = 10$$
$$y = \frac{3x-10}{4}$$

Step 2 *Replace x by $\bar{x}$ and y by $\bar{y}$* & Solve $\bar{y}$

So $\bar{y} = \frac{3\times15-10}{4} = \frac{35}{4} = 8.75$

Q) If two variables 'x' and 'y' are related as $2x - y = 3$, if the median of 'x' is 10, what is the median of 'y'?

(a) 4
(b) 17
(c) 5
(d) 6

$2x - y = 3$
$2\times10 - y = 3$
$20 - y = 3$
$y = 17$

Q) The relationship between two variables x and y is given by $4x - 10y = 20$. If the median value of the variable x is 10, then what is the median value of variable y?

(a) 1.0
(b) 2.0
(c) 3.0
(d) 4.0

$4x - 10y = 20$
$4\times10 - 10\times y = 20$
$y = 2$

Q) If $y = 3 + 1.9x$, and mode of x is 15 then the mode of y is:

(a) 15.9
(b) 27.8
(c) 35.7
(d) 31.5

$y = 3 + 1.9\times15$
$= 31.5$

Mathematical Properties of AM, GM, and HM

Weighted Arithmetic Mean (Weighted AM)

The **Weighted AM** is calculated as:

$$\text{Weighted AM} = \frac{\sum w \cdot x}{\sum w}$$

$\frac{\Sigma fx}{\Sigma f}$

$\frac{\Sigma wx}{\Sigma w}$

where w **represents the weight** assigned to each value x.

Weighted Harmonic Mean (Weighted HM)

The **Weighted HM** is given by:

$$\text{Weighted HM} = \frac{\sum w}{\sum \left(\frac{w}{x}\right)}$$

This is useful in scenarios where rates and ratios are involved.

Weighted Geometric Mean (Weighted GM)

The **Weighted GM** is calculated as:

$$\text{Weighted GM} = \text{Antilog}\left(\frac{\sum w \cdot \log x}{\sum w}\right)$$

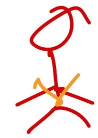
This is useful for **percentage growth rates and multiplicative processes.**

Q) When each value does not have equal importance, then

(a) A M
(b) G M
(c) H M
(d) Weighted Average

Dispersion

Dispersion:	• Dispersion in statistics refers to the extent to which a set of values is spread out or clustered together. • It is a measure of how the data points in a dataset are



5	7	7		8
6	6	8	8	
7	9	9	8	
6	7.3	8	8	

	distributed around the central tendency (mean, median, or mode). • Central tendency measures are said to be first order measures , whereas Dispersion measures are said to be 2nd order measures
Purpose of Dispersion:	• Dispersion helps to understand the variability or consistency within a dataset. • It can reveal whether the data points are generally close to the mean or if they vary widely, which could significantly impact interpretations and decisions based on the data.
Measures of Dispersion	• In statistics, measures of dispersion are categorized into two main types: **absolute measures** and **relative measures.**

Absolute Measures	**Relative Measures** %
• Absolute measures of dispersion are expressed in the same units as the data. • They provide an actual value of spread without context to the mean or any other related measure.	• Relative measures, also known as measures of relative variation, are unitless. • They provide a measure of spread in relation to the size of the mean or another average value.
Examples 1) Range 2) Mean Deviation (MD) 3) Standard deviation (SD) 4) Quartile Deviation (QD)	**Examples** 1) Coefficient of Range. 2) Coefficient of MD 3) Coefficient of variation (cv) 4) Coefficient of QD

Range & Coefficient of Range

Range Narration with an example	**Example:** Let's say you have a dataset representing the ages of a group of people: 23 , 29 , 31 , 44 , 52 , 58 To find the range: **Step 1** Identify the **largest observation (L)**, which is 58 in this case. **Step 2** Identify the **smallest observation (S)**, which is 23 here.

	Step 3 Apply the Range formula: $$R = L - S$$ $$R = 58 - 23 = 35$$

Q) If the range of a set of values is 65 and the maximum value in the set is 83, then the minimum value in the set is:
(a) 74
(b) 9
(c) 18
(d) None of the above

L

R= L - S

65= 83 - S

Theory & Generalisation	The range is a measure of dispersion that indicates the spread between the largest and smallest observation in a dataset. For both grouped and ungrouped data set. It's calculated by subtracting the smallest value (S) from the largest value (L) in the data set. $$R = L - S$$ L is the largest Observation S is the smallest Observation
Coefficient of Range	**Example:** Let's consider the same dataset representing the ages of a group of people: 23 , 29 , 31 , 44 , 52 , 58 To find the coefficient of range: **Step 1** Identify the **largest observation (L)**, which is 58 in this case. **Step 2** Identify the **smallest observation (S)**, which is 23 here. **Step 3** Apply the Coefficient of Range formula: $$Coefficient\ of\ Range = \boxed{\left(\frac{L-S}{L+S}\right) \times 100}$$ $$Coefficient\ of\ Range = \left(\frac{58-23}{58+23}\right) \times 100 = 43.2$$

Q) If L_1 = highest observation and L_2 = smallest observation, then Coefficient of Range =
(a) $\frac{L_1 \times L_2}{L_1 / L_2} \times 100$
(b) $\frac{L_1 - L_2}{L_1 + L_2} \times 100$
(c) $\frac{L_1 + L_2}{L_1 - L_2} \times 100$
(d) $\frac{L_1 / L_2}{L_1 \times L_2} \times 100$

$x^2 \to 10 \to 20 \to 30$ $30 - 10 = 20$

5, 10, 15   10

6   11   16   10

Q) The marks secured by 5 students in a subject are 82, 73, 69, 84, 66. What is the coefficient of Range? (a) 0.12 (b) 12 (c) 120 (d) 0.012 $\left(\frac{L-S}{L+S}\right) \times 100 = \left(\frac{84-66}{84+66}\right) \times 100 = 12$

Mean Deviation (MD) & Coefficient of Mean Deviation

Mean Deviation (MD)	• The concept of Mean Deviation (MD) is a statistical measure that captures the average distance between each data point in a set and a central point of the dataset. • The central point can be the mean or the median of the dataset. • This measure is also referred to as the average deviation. • The Mean Deviation about the mean tends to be larger than the Mean Deviation about the median.
Mean Deviation (MD) For ungrouped data:	**Mean Deviation about the mean** is calculated by taking the sum of the absolute differences between each data point (x) and the mean ($\bar{x}$) and then dividing by the number of observations (n). $MD\ about\ mean\ = \frac{\Sigma \lvert x - \bar{x} \rvert}{n}$ $\frac{\Sigma \lvert x - \bar{x} \rvert}{n}$ **Mean Deviation about the median** is similar, but uses the median (M) instead of the mean. This can sometimes give a better representation of MD, especially for **skewed distributions**, as the median is less affected by extreme values. $MD\ about\ median\ = \frac{\Sigma \lvert x - M \rvert}{n}$
Mean Deviation (MD) For grouped (both for discrete & Continuous)	$MD\ about\ mean\ = \frac{\Sigma f. \lvert x - \bar{x} \rvert}{n}$ $MD\ about\ median\ = \frac{\Sigma f. \lvert x - M \rvert}{n}$
Coefficient of Mean Deviation	The coefficient of mean deviation is a relative measure of dispersion that standardizes the mean deviation by comparing it to the central measure. $Coefficient\ of\ MD\ about\ the\ Mean\ = \frac{MD\ about\ mean}{mean} \times\ 100$

	$Coefficient\ of\ MD\ about\ Median = \frac{MD\ about\ Median}{Median} \times 100$
Question 1	What is the value of mean deviation about mean & coefficient of MD about mean for the numbers 3 ,7 , 6 , 8 , 12 $Mean,\ \bar{x} = \frac{3+7+6+8+12}{5} = 7.2$ $MD\ about\ mean = \frac{\Sigma\lvert x-\bar{x}\rvert}{n} = \frac{\lvert 3-7.2\rvert+\lvert 7-7.2\rvert+\lvert 6-7.2\rvert+\lvert 8-7.2\rvert+\lvert 12-7.2\rvert}{5}$ $= \frac{4.2+.2+1.2+.8+4.8}{5} = 2.24$ $Coefficient\ of\ MD\ about\ the\ Mean = \frac{MD\ about\ mean}{mean} \times 100$ $= \frac{2.24}{7.2} \times 100 = 31.11$

Q) The coefficient of mean deviation about the mean for the first 9 natural numbers is:
(a) $\frac{200}{9}$
(b) 80
(c) $\frac{400}{9}$ = 44.44
(d) 50

1, 2, 3, 4, 5, 6, 7, 8, 9 $\bar{x} = \frac{1+9}{2} = 5$

M.D about mean $= \frac{\Sigma\lvert x-\bar{x}\rvert}{n} = 2.2222$

1-5 M⁻   7-5 M⁺
2-5 M⁻   8-5 M⁺
3-5 M⁻   9-5 M⁺
4-5 M⁻   MRC ÷ 9
6-5 M⁺

Coef of M.D $= \frac{MD}{\bar{x}} \times 100$
$= \frac{2.222}{5} \times 100$
$= 44.44$

Q) What is the value of mean deviation about the mean from the numbers 5, 8, 6, 3, 4?
(a) 5.20
(b) 7.20
(c) 1.44
(d) 2.23

M.D about Mean $= \frac{\Sigma\lvert x-\bar{x}\rvert}{n}$

$\bar{x} = \frac{5+8+6+3+4}{5} = 5.2$

5-5.2 M⁻   8-5.2 M⁺   6-5.2 M⁺   3-5.2 M⁻   4-5.2 M⁻
MRC ÷ 5

Q) Mean deviation is the least when deviations are taken from:
(a) Mean
(b) Median
(c) Mode

(d) Harmonic mean

Q) The mean deviation of the numbers 3, 10, 6, 11, 14, 17, 9, 8, 12 about the mean is (correct to one decimal place):
(a) 8.7
(b) 4.2
(c) 3.1
(d) 9.8

$\bar{x} = 3+10+6+11+14+17+9+8+12 \div 9 = 10$

3−10 m−
6−10 m−
11−10 m+
14−10 m+
17−10 m+
9−10 m−
8−10 m−
12−10 m+
MRI ÷ 9 = 3.1

Standard deviation (SD) & Coefficient of Variation (CV)

*

Standard deviation (SD)	Standard deviation is the positive square root of the average of the deviations of all the observations taken from the mean $SD = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$
Variance	It is the **square of Standard deviation**
Coefficient of Variation (CV)	The Coefficient of Variation (CV) is a statistical metric used to assess the relative variability of a dataset in relation to the mean of the dataset. It is a useful tool for comparing the degree of variation from one dataset to another, even if the means are drastically different. $CV = \frac{SD}{AM} \times 100$ SD is the standard deviation of the dataset. AM is the arithmetic mean (average) of the dataset.
Formula for Standard deviation (SD) For Ungrouped data	$SD = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$ $\bar{x} = \frac{\Sigma x}{n}$ Or $SD = \sqrt{\frac{\Sigma x^2}{n} - (\bar{x})^2}$

$SD = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$

or

$SD = \sqrt{\frac{\Sigma x^2}{n} - (\bar{x})^2}$

Variance $SD^2 = \frac{\Sigma x \times x}{n} - (\bar{x} \times \bar{x})$

$Cov(x,y) = \frac{\Sigma x y}{n} - \left(\frac{\Sigma x}{n} \times \frac{\Sigma y}{n}\right)$

Σa^3

Q) The sum of squares of deviation from the mean of 10 observations is 250. The mean of the data is 10. Find the coefficient of variation.

(a) 10%

(b) 25%

(c) 50%

(d) 0%

$\Sigma(x-\bar{x})^2 = 250$ $n = 10$ $\bar{x} = 10$

$SD = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{250}{10}} = 5$

$CV = \frac{SD}{AM} \times 100 = \frac{5}{10} \times 100$

Q) The variance of data: 3, 4, 5, 8 is

(a) 4.5

(b) 3.5

(c) 5.5

(d) 6.5

$\bar{x} = \frac{3+4+5+8}{4} = 5$

$\text{Variance} = SD^2 = \left(\sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}\right)^2 = \frac{\Sigma(x-\bar{x})^2}{n}$

$3-5 \ x= \ M+$

$4-5 \ x= \ M+$

$8-5 \ x= \ M+$

$MRC \div 4 = 3.5$

Q) The standard deviation of the weights (in kg) of the students of a class of 50 students was calculated to be 4.5 kg. Later on, it was found that due to some fault in the weighing machine, the weight of each student was under-measured by 0.5 kg. The correct standard deviation of the weight will be:

(a) Less than 4.5

(b) Greater than 4.5

(c) Equal to 4.5

(d) Cannot be determined

Not effected by origin

Q) If the sum of squares of the values = 3390, $N = 30$, and standard deviation = 7, find out the mean.

(a) 113

(b) 210

(c) 8

(d) None of these

$\Sigma x^2 = 3390$ $SD = 7$

$N = 30$

$SD = \sqrt{\frac{\Sigma x^2}{n} - (\bar{x})^2}$

$7 = \sqrt{113 - (\bar{x})^2}$

option Hit using (c)

$LHS = RHS = \sqrt{49} = 7$

Q) If the standard deviation for the marks obtained by a student in a monthly test is 36, then the variance is:

(a) 6

(b) 36

(c) 1296

(d) None of the above

Variance = $(SD)^2$

$= (36)^2$

$= 1296$

Q) Coefficient of variation is 80. Mean is 20. Find variance:

(a) 640

(b) 256

(c) 16

(d) 250

$CV = \frac{SD}{AM} \times 100$

$80 = \frac{SD}{20} \times 100$

SD = 16

Variance = 256

Standard deviation (SD) For grouped data

Example: Find the SD, Variance & Coefficient of Variation for the following

Class intervals (CI) in hours	1-3	4-6	7-9	10-12
Frequency (f)	5	8	12	5

Step 1: Find $\bar{x}$

$\bar{x} = \frac{\Sigma fx}{\Sigma f}$

Midpoints x	2	5	8	11
Frequency (f)	5	8	12	5

The AM is then calculated as:

$SD = \sqrt{\frac{\Sigma f(x-\bar{x})^2}{N}}$

$$\bar{x} = \frac{\sum f \times x}{N}$$

$$N = \Sigma f$$

$$\bar{x} = \frac{(5\times2)+(8\times5)+(12\times8)+(5\times11)}{5+8+12+5} = 6.7$$

$2 - 6.7$ x = x 5 M+

$5 - 6.7$ x = x 8 M+

$8 - 6.7$ x = x 12 M+

$11 - 6.7$ x = x 5 M+

MRC ÷ 30

Step 2 Use Standard deviation (SD) For grouped data

Click T

SD = 2.86

CV = SD/AM × 100

= 2.86/6.7 × 100

= 42.6

	$SD = \sqrt{\frac{\Sigma f.(x-\bar{x})^2}{N}}$ $= \sqrt{\frac{5\times1.7^2+8\times1.3^2+12\times5.3^2+5\times1.7^2}{30}} = \sqrt{12.65}$ $Variance = SD^2 = 12.65$ $Coefficient\ of\ Variation\ (CV) = \frac{SD}{AM} \times 100 = \frac{\sqrt{12.65}}{6.7} \times 100$
Formula for Standard deviation (SD) For grouped data	$Standard\ deviation\ (SD) For\ grouped\ data$ $SD = \sqrt{\frac{\Sigma f.(x-\bar{x})^2}{N}}$ where $N = \Sigma f$ Or $SD = \sqrt{\frac{\Sigma f x^2}{N} - (\bar{x})^2}$

Question ✗

Determine the SD , Variance & Coefficient of Variation on time spent on homework each week by students in a high school from the following data:

Time in Hours	0-2	3-5	6-8	9-11	12-14
No. of students	5	12	20	8	4

Shift of Origin: If you add or subtract a constant from all data points (shifting the data set on the number line), the standard deviation does not change.

Change of Scale: If you multiply or divide all data points by a constant (changing the scale of measurement), the standard deviation will change proportionally.

Combined SD

Example:Let's say we have two groups of data:

Group 1: $n_1 = 10,\ \bar{x}_1 = 70,\ \&\ S_1 = 5$

Group 2: $n_2 = 15,\ \bar{x}_2 = 80,\ \&\ S_2 = 7$

Here n_1, n_2 *are no. of observations* $\bar{x}_1 \& \bar{x}_2$ *are AM*, S_1, S_2 *are SD*

To Find combined Standard Deviation

Formula for Combined SD,

KV TD

$\bar{x}_1$ $\bar{x}_2$

$\bar{x}_c$

$$\text{Combined SD} = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2 + n_1 d_1^2 + n_2 d_2^2}{n_1 + n_2}} =$$

$$d_1^2 = \left(\bar{x}_1 - \bar{x}_c\right)^2 \quad \& \quad d_2^2 = \left(\bar{x}_2 - \bar{x}_c\right)^2$$

$\bar{x}_c$ *is the combined mean*

$$\bar{x}_c = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

Step 1: Find $\bar{x}_c$

$$\bar{x}_c = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} = \frac{10\times70 + 15\times80}{25} = 76$$

Step 2: Find $d_1^2 \& d_2^2$

$$d_1^2 = \left(\bar{x}_1 - \bar{x}_c\right)^2 = (70 - 76)^2 = 36$$

$$d_2^2 = \left(\bar{x}_2 - \bar{x}_c\right)^2 = (80 - 76)^2 = 16$$

Step 3: Apply Combined SD Formula and substitute the values

$$\text{Combined SD} = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2 + n_1 d_1^2 + n_2 d_2^2}{n_1 + n_2}}$$

$$\text{Combined SD} = \sqrt{\frac{10\times5^2 + 15\times7^2 + 10\times36 + 15\times16}{25}} = 7.96$$

Q) The mean and variance of a group of 100 observations are 8 and 9 respectively. The mean and standard deviation of 60 observations are 10 and 2, respectively. Find the standard deviation of the remaining 40 observations.

(a) 4.5

(b) 3.5

(c) 2.5

(d) 1.5

$\bar{x}_c = 8$ $\quad SD_c = 3$

$n_1 = 60$ $\quad \bar{x}_1 = 10$ $\quad s_1 = 2$ $\quad s_1^2 = 4$

$n_2 = 40$ $\quad \bar{x}_2 = ?$ $\quad s_2 = ?$

$d_1^2 = (\bar{x}_1 - \bar{x}_c)^2 = 2^2 = 4$

$d_2^2 = (\bar{x}_2 - \bar{x}_c)^2 = 9$

$$\bar{x}_c = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

$$8 = \frac{600 + 40 \times \bar{x}_2}{100}$$

$\bar{x}_2 = 5$

$$SD_c = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2 + n_1 d_1^2 + n_2 d_2^2}{n_1 + n_2}}$$

$$3 = \sqrt{\frac{60 \times 4 + 40 \times s_2^2 + 60 \times 4 + 40 \times 9}{100}}$$

9×100 M+
60×4 M−
60×4 M−
40×9 M−
MRC ÷ 40

$s_2^2 = 1.5$

$s_2 = \sqrt{1.5}$

Standard deviation (SD) of the first n natural numbers	The standard deviation (SD) of the first n natural numbers, or the formula for the standard deviation of any sequence of n consecutive natural numbers. $SD\ of\ the\ first\ n\ natural\ numbers = \boxed{\sqrt{\frac{n^2-1}{12}}}$

*

Q) If the standard deviation of the first 'n' natural numbers is 2, then the value of 'n' is:

(a) 10

(b) 7

(c) 6

(d) 5

$$2 = \sqrt{\frac{n^2-1}{12}}$$

$$4 = \frac{n^2-1}{12}$$

$48 = n^2 - 1$

$n^2 = 49$

$n = \sqrt{49} = 7$

Quartile Deviation (QD) & Coefficient of Quartile Deviation

Quartile Deviation (QD) QD May25	It is the average of the distances from the median to the first quartile (Q1, the 25th percentile) and the third quartile (Q3, the 75th percentile). It essentially measures the spread of the middle half of the data. It is particularly useful for open-ended classifications where you don't have detailed data about the lower and upper extremes of the distribution. $\frac{Q_3 - Q_1}{2}$
Formula for Quartile Deviation (QD) Semi-interquartile range	$QD = \frac{Q_3 - Q_1}{2}$, QD is also known as the Semi-Interquartile Range, $Q_3 - Q_1$ is called as the inter quartile range Q_3, Q_1 *are Third & First Quartile* $\frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$
Coefficient of Quartile Deviation	$Coefficient\ of\ QD = \frac{Q_3 - Q_1}{Q_3 + Q_1} x100 = \frac{QD}{M} x100$ $M\ is\ the\ median\ = \frac{Q_3 + Q_1}{2}$ $M = \frac{Q_3 + Q_1}{2}$ $\boxed{\frac{QD}{M} \times 100}$

Q) Inter Quartile Range is _____ of Quartile Deviation.

(a) Half

(b) Double

(c) Triple

(d) Equal

$QD = \frac{Q_3 - Q_1}{2}$

$2 \times QD = \boxed{Q_3 - Q_1}$

Q) If mean = 5, standard deviation = 2.6, median = 5, and quartile deviation = 1.5, then the coefficient of quartile deviation equals:

(a) 35

(b) 39

(c) 30

(d) 32

$$\text{Coef } QD = \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$$

$$= \frac{QD}{M} \times 100$$

$$= \frac{1.5}{5} \times 100 = 30$$

Q) The formula for the range of middle 50% items of a series is: (a) $Q_3 - Q_1$ (b) $Q_3 - Q_2$ (c) $Q_2 - Q_1$ (d) $\frac{Q_3 - Q_1}{2}$
Q) If the first quartile is 142 and the semi-interquartile range is 18, then the value of the median is: (a) 151 (b) 160 (c) 178 (d) None of these $Q_1 = 142$ $\frac{Q_3 - Q_1}{2} = 18$ $Q_3 = 18 \times 2 + 142 = 178$ $M = \frac{Q_3 + Q_1}{2} = 160$
Q) The Q.D. (Quartile Deviation) of 6 numbers 15, 8, 36, 40, 38, 41 is equal to: (a) 12.5 (b) 25 (c) 13.5 (d) 37 $QD = \frac{Q_3 - Q_1}{2}$ $\frac{40.25 - 13.25}{2} = 13.5$ 8, 15, 36, 38, 40, 41 $Q_1 = \left(\frac{1}{4}(n+1)\right)^{th}$ value $= \frac{7}{4}^{th}$ value $= 1.75^{th}$ value $= 8 + 0.75$ gap $= 8 \times 0.75 \times 7$ $= 13.25$ $Q_3 = \frac{3}{4}(7) = 5.25^{th}$ value $= 40 + 0.25 = 40.25$

Relationship between SD, MD, & QD for Normal or symmetrical distribution

Relationship between SD, MD, & QD for Normal or symmetrical distribution	$2SD = 2.5\,MD = 3QD$ $2.5MD = 3QD$ $Note{:}\ SD > MD > QD$ • The **standard deviation** is usually the **largest** because it squares the differences from the mean, giving more weight to extreme values. • The **mean deviation** does not square the differences, so it is typically **smaller**. • Finally, the **quartile deviation** only looks at the middle 50% of the data, thus excluding any outliers or extreme values, which usually makes it the **smallest** of the three.

Q) What will be the probable value of mean deviation? When $Q_3 = 40$ and $Q_1 = 15$:

(a) 17.50

(b) 18.75

(c) 15.00

(d) None of the above

$QD = \frac{Q_3 - Q_1}{2} = 12.5$

$2.5\ MD = 3QD$

$2.5 \times MD = 3 \times 12.5$

$MD = 15$

Q) The quartile deviation is:

(a) $\frac{2}{3}$ of S.D

(b) $\frac{4}{5}$ of S.D.

(c) $\frac{5}{6}$ of S.D.

(d) None of these

$2SD = 3QD$

$QD = \frac{2}{3} SD$

Q) The approximate ratio of SD, MD, QD is:

(a) ~~3 : 4~~ : 5

(b) ~~2 : 3 : 4~~

(c) 15 : 12 : 10

(d) 5 : 6 : 7

$\boxed{2SD = 2.5MD} = 3QD$

$2SD = 2.5MD$

$\frac{SD}{MD} = \frac{2.5}{2} = 1.25$

Dispersion Relationship in Linear Variables:

Q) The equation of a line is $5x + 2y = \cancel{17}$ Mean deviation of y about mean is 5 Calculate mean deviation of x about mean.

(a) -2

(b) 2

(c) -4

(d) None

$5x = 2y$

$\cancel{5}x = 2 \times \cancel{5}$

$x = 2$

$y = 2 - 3x$

Q) If the variance of x is 5, then find the variance of $2 - 3x$:

(a) 10

(b) 45

(c) 5

(d) -13

SD of $x = \sqrt{\text{variance}} = \sqrt{5}$

$y = 3x$

$SD_y = 3\sqrt{5}$

$SD_y^2 = (3\sqrt{5})^2 = 45$

Q) If the arithmetic mean and coefficient of variation of x are 10 and 40, respectively, then the variance of $-15 + \frac{3x}{2}$ will be:

(a) 64

(b) 81

(c) 49

(d) 36

$CV = \frac{SD}{\bar{x}} \times 100$

$40 = \frac{SD_x}{10} \times 100$

SD of $x = 4$

$y = \frac{3x}{2}$

SD of $y = 6$

Variance $= 6^2 = 36$

Q) If x and y are related as $4x + 3y + 11 = 0$ and the mean deviation of y is 7.2, then the mean deviation of x is:

(a) 2.70

(b) 7.20

(c) 4.50

(d) 5.40

$4x = 3y$

$4x = 3 \times 7.2$

$x = 5.4$

Q) If x and y are related as $3x - 4y = 20$ and the quartile deviation of x is 12, then the quartile deviation of y is:

(a) 9

(b) 8

(c) 7

(d) 6

$3x = 4y$

$3 \times 12 = 4y$

$y = 9$

Q) Which of the following is a relative measure of dispersion?

(a) Range

(b) Mean deviation

(c) Standard deviation

(d) Coefficient of quartile deviation

Q) If every observation is increased by 7, then:

(a) Standard deviation increased by 7

(b) Mean deviation increased by 7

(c) Not affected at all

(d) Quartile deviation increased by 7

Q) Which of the following is based on absolute deviation?

(a) Standard deviation

(b) Mean deviation

(c) Range

(d) Quartile deviation

Q) __________ is based on all the observations and __________ is based on the central fifty percent of the observations.

(a) Mean deviation, Range

(b) Mean deviation, Quartile deviation

(c) Range, Standard deviation

(d) Quartile deviation, Standard deviation

Chapter 13

Statistics is a branch of mathematics that focuses on the **collection, analysis, interpretation, and presentation** of numerical data in a systematic manner.

Etymology (Word Origin):

- The word "Statistics" is derived from:
  - Italian word "Statista", meaning *Statesman.*
  - Latin word "Status", meaning *Political State.*

Different Senses of Statistics:

Statistics can be understood in two different ways: **Singular Sense** and **Plural Sense**.

1. Statistics in Singular Sense:

- In a singular sense, statistics refers to **the science** that provides **methods and techniques** for:
  - **Collecting** data
  - **Analyzing** data
  - **Interpreting** data
  - **Presenting** data in a meaningful way

2. Statistics in Plural Sense:

- In a plural sense, statistics refers to **numerical statements of facts** related to various fields of study.
- It involves **data related to different sectors**, such as:

- It involves **data related to different sectors**, such as:

  - **Production**
  - **Income**
  - **Population**
  - **Prices**
  - **Other economic and social aspects**

10%

Types of Data

Data is classified into two basic types:

1. Primary Data
2. Secondary Data

Key Differences Between Primary and Secondary Data

Aspect	Primary Data	Secondary Data
Source	Directly collected	Collected from existing records
Accuracy	More accurate and reliable	Less accurate, depends on source credibility
Cost & Time	Expensive & time-consuming	Less costly & readily available
Examples	Surveys, interviews, experiments	Government reports, books, websites

Data Classification

Data can be classified into different types based on its nature and how it is collected.

1. Temporal (Chronological) Data

- **Definition:** Data that is related to time.
- **Example:** Sales data of a company recorded over multiple years.

2. Spatial (Geographical) Data

- **Definition:** Data that is related to a specific location or area.
- **Example:** Weather reports of different cities.

3. Qualitative Data

- **Definition:** Data that describes characteristics, attributes, or qualities but **cannot be measured** numerically.
- **Examples:**
  - Knowledge level of a person
  - Habits and skills
  - Nationality

4. Quantitative Data (Measurable Data)

- **Definition:** Data that can be measured numerically.
- **Examples:**
  - Marks obtained in an exam
  - Height and weight of an individual
  - Age of a person

Key Differences Between Qualitative and Quantitative Data

Feature	Qualitative Data	Quantitative Data
Nature	Descriptive	Measurable
Representation	Text-based or categorical	Numeric values
Examples	Nationality, habits, skills	Height, weight, age, marks

Frequency Distribution

Definition:

- A **systematic presentation** of the values taken by a variable along with their corresponding **frequencies** is called a **frequency distribution** of that variable.
- It helps in organizing **raw data** into a **structured format**.
- The frequency of a value refers to **how many times** it occurs in a dataset.

1. Continuous (Grouped) Frequency Distribution

a) Inclusive Classes Example

- In **inclusive classes**, both the lower and upper class limits are included.
- Example: If the range is **0-10**, a score of **10** belongs to this class.

Marks Range CI	0 - 10	11 - 20	21 - 30	31 - 40	41 - 50
No. of Students	5	12	18	10	7

b) Exclusive Classes Example

- In **exclusive classes**, the lower limit is **included**, but the upper limit is **excluded**.
- Example: If a score is **20,** it falls into the **20-40** category, **not** 0-20.

Marks Range	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100
No. of Students	8	15	12	10	5

Key Differences Between Inclusive and Exclusive Classes

Feature	Inclusive Classes	Exclusive Classes
Class Limit Inclusion	Both lower & upper limits are included	Only the lower limit is included, upper is excluded
Example of Class Interval	0-10, 11-20, 21-30	0-20, 20-40, 40-60
Suitability	Used for grouped data where clear upper/lower limits are needed	Common in scientific and statistical analysis

2. Discrete Frequency Distribution

- Used for **countable** values (e.g., number of pets, number of family members).
- Each value occurs **individually** instead of in a range.

No. of Pets (X)	0	1	2	3	4
No. of Students (F)	4	9	14	7	5

Terminologies Used in Frequency Distribution

Understanding the key terms in frequency distribution helps in interpreting and organizing statistical data effectively.

1. Class Interval

- When the range of a dataset is **large**, it is divided into **smaller sub-ranges** called **class intervals.**
- Example: If the dataset ranges from **0 to 100**, it can be divided into **intervals like 0-20, 20-40, 40-60**, etc.

2. Class Limit

- Each class interval has **two limits**:
  - **Lower Class Limit (LCL)**: The smallest value in the class.
  - **Upper Class Limit (UCL)**: The largest value in the class.

3. Class Width (Class Length)

- The **difference between the class limits** of a class interval.
- Formula:

$$\text{Class Width} = \text{Upper Class Limit} - \text{Lower Class Limit}$$

- Example: In the class **10-20**, the width is **20 - 10 = 10**.

4. Class Mark (Class Mid-Value)

- The central value of a class interval.
- Formula:

$$\text{Class Mark} = \frac{\text{Lower Limit} + \text{Upper Limit}}{2}$$

- Example: For the class **10-20**, the class mark is **(10+20)/2 = 15**.

5. Class Frequency

- The **number of observations** in a particular class interval.
- Example:

Marks Range	Frequency
0 - 10	5
10 - 20	8

Here, **8** students scored between **10-20**, so the frequency is **8**.

6. Total Frequency (N)

- The **sum of all frequencies** in a dataset.
- Formula:

$$N = f_1 + f_2 + f_3 + \ldots + f_n$$

- Example: If frequencies are 5, 8, 12, then $N = 5 + 8 + 12 = 25$.

7. Inclusive vs. Exclusive Class Intervals

Type	Definition	Example
Inclusive Class Interval	Both **upper and lower** limits are included.	10 - 20 includes **10 and 20**.
Exclusive Class Interval	Lower limit is included, but **upper limit is** excluded.	10 - 20 includes **10** but not **20** (20 is in the next class).

8. Class Boundaries

- **Upper Class Boundary (UCB):** The upper limit extended slightly to separate adjacent classes.
- **Lower Class Boundary (LCB):** The lower limit adjusted similarly.

9. Frequency Density *

- The **ratio of class frequency to class width.**
- Formula:

$$\boxed{\text{Frequency Density} = \frac{\text{Class Frequency}}{\text{Class Width}}}$$

10. Relative Frequency (RF)

- The **proportion of a class frequency** to the **total frequency**.
- Formula:

$$\boxed{RF = \frac{\text{Class Frequency}}{\text{Total Frequency}}}$$

$$\frac{f_i}{\Sigma f}$$

- RF values range between 0 and 1.

11. Percentage Frequency

- The **relative frequency** converted to percentage.
- Formula:

$$\boxed{\text{Percentage Frequency} = RF \times 100}$$

Basics Rules for Frequency Distribution

To ensure an **effective frequency distribution**, certain rules must be followed:

1. **Equal Class Lengths:**
   - As far as possible, class intervals should have **equal lengths** for consistency in analysis.
2. **Unambiguous Definition:**
   - Each class interval must be **clearly defined** to avoid confusion.
3. **Homogeneous Data:**
   - The data within the frequency distribution should be of the **same type**.
4. **Mutually Exclusive Classes:**
   - No data value should belong to more than **one class interval**.
5. **Exhaustive Classes:**
   - The frequency distribution should cover **all possible values** in the dataset.

Cumulative Frequency

Cumulative frequency represents the **running total of frequencies**. It is useful for **finding medians, quartiles**, and **plotting ogives (cumulative frequency graphs)**.

1. Less Than Cumulative Frequency (LCF)

- **Definition:** It is the **cumulative sum of frequencies** up to a certain class limit.
- **Used to find:**
  - **Median** and other statistical **partitions**.
  - **Class boundaries** (Upper Class Boundary - **UCB**).
- **Final cumulative frequency** is **N** (total number of observations).

Example Table for LCF

Marks Range	Frequency (F)	Less Than Cumulative Frequency (LCF)	MCF
0 - 10	5	5	52
10 - 20	12	5 + 12 = 17	40
20 - 30	18	17 + 18 = 35	22
30 - 40	10	35 + 10 = 45	12
40 - 50	7	45 + 7 = 52	

2. More Than Cumulative Frequency (MCF)

- **Definition:** It is the **cumulative sum of frequencies** starting from the highest class and moving downward.
- **Used for:**
  - **Ogive plotting (along with LCF).**
  - **Class boundaries** (Lower Class Boundary - **LCB**).
- **First cumulative frequency** is **N**.

0–10

How to Determine the Number of Classes in a Frequency Distribution

When creating a **frequency distribution table**, the **number of classes** is determined using either of the two methods:

Method 1: Using Range and Class Length

$$\text{Number of Classes} = \frac{\text{Range}}{\text{Class Length}}$$

$$\text{Range} = \text{Largest Observation} - \text{Smallest Observation}$$

5

3, 4, 5, 10, 11, 12, 13, 14, 15

No: of classes = $\frac{12}{5}$

$4 = \frac{20}{\text{class length}}$

class length = 5

Method 2: Sturges' Rule

$$\text{Number of Classes} = 1 + 3.322 \log N$$

Where:

- N = Total number of observations
- log N = Logarithm of the total number of values

Methods of Data Presentation

Data can be presented in various forms to ensure clarity and ease of analysis. The three main methods are:

1. Textual Presentation (Descriptive Method)
   - Data is presented in the form of a **written explanation**.
   - Best suited for **small datasets** or **reports**.
   - Example:
     - "In 2023, the company's revenue increased by 20% compared to 2022."
2. **Tabulation (Data Arranged in Rows & Columns)**
   - The **best method** for data presentation.
   - Organizes data into **structured tables** for better understanding.
   - Helps in **comparison and analysis**.
   - Example:

Year	Sales (in million $)
2021	50
2022	60
2023	72

3. **Diagrammatic (Graphical) Presentation**
   - The **most attractive** and easy-to-understand method.
   - Uses **graphs, charts, and diagrams** to present data.
   - Common types:
     - **Bar Graphs** (for categorical data)
     - **Pie Charts** (for proportions)
     - **Line Graphs** (for trends over time)

Tabulation

Definition

Tabulation is the **systematic arrangement** of classified data into **rows and columns** within a table.

Advantages of Tabular Presentation

✅ **Facilitates comparison** between different data points.

✅ **Simplifies complicated data** for better analysis.

✅ **Essential for graphical representation** (diagrams and charts).

✅ **Enables statistical analysis**, which is impossible without tables.

Parts of a Table in Tabulation

A well-structured table consists of **four main parts:**

1. **Caption:**
   - The **topmost** part of the table.
   - Describes **columns or sub-columns.**
   - Also called the **Column Heading**.
2. **Box Head:**
   - The **entire upper part** of the table.
   - Includes the **caption, column names, and units of measurement**.

	A	B
[illegible]	5	10
P%	100	35

3. **Stubs:**
   - The **leftmost** part of the table.
   - Lists **row headings** (e.g., names, categories).

4. **Body:**
    - The **main section** where the actual **data (numbers, values)** are arranged in rows and columns.
5. **Footnote (Optional):**
    - Located at the **bottom** of the table.
    - Provides **source information** or any **missed details**.

Diagrammatic Presentation of Data

Median

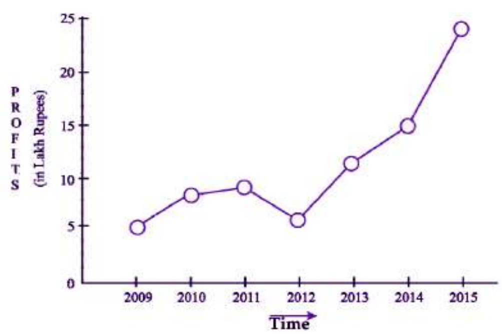
Diagrammatic presentation refers to the **visual representation** of statistical data using charts, diagrams, and pictures. It makes data **easier to understand, analyze, and compare**.

- It can be:
    - **One-dimensional** (e.g., bar charts, line graphs)
    - **Two-dimensional** (e.g., histograms, pie charts)
    - **Three-dimensional** (e.g., 3D bar charts, 3D surface plots)

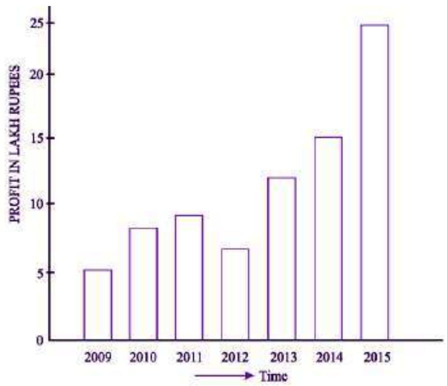
Types of Diagrams

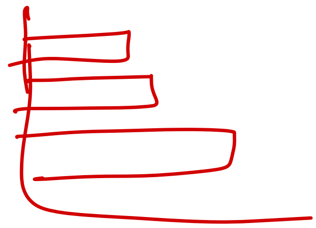
Here are the different types of **graphical data representations**:

1. Line Diagram
    - Used to show **trends** over time.
    - Commonly used for **stock prices, population growth, and sales trends.**



2. **Bar Diagram (Bar Chart)**
    - Represents **categorical data** with rectangular bars.
    - Bars can be **vertical or horizontal.**
    - Example: Number of students in different departments.





Types of Bar Graphs & Their Uses

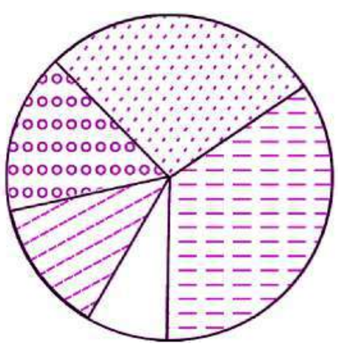
A **bar graph** is a chart that uses **rectangular bars** to represent data. The length of each bar is proportional to the **value it represents.**

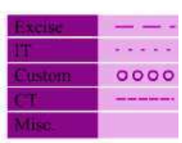
Comparison of Bar Graph Types

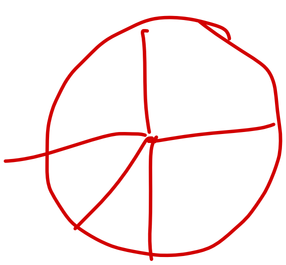
Type	Used for	Example
Horizontal Bar Graph	Qualitative data	Favorite sports of students
Vertical Bar Graph	Quantitative data	Annual sales report
Multiple Bar Graph	Comparison of multiple datasets	Monthly sales of two brands
Divided Bar Graph	Proportions within a whole	Budget distribution in a family

3. **Pie Chart**

- A **circular graph** that shows proportions.
- The **entire circle represents 100%**, and each slice corresponds to a proportion of the whole.
- Example: Budget allocation in a company.







Formula for Central Angle

Each segment (category) in a pie chart is represented as a **sector** with a central angle given by:

$$\boxed{\text{Central Angle}} = \left(\frac{x}{\sum x}\right) \times 360$$

$$\boxed{\frac{x}{\Sigma x} \times 360^\circ}$$

Where:

- x = Value of the category
- $\sum x$ = Total of all values
- 360° represents the entire circle

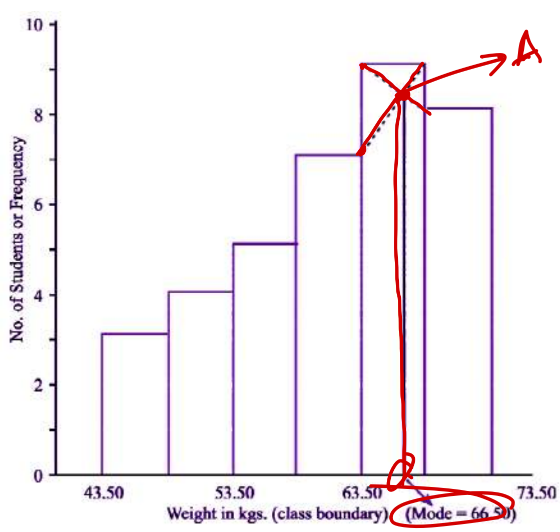
Uses of Pie Charts

- ✅ Family's monthly budget distribution
- ✅ Five-year economic planning of a country
- ✅ Market share comparison
- ✅ Population distribution by age or region

4. Histogram

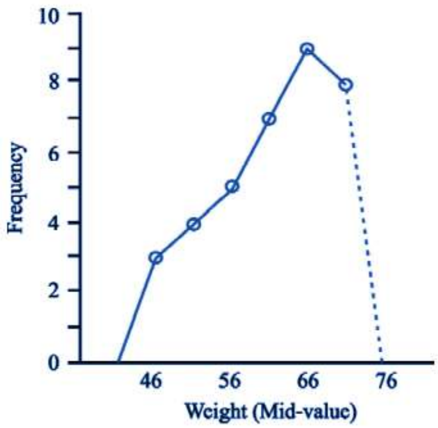
- A **graphical representation of a frequency distribution.**
- Used for **continuous data.**
- Bars are adjacent (no gaps between them).
- Example: Distribution of student marks.

A → Mod



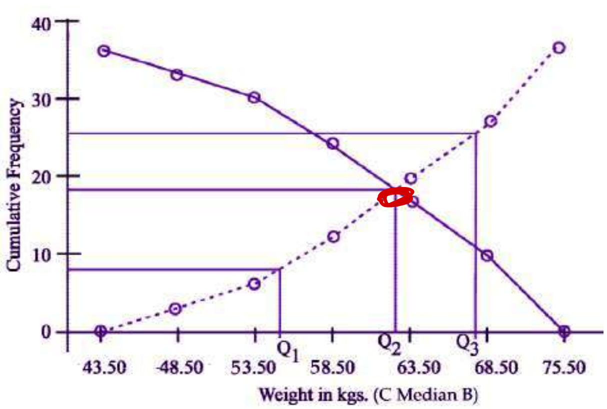
5. **Frequency Polygon**

- A **line graph** that represents a frequency distribution.
- Plotted by joining the midpoints of histogram bars.
- Example: Comparing the performance of different groups of students.



6. **Ogives (Cumulative Frequency Curve)**

- A graph used to find **median, quartiles, and percentiles.**
- Two types: **Less than Ogive & More than Ogive.**

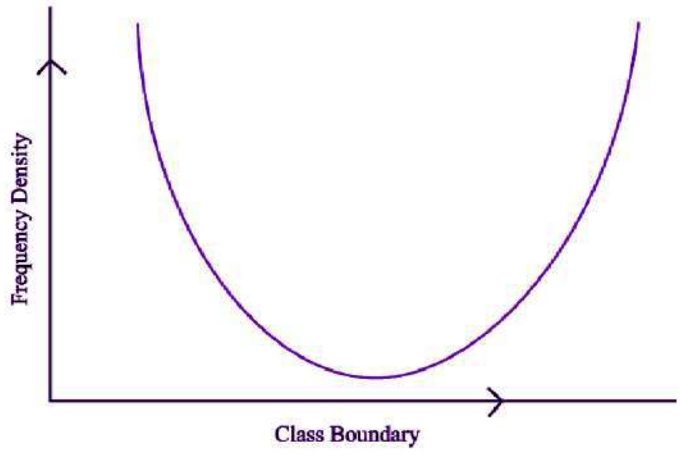


7. **Frequency Curves**

- A **smooth curve** drawn through frequency distribution points.
- Used for **probability distributions.**
- Example: Bell curve (Normal Distribution).

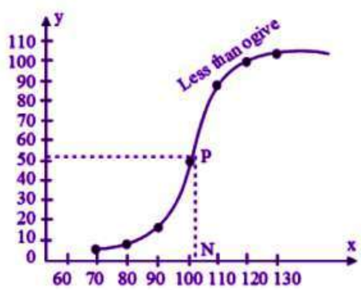
1. U-Shaped Curve

- **Definition:** A curve that has a dip in the middle and rises at both ends.
- **Indicates:**
  - Two extreme values occur frequently.
  - Middle values occur less frequently.



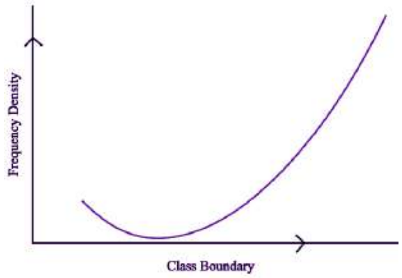
2. Inverted J-Shaped (S-Shaped) Curve

- **Definition:** A curve that starts high and gradually decreases or an "S" pattern showing slow and then rapid increase.
- **Indicates:**
  - More frequent lower values and fewer higher values.



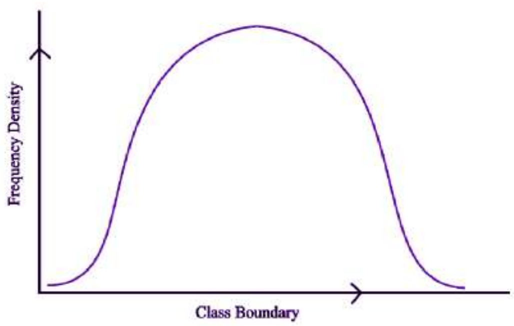
3. J-Shaped Curve

- **Definition:** A curve that **starts low and increases steeply.**
- **Indicates:**
  - **Rapid growth** after a slow start.



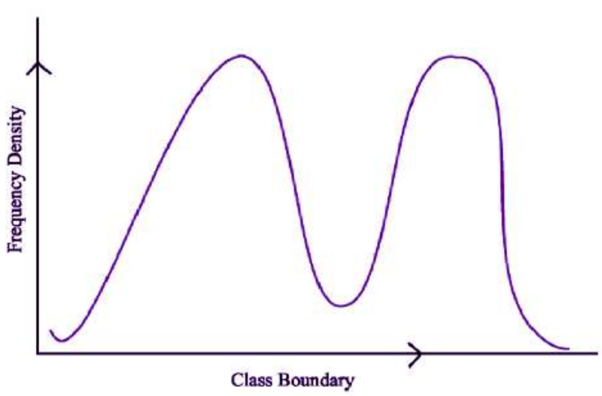
4. Bell-Shaped Curve (Normal Distribution)

- **Definition:** A symmetrical, peak-centered curve (also called a **Gaussian curve**).
- **Indicates:**
  - Data is **normally distributed**.
  - Mean, median, and mode are **equal**.



5. Mixed Curve

- **Definition:** A curve with multiple peaks and valleys.
- **Indicates:**
  - Data is **not normally distributed**.
  - There are multiple sub-groups in the dataset.



Curve Type	Shape	Example
U-Shaped	Dips in the middle, rises at ends	Income distribution
Inverted J-Shaped (S-Shaped)	High at start, decreases or curves like "S"	Age distribution
Ogive	Smooth increasing curve	Cumulative exam scores
Bell-Shaped	Peak in the center, tapering ends	IQ scores, height distribution
Mixed Curve	Irregular pattern with multiple peaks	Seasonal sales trends

Q: Out of the following, the one which affects the regression coefficient is

(a) Change of origin only
(b) Change of scale only
(c) Change of scale and origin both
(d) Neither change in origin nor change of scale

Q: Which of the following is not a two-dimensional figure?

(a) Line Diagram
(b) Pie Diagram
(c) Square Diagram
(d) Rectangle Diagram

Q: Less than type and more than type Ogives meet at a point known as:

(a) Mean
(b) Median
(c) Mode
(d) None

Q: Arrange the dimensions of Bar diagram, Cube diagram, Pie diagram in sequence.

(a) 1, 3, 2
(b) 2, 1, 3
(c) 2, 3, 1
(d) 3, 2, 1

Q: With the help of a histogram, one can find:

(a) Mean

(b) Median

(c) Mode

(d) First Quartile

Q: Nationality of a person is:

(a) Discrete variable

(b) An attribute

(c) Continuous variable

(d) None

Q: If we plot less than and more than type frequency distribution, then the graph plotted is ____

(a) Histogram

(b) Frequency Curve

(c) Ogive

(d) None of these

Q: Using Ogive Curve, we can determine

(a) Median

(b) Quartile

(c) Both (a) and (b)

(d) None.

Q: Mode can be obtained from

(a) Frequency polygon.

(b) Histogram.

(c) Ogive

(d) All of the above.

Q: The data obtained by the internet are

(a) Primary data

(b) Secondary data

(c) Both (a) and (b)

(d) None of these.

Q: The chronological classification of data is classified on the basis of

(a) Attributes

(b) Area

(c) Time

(d) Class Interval

Q: The frequency of class 20-30 in the following data is

Class	0-10	10-20	20-30	30-40	40-50
Cumulative Frequency	5	13	28	34	38
f	5	8	15	6	4

(a) 5

(b) 28

(c) 15

(d) 13

Q: The data given below refers to the marks gained by a group of students:

Marks	Below 10	Below 20	Below 30	Below 40	Below 50
No. of Students	15	38	65	84	100

The number of students getting marks more than 30 would be?

(a) 50

(b) 53

(c) 35

(d) 62

Total students − Below 30

$100 - 65 = 35$

Q: Cost of Sugar in a month under the heads raw materials, labour, direct production, and others were 12, 20, 35, & 23 units respectively. The difference between their central angles for the largest & smallest components of the cost of Sugar is

(a) 92°

(b) 72°

(c) 48°

(d) 56°

$\Sigma x = 12 + 20 + 35 + 23$

$= 90$

$\frac{x_L}{\Sigma x} \times 360^\circ = \frac{35}{90} \times 360$

$\frac{x_s}{\Sigma x} \times 360 = \frac{12}{90} \times 360$

$\frac{12}{90} \times 360 + \frac{20}{90} \times 360 = 128^\circ$

$\frac{(35-12)}{90} \times 360 = 92^\circ$

Q: For data on frequency distribution of weights:

70, 73, 49, 57, 55, 44, 56, 71, 65, 62, 60, 50, 55, 49, 63, and 45

If we assume class length as 5 the number of class intervals would be

(a) 5

(b) 6

(c) 7

(d) 8

$$\text{No. of classes} = \frac{\text{Range}}{C} = \frac{73-44}{5}$$

Q: A pie diagram is used to represent the following data:

$$Ex = 120+180+240+180$$

Source	Customs	Excise	Income tax	Wealth tax
Revenue in million rupees	120	180	240	180

The central angles in the pie diagram corresponding to income tax and wealth tax respectively:

(a) (120°, 90°)

(b) (90°, 120°)

(c) (60°, 120°)

(d) (90°, 60°)

$$\frac{240}{720} \times 360 = 120^\circ$$

Q: If class interval is **10 - 14**, **15 - 19**, **20 - 24**, then the first class is

(a) **10 - 15**

(b) **9.5 - 14.5**

(c) **10.5 - 15.5**

(d) **9 - 15**

9.5 — 14.5

Q: The following data relates to the marks of a group of students:

Marks	No. of Students
More than 70%	07
More than 60%	18
More than 50%	40
More than 40%	60
More than 30%	75
More than 20%	100

$$100 - 40 = 60$$

How many students have got marks less than 50%?

(a) 60

(b) 82

(c) 40

(d) 53

Q: The number of car accidents in seven days in a locality are given below:

No. of accidents	0	1	2	3	4	5	6	7
Frequency	12	9	11	13	8	9	6	3

What will be the number of cases when 4 or more accidents occurred?

(a) 32

(b) 41

(c) 26

(d) 18

8+9+6+3

=

Q: Frequency density corresponding to class interval is the ratio of:

(a) **Class frequency to the total frequency**

(b) **Class frequency to the class length**

(c) **Class length to the class frequency**

(d) **Class frequency to the cumulative frequency**

Q: 'Stub' of a table is the

(a) **Left part of the table describing the columns**

(b) **Right part of the table describing the columns**

(c) **Right part of the table describing the rows**

(d) **Left part of the table describing the rows**

Q: In a graphical representation of data, the largest numerical value is 45, the smallest numerical value is 25. If the classes desired are 4, then which class interval is:

(a) 45

(b) 5

(c) 20

(d) 7.5

L = 45

S = 25

no: of classes = 4

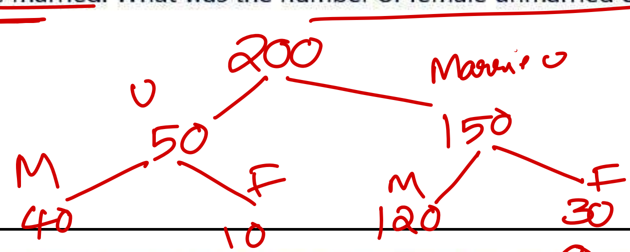
Q: There were 200 employees in an office in which 150 were married. Total male employees were 160, out of which 120 were married. What was the number of female unmarried employees?

(a) 30

(b) 40

(c) 50

(d) 10



Q: A student's marks in five subjects S1, S2, S3, S4, and S5 are **86, 79, 90, 88, and 89**. If we need to draw a **Pie chart** to represent these marks, then what will be the **Central angle** for **S3**?

(a) 103.2°

(b) 75°

(c) 105.6°

(d) 94.8°

$$\frac{90}{432} \times 360 = 75^\circ$$

Q: The suitable formula for computing the number of class intervals is:

(a) 3.322 logN

(b) 0.322 logN

(c) 1 + 3.322 logN

(d) 1 - 3.322 logN

Sampling

1. Need of Sampling

Sampling is required because:

- In many situations, we want to study a **large or infinite population.**
- Practical issues like **time, cost, efficiency**, and **vastness** of population make complete enumeration impossible.
- Therefore, we select a **representative part of the population (sample)**.
- Using information from the sample, we **infer** characteristics of the **unknown population**.

2. Term – Population

Population / Universe

- Aggregate of **all units** under consideration.
- Number of units in population = **Population Size**.
- Population may be:
  - **Finite or Infinite**
  - **Existent or Hypothetical**

Types of Population

a) Existent Population

- Consists of **real, tangible units**.
- **Example:** Population of lamps produced by a manufacturer.

b) Hypothetical Population

- Does **not physically exist**, only imagined.
- **Example:** Population of heads when a coin is tossed infinitely.

---

3. Term – Sample

- A **subset** of population selected to represent the entire population.
- Choosing a proper representative sample is **very important** because inferences are based only on sample data.
- Number of units in a sample = **Sample Size**.
- Units in sample = **Sampling Units**.
- Complete list of all sampling units = **Sampling Frame**.

---

4. Term – Parameter

- A **characteristic of a population** (based on all units).
- Inferences are drawn about population parameters using **sample observations**.
- Examples:
  - **Population Mean**

- **Population Variance**

---

5. Term – Statistic / Estimator

- A **statistical measure** computed from sample observations.
- A statistic is a **function** of sample values.
- If sample values are $x_1, x_2, ..., x_n$, then:

$$T = f(x_1, x_2, ..., x_n)$$

- Used to **estimate** population parameters.
- Examples:
  - Sample mean, sample variance, sample proportion.

---

6. Sampling Fluctuations

- Sample statistics vary from one sample to another.
- This variation is called **Sampling Fluctuation**.
- Occurs because sample units change from sample to sample.

---

7. Sampling Distribution, Expectation, and Standard Error

- If we list values of a statistic from **all possible samples** of fixed size, we get its **sampling distribution**.
- **Expectation** = Mean of the sampling distribution.
- **Standard Error (SE)** = Standard deviation of the sampling distribution.

Properties of SE

- Indicates **precision** of sampling.
- SE is **inversely proportional** to √(sample size).

$$\boxed{SE = \frac{\sigma}{\sqrt{n}}}$$

$$SE \propto \frac{1}{\sqrt{n}}$$

$$SE = \frac{1}{\sqrt{10000}} = 0.01$$

$$SE = \frac{1}{\sqrt{100}} = 0.1$$

8. Standard Error – Formulas

n → Sample Size

A. When Statistic is Mean

(i) Simple Random Sampling With Replacement (SRS WR)

$$\boxed{SE(\bar{x}) = \frac{\sigma}{\sqrt{n}}}$$

(ii) Simple Random Sampling Without Replacement (SRS WOR)

$$SE(\bar{x}) = \frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}$$

B. When Statistic is Proportion

(i) Simple Random Sampling With Replacement (SRS WR)

$$SE(p) = \sqrt{\frac{pq}{n}}$$

(ii) Simple Random Sampling Without Replacement (SRS WOR)

$$SE(p) = \sqrt{\frac{pq}{n}}\sqrt{\frac{N-n}{N-1}}$$

N → Population Size

Finite Population Correction (fpc)

$$fpc = \sqrt{\frac{N-n}{N-1}}$$

Used when sampling is **without replacement**.

Sample Survey

---

1. Meaning of Sample Survey

- Sample Survey is the **study of an unknown population** based on a **representative sample** drawn from it.
- It answers the question: *How can a part of the universe reveal the characteristics of the whole universe?*
- This is possible due to the **basic principles of sample survey**.

---

2. Basic Principles of Sample Survey

A. Law of Statistical Regularity

- If a **fairly large random sample** is selected from a population, then **on average** the sample will possess the characteristics of that population.
- Foundation principle for sample representativeness.

B. Principle of Inertia of Large Numbers

- Larger samples provide **more reliable, accurate, and precise results**, assuming other factors remain constant.
- This is a consequence of the Law of Statistical Regularity.

C. Principle of Optimization

- Ensures **optimum efficiency** at:
  - **Minimum cost**, or
  - **Maximum output** at a given cost.
- Achieved by selecting an **appropriate sampling design**.

D. Principle of Validity

- A sampling design is **valid only if**:
  - It provides **unbiased estimates**.
  - It enables valid tests about population parameters.
- **Only probability sampling** ensures validity.

---

3. Complete Enumeration

- When **every unit** of the population is studied, it is called **complete enumeration** or **census**.
- Provides full information, but often impractical.

---

4. Preference for Sample Survey over Complete Enumeration

A. Speed

- Much faster because only a **portion** of the population is studied.

B. Cost

- Cost per unit is higher in sample survey (requires trained personnel).
- But **overall cost is lower** because **fewer units** are covered.

C. Reliability

140

- More reliable results due to:
    - Trained enumerators
    - Better supervision
    - Use of modern techniques

D. Accuracy

7.5 8

- **Complete Enumeration**:
    - No sampling error
    - But **non-sampling errors** may occur (recording errors, bias, wrong interpretation).
- **Sample Survey**:
    - Sampling errors exist but can be **reduced** by:
        - Increasing sample size
        - Using proper sampling design
    - Non-sampling errors can also be controlled.

E. Necessity

- Sampling is **mandatory** when:
    - The process is **destructive** (e.g., testing bulb lifespan).

- Population is **hypothetical** (e.g., infinite coin tosses).
- In such cases, census is impossible.

Note: When population size is small, sampling is not needed.

Errors in Sample Survey & Types of Sampling

---

1. Errors in Sample Survey

Meaning

- Errors or biases in a survey are defined as the **difference between the population parameter** and the **value obtained from the sample**.
- Since only a **part** of the population is studied, sampling errors are unavoidable.

A. Sampling Errors

These errors occur **only in sample surveys**, not in complete enumeration.

1. Errors due to Defective Sampling Design

- If a **non-probabilistic sampling design** is used, sampler's bias or prejudice affects the results.

2. Errors due to Substitution

- Enumerators sometimes replace the selected unit with another unit for convenience.
- Since sampling design is not strictly followed, this leads to **bias**.

3. Errors from Wrong Choice of Statistic

- Incorrect statistic chosen for estimating a population parameter results in wrong conclusions.
- Example: using mean when median is more appropriate.

4. Variability in the Population

- When population units vary widely, errors may increase.
- Can be reduced by using complex sampling designs:
  - **Stratified Sampling**

- Multistage Sampling

B. Non-Sampling Errors

These errors occur in **both** sample surveys and complete enumeration.

Causes of Non-Sampling Errors

- Lapse of memory
- Ignorance, carelessness
- Biases of enumerators/interviewers
- Psychological factors (vanity, preference for certain answers)
- Non-responses from interviewees
- Wrong measurement of units
- Communication gaps
- Incomplete coverage

📘 2. Types of Sampling

Sampling techniques are classified as:

A. Probability Sampling

- Each member of population has a **known and non-zero probability** of being selected.
- Common types:
  - Simple Random Sampling
  - Stratified Sampling
  - Multi-Stage Sampling
  - Cluster Sampling
  - Systematic Sampling (partly)

B. Non-Probability Sampling

- Units are selected based on **judgment** or **convenience** of the sampler.
- No known probability of inclusion.
- Also called **Purposive Sampling** or **Judgement Sampling**.

C. Mixed Sampling

- Combines both probability and non-probability elements.
- Systematic sampling often falls under this category.

📘 3. Commonly Used Sampling Processes

A. Simple Random Sampling (SRS)

Meaning

Each unit in the population has an **equal chance** of selection.

Characteristics

- May be **with** or **without** replacement.
- Very simple and effective when:
    a. Population is not very large
    b. Sample size is not very small
    c. Population is homogeneous
- Free from sampler's bias.
- All significance tests are based on SRS theory.

B. Stratified Sampling

Used when the population is **large and heterogeneous**.

Steps

- Divide population into **strata** (homogeneous subgroups).

- Select samples from each stratum.

Purpose

1. To represent all sub-populations.
2. To obtain estimates for each stratum as well as the whole population.
3. To reduce variability → increases precision.

Allocations

- **Proportional Allocation:** Sample sizes in strata proportional to population size.
- **Bowley's Allocation:** Variation of proportional allocation.
- **Neyman's Allocation:** When strata variances differ significantly; sample sizes based on population size × standard deviation.

Not advisable when:

1. Population is small
2. No prior information available
3. No clear differences among population units

C. Multistage Sampling

Used for large, widely spread populations.

Features

- Sampling done in **stages** (first-stage units → second-stage → third-stage, etc.).
- Large coverage.
- Saves cost and effort.
- More flexible than stratified sampling.
- But usually **less accurate** than stratified sampling.

D. Systematic Sampling

Sampling at **regular intervals** after selecting the first unit randomly.

Features

- First unit selected randomly → next units selected at fixed intervals.

- Partly probability (first unit random) and partly non-probability (subsequent units fixed-rule).
- Very simple and cost-effective.
- Requires updated sampling frame.
- **Drawback:** If sampling interval coincides with hidden periodicity in population → very biased results.

E. Purposive / Judgement Sampling

Meaning

Sampler selects units **based on judgment** and belief.

Characteristics

- Completely subjective.
- Non-probability method.
- Cannot be used for statistical testing or drawing valid population conclusions.
- Results differ from person to person.

3. Parameter

- A **parameter** is a characteristic of a population based on all its units.
- Parameters are estimated from **sample statistics.**

(a) Population Mean (μ)

- Represents the **average value** of a population characteristic.
- **Formula:**

$$\mu = \frac{\sum_{a=1}^{n} x_a}{N}$$

Where:

- **N** = Population size
- x_a = Value of the a-th unit in the population

(b) Population Proportion (P)

- Represents the **ratio of units with a particular attribute** in the population.
- **Formula:**

$$P = \frac{X}{N}$$

Where:

- **X** = Number of individuals with the attribute
- **N** = Total population size

(c) Population Variance (σ^2)

- Measures the **spread of data points** in a population.
- **Formula:**

$$\sigma^2 = \frac{\sum(x_a - \mu)^2}{N}$$

(d) Population Standard Deviation (σ)

- Represents the **average deviation from the mean.**
- **Formula:**

$$\sigma = \sqrt{\frac{\sum(x_a - \mu)^2}{N}}$$

Sampling Distribution and Standard Error of a Statistic

1. Concept of Sampling Distribution

- When selecting a **sample of fixed size (n)** from a **population of size (N)**:
  - **With Replacement (SRSWR):** Total possible samples = **N^n**.
  - **Without Replacement (SRSWOR):** Total possible samples = **N_C_n (combinations of N taken n at a time).**

- **Sampling Fluctuations:**
  - The value of a sample statistic (e.g., mean) varies from sample to sample.
  - This variation creates a **sampling distribution**, a probability distribution of the statistic.

2. Properties of Sampling Distribution

- The **mean** of the sampling distribution is called **Expectation (E)**.
- The **Standard Deviation** of the sampling distribution is called the **Standard Error (SE)**.
- **SE measures the precision** of a sample statistic.
- **SE is inversely proportional to the square root of the sample size (n).**

3. Standard Error (SE) Formulas

(a) Standard Error of Mean (SE of x̄)

1. **For Simple Random Sampling With Replacement (SRSWR):**

$$SE(\bar{x}) = \frac{\sigma}{\sqrt{n}}$$

2. **For Simple Random Sampling Without Replacement (SRSWOR):**

$$SE(\bar{x}) = \frac{\sigma}{\sqrt{n}} \times \sqrt{\frac{N-n}{N-1}}$$

- The term √((N-n)/(N-1)) is the **finite population correction (fpc)**.
- If **n is very large** or **N is infinite**, the fpc ≈ 1 and can be ignored.

(b) Standard Error for Proportion (SE of P)

1. For Simple Random Sampling With Replacement (SRSWR):

$$SE(P) = \sqrt{\frac{Pq}{n}}$$

2. For Simple Random Sampling Without Replacement (SRSWOR):

$$SE(P) = \sqrt{\frac{Pq}{n} \times \frac{N-n}{N-1}}$$

- Where:
  - P = Sample proportion
  - q = 1 - P (Complement of P)
  - n = Sample size

4. Importance of Standard Error

- **Indicates precision:** Smaller SE means higher precision.
- **Helps in hypothesis testing:** Used in confidence intervals and significance tests.
- **Accounts for finite population effects:** The **finite population correction (fpc)** is applied for **sampling without replacement** but can be ignored for very large populations.

Chapter 17: Correlation and Regression

(5

$x \rightarrow$ Her interest ↑ ↓

$y \rightarrow$ Him talking ↑ ↑

Correlation

Correlation is a statistical tool used to measure and evaluate the extent to which two variables are related. For example, it helps in understanding how the value of one variable (e.g., y) changes when there is a change in another variable (e.g., x).

↑ ↓

↑ ↑

↑ ↑

1 0

-1

Correlation Coefficient (r)

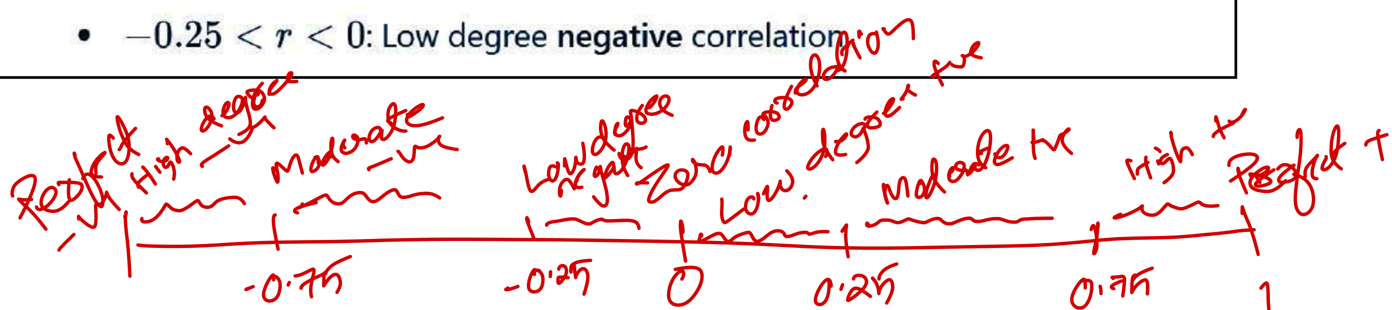
The correlation coefficient (r) is a statistical measure used to determine the strength and direction of a linear relationship between two variables.

Key Features:

- It helps to understand whether there exists a **high**, **moderate**, or **low** degree of correlation between two variables.
- The value of r ranges between **-1** and **+1**, where:
  - $r = 1$: Perfect positive correlation.
  - $r = -1$: Perfect negative correlation.
  - $r = 0$: No correlation or zero correlation.

Range and Interpretation of r:

1. **Perfect Positive Correlation ($r = 1$)**:
   - A unit increase in one variable results in an equal unit increase in the other variable.
   - The data points lie perfectly on a straight, upward-sloping line.
2. **Perfect Negative Correlation ($r = -1$)**:
   - A unit increase in one variable results in an equal unit decrease in the other variable.
   - The data points lie perfectly on a straight, downward-sloping line.
3. **Low Degree Correlation**:
   - $0 < r < 0.25$: Low degree **positive** correlation.
   - $-0.25 < r < 0$: Low degree **negative** correlation.



4. **Moderate Degree Correlation:**
    - $0.25 < r < 0.75$: Moderate degree **positive** correlation.
    - $-0.75 < r < -0.25$: Moderate degree **negative** correlation.
5. **High Degree Correlation:**
    - $0.75 < r < 1$: High degree **positive** correlation.
    - $-1 < r < -0.75$: High degree **negative** correlation.
6. **Zero Correlation ($r = 0$):**
    - No linear relationship exists between the variables.

Significance of Correlation:

- A high correlation (r close to 1 or -1) indicates a strong relationship.
- A low correlation (r near 0) indicates a weak relationship.
- The sign of r ($+$ or $-$) indicates the direction of the relationship.

Spurious Correlation

Definition:

Spurious correlation refers to a statistical relationship between two variables that appear to be related but lack a direct causal connection. The observed correlation exists due to coincidence, the influence of a third variable, or data anomalies, rather than any real association between the variables.

Examples:

1. **Age and Height:** There might be a correlation between age and height during childhood due to natural growth. However, for adults, this relationship does not imply causation. The observed correlation in such cases is incidental.
2. **Height and Weight:** While taller individuals might weigh more on average, this does not imply a direct causal relationship. The correlation might result from other underlying factors such as body composition or health.

Q) Which of the following is spurious correlation?

(a) Correlation between two variables having no causal relationship

(b) Negative correlation

(c) Bad relation between two variables

(d) Very low correlation between two variables

Measures of Correlation

The following are the primary methods used to measure the strength and nature of the correlation between variables:

(a) Scatter Diagram (Not finding the value of r)

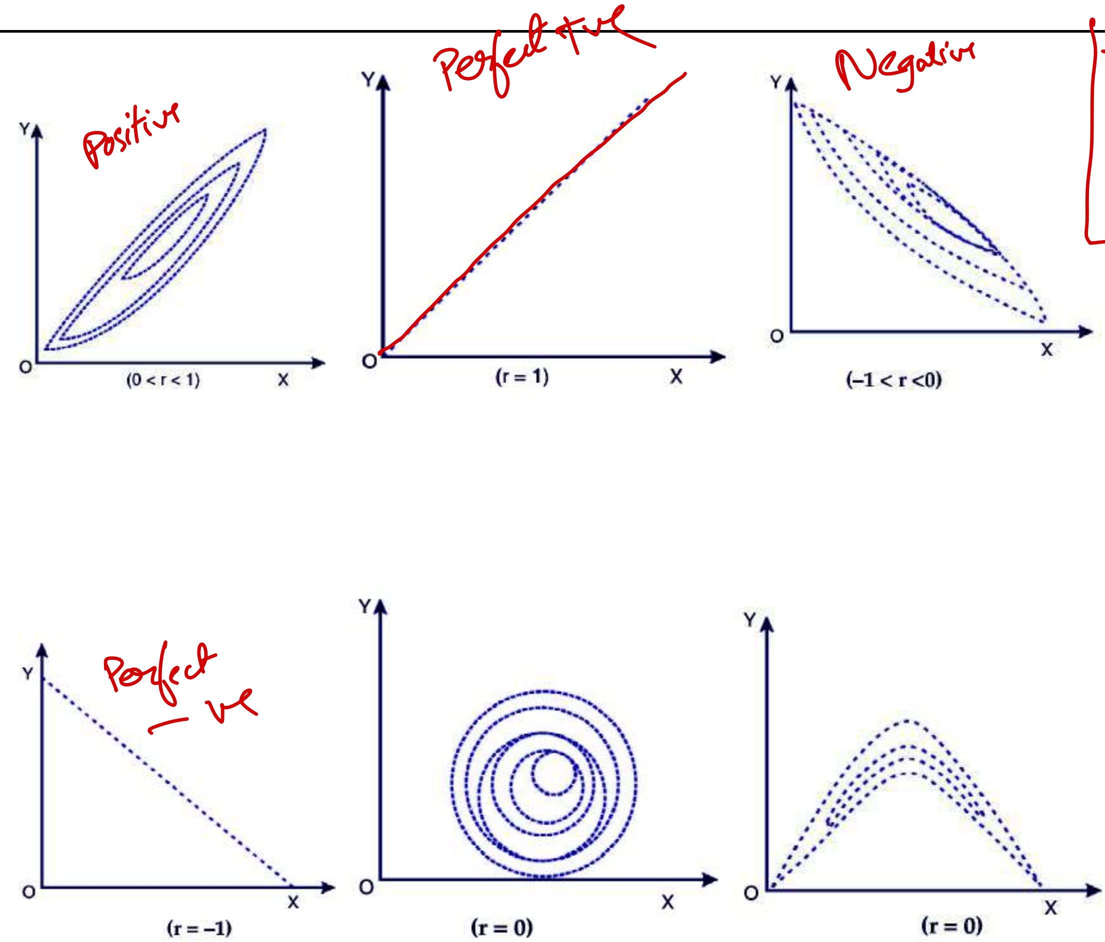
(b) Karl Pearson's Product-Moment Correlation Coefficient (r)

(c) Spearman's Rank Correlation Coefficient (r)

(d) Coefficient of Concurrent Deviations (r)

(a) Scatter Diagram

- A graphical representation of the relationship between two variables.
- Each point on the graph represents an observation.
- Patterns in the points (e.g., upward or downward trends) indicate the type and strength of the correlation.
  - Positive slope: Positive correlation.
  - Negative slope: Negative correlation.
  - Random distribution: No correlation.



Q) Scatter diagram does not help us to:

(a) Find the type of correlation

(b) Identify whether variables are correlated or not

(c) Determine the linear or non-linear correlation

(d) Find the numerical value of the correlation coefficient

Q) Scattered diagram is used to plot:

(a) Quantitative data

(b) Qualitative data

(c) Discrete data

(d) Continuous data

Q) Correlation coefficient between X and Y will be negative when:

(a) X and Y are decreasing

(b) X is increasing, Y is decreasing

(c) X and Y are increasing

(d) None of these

Q) Price and Demand is the example for:

(a) No correlation

(b) Positive correlation

(c) Negative correlation

(d) None of the above

P ↑ D ↓

Perfect +ve

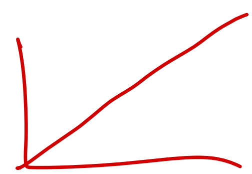
Q) When the correlation coefficient r is equal to $+1$, all the points in a scatter diagram would be:

(a) On a straight line directed from upper left to lower right

(b) On a straight line directed from lower left to upper right

(c) On a straight line

(d) Both (a) and (b)



Q) The coefficient of correlation between the temperature of the environment and power consumption is always:

(a) Positive

(b) Negative

(c) Zero

(d) Equal to 1

T ↑ P ↑

Q) If the plotted points in a scatter diagram are evenly distributed, then the correlation is:

Symmetrical

(a) Zero

(b) Negative

(c) Positive

(d) (a) Or (b)

(b) Karl Pearson's Product-Moment Correlation Coefficient

($r = ?$)

Key Concepts:

1. **Definition**: It measures the strength and direction of the linear relationship between two variables.
2. **Linear Relation**: If a linear relationship exists, KPCC is the best measure.
   - If $r_{xy} = 0$, there is no linear relationship between x and y.

$$\boxed{r_{xy} = \frac{\text{Cov}(x,y)}{S_x \cdot S_y}}$$

variance = $\frac{\Sigma x \cdot x}{n} - \frac{\Sigma x}{n} \times \frac{\Sigma x}{n}$

$\text{Cov}(x,y) = \frac{\Sigma xy}{n} - \frac{\Sigma x \Sigma y}{n^2}$

$S_x = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$

$S_y = \sqrt{\frac{\Sigma y^2}{n} - \left(\frac{\Sigma y}{n}\right)^2}$

- r_{xy}: Correlation coefficient.
- $\text{Cov}(x, y)$: Covariance between x and y.
- S_x, S_y: Standard deviations of x and y, respectively.

Expanded formula:

$$r_{xy} = \frac{n\sum xy - (\sum x)(\sum y)}{\sqrt{[n\sum x^2 - (\sum x)^2][n\sum y^2 - (\sum y)^2]}}$$

When Deviations Are Taken From the Mean:

$$r_{uv} = \frac{\sum dx \cdot dy}{\sqrt{\sum dx^2 \cdot \sum dy^2}}$$ Where:

- dx: Deviation of x from its mean $(x - \bar{x})$.
- dy: Deviation of y from its mean $(y - \bar{y})$.
- $\sum dx \cdot dy$: Sum of the product of deviations.
- $\sum dx^2$: Sum of squared deviations in x.
- $\sum dy^2$: Sum of squared deviations in y.

Properties of Correlation Coefficient

1. **Independence from Units of Measurement**:
   - The correlation coefficient (r_{xy}) does not depend on the units of measurement of the variables x and y. For example, whether the variables are measured in meters, kilograms, or any other unit, the value of r_{xy} remains the same.
2. **Shift of Origin and Scale**:
   - The correlation coefficient remains **unchanged in magnitude** when there is a shift in the origin (adding or subtracting a constant to the values of x or y).
   - It may or may not change in terms of **sign** due to a change in scale (multiplying the values of x or y by a positive or negative constant).
3. **Relationship Transformation**:
   - If x and u are related by $u = ax + b$ (linear transformation, equation 1) and y and v are related by $v = cy + d$ (linear transformation, equation 2):
   - The correlation between u and v remains the same as that between x and y if the signs of the constants a and c are **the same**.

$$r_{uv} = r_{xy}$$

   - If the signs of a and c are **different**, the correlation coefficient between u and v becomes the negative of the correlation coefficient between x and y:

$$r_{uv} = -r_{xy}$$

Implications:

- The correlation coefficient is robust to linear transformations, which makes it a versatile measure to evaluate relationships in datasets where variables are scaled or shifted.
- The sign of the correlation may flip if the transformations involve a reversal of signs in the scaling factors.

Q) If the sum of the product of deviations of x and y series from their means is zero, then the coefficient of correlation will be

(a) 1
(b) -1
(c) 0
(d) None of these

$\Sigma dxdy = 0$

$r = \frac{\Sigma dxdy}{\sqrt{\Sigma dx^2 dy^2}} = 0$

Q) The covariance between two variables X and Y is 8.4 and their variances are 25 and 36 respectively. Calculate Karl Pearson's coefficient of correlation between them.

(a) 0.82
(b) 0.28
(c) 0.01
(d) 0.09

$S_x = \sqrt{25} = 5$

$r = \frac{cov(x,y)}{S_x \times S_y} = \frac{8.4}{5 \times 6} = 0.28$

Q) The coefficient of correlation between two variables x and y is 0.28. Their covariance is 7.6. If the variance of x is 9, then the standard deviation of y is:

(a) 8.048
(b) 9.048
(c) 10.048
(d) 11.048

$r = 0.28$

$cov(x,y) = 7.6$

$S_x = \sqrt{9} = 3$ $S_y = ?$

$r = \frac{cov(x,y)}{S_x S_y}$ $0.28 = \frac{7.6}{3 \times S_y}$

$S_y = 9.048$

Q) Determine the coefficient of correlation between x and y series:

	x Series	y Series
No. of items	15	15
Arithmetic Mean	25	18
Sum of Squares of Deviations from Mean	136	138

Sum of products of Deviations of x and y series from Mean = 122

(a) -0.89
(b) 0.89
(c) 0.69
(d) -0.69

$\Sigma dx^2 = 136$ $\Sigma dy^2 = 138$

$\Sigma dxdy = 122$

$r = \frac{\Sigma dxdy}{\sqrt{\Sigma dx^2 \Sigma dy^2}} = \frac{122}{\sqrt{136 \times 138}} = 0.89$

$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{(n\sum x^2 - (\sum x)^2)(n\sum y^2 - (\sum y)^2)}}$

$n = 5$

Q) What is the coefficient of correlation from the following data?

x	1	2	3	4	5
y	5	4	3	2	6

$\sum xy = 1\times5$ M+ 2×4 M+ 3×3 M+ 4×2 M+ 5×6 M+ MRC $= 60$

$\sum x = 15$

$\sum y = 20$

$5\times60 - 15\times20 = 0$

(a) 0
(b) -0.75
(c) -0.85
(d) 0.82

Q) For the set of observations $\{(1,2),(2,5),(3,7),(4,8),(5,10)\}$, the value of Karl Pearson's coefficient of correlation is approximately given by: $n = 5$

x	1	2	3	4	5
y	2	5	7	8	10

$\sum xy = 115$ $\quad \sum x = 15$ $\quad \sum y = 32$

$\sum x^2 = 55$ $\quad (\sum x)^2 = 225$ $\quad \sum y^2 = 242$ $\quad (\sum y)^2$

1×2 M+ 2×5 M+ MRC 3×7 M+ 4×8 M+ 5×10 M+

(a) 0.755
(b) 0.655
(c) 0.525
(d) 0.985

Q) _____ may be defined as the ratio of covariance between the two variables to the product of the standard deviations of the two variables.

$\frac{cov}{S_x S_y}$

(a) Scatter diagram
(b) Karl Pearson's correlation coefficient
(c) Spearman's correlation coefficient
(d) Coefficient of concurrent deviations

Nature of Correlation (r) from Linear Equation

Concept:

The relationship between two variables x and y in the form of a linear equation $ax + by + c = 0$ can help determine whether the correlation between x and y is **perfect positive** or **perfect negative**.

Key Points:

1. **Perfect Positive Correlation**:
   - If increasing the value of x causes y to increase, the correlation is perfect positive.
   - This happens when the signs of a and b are **similar** (both positive or both negative).
2. **Perfect Negative Correlation**:
   - If increasing the value of x causes y to decrease, the correlation is perfect negative.
   - This happens when the signs of a and b are **opposite** (one positive and one negative).

Shortcut Rule:

- **If the signs of x and y in the equation $y = a + bx$ are the same**, the correlation is **perfect positive.**
- **If the signs are different**, the correlation is **perfect negative.**

Q) If the correlation coefficient between x and y is r, then between $U = \frac{x-5}{10}$ and $V = \frac{y-7}{2}$ is:

(a) r

(b) $-r$

(c) $\frac{r-5}{2}$

(d) $\frac{r-7}{10}$

$U = \frac{x-5}{10}$

$V = \frac{y-7}{2}$

$U = 5x + 10$

$V = -5x + 9$

(c) Spearman's Rank Correlation Coefficient

$(r = ?)$

Spearman's rank correlation coefficient (r_r) is a non-parametric measure used to assess the strength and direction of the association between two variables. Instead of using raw data, it uses ranks assigned to the data values.

Key Uses:

1. Measures agreement or disagreement between variables.
2. Especially useful for ordinal data or when the relationship is monotonic (but not necessarily linear).
3. Easy to compute compared to Karl Pearson's correlation coefficient.

Formula for r_r:

1. With Ties:

$$r_r = 1 - \left[\frac{6\left(\sum d^2 + \boxed{\frac{\sum(t^3-t)}{12}}\right)}{n^3 - n}\right]$$

- $\sum d^2$: Sum of the squared rank differences.
  $d^2 = (x_R - y_R)^2$, where x_R and y_R are the ranks of variables x and y.
- t: Length of the tie.
- n: Number of observations.

2. Without Ties:

$$\boxed{r_r = 1 - \frac{6\sum d^2}{n^3 - n}}$$

- The term for tie correction $\left(\frac{\sum(t^3-t)}{12}\right)$ is omitted.

$$1 - \frac{6\sum d^2}{n^3 - n} = 1 - \frac{6 \times 38}{120} = -0.9$$

$n = 5$

x	50	100	70	20	80
y	78	40	90	95	45
x_R	4	1	3	5	2
y_R	3	5	2	1	4

$d = x_R - y_R$

$\sum d^2 =$ 4−3 x = m+
1−5 x = m+
3−2 x = m+
5−1 x = m+
2−4 x = m+
MRC

1	2	3	4	5
5	4	3	2	1

1. $r_r = -1$ **When the Ranks are in Reverse Order**
   - This happens when one variable's ranks are in exactly the opposite order to the other variable's ranks.
   - Example: If Variable X has ranks [1, 2, 3, 4] and Variable Y has ranks [4, 3, 2, 1], the rank correlation coefficient (r_r) will be -1, indicating a perfect negative correlation.
2. **The Association Between the Two Variables Need Not Be Linear**:
   - Unlike Pearson's correlation coefficient, Spearman's rank correlation coefficient does not require the relationship between the two variables to be linear.
   - It only measures the degree of monotonic association (a consistent direction of increase or decrease) between variables.
3. **The Sum of the Rank Differences Will Always Be Zero**:
   - In ranking, when you calculate the differences in ranks ($d_i = x_R - y_R$), the sum of these differences across all observations will always be zero.
   - Mathematically, $\sum d_i = 0$. This is because ranks are assigned relative to the same set of observations, and any increase in one rank is offset by a corresponding decrease in another.

Q) Ranks of ~~two characteristics~~ by two judges are in reverse order. Then find the value of Spearman rank correlation coefficient.

(a) -1

(b) 0

(c) 1

(d) 0.75

Q) If the rank correlation coefficient between marks in Management and Mathematics for a group of students is 0.6 and the sum of the squares of the difference in ranks is 66, then what is the number of students in the group?

(a) 9

(b) 10

(c) 11

(d) 12

$r_R = 1 - \frac{6\Sigma d^2}{n^3-n}$

$0.6 = 1 - \frac{6\times 66}{n^3-n}$

$\frac{396}{n^3-n} = 0.4$

option Hit using (b)

Q) The ranks of five participants given by two judges are:

$n=5$

Participants	A	B	C	D	E
Judge 1	1	2	3	4	5
Judge 2	2	5	4	3	1

Rank correlation coefficient between ranks will be:

(a) 1

(b) 0

(c) -1

(d) $\frac{1}{2}$

$r = 1 - \frac{6\Sigma d^2}{n^3-n}$

$= -0.4$

nearest -ve correlated value is -1

$\Sigma d^2 =$ 1-2 x = m+, 2-5 x = m+, 3-4 x = m+, 4-3 x = m+, 5-1 x = m+

MRC

$\times 6$

$\div 120$

+/-

+1

Q) When each individual gets the exactly opposite rank by the two judges, then the rank correlation will be _____.

(a) 0

(b) -1

(c) +1

(d) $\frac{1}{2}$

$\frac{4+5}{2} = 4.5$

x	20	30	40	30	70	99
y	100	51	51	81	65	96
x_R	6	4.5	3	4.5	2	1
Y_R	1	5.5	5.5	3	4	2

$r = 1 - \frac{6\left(\Sigma d^2 + \frac{\Sigma t^3 - t}{12}\right)}{n^3-n}$

$\boxed{\frac{\Sigma t^3 - t}{12}} = \frac{2^3-2}{12} + \frac{2^3-2}{12}$

$n=10$

Q) In a beauty contest, there were 10 competitors. Ranks of these candidates are assigned by two judges A and B. The sum of squares of differences of ranks is 44. The value of rank correlation is:

(a) 0.70

(b) 0.73

(c) 0.80

(d) 0.60

$$r = 1 - \frac{6\Sigma d^2}{n^3 - n}$$

$$= 1 - \frac{6 \times 44}{990}$$

Q) Rank correlation coefficient lies between:

(a) 0 to 1

(b) -1 to $+1$ inclusive of these values

(c) -1 to 0

(d) Both

(d) Coefficient of Concurrent Deviations

The **Coefficient of Concurrent Deviation** is one of the simplest methods to measure the relationship between two variables when we are not concerned about their magnitude but only about their direction of change. Here's the explanation:

Key Points:

1. **Concept**:
   - The method focuses on whether the two variables increase or decrease together, disregarding the magnitude of change.
   - A **positive sign** is assigned if the value is increasing compared to the previous value.
   - A **negative sign** is assigned if the value is decreasing compared to the previous value.
   - An **equal sign** (=) is assigned if there is no change.

2. **Concurrent Deviations:**
   - If the signs of deviations for both variables (X and Y) are the same, it is called a **concurrent deviation**.
   - For example, if both variables increase or decrease simultaneously, their deviations are concurrent.

Formula:

$$\boxed{r_{cd} = \pm\sqrt{\frac{2c - m}{m}}}$$

Where:

- c: Number of concurrent deviations (when signs match for X and Y).
- m: Total number of pairs of deviations ($m = n - 1$).
- n: Total number of observations.

How to Use:

1. **Count Concurrent Deviations:**
   - Check the direction of change for each observation pair and count the instances where X and Y have the same direction (both increase or decrease).
   - Assign c as the count of such concurrent deviations.
2. **Substitute Values:**
   - Calculate $m = n - 1$.
   - Substitute c and m into the formula.
3. **Interpret Sign:**
   - If $2c - m$ is **positive**, the sign of r_{cd} is **positive**.
   - If $2c - m$ is **negative**, the sign of r_{cd} is **negative**.

C=12

n=?

Interpretation:

- $r_{cd} = 1$: Perfect positive correlation (both variables consistently change in the same direction).
- $r_{cd} = -1$: Perfect negative correlation (one variable increases while the other decreases).
- $r_{cd} = 0$: No correlation between the variables.

Note:

- This method is less precise compared to Pearson's or Spearman's correlation coefficients.
- It is mostly used for quick or preliminary analysis where only the direction of change matters.

$m = n - 1$ $\boxed{n = m + 1}$

Q) If the concurrent coefficient is $\frac{1}{\sqrt{3}}$, and the sum of deviations is 6 for n pairs of data, what is the value of n?

LHS $= \frac{1}{\sqrt{3}} = 0.577$

3 Click $\sqrt{}$ ÷ =

$r = \frac{1}{\sqrt{3}}$ $c = 6$ $n = ?$

(a) 9
(b) 8
(c) 10
(d) 11

$r = \sqrt{\frac{2c - m}{m}} = \sqrt{\frac{2c - (n-1)}{n-1}} = \sqrt{\frac{12 - n + 1}{n-1}} = \sqrt{\frac{13-n}{n-1}} = 0.577 = \text{LHS}$

$\frac{1}{\sqrt{3}} = \sqrt{\frac{12 - n + 1}{n-1}}$

$\frac{1}{\sqrt{3}} = \sqrt{\frac{13-n}{n-1}}$

$m = n - 1$

Q) Find the coefficient of concurrent deviations for the data given:

Year	2015	2016	2017	2018	2019	2020
Demand	60	65	55	60	70	65
Supply	50	55	50	50	60	60
		+	−	+	+	−
		+	−	=	+	=

$n = 6$

(a) 0.447
(b) 0.500
(c) 0.400
(d) 0.600

$c = 3$ $m = 5$

$r = \pm\sqrt{\frac{2c - m}{m}} = \sqrt{\frac{6-5}{5}} = \sqrt{0.2} = 0.447$

Q) The number of concurrent deviations for the 10 pairs of observations is 4. Find the coefficient of concurrent deviation.

$n = 10 \quad m = 9$

$c = 4$

(a) $-\frac{1}{3}$

(b) $\frac{1}{3}$

(c) 0

(d) 0.5

$r = \pm\sqrt{\pm\frac{2c-m}{m}} = \sqrt{\frac{8-9}{9}}$

$= -\sqrt{\frac{1}{9}} = -\frac{1}{3}$

Probable Error (PE):

- **Definition:** It is used to determine the limits of the correlation coefficient.
- **Formula:**

$$\boxed{PE = 0.6745 \frac{(1 - r^2)}{\sqrt{n}}}$$

Where:

- r: Correlation coefficient
- n: Number of observations

- **Properties:**
  - PE is never negative.
  - If $r < PE$, there is no significant correlation.

Standard Error (SE):

- **Definition:** It provides an extended range for the probable error.
- **Formula:**

$$\boxed{SE = 1.5\,\text{PE}}$$

Significant Value of r:

- r is said to be significant if:

$$r > 6 \times PE$$

Coefficient of Determination (r^2): *

r

- **Definition**: Represents the proportion of explained variation in the dependent variable.
- **Value**: The square of the correlation coefficient, r^2.

Coefficient of Non-Determination:

- **Definition**: Represents the unexplained variation.
- **Formula**:

$$1 - r^2$$

May 25

Q) Find the probable error if $r = \frac{2}{\sqrt{10}}$ and $n = 36$.

(a) 0.6745

(b) 0.067

(c) 0.5287

(d) None

$PE = 0.6745 \frac{(1-r^2)}{\sqrt{n}}$

r 10 click √ ÷ = × 2

r² × = click +/- +1

$-r^2$

× 0.6745 ÷ 6

Q) A relationship $r^2 = 1 - \frac{500}{300}$ is not possible.

(a) True

(b) False

(c) Both

(d) None

$r^2 = 1 - \frac{500}{300}$

$r^2 = -0.66666$

$= \sqrt{-0.6666}$

imaginary no:

Q) The coefficient of determination is defined by the formula:

(a) $r^2 = 1 - \frac{\text{unexplained variance}}{\text{total variance}}$

(b) $r^2 = \frac{\text{explained variance}}{\text{total variance}}$

(c) Both (a) and (b)

(d) None

Q) If $r = 0.6$, then the coefficient of non-determination will be:

(a) 0.40

(b) -0.60

(c) 0.36

(d) 0.64

$1 - r^2$

$= 1 - 0.6^2$

$= 0.64$

Q) If $r = 0.6$, then the coefficient of determination is:

(a) 0.4

(b) -0.6

(c) 0.36

(d) 0.64

$r^2 = 0.6^2$

$= 0.36$

Regression Analysis

- **Purpose**: To find a linear relationship between two variables using the least squares method. This helps predict the value of one variable based on the value of another.

0.47

Love 100

x, y

$y =$

$x =$

$b_{yx} =$

$y = a + \boxed{b_{yx}}\, x$

$x = a + \boxed{b_{xy}}\, y$

$b_{xy} =$

Regression Line of y on x:

- **Formula:** $y = a + b_{yx} \cdot x$
- **Use:** To determine the probable value of y when x is known.
- **Explanation:**
  - Comparable to the equation of a straight line: $y = mx + c$.
  - b_{yx} is the slope (rate of change of y with respect to x).
  - a is the y-intercept.

$r \times \frac{S_y}{S_x} = \frac{Cov(x,y)}{S_x \cancel{S_y}} \times \frac{\cancel{S_y}}{S_x} = \frac{Cov(x,y)}{(S_x)^2}$

- **Calculation of b_{yx}:**

$$b_{yx} = r \cdot \frac{s_y}{s_x} = \frac{n\sum xy - (\sum x)(\sum y)}{n\sum x^2 - (\sum x)^2}$$

  - r: Correlation coefficient.
  - s_x, s_y: Standard deviations of x and y.
- **Note:** The slope of the x-axis is zero.

Regression Line of x on y:

- **Formula:** $x = a + b_{xy} \cdot y$
- **Use:** To determine the probable value of x when y is known.
- **Explanation:**
  - b_{xy} is the slope (rate of change of x with respect to y).
- **Calculation of b_{xy}:**

$$b_{xy} = r \cdot \frac{s_x}{s_y} = \frac{n\sum xy - (\sum x)(\sum y)}{n\sum y^2 - (\sum y)^2}$$

Key Notes:

1. Regression lines represent the line of best fit.
2. Regression coefficients (b_{yx} and b_{xy}):
   - They are derived from the correlation coefficient (r).
   - The regression coefficients are proportional to r and the ratio of standard deviations. $r = \pm\sqrt{b_{xy} b_{yx}}$

$b_{xy} = 4$ $\quad r = \sqrt{b_{xy} b_{yx}}$
$b_{yx} = \frac{1}{2}$ $\quad = \sqrt{4 \times \frac{1}{2}}$
$= \sqrt{2}$
$= 1.4$

3. $b_{yx} \cdot b_{xy} = r^2$, which shows the relationship between both regression coefficients and the correlation coefficient.

Properties of Regression Coefficients:

1. **Effect of Origin and Scale:**
   - Regression coefficients $\boxed{\text{remain unchanged}}$ when the origin is shifted (additive constant).
   - Regression coefficients change when the scale (multiplicative constant) is altered.

- If transformations $u = \frac{x-a}{b}$ and $v = \frac{y-c}{d}$ are applied:

$$b_{uv} = \frac{d}{b} b_{xy}$$

Finding Regression Line of y on x:

- When regression coefficients b_{yx}, b_{xy}, and means $\overline{x}, \overline{y}$ are given:
  - The regression equation of y on x is:

$$(y - \overline{y}) = b_{yx}(x - \overline{x})$$

  - This is useful when we want to predict y based on the values of x.

Relationship Between Regression Coefficients and Correlation Coefficient:

1. **Correlation Coefficient as the Geometric Mean:**
   - The correlation coefficient (r) is the geometric mean (GM) of the two regression coefficients b_{xy} and b_{yx}:

$$r = \pm\sqrt{b_{xy} \cdot b_{yx}}$$

2. **Sign of r:**
   - r is positive (+) if both b_{xy} and b_{yx} are positive.
   - r is negative (−) if both b_{xy} and b_{yx} are negative.

Key Notes:

1. Regression coefficients indicate the slope or the rate of change of one variable with respect to another.
2. The correlation coefficient measures the strength and direction of the linear relationship between two variables.
3. While regression coefficients are directional (dependent on which variable is independent or dependent), the correlation coefficient is symmetrical and remains the same regardless of the choice of x or y.

Finding r When Regression Lines Are Given

Procedure:

1. **Assumption:**
   - Assume one regression line represents y on x, given by $y = a + b_{yx} \cdot x$.
   - Assume the other regression line represents x on y, given by $x = a + b_{xy} \cdot y$.

2. **Determine Regression Coefficients:**
   - Compute b_{yx} and b_{xy} from the respective regression equations.
3. **Calculate Correlation Coefficient r:**
   - Use the formula:

$$r = \pm\sqrt{b_{xy} \cdot b_{yx}}$$

If b_{xy} and b_{yx} are negative $r = -$

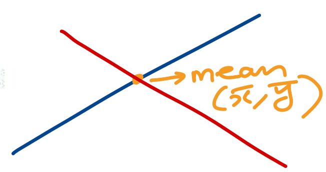
4. **Verify Assumptions:**
   - If r lies within the valid range $[-1, 1]$, the assumptions are correct.
   - If not, reverse the assumptions:
     - Swap $b_{yx} = \frac{1}{b_{xy}}$ and $b_{xy} = \frac{1}{b_{yx}}$.

Point of Intersection of Regression Lines

- The point where the regression lines intersect represents the **mean values** of x and y:

$$(\overline{x}, \overline{y})$$

Value of r Based on the Angle Between Regression Lines



1. **Orthogonal Regression Lines (90°):**
   - If the angle between regression lines is 90°, $r = 0$, indicating no correlation.
2. **Coincident or Identical Regression Lines:**
   - If the regression lines overlap (angle is 0°), $r = \pm 1$, indicating perfect positive or negative correlation.

$r = 0$

$r = \pm 1$

Key Notes:

1. The relationship between r and the slopes b_{yx}, b_{xy} depends on their product.
2. $r = 0$ represents no linear relationship, while $r = \pm 1$ represents a perfect linear relationship.
3. Intersection points of regression lines provide insights into central tendencies of the data.

Q: The two regression equations are:

y on x

$2x + 3y + 18 = 0$

x on y

$x + 2y - 25 = 0$

Find the value of y if $x = 9$.

$b_{yx} = -\frac{2}{3}$ $\quad r = -\sqrt{\frac{4}{3}}$

$= -1.15$

$b_{xy} = -\frac{2}{1}$

our assumption is wrong

(a) -8

(b) 8

(c) -12

(d) 0

So $x + 2y - 25 = 0$ is y on x

In this eq substitute $x = 9$

Find y

$9 + 2y - 25 = 0$

$2y = 16$

$y = 8$

Q: Which of the following regression equations represent the regression line of Y on X:

x on y $\quad$ y on x

$7x + 2y + 15 = 0, 2x + 5y + 10 = 0$

b_{yx}

(a) $7x + 2y + 15 = 0$

(b) $2x + 5y + 10 = 0$

(c) Both (a) and (b)

(d) None of these

Assume $7x + 2y + 15$ is y on x

$b_{yx} = -\frac{7}{2}$ $\quad b_{xy} = -\frac{5}{2}$

$r = \sqrt{\frac{35}{4}} = 8.75$

Q: The two regression lines are $7x - 3y - 18 = 0$ and $4x - y - 11 = 0$. Find the values of b_{yx} and b_{xy}.

y on x $\quad$ x on y

(a) $\frac{7}{3}, \frac{1}{4}$

(b) $-\frac{7}{3}, -\frac{1}{4}$

(c) $-\frac{3}{7}, -\frac{1}{4}$

(d) None of these.

$b_{yx} = -\frac{7}{-3} = \frac{7}{3}$

$b_{xy} = -\frac{(-1)}{4}$

$b_{xy} = \frac{1}{4}$

$r = \sqrt{\frac{7}{3} \times \frac{1}{4}} = 0.763$

$\frac{\text{Coef of known}}{\text{Coef of unknown}}$

$\frac{b_{xy}+b_{yx}}{2}$ $> b_{xy} b_{yx}$

Q: ____ of the regression coefficients is greater than the correlation coefficient

(a) Combined mean

(b) Harmonic mean

(c) Geometric mean

(d) Arithmetic mean

A > G > H

Q: Regression coefficients are ____

(a) dependent on the change of origin and of scale.

(b) independent of both change of origin and of scale.

(c) dependent on the change of origin but not of scale.

(d) independent of the change of origin but not of scale.

Q: For a bivariate data, two lines of regression are

$40x - 18y = 214$ and $8x - 10y + 66 = 0$, then find the values of x and y.

(a) 17 and 13

(b) 13 and 17

(c) 19 and -17

(d) -13 and 17

Substitute options on equati–

option (b) $x=13$ $y=17$

$40\times13-18\times17=214$

$214=214$

$8\times13-10\times17+66$

$0=0$

Q: Out of the following, which one affects the regression coefficient?

(a) Change of origin only

(b) Change of scale only

(c) Change of scale & origin both

(d) Neither change of origin nor change of scale

Q: For a bivariate data, the lines of regression of Y on X, and of X on Y are respectively

$2.5Y - X = 35$ and $10X - Y = 70$, then the correlation coefficient r is equal to:

y on x x on y

(a) 0.2

(b) -0.2

(c) 0.5

(d) -0.5

$b_{yx} = \frac{-(-1)}{2.5} = 0.4$ $b_{xy} = \frac{1}{10}$

$r = \sqrt{0.4 \times 0.1} = 0.2$

Q: If 2 variables are uncorrelated their regression lines are:

$r = 0$

(a) Parallel

(b) Perpendicular

(c) Coincident

(d) Inclined at 45 degrees.

Q: Two regression lines for a bivariate data are:

$2x - 5y + 6 = 0$ and $5x - 4y + 3 = 0$.

Then the coefficient of correlation should be:

(a) $\frac{-2\sqrt{2}}{5}$

(b) $\frac{2}{5}$

(c) $\frac{+2\sqrt{2}}{5}$

(d) $\frac{\sqrt{2}}{5}$

Q: If the mean of the two variables 'x' and 'y' are 3 and 1 respectively, then the equation of the two regression lines are ____.

(a) $5x + 7y - 22 = 0, \quad 6x + 2y - 20 = 0$

(b) $5x + 7y - 22 = 0, \quad 6x + 2y + 20 = 0$

(c) $5x + 7y + 22 = 0, \quad 6x + 2y - 20 = 0$

(d) $5x + 7y + 22 = 0, \quad 6x + 2y + 20 = 0$

$x = 3 \quad y = 1$

$5 \times 3 + 7 \times 1 - 22 = 0$

$6 \times 3 + 2 - 20 = 0$

10:30 Am

Q: Two regression equations are as follows:

Regression equation of x on y: $5x - y = 22$

Regression equation of y on x: $64x - 45y = 24$

What will be the mean of x and y?

Option Hit

(a) $\bar{x} = 8, \bar{y} = 6$

(b) $\bar{x} = 6, \bar{y} = 6$

(c) $\bar{x} = 6, \bar{y} = 8$

(d) $\bar{x} = 8, \bar{y} = 8$

using (c)

Put $x=6$ $y=8$

$5 \times 6 - 8 = 22$

$64 \times 6 - 45 \times 8 = 24$

Q: The two lines of regression become identical when

(a) $r = 1$

(b) $r = -1$

(c) $r = 0$

(d) (a) or (b)

± 1

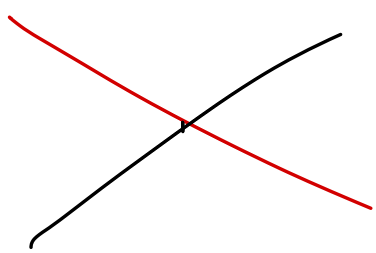
Q: The two regression lines passing through

(a) Represent means

(b) Represent S.Ds

(c) (a) and (b)

(d) None of these.



Q: Out of the following, the one which affects the regression coefficient is

(a) Change of origin only

(b) Change of scale only

(c) Change of scale and origin both

(d) Neither change in origin nor change of scale

Q: If the Regression coefficient (r_{yx}) of **y on x** is **greater than unity**, then the other Regression coefficient (r_{xy}) of **x on y** is:

(a) **Less than one**

(b) **Greater than one**

(c) **Equal to one**

(d) **Equal to zero**

upto 16

Chapter 4: Mathematics of Finance

Time Value of Money

- Money's value changes over time.
- Money received in the future is worth less than money received today.
- Today's money is more valuable than future money.
- Investors prefer to get money now rather than later.
- Delaying payments means paying extra, called interest.

Interest

SI and CI

- Interest is the extra money paid by a borrower to a lender for using their money.
- If you borrow (or lend) ₹50,000 for a year and pay (or receive) ₹55,000, the ₹5,000 difference is the interest.

Principal

- The principal is the original amount of money lent or borrowed.

Accumulated Amount (or Balance) A = P + I

- The accumulated amount is the total of the principal plus earned interest.
- If you deposit ₹50,000 at 5% interest for a year, you earn ₹2,500 in interest.
- After one year, the accumulated amount is ₹52,500 (principal + interest).

Basic Understanding of Accumulated Amount or Balance

- **Formula:** A = Principal + Interest → SI or CI
- **Interest Types:**
  - **Simple Interest:** Interest is calculated on the principal amount only.
  - **Compound Interest:** Interest is calculated on the principal amount and any previously earned interest.

Simple Interest

Definition

Simple interest is the additional money paid for using somebody else's money (principal) over a period of time at a simple rate.

Simple Interest (SI) =

$$\frac{P \times R \times T}{100} = PTR\%$$

$$A = P + PTR\%$$

$$A = P(1 + TR\%)$$

Where:

- **P** = Principal amount
- **R** = Rate of interest per annum
- **T** = Time period in years

$$\text{Amount}(A) = P + SI$$

$$\text{Amount}(A) = P + \frac{P \times R \times T}{100}$$

Basic Questions on Simple Interest (SI)

Type 1:

To find Simple Interest (SI) and Amount (A) when Principal (P), Time (T), and Rate (R) are given.

Type 2:

To find Principal (P) when Amount (A), Time (T), and Rate (R) are given.

Type 3:

a) To find Rate (R) when Amount (A), Principal (P), and Time (T) are given.

b) To find Time (T) when Amount (A), Principal (P), and Rate (R) are given.

Q) Simple interest on $\boxed{₹2,000}$ for 5 months at $\boxed{16\%}$ p.a. is:

(a) ₹133.33
(b) ₹133.26
(c) ₹134.00
(d) ₹132.09

$T = \frac{5}{12}$ $P = 2000$ $R = 16\%$

$SI = PTR\ \%$

$= 2000 \times 5 \div 12 \times 16$

Click %.

12 months = 1 year

1 month = $\frac{1}{12}$ year

5 month = $\frac{5}{12}$

$= 133.33$

SI=420

Q) How much investment is required to yield an annual income of ₹420 at 7% p.a. simple interest:

R

(a) ₹6,000

(b) ₹6,420

(c) ₹5,580

(d) ₹5,000

T=1

PTR% = SI

$\frac{420}{7\%} = 6000$

Option Hit

$6000 \times 7\% \times 1 = 420$

Q) If a simple interest on a sum of money at 6% p.a. for 7 years is equal to twice the simple interest on another sum for 9 years at 5% p.a., the ratio will be:

(a) 2 : 15

(b) 7 : 15

(c) 15 : 7 =2.14

(d) 1 : 7

$SI_1 = 2 SI_2$

$P_1T_1R_1 = 2 P_2T_2R_2$

$P_1 \times 7 \times 6 = 2 \times P_2 \times 9 \times 5$

$\frac{P_1}{P_2} = \frac{2 \times 9 \times 5}{7 \times 6} = 2.14$

Q) An amount is lent at **R**% simple interest for **R** years, and the simple interest amount was one-fourth of the principal amount. Then **R** is _____

(a) **5**

(b) **6**

(c) **5**$\frac{1}{2}$

(d) **6**$\frac{1}{2}$

$SI = \frac{1}{4}P$

$\frac{\cancel{P}TR}{100} = \frac{1}{4}\cancel{P}$

$R \times R = \frac{100}{4}$

$R^2 = 25 \quad R = 5$

$A = 10000$ $SI = 7000$

$P = 3000$ $R\% = ?$

$T = 5$ years

$A = P\left(1 + TR\%\right)$

$10000 = 3000\left(1 + 5 \times R\%\right)$

$$7000 = 3000 \times 5 \times \frac{R}{100}$$

$R = 46.66$

$R\% = 10\%$ $P = 1000$

$= \frac{10}{100}$

$= \frac{1}{10}$ $\xrightarrow{}$ SI

$\times 100$

11 is my A

$\times 100$

1100

100

$10x = 1000$

$x = 100$

$11x = 11 \times 100$

$= 1100$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ 5 and 6

Q) In simple interest, if the principal is ₹2,000 and the rate and time are the roots of the equation

Sum = 11
Product = 30

$$x^2 - 11x + 30 = 0$$

then the simple interest is:

$x^2 - (\text{Sum of roots})x + \text{Product of roots} = 0$

(a) ₹500

(b) ₹600

(c) ₹700

(d) ₹800

R = 5 and T = 6

SI = PTR%
= 2000 × 6 × 5%
= 600

Type 4

When two amounts, given in terms of the principal, along with the time for the first case are provided, the objective is to determine the time for the second case.

Q) A certain money doubles itself in 10 years when deposited on simple interest. It would triple itself in:

(a) 20 years

(b) 15 years

(c) 25 years

(d) 30 years

P 100 $\xrightarrow[SI_1 = 100]{T_1 = 10 \text{ year}}$ A 200

P 100 $\xrightarrow[SI_2 = 200]{T_2 = ?}$ A 300

$T_2 = \frac{SI_2}{SI_1} \times T_1 = \frac{200 \times 10}{100} = 20$

Q) A certain sum of money doubles itself in 6 years as per Simple Interest (SI). In what time will it become 4 times of itself?

A) 12 years

B) 18 years

C) 24 years

D) 30 years

6 ×3 → 18

100 $\xrightarrow[SI_1 = 100]{T_1 = 6}$ 200

×3

100 $\xrightarrow[SI_2 = 300]{T_2 = ?}$ 400

$T_2 = \frac{300}{100} \times 6 = 18$

$SI = \frac{PTR}{100}$

Type 5

When two amounts (in terms of Rs) and their respective time durations are given, the objective is to determine the principal (P) and the simple interest (SI) rate per annum.

Note:

SI rate per annum = $\frac{\text{SI for 1 year}}{P} \times 100$

$SI = \frac{P \times R}{100}$

$R = \frac{SI}{P} \times 100$

Q) What is the rate of simple interest if a sum of money amounts to **₹2,784** in **4 years** and ₹2,688 in **3 years**?

(a) 1% **p.a.**

(b) **4% p.a.**

(c) **5% p.a.**

(d) **8% p.a.**

$P \xrightarrow{3\text{ year}} 2688 \xrightarrow{1\text{ year}} 2784$ (4 years)

$SI_{1 year} = \frac{A_2 - A_1}{T_2 - T_1}$

$= 2784 - 2688$

$SI_1 = 96$

$P = A_1 - T_1 \times SI_1$

$= 2688 - 3 \times 96$

$= 2400$

$R = \frac{96}{2400} \times 100 = 4\%$

Q) The certain sum of money became **₹692 in 2 years** and **₹800 in 5 years**. Then the principal amount is:

(a) **₹520**

(b) **₹620**

(c) **₹720**

(d) **₹820**

$P \xrightarrow{2\text{ year}} 692 \longrightarrow 800$ (5 years)

$SI_1 \text{ year} = \frac{800 - 692}{3}$

$= 36$

$P = A_1 - T_1 \times SI_1$

$= 620$

1000 4 5%, 2000 6, 11%.
7000 7% 5.

Type 6

When the simple interest (SI) rate per annum varies for different years, the total SI is calculated as follows:

Total SI

(i) When the principal is the same

$$\text{Total SI} = \frac{P}{100}(T_1R_1 + T_2R_2 + T_3R_3 + \ldots)$$

(ii) When the principal amounts are different

$$\text{Total SI} = \frac{P_1T_1R_1 + P_2T_2R_2 + \ldots}{100}$$

Q) 1/7 of a money is deposited at 4% per annum, 1/2 of a money is deposited at 5% per annum, and the remaining at the rate of 6%. If the total interest gained is ₹730, find the deposit amount.

option Hit

(a) ₹14,000
(b) ₹215,500
(c) ₹212,800
(d) ₹214,500

If total principal = 14000

$P_1 = 2000$ $R_1 = 4\%$. $T_1 = T_2 = T_3 = 1$

$P_2 = 7000$ $R_2 = 5\%$.

$P_3 = 5000$ $R_3 = 6\%$.

RHS

$P_1T_1R_1\% + P_2T_2R_2\% + P_3T_3R_3\% = 730$

Type 7

To find the time (T) when an amount (in terms of principal, P) and the simple interest (SI) rate per annum are given, use the following formula:

$$T = \frac{\text{Total SI rate}}{\text{SI rate per annum}}$$

Q) In how much time does a sum of amount double at simple interest at a **12.5%** rate?

(a) **7 years**

(b) **8 years**

(c) **9 years**

(d) **10 years**

$100 \xrightarrow{R=12.5} 200$

$SI = 100$

$T = \frac{100}{12.5} = 8$ years

Q) In how many years will a sum of money become **four times** at **12% p.a.** simple interest?

(a) **18 years**

(b) **21 years**

(c) **25 years**

(d) **28 years**

P A

$100 \xrightarrow{R=12\%} 400$

$SI = 300$

$T = \frac{300}{12} = 25$

EXTRA QUESTIONS

Q) A man invests an amount of **₹15,860** in the names of his three sons **A, B, and C** in such a way that they get the **same interest** after **2, 3, and 4 years** respectively. If the rate of interest is **5%**, then the ratio of the amount invested in the name of **A, B, and C** is:

T_1 T_2 T_3

(a) **6 : 4 : 3**

(b) **3 : 4 : 6**

(c) **30 : 12 : 5**

(d) **None of the above**

$SI_1 = SI_2 = SI_3$

$P_1 T_1 \cancel{5\%} = P_2 T_2 \cancel{5\%} = P_3 T_3 \cancel{5\%}$

$P_1 \times 2 = P_2 \times 3 = P_3 \times 4$

$\frac{P_1}{P_2} = \frac{3}{2} = 1.5$

Q) A farmer borrowed ₹3,600 at the rate of 15% **simple interest per annum**. At the end of **4 years**, he cleared this account by paying **₹4,000** and a cow. The cost of the cow is:

(a) **₹1,000**

(b) **₹1,200**

(c) **₹1,550**

(d) **₹1,760**

P = 3600 R = 15% T = 4

A = 4000 + cow

A = P (1 + TR%)

4000 + cow = 3600 (1 + 0.6)

cow = 5760 − 4000

= 1760

Compound Interest

1000 $\xrightarrow{10\%}$ 1000 + 100 → 1000 100 10 → 1000 100 10 1

Definition

Compound interest is the interest calculated on the initial principal and also on the accumulated interest of previous periods. This means that interest is earned on both the initial principal and the interest that has been added to it over time.

Formula for Amount (A) in Compound Interest

$$\boxed{A = P(1 + i)^n}$$

1000 + 10% + 10% + 10%

Where:

R = 10% i = 0.1

R = 20% i = 0.2

- A = Final Amount
- P = Principal
- i = Rate of interest per period
- n = Number of periods

A = P + I

Formula for Compound Interest (CI)

$$\boxed{CI = P\left[(1+i)^n - 1\right]}$$

Where:

- P → Principal amount
- i → Interest rate (as a decimal or fractional value)
- n → Number of compounding periods or conversion periods

Types of Compoundings & Corresponding Conversions

(i) Annual (Yearly) Compounding

- Principal (P) changes once in a year.
- n = Number of years
- i = Interest rate per annum

(ii) Semi-Annual (Half-Yearly) Compounding

- Principal (P) changes twice in a year.
- $n = 2\times$ Number of years
- $i = \frac{\text{Interest rate per annum}}{2}$

(iii) Quarterly Compounding

- Principal (P) changes four times in a year.
- $n = 4\times$ Number of years 4n
- $i = \frac{\text{Interest rate per annum}}{4}$

(iv) Monthly Compounding

- Principal (P) changes twelve times in a year.
- $n = 12\times$ Number of years
- $i = \frac{\text{Interest rate per annum}}{12}$

(v) Daily Compounding

- Principal (P) changes daily.
- $n = 365\times$ Number of years (assuming 365 days in a year)
- $i = \frac{\text{Interest rate per annum}}{365}$

Type 1: Problems on Compound Interest (CI) and Amount (A)

This section deals with solving various types of problems related to **Compound Interest (CI)** and the **Final Amount (A)** accumulated over time. The problems typically involve calculations based on the compound interest formula and understanding different compounding periods.

Q) If compound interest on any sum at the rate of 5% for two years is ₹512.50, then the sum would be:

(a) ₹3,000

(b) ₹4,000

(c) ₹5,000

(d) ₹6,000

$$CI = P\left((1+i)^n - 1\right)$$

$$512.5 = P\boxed{\left((1.05)^2 - 1\right)}$$

$$1.05 \times = \; -1$$

$$\div =$$

$$\times 512.5$$

$$= 5000$$

Q) If ₹10,000 is invested at 8% per year compounded quarterly, then the value of the investment after 2 years is:

(given $(1+0.02)^8 = 1.171659$)

(a) ₹11,716.59

(b) ₹10,716.59

(c) ₹117.1659

(d) None of the above

$i = \frac{8}{400} = 0.02$

$n = 4 \times 2 = 8$

$A = P(1+i)^n = 10000\,(1.02)^8$

$= 11716.59$

Q) The sum invested at 4% per annum compounded semi-annually amounts to ₹7,803 at the end of one year. The sum is:

(a) ₹7,000

(b) ₹7,500

(c) ₹7,225

(d) ₹8,000

$A = P\left((1+i)^n\right)$

$i = \frac{4}{200} = 0.02$

$n = 2 \times 1 = 2$

$7803 = P(1.02)^2$

$1.02 \times = \div = \times 7803$

$= 7500$

Q) Mr. X bought an electronic item for ₹1,000. What would be the future value of the same item after 2 years, if the value is compounded semi-annually at 22% per annum?

(a) ₹1,488.40

(b) ₹1,518.07

(c) ₹2,008.07

(d) ₹2,200.00

$R = \frac{22}{2} = 11\%$

$n = 2 \times 2 = 4$

On Calulator

$1000 + 11\% + 11\% + 11\% + 11\%$

$= 1518.07$

Q) If in two years' time a principal of ₹100 amounts to ₹121 when the interest at the rate of r% is compounded annually, then the value of r will be:

(a) 10.5

(b) 10%

(c) 15

(d) 14

Q) At what % rate of compound interest (C.I) will a sum of money become 16 times in four years, if interest is being calculated compounding annually:

(a) r = 100%

(b) r = 10%

(c) r = 200%

(d) r = 20%

P

1 ⟶ $\boxed{16}$

option Hit

On calculator

1

+100%

+100%

+100%

+100%

=16 = RHS

Type 2: Finding A When P and Different Interest Rates for Different Years Are Given

In some cases, the interest rate varies from year to year instead of remaining constant throughout the investment period. To calculate the final amount (A) when different interest rates apply for different years, we use the following formula:

Formula:

$$A = P(1+i_1)(1+i_2)(1+i_3)\dots(1+i_n)$$

Where:

- A = Final amount
- P = Principal amount (initial investment)
- $i_1, i_2, i_3, \dots$ = Interest rates for each corresponding year (expressed as decimals)
- n = Total number of years

Q) How much will ₹25,000 amount to in 2 years at compound interest if the rates for the successive years are 4% and 5% per year?

P

(a) ₹27,300
(b) ₹27,000
(c) ₹27,500
(d) ₹27,900

25000
+4%
+5%
= 27300

Q) What will be the population after 3 years when the present population is ₹25,000 and the population increases at the rate of 3% in I year, at 4% in II year, and at 5% in III year?

(a) ₹28,119
(b) ₹29,118
(c) ₹27,000
(d) ₹30,000

25000
+3%
+4%
+5%
= 28119

P=1000
2years 11%
3years 12%
1000
+11%
+11%
+12%
+12%
+12%

Q) The present population of a town is 70,000. If it grows at 4%, 5%, and 6% per annum for the 1st, 2nd, and 3rd years respectively, then find the population of the town at the end of the 3rd year.

(a) **81,026**

(b) 79,856

(c) 80,450

(d) 78,920

70000

+4%

+5%

+6%

CI−SI

$= P((1+i)^n - 1) - PTR\%$

$= P((1+i)^n - 1 - TR\%)$

Type 3: Finding the Time Period for the Second Case

This type of problem involves determining the time period (T_2) for a second scenario when two amounts (expressed in terms of principal, P) and the time period of the first case (T_1) are given.

Q) A sum of money doubles itself in 4 years at a certain compound interest rate. In how many years will this sum become 8 times at the same compound interest rate?

(a) 12 Years

(b) 14 Years

(c) 16 Years

(d) 18 Years

Doubles 2 times → 4 year

8 times 2^3 times → 3×4 = 12

extra

16 times 2^4 times = 4×4 = 16

Q) A sum of money invested at compound interest ~~doubles itself in four years~~. It becomes 32 times of itself at the same rate of compound interest in:

(a) 12 years

(b) 16 years

(c) 20 years

(d) 24 years

2 times → 4 year

32 times 2^5 times = 5×4 = 20

EXTRA QUESTIONS

$\frac{114}{}$, $\frac{144}{R}$

Q) A sum amounts to ₹1,331 at a principal of ₹1,000 at 10% compounded annually. Find the time.

(a) 3.31 years
(b) 4 years
(c) 3 years
(d) 2 years

1000 +10%
+10%
+10%

Q) In how many years will a sum become double at 5% p.a. compound interest?

(a) 14.0 years
(b) 15 years
(c) 16 years
(d) 14.3 years

Rule of 72

$\frac{72}{5} = 14.4$

Rule of 114 Money triples $= \frac{114}{R}$

Rule of 144 Money 4times $= \frac{144}{R}$

Q) A sum of money compounded annually becomes ₹1,140 in two years and ₹1,710 in three years. Find the rate of interest per annum:

(a) ~~30%~~
(b) ~~40%~~
(c) 50%
(d) 60%

$P \xrightarrow{2\text{ year}} 1140 \xrightarrow{1\text{ year}} 1710$

3

Option Hit using (c)

$1140 + 50\% = 1710 = \text{RHS}$

$P \xrightarrow{2} 1400 \xrightarrow{1\text{ year}} 1700$

3

$A = P(1+i)^1$

$1700 = 1400(1+i)$

$\frac{1700}{1400} = (1+i)$

$1400 = P(1+i)^2$

Difference Between CI and SI

The difference between **Compound Interest (CI)** and **Simple Interest (SI)** is given by:

$$\boxed{\text{Difference} = P\left[(1+i)^n - 1 - TR\%\right]}$$

Where:

- P = Principal amount
- i = Interest rate per period (as a decimal)
- n = Number of years (or compounding periods)
- $TR\%$ = Total Simple Interest percentage over n years

2year
$CI - SI = Pi^2$

P T

Q) The difference between compound and simple interest on a certain sum of money for 2 years at 4% p.a. is ₹1. The sum (in ₹) is:

(a) 625
(b) 630
(c) 640
(d) 635

$i = 0.04$

$CI - SI = 1$

$Pi^2 = 1$

$P \times (0.04)^2 = 1$

$P =$ $0.04 \times =$ $\div =$ $= 625$

Q) The difference between compound interest and simple interest on a certain sum of money invested for three years at 6% per annum is ₹1,016. The principal is:

(a) ₹3,000
(b) ₹3,700
(c) ₹12,000
(d) ₹10,000

$CI - SI = P\left((1+i)^n - 1 - TR\%\right)$

$1016 = P\left((1.06)^3 - 1 - 3 \times 6\%\right)$

$1.03 \times (= 2\text{times}) - 1$ M+

$3 \times 6\%$ M−

MRC

$\div = \times 1016$ $= 12000$

Q) The difference between the compound interest and simple interest at 10% per annum for 4 years on ₹10,000 is ₹_______.

(a) 650
(b) 640
(c) 641
(d) 600

$P = 10000$

$CI - SI = ?$

$CI - SI = 10000\left((1.1)^4 - 1 - 4 \times 10\%\right)$

1.1 × (= 3 times)
−1 M+
4×10 M−
MRC ×10000

Q) If the simple interest on a sum of money at 12% p.a. for two years is ₹3,600, the compound interest on the same sum for two years at the same rate is:

(a) ₹3,816
(b) ₹3,806
(c) ₹3,861
(d) ₹3,860

$SI = PTR\%$

$3600 = P \times 2 \times 12\%$

$2 \times 12\% \div = \times 3600$

$P = 15000$

$CI = P\left((1+i)^n - 1\right) = 15000 \times \left((1.12)^2 - 1\right)$

$= 3816$

Q) If the nominal rate of growth is 17% and inflation is 9% for five years, let P be the Gross Domestic Product (GDP) amount at the present year, then the projected real GDP after 6 years is:

(a) 1.587 P
(b) 1.921 P
(c) 1.403 P
(d) 2.51 P

Real Rate = GR − Inflation

$= 8\%$

$i = 0.08 \quad n = 6$

$A = P(1+i)^n$

$= P(1.08)^6 = 1.586\,P$

Effective Rate of Interest (ERI)

Definition

The **Effective Rate of Interest (ERI)** is the **actual interest rate** earned or paid in one year, accounting for the effects of compounding. It is used to compare the true cost or return of interest rates when different compounding periods are involved.

Key Concepts

- NRI (Nominal Rate of Interest):
  - The stated interest rate without considering compounding.
  - Example: A bank advertises a 10% annual interest rate, but it may be compounded quarterly or monthly.
- ERI (Effective Rate of Interest):
  - The **actual** interest rate after considering the compounding effect.
  - ERI is always **equal to or greater than** the NRI.
  - More frequent compounding leads to a higher ERI.

Relationship Between NRI and ERI

- **Annual Compounding:** $\boxed{ERI = NRI}$ (No compounding effect)
- **More Frequent Compounding:** $\boxed{ERI > NRI}$
  - Semi-annual, quarterly, and monthly compounding increase the ERI.

Formula for ERI

$$ERI = (1+i)^n - 1$$

Where:

- i = Interest rate per compounding period (fractional value)
- n = Number of compounding periods in a year

Formula for Percentage ERI

$$\%ERI = [(1+i)^n - 1] \times 100$$

This formula converts the effective rate into percentage form.

Q) The effective rate of interest equivalent to the nominal rate of 7% converted monthly is:

(a) 7.26%

(b) 7.22%

(c) 7.02%

(d) 7.20%

$i = \frac{7}{1200}$

$n = 12 \times 1 = 12$

$ERI = ((1+i)^n - 1) \times 100$

$7 \div 1200 + 1 \times (= 11\ \text{times}) - 1 \times 100 = 7.22$

Q) Which is a better investment: 9% p.a. compounded quarterly or 9.1% p.a. simple interest?

(a) 9% compounded

(b) 9.1% S.T.

(c) Both are the same

(d) Cannot be said

$ERI = \left(\left(1 + \frac{9}{400}\right)^4 - 1\right) \times 100$

$= 9.308 > 9.1\%\ SI$

10% compounding quaterly

$10 \div 400 + 1 \times (= 3\ \text{times}) - 1 \times 100$

Q) Effective rate of interest does not depend upon:

(a) Amount of Principal

(b) Amount of Interest

(c) Number of conversion periods

(d) None of these

Depreciation

Definition

Depreciation refers to the **reduction in the value of assets** such as machinery, furniture, buildings, and equipment over time due to factors like **wear and tear, obsolescence, or usage**. Assets that lose value over time are called **depreciating assets**.

Key Concepts

- **Depreciating Assets:** Assets whose value decreases over time due to factors like use, aging, or technological advancements.
- **Future Value (A):** The value of the asset after a certain period, also known as the **scrap value** or **residual value**.
- **Present Value (P):** The **initial cost** or current value of the asset.

For depreciating assets, the future value (A) will always be **less than** the present value (P), i.e.,

$$A < P$$

Formula for Depreciation

$$\boxed{A = P \times (1 - i)^n}$$

Where:

- A = Future value or scrap value
- P = Present value or cost price
- i = Depreciation rate per period (expressed as a decimal)
- n = Number of periods (years, months, etc.)

Q) The cost of machinery is ₹1,25,000. If its useful life is estimated to be 20 years and the rate of depreciation of its cost is 10% p.a. then the scrap value of the machinery is [given that $(0.9)^{20} = 0.1215$]:

(a) ₹15,187
(b) ₹15,400
(c) ₹15,300
(d) ₹15,250

$A = P(1-i)^n$

$A = 125000\ (0.1215)$

$= 15187$

Q) The value of furniture depreciates by 10% a year. If the present value of the furniture in an office is ₹21,870, calculate the value of furniture 3 years ago:

(a) ₹30,000
(b) ₹35,000
(c) ₹40,000
(d) ₹50,000

$P = ?$

$A = 21870$

30000 − 10%
− 10%
− 10% = 21870 = RHS

Q) A machine worth ₹4,90,740 is depreciated at 15% on its opening value each year. When will its value reduce to ₹2,00,750?

~~(a) 5 years 5 months~~
(b) 5 years 6 months
(c) 5 years 7 months
(d) 5 years 8 months

$A = P(1-i)^n$

$200750 = 490740\ (0.85)^n$

$0.4090 = (0.85)^n$

$\frac{200750}{490740}$

option Hit with option (b)
$n = 5.5$

0.85

0.85
Γ 12 times
−1
× 5.5
+1
x = 12 times

5 + 0.5
5.5

Γ 12 times
−1
× 5.5
+1
x = (12 times)
= 0.409

Annuity

An annuity is a fixed amount of money paid or received regularly over a specified number of periods.

Types of Annuity

1. Annuity Regular (Ordinary Annuity):

- Payments are made at the end of each period (e.g., end of year, half-year, quarter, or month).
- **Examples:** Rent, EMI, etc.

2. Annuity Immediate (Annuity Due):

- Payments are made at the beginning of each period.
- **Examples:** Recurring Deposits (RD), insurance premiums, etc.

FV
PV

Future Value of an Annuity

If a is the fixed amount paid regularly for n periods at an interest rate i per annum, then the future value can be calculated as:

For Annuity Regular:

$$\text{Future Value (FV) of AR} = \frac{a\left((1+i)^n - 1\right)}{i} = \frac{a}{i}\left((1+i)^n - 1\right)$$

For Annuity Due (AD):

$$\text{Future Value (FV) of AD} = \frac{a\left((1+i)^n - 1\right)}{i} \times (1+i)$$

AI

AR	AI
End of the period They will not mention end or Beginning	From today From Today eve Beginning of period

10000 for 5 years at the begining of period at 3% p.a CI.
Find FV.

$$FV = \frac{a}{i}\left((1+i)^n - 1\right)(1+i)$$

$$= \frac{10000}{0.03}\left((1.03)^5 - 1\right)(1.03)$$

$$= 54684.09$$

$A = P(1+i)^n$

$FV = PV(1+i)^n$

$\boxed{PV = \frac{FV}{(1+i)^n}}$ *

Relationship Between Future Value and Present Value

- Future Value (FV) and Present Value (PV) are related by the following formulas:

$$FV = PV \times (1+i)^n$$

$$PV = \frac{FV}{(1+i)^n}$$

Note

- If the type of annuity is not specified, the default assumption is an Annuity Regular (AR).

Present Value of an Annuity (Regular Annuity / Ordinary Annuity)

The formula provided is for the **Present Value (PV)** of a **regular annuity** (also known as an **ordinary annuity**), where payments are made **at the end of each period**.

$$PV = A\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$PV = a \times \boxed{GT}$

$a = 1000$

$n = 10$

$i = 0.1$

1.1 ÷ = 10 times GT

$= \frac{a}{i}\left(1 - (1+i)^{-n}\right)$

Where:

- **PV** = Present Value of the annuity
- **A** = Periodic payment (annuity amount)
- **i** = Interest rate per period
- **n** = Number of periods

Q) The future value of an annuity of ₹5,000 made annually for 8 years at an interest rate of 9% compounded annually [Given that $(1.09)^8 = 1.99256$] is:

(a) ₹55,142.22

(b) ₹65,142.22

(c) ₹65,532.22

(d) ₹57,425.22

$FV = \frac{a}{i}\left((1+i)^n - 1\right)$

$= \frac{5000}{0.09} \times (1.99256 - 1)$

$= 55142.22$

$a = 2000$

$R\% = 5\%$

$n = 20$

Find PV_{AR}

1.05 ÷ = 20 times GT × 2000

= 24924.42

a

Q) The future value of an annuity of ₹1,000 made annually for 5 years at the interest of 14% compounded annually is:

(Given $(1.14)^5 = 1.92541$)

(a) ₹5,610

(b) ₹6,610

(c) ₹6,160

(d) ₹5,160

$FV_{AR} = \frac{a}{i}\left((1+i)^n - 1\right)$

$= \frac{1000}{0.14}(1.92541 - 1)$

$= 6610$

Q) ₹200 is invested at the end of each month in an account paying interest **6% per year compounded monthly**. What is the future value of this annuity after the 10th payment?

$FV_{AR} = \frac{a}{i}\left((1+i)^n - 1\right)$ $n = 10$

$a = 200$ $i = \frac{6}{1200}$

$= \frac{200}{6/1200}\left(\left(1 + \frac{6}{1200}\right)^{10} - 1\right)$

(a) ₹2,044

(b) ₹12,044

(c) ₹2,040

(d) ₹12,000

Q) How much amount is required to be invested every year so as to accumulate ₹5,00,000 at the end of 12 years if interest is compounded annually at **10% {Where A(12, 0.1) = 21.384284}?**

AR

(a) ₹23,381.65

(b) ₹24,385.85

(c) ₹26,381.65

(d) ₹28,362.75

$FV = 500000$ $n = 12$

$R = 10\%$ $a = ?$

$FV = \frac{a}{i}\left((1+i)^n - 1\right)$

FVAF $FV = a \times \boxed{FVAF}$

$500000 \div 21.384284 = a$

Q) Find the present value of an annuity of ₹1,000 payable at the end of each year for 10 years. If the rate of interest is **6% compounding per annum** (given $(1.06)^{-10} = 0.5584$):

(a) ₹7,360

(b) ₹8,360

(c) ₹12,000

(d) None of these.

GT method

1.06 ÷ = 10 GT × 1000

7360

$$PV_{AR} = \frac{a}{i}\left(1-(1+i)^{-n}\right)$$

$$= \frac{1000}{0.06} \times \left(1-(1.06)^{-10}\right)$$

$$= \frac{1000}{0.06} \times (1-0.5584)$$

Q) Mrs. X invests in an annuity immediately that promises annual payments of ₹50,000 for the next **16 years**. If the interest rate is **6% compounded annually**, the approximate present value of this annuity is ____, where **(1.06)^{15} = 2.3965**.

(a) ₹5,51,217.75

(b) ₹5,75,900.00

(c) ₹5,05,288.08

(d) ₹5,35,612.45

1.06 ÷ = 16 $\boxed{GT}$ × 50000 × 1.06

Since AD

Q) ₹2,500 is paid every year for 10 years to pay off a loan. What is the loan amount if the interest rate is **14% per annum compounded annually**?

(a) ₹15,847.90

(b) ₹13,040.27

(c) ₹14,674.21

(d) ₹16,345.11

PV → Since loan is mention

1.14 ÷ = 10 times GT ×2500

PV = 13040.2

PV

Q) Anshika took a loan of **₹1,00,000 @ 8% for 5 years**. What amount will she pay if she wants to pay the whole amount in **five equal installments**?

(a) ₹25,045.63

(b) ₹26,045.68

(c) ₹28,045.50

(d) None

$n = 5 \quad a = ?$

$i = 0.08$

$PV = a \times \boxed{GT}$

$\boxed{1.08 \div = 5\ GT}$ $\div =$ $\times 100000$ $= 25045.63$

$PV = \frac{a}{i}\left(1-(1+i)^{-n}\right)$

$100000 = \frac{a}{0.08}\left(1-(1.08)^{-5}\right)$

1.08 × = 4 times

÷ =

Click +/−

+1

÷ 0.08

÷ =

× 100000

PV

Q) A loan of **₹1,02,000** is to be paid back in **two equal annual installments**. If the rate of interest is **4% p.a., compounded annually**, then the total interest charged (in ₹) under this installment plan is:

(a) 6,160

(b) 8,120

(c) 5,980

(d) 7,560

2:10PM

$\boxed{GT}$

$\boxed{1.04 \div = 2\text{ times } GT}$ $\div =$ $\times 102000$

$a = 54080$

Total amount paid

$= 2 \times 54080$

$= 108160$

$I = 108160 - 102000$

$= 6160$

Q) A car is available for **₹4,98,200 cash payment** or **₹60,000 cash down payment** followed by **three equal annual installments**. The rate of interest charged is **14% per annum compounded yearly**. The total interest charged in the installment plan is **(Given P(3, 0.14) = 2.32163)**:

(a) ₹1,46,314

(b) ₹1,46,137

(c) ₹1,28,040

(d) ₹1,58,040

$PV = 498200 - 60000$

$PV = 438200$

$a = ?$

$PV = a \times \boxed{GT}$

$\boxed{GT}$

1.14 ÷ = 3 GT

÷ =

× 438200

a = 188746.5

Total amount paid

$= 3 \times 188746.5$

$= 566239.6$

$566240 - 438200$

$= 128040$

Q) Suppose you have decided to make a **systematic investment plan (SIP)** in a mutual fund with **₹1,00,000 every year** from today for the next **10 years** at the rate **10% per annum compounded annually**. What is the future value of this annuity? **Given** $1.1^{10} = 2.59374$.

(a) ₹17,35,114

(b) ₹17,53,411

(c) ₹17,35,411

(d) ₹17,53,114

Q) Mr. A borrows ₹5,00,000 to buy a house. If he pays equal instalments for 20 years and 10% interest on the outstanding balance, what will be the equal annual instalment?

(a) ₹58,239.84

(b) ₹58,729.84

(c) ₹68,729.84

(d) None of these

PV = 500000
n = 20
i = 0.1.
a = ?
1.1 ÷ (= 20 times) GT
÷ =
× 500000
= 58729

CI ✓

Sinking Fund

Purpose of a Sinking Fund:

- A sinking fund is established to accumulate money for a specific purpose. This purpose could be to **repay bonds, replace assets, or cover any large future expense.**

Mechanism:

- Contributions are made into this fund in periodic payments over a defined time period. These payments are known as **deposits to the sinking fund.**

Q) A company establishes a sinking fund to provide for the payment of **₹2,00,000 debt maturing in 20 years**. Contribution to the fund is to be made **at the end of every year**. Find the amount of each deposit if the interest is **10% per annum**?

(a) ₹3,592.11

(b) ₹3,491.92

(c) ₹3,392.11

(d) None

$i = 0.1$ $FV = 200000$ $n = 20$ $a = ?$

$$FV_{AR} = \frac{a}{i}\left((1+i)^n - 1\right)$$

$$200000 = \frac{a}{0.1}\left((1.1)^{20} - 1\right)$$

$a = 3491.92$

Applications of Annuity

1. Leasing (Renting)

- **Definition**: Leasing is essentially renting.
- **Decision Making**: When deciding between purchasing a machine or leasing it, compare the present values of both options.
  - If the **PV of leasing < PV of purchasing**, leasing is preferred.
  - If the **PV of leasing > PV of purchasing**, purchasing is preferred.
  - If the **PV of leasing = PV of purchasing**, both options are equally good.

Lessor – Owner

Lessee –

$PV_{purchase} = 500000$

Q) A person wants to lease out a machine costing ₹5,00,000 for a **10-year period**. It has fixed a rental of **₹51,272 per annum** payable annually starting from the end of the first year. Suppose the rate of interest is **10% per annum compounded annually** on which money can be invested. To whom is this agreement favorable?

a　AR

$i = 0.1$

(a) Favour of Lessee

(b) Favour of Lessor

(c) Not for both

(d) Can't be determined

GT method

$PV_{Leasing}$

1.1 ÷ = 10 GT ×51272

$PV_{Leasing} = 315044$

$PV_{Leasing} < PV_{Purchasing}$

2. Investment Decision

- **Rule**: If the **PV of cash inflows (returns)** is **greater than or equal** to the **PV of cash outflows (investment)**, the investment is considered good; otherwise, it's a loss.

Net Present Value (NPV)

Formula:

$$NPV = PV \text{ of Cash Inflows} - PV \text{ of Cash Outflows}$$

- An investment is considered **good** if $NPV \geq 0$.

Q) A machine can be purchased for Rs 65,000. The machine will contribute Rs 15,000 per year for the next 6 years. Assume borrowing cost is 10% per annum. Determine whether the machine should be purchased or not:

$a = 15000 \quad n = 6 \quad i = 0.1$

(a) Should be purchased

(b) Should not be purchased

(c) Can't say about purchase

(d) None of the above

PV_{AR}

1.1 ÷ (= 6 times) GT ×15000

$PV_{inflow} = 65329$

$NPV = 65329 - 65000 = 329$

Q) If the cost of capital is 12% per annum, then the net present value (in nearest ₹) from the given cash flow is given as:

$\frac{FV}{(1+i)^n}$

Years	0	1	2	3
Operating profit (in thousands ₹)	(100)	60	40	50

(a) 31,048

(b) 34,185

(c) 21,048

(d) 24,187

$NPV = PV_{inflow} - PV_{outflow}$

$= \frac{60}{1+i} + \frac{40}{(1+i)^2} + \frac{50}{(1+i)^3} - 100$

$= \frac{60}{1.12} + \frac{40}{1.12^2} + \frac{50}{(1.12)^3} - 100$

60 ÷ 1.12 M+   40 ÷ 1.12 ÷ 1.12 M+   50 ÷ 1.12 ÷ 1.12 ÷ 1.12 M+   100 M−   MRC

3. Valuation of Bonds

- **Definition:** A bond is a debt security where the issuer owes the holder a debt and is obligated to **repay the principal and interest.** Bonds are usually issued for a term **longer than one year**.

SBI GOI

- **Issuer's Obligation:** The bond issuer enters into a **contract with the bondholder** to **pay interest** over the term of the bond.

Bond Valuation Formula

PV GOI

Present Value (PV) of the bond:

$$PV = a\left(\frac{(1+i)^n - 1}{i(1+i)^n}\right) + \frac{\text{Par value}}{(1+i)^n}$$

M+   M+

Where:

- a = Coupon Payment = Par Value × Nominal Rate
- i = Required Rate of Return
- n = Number of periods to maturity

MRC

$m=3$ Parvalue $=1000$

Q) An investor intends to purchase a three-year ₹1,000 par value bond having a nominal interest rate of 10%. At what price the bond may be purchased now if it matures at par and the investor requires a rate of return of 14%?

nominal rate $=10\%$

Options:

(a) ₹907

(b) ₹950

(c) ₹980

(d) ₹920

$i=$ Rate of return $=14\%=0.14$

$a=$ Parvalue $\times$ nominal rate

$a=1000\times 10\%=\underline{100}$

$\underline{PV}$ $+\dfrac{1000}{(1.14)^3}$

1.14

$\div$

$=3$ times

GT

$\times 100$

M+

$1000\div 1.14\div 1.14\div 1.14$

M+

MRC $=\underline{907}$

Perpetuity

Definition

Perpetuity is a type of annuity where a **fixed amount (receipt amount) is paid or received regularly and indefinitely**, without an end date.

$$PV=\frac{a}{i}$$

Types of Perpetuity

1. Normal Perpetuity (Multi-Period Perpetuity):

- **Formula:**

$$\boxed{PV \text{ of Multiperiod Perpetuity}} = \frac{R}{i} = \frac{a}{i}$$

- **Where:**
  - R = Fixed payment (Receipt amount)
  - i = Interest rate

2. Growing Perpetuity:

- A stream of cash flows that **grows at a constant rate forever.**
- **Formula:**

$$\boxed{PV \text{ of Growing Perpetuity} = \frac{R}{i-g}} = \frac{a}{i-g}$$

- **Where:**
  - g = Growth rate

Q) Assuming that the discount rate is 7% p.a., how much would you pay to receive **₹200 growing at 5% annually forever?**

(a) ₹2,500

(b) ₹5,000

(c) ₹7,500

(d) ₹10,000

$PV = \frac{a}{i-g} = \frac{200}{2\%}$

$= 10000\%$

$200 \div 2$

click %

Q) If the **discount rate is 14% per annum**, then how much does a company have to pay to receive **₹280 growing at 9% annually forever**?

(a) ₹5,600

(b) ₹2,800

(c) ₹1,400

(d) ₹4,200

$\frac{280}{5\%} = \underline{5600}$

Q) If a person bought a house by paying **₹45,00,000 down payment** and **₹80,000 at the end of each year till the perpetuity**. Assuming the **rate of interest as 16%**, the **present value of the house (in ₹)** is given as:

(a) **₹47,00,000**

(b) **₹45,00,000**

(c) **₹57,80,000**

(d) **₹50,00,000**

$PV = \frac{80000}{16\%}$

$= \underline{500000}$

45 Lakh + 5 Lakh

50 Lakh

Compound Annual Growth Rate (CAGR)

Definition

CAGR, or **Compound Annual Growth Rate**, is a measure used to describe the growth of a certain element of a business (e.g., revenue, income, etc.) over a specified period. It represents the **mean annual growth rate of an investment over a specified time period**, assuming the investment grows at a steady rate.

Formula for CAGR

$$CAGR = \left(\frac{V_n}{V_0}\right)^{\frac{1}{t_n - t_0}} - 1$$

Where:

- t_n = End period
- t_0 = Beginning period
- V_n = Value at the end period (t_n)
- V_0 = Value at the beginning period (t_0)

Percentage CAGR

$$\%CAGR = \left[\left(\frac{V_n}{V_0}\right)^{\frac{1}{t_n - t_0}} - 1\right] \times 100$$

Short Trick for Solving CAGR Based Questions

$$(1 + \overset{\text{CAGR}}{option})^{t_n - t_0} = \frac{V_n}{V_0}$$

Q) Let the operating profit of a manufacturer for five years be given as:

$t_n = 6$

Years	1	2	3	4	5	6
Operating profit (in lakh ₹)	90	100	106.4	107.14	120.24	157.34

$t_0 = 2$, $V_0 = 100$, $V_n = 157.34$

Then the operating profit of Compound Annual Growth Rate (CAGR) for year 6 with respect to year 2 is given that:

(a) 9%

(b) 12%

(c) 11%

(d) 13%

$$RHS = \frac{V_n}{V_0} = \frac{157.34}{100} = 1.5734$$

$$LHS = (1 + option)^{6-2}$$

- Option Hit

$$(1 + 12\%)^4 = 1.573 = RHS$$

$t_n - t_0 = 3$

Q) The CAGR of the initial value of an investment of ₹15,000 (V_0) and the final value of ₹25,000 (V_n) in 3 years is:

(a) 19%

(b) 18.56%

(c) 17.56%

(d) 17%

$$(1+\text{option})^{t_n - t_0} = \frac{V_n}{V_0}$$

$$RHS = \frac{25000}{15000} = 1.67$$

$$LHS = (1+18.56\%)^3 = 1.666 \approx RHS$$

Q) Ravi made an investment of ₹15,000 in a scheme and at the time of maturity the amount was ₹25,000. If Compound Annual Growth Rate (CAGR) for this investment is 8.88%, calculate the approximate number of years for which he has invested the amount.

(a) 6

(b) 7.7

(c) 5.5

(d) 7

$$(1+CAGR)^{t_n - t_0} = \frac{V_n}{V_0}$$

$$RHS = \frac{25000}{15000} = 1.666$$

$$LHS = (1+8.88\%)^{t_n - t_0}$$

option Hit

1.888 × (= 5 times

Chapter 10: Direction Sense Tests

Right of North
East

Right of East
South

Right of South
west

Right of west
North


VIRAT
18


DHONI
7

Type 1 - Determining Final Direction Based on Left and Right Turns

This type involves finding the final direction a person is facing after following a sequence of left and right turns starting from an initial direction (e.g., North).

Notes:

N R L R L R L

N L L
S

N R R
S

Q) Jairaj started from her house, walked 20 km towards North. Now, he took a right turn and moved 2 km. Again he took a right turn and walked for 10 km. In which direction is he going?

~~(a) North~~

(b) South

(c) East

(d) West

N R R
S

Q) One morning Sujata started to walk towards the Sun. After covering some distance she turned to the right, then again to the right, and after covering some distance she again turns to the right. Now in which direction is she facing?

(a) North

(b) South

(c) N-E

(d) S-W

E R R R
W R
North

Q) X walks southwards and then turns right, then left, and then right. In which direction is he moving now?

(a) South

(b) North

(c) West

(d) South-west

SRLR

West

Q) You go North, turn right, then right again, and then go to the left. In which direction are you now?

(a) South

(b) East

(c) West

(d) North

NRRL

East

Q) A man started to walk East. After moving a distance, he turned to his right. After moving a distance, he turned to his right again. After moving a little, he turned in the end to his left. In which direction was he going now?

(a) East

(b) West

(c) North

(d) South

ERRL

South

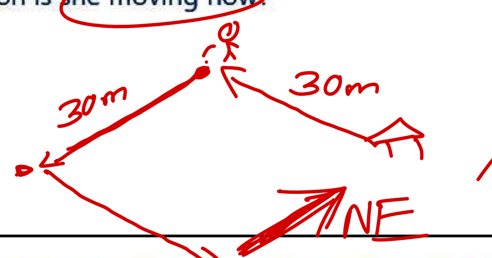
Q) Sangeeta leaves from her home. She first walks 30 meters in the north-west direction and then 30 m in the south-west direction. Next, she walks 30 meters in the south-east direction. Finally, she turns towards her house. In which direction is she moving now?

(a) North-West

(b) North-East

(c) South-East

(d) South-West



Q) A man stands on a point and starts walking towards north. He then turns left, then turns right, and then left. In which direction is he moving now?

(a) West

(b) North

(c) East

(d) South

NLRL

W

Q) R's office is 4 km in the East direction from his home, and the club is 4 km in the North direction from his home. On the midway from office to club, R starts moving towards his home. In which direction is he facing his back?

(a) South-East

(b) North-West

(c) North-East

(d) South-West

Northeast

Office

Type 2 - Determining Final Direction Based on Angular Turns

This type involves calculating the final direction a person is facing after following instructions that specify angular turns (e.g., 90°, 180°, clockwise or counterclockwise).

Notes:

$+$

$-$

$N +90^\circ -180^\circ +90^\circ -45^\circ$

$N -45^\circ$

NW

Q) Madhuri moved a distance of 75 meters toward north. She then turned to her left and walked for about 25 m, turned left again and walked 80 m. Finally, she turned to her right at an angle of 45°. In which direction was she moving now?

(a) South-East

(b) South-West

(c) North-West

(d) North-East

N L L $+45^\circ$

S $+45^\circ$

SW

Q) Srikant is facing east and turns 120 degrees in the clockwise direction and then turns 180 degrees in the anticlockwise direction. Which direction is Srikant facing now?

(a) East

(b) North-East

(c) North

(d) South-West

$E +120^\circ - 180^\circ$

$E - 60^\circ$

NE

60°

Q) A man is facing north. He turns 45 degree in the clockwise direction and then another 180 degree in the same direction and then 45 degree in the anticlockwise direction. Find which direction he is facing now?

(a) North

(b) East

(c) West

(d) South

$N + 45^\circ + 180^\circ - 45^\circ$

$N + 180^\circ$

Type 3 - Determining Directions Based on Clock Hands

Notes:

12:45PM

Q) It is 3 o'clock on a watch. If the minute hand points towards the North-East, then the hour hand will point towards the:

(a) South

(b) South-West

(c) North-West

(d) South-East

Q) Mr. Kartik puts his timepiece on the table in such a way that at 6 p.m. the hour hand points to the north. In which direction will the minute hand point at 9.15 p.m.?

(a) South-East

(b) East

(c) West

(d) South-West

Q) The hour hand of a clock is in the west direction when the time is 3 o'clock. What is the direction of the minute hand when the time is 6:45?

(a) East
(b) West
(c) North
(d) South

S 12 E 9 3W 6 N

Q) In a clock at 12:30, hour needle is in North direction while minute needle is in South direction. In which direction would be minute needle at 12:45?

(a) North-West
(b) South-East
(c) West
(d) East

Type 4 - Determining Final Position with Respect to Initial Position

This type involves calculating the final position of a person or object after a series of movements (e.g., North, South, East, West) relative to their starting point.

Q) Raj started from point A and walked towards West 4 m, then he turned towards South up to 4 m. In which direction is he from the start point?

(a) W-S
(b) N-E
(c) South
(d) West

W_4 S_4

Q) Laxman went 15 km to North, then he turned West and covered 10 km. Then he turned South and covered 5 km. Finally turning to East he covered 10 km. In which direction is he from his house?

(a) East
(b) West
(c) North
(d) South

N_{15} W_{10} S_5 E_{10}

N_{10} S_5 E_{10} W_5

N_5 E_5

N_{15-5}

N_{10}

Q) Manu wants to go to the market. He starts from his house towards North and reaches a crossing after 30 m. He turns towards East, goes 10 m till the second crossing and turns again, moves towards South straight for 30 m where the marketing complex exists. In which direction is the market from his house?

(a) North

(b) South

(c) East

(d) West

$N_{30}\ E_{10}\ S_{30}$

E_{10}

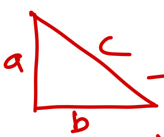
Q) When a person faces north and walks 25 m right, then turns left and walks 20 m, and again turns right and walks 25 m, and turns right, and walks 25 m, and turns right and walks 40 m, in which direction is he now from his starting point?

(a) North-West

(b) North-East

(c) South-East

(d) South-West



$c^2 = a^2 + b^2$

$c = \sqrt{a^2+b^2}$

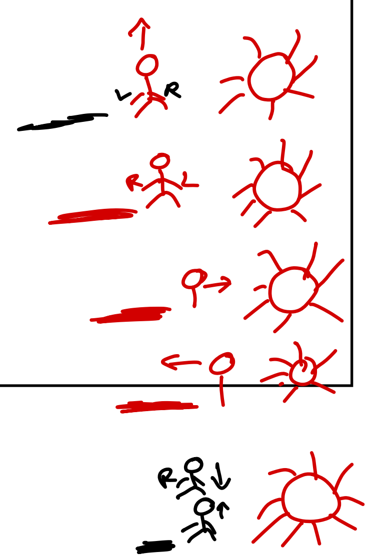
$E_{25}\ N_{20}\ E_{25}\ S_{25}\ W_{40}$

$E_{10}\ S_{5}$

$\sqrt{10^2 + 5^2}$

Type 5 - Determining Directions Based on Sunset, Sunrise, and Shadows

This type involves using the Sun's position during sunrise and sunset, or the direction of shadows, to identify directions.



Q) Raman starts walking in the morning facing the Sun. After some time, he turned to the left, later again he turned to his left. In what direction is Raman moving now?

(a) East

(b) West

(c) South

(d) North

E L L

W

Q) The Sun rises behind the tower and sets behind the railway station. In which direction is the tower from the railway station?

(a) North

(b) South

(c) East

(d) West

E

Q) One day morning after sunrise, Vimal started to walk. During this, he met Sheru who was coming from the opposite direction. Vimal watched the shadow of Sheru to the right of him (Vimal). To which direction was Vimal facing?

(a) South

(b) North

(c) East

(d) West

N
R L
L R

Q: If Kiran sees the rising sun behind the Hill and the setting sun behind the Airport from his house, what is the direction of the Airport from the Hill?

(a) South

(b) North

(c) West

(d) East

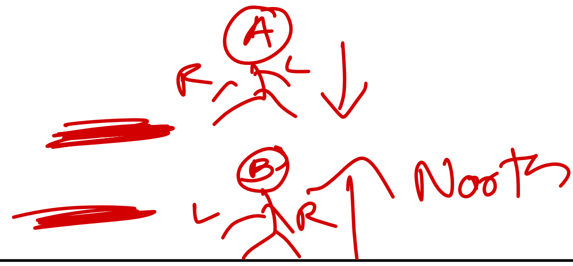
Q) One morning after sunrise, A and B were talking to each other face to face very closely at a crossing point. If B's shadow was exactly to the right of A in which direction B was facing?

(a) East

(b) West

(c) North

(d) South



Type 6 - Determining Distance Between Starting Point and Final Point

This type involves calculating the shortest distance (straight-line distance) between the starting and final positions after a series of movements in different directions.

Q) Starting from the point O, Vaibhav walked 15 m towards West. He turned left and walked 20 m. Again, he turned left and walked 15 m. Now he turned to his right and walked 12 m. How far is he now from the point O?

(a) 32 m

(b) 47 m

(c) 42 m

(d) 27 m

W ~~15~~ S 20 E ~~15~~ S 12

S 32

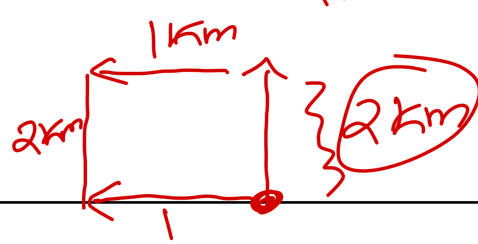
Q) A boy rode his bicycle Northward, then turned left and rode 1 km and again turned left and rode 2 km. He found himself 1 km West of his starting point. How far did he ride Northward initially?

(a) 1 km

(b) 2 km

(c) 3 km

(d) 4 km



Q) One day, Ram left home and cycled 10 km southward, turned right and cycled 5 km, turned right and cycled 10 km, then turned left and cycled 10 km. How many kilometers will he have to cycle to reach his home straight?

(a) 10

(b) 15

(c) 20

(d) 25

$\cancel{S_{10}}\ W_5\ \cancel{N_{10}}\ W_{10}$

W_{15}

Q) A person walks 1 km towards West and then he turns to South and walks 5 km. Again, he turns to West and walks 2 km. After this, he turns to North and walks 9 km. How far is he from his starting point?

(a) 3 km

(b) 4 km

(c) 5 km

(d) 7 km

$W_1\ S_5\ W_2\ N_9$

$W_3\ N_4$

$\sqrt{3^2+4^2}$

$= \sqrt{25} = 5$

Q) If Ramu faces West and moves 5 km in that direction, then takes a left turn and moves 10 km, then takes another left turn and moves 15 km in the same direction, then moves 10 km in the North direction and reaches point A. What is the distance between the starting point and A, and in which direction is Ramu facing now?

(a) 10 km, North

~~(b) 5 km, South~~

(c) 10 km, South

~~(d) 5 km, North~~

$W_5\ \cancel{S_{10}}\ E_{15}\ \cancel{N_{10}}$

E_{10}

Q: Disha walks 2 km towards South, then she turns west and walks 10 km. After this, she turns South and walks 3 km. Again she turns towards west and walks 2 km. How far is Disha from the starting point?

(a) 10 km

(b) 13 km

(c) 15 km

(d) 17 km

$S_2\ W_{10}\ S_3\ W_2$

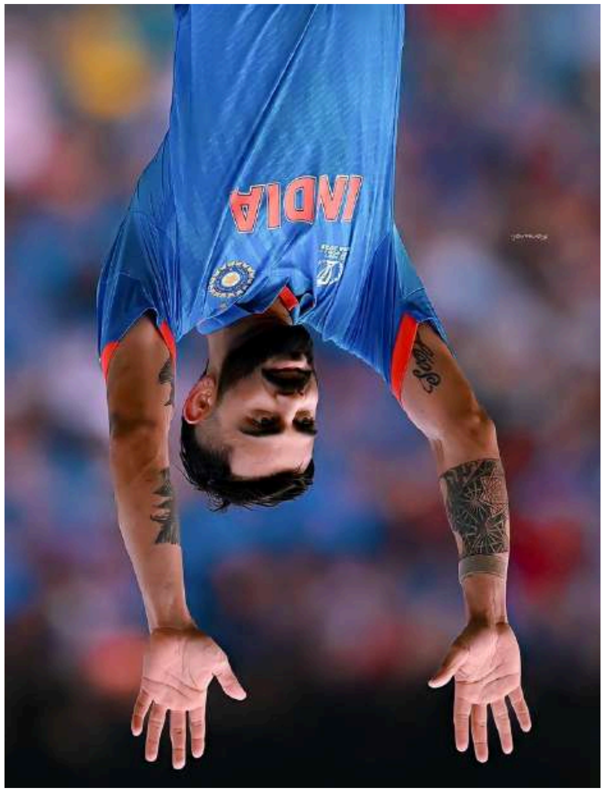
$S_5\ W_{12}$

$\sqrt{5^2+12^2}$

$= \sqrt{169}$

$= 13$

Type 7 - Based on Standing on Head



Q) If Ajay stands on his head with his face towards North, in which direction will his left-hand point?

(a) North-East

(b) North

(c) East

(d) North-West

Q: If Mr. Virat stands on his head with his face towards West, in which direction will his right hand point?

(a) South

(b) North

(c) East

(d) None

Type 8 - Calculating Total Distance Covered During the Journey

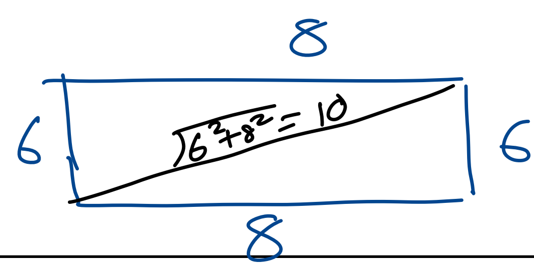
Q) Anil started walking **5 kms** towards north, then he turned **left** and walked **3 kms**. Again he turned **left** and walked **5 kms**. Then the total number of kilometers he walked is:

(A) 13 kms
(B) 8 kms
(C) 3 kms
(D) 5 kms

5 + 3 + 5
= 13

Q) The length and breadth of a room are 8 metre and 6 metre respectively. A cat runs along all four walls and finally along diagonal order to catch a rat. How much total distance is covered by the cat?

(a) 10
(b) 14
(c) 38
(d) 48



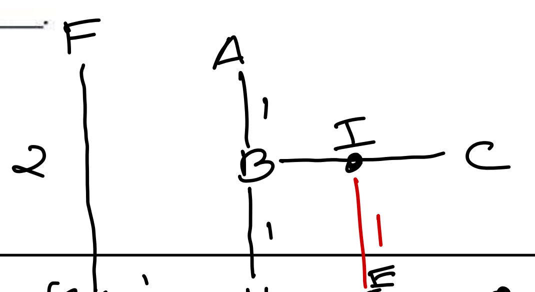
Extra Question

6+8+6+8+10 = 38

A, B, C, D, E, F, G, H and J are nine houses.
C is 2 km east of B.
A is 1 km north of B and H is 2 km south of A.
G is 1 km west of H while D is 3 km east of G and F is 2 km north of G.
I is situated just in the middle of B and C while E is just in the middle of H and D.

Q) Distance between E and I is ________.

(a) 4 km
(b) 2 km
(c) 1 km
(d) 3 km



Chapter 18: Index Numbers

8

8 hours

6 hours

Index numbers are statistical tools used to compare the **price, quantity, or value** of a commodity over **two different time periods** (Base year and Current year).

Jan 26
May 27
May 29
Jan 30

P_0 1947 2 | P_1 2025 10

Key Features of Index Numbers

- **Expressed in Percentages:** Index numbers represent changes as percentages.
- **Use of Averages:**
  - **Arithmetic Mean (AM)**
  - **Geometric Mean (GM)**
  - GM is considered the best average for constructing index numbers.

1947 | 2025

Relatives:

- One of the simplest forms of an index number is a **price relative**.
- A **price relative** is the **ratio of the price of a commodity in a given period to its price in another (base) period.**

Price Relative Formula:

$$Price\ Relative = \frac{P_n}{P_0} = \frac{P_1}{P_0} \times 100$$

- To express it as a percentage, it is multiplied by 100:

$$\boxed{Price\ Relative = \left(\frac{P_n}{P_0}\right) \times 100}$$

Methods for Constructing Index Numbers

Index numbers can be constructed using two main methods:

1) Simple Index Numbers Method

- Simple Aggregative Price Index
- Simple Relative Price Index

2) Weighted Index Numbers Method

- Laspeyres' Index (L)
- Paasche's Index (P)
- Bowley's Index (B)
- Fisher's Index (F)
- Marshall's Index (M)

	P_0 1947	P_1 2025
Gold	5	100
[illegible]	4	25
Cu	1	21
Dicn	4	30
[illegible]	6	71

Simple Aggregative Price Index (SAPI)

$$SAPI = \left(\frac{\sum P_1}{\sum P_0}\right) \times 100$$

where:

- P_1 = Current Year Price
- P_0 = Base Year Price

Simple Relative Price Index (SRPI)

$$SRPI = \left(\frac{\sum \left(\frac{P_1}{P_0}\right)}{N}\right) \times 100$$

where:

- N = Number of Commodities

Important Note:

- SAPI is unit-dependent, meaning its values depend on the units of measurement.
- SRPI is unit-independent, making it a more standardized measure.

Q) The simple index number for the current year using the simple aggregative method for the following data is ______.

SAPI

Commodity	Base Year Price (P_0)	Current Year Price (P_1)
Wheat	80	100
Rice	100	150
Gram	120	250
Pulses	200	300

(a) 200

(b) 150

(c) 240

(d) 160

$SAPI = \frac{\Sigma P_1}{\Sigma P_0} \times 100 = \frac{800}{500} \times 100 = 160$

Weighted Index Numbers

- Weight refers to quantity.
- Weighted index numbers consider the importance of different commodities by assigning them weights.

Types of Weighted Index Numbers

1. Laspeyres' Index (L)

$\frac{\Sigma P_1 q_0}{\Sigma P_0 q_0}$

$$L = \left(\frac{\sum(P_1 q_0)}{\sum(P_0 q_0)}\right) \times 100$$

where:

- P_1 = Current year price
- P_0 = Base year price
- q_0 = Base year quantity

1947		2020	
q_0	P_0	q_1	P_1
5	20	8	100
6	30	6	200
7	40	9	300
8	50	7	400

100×5 mt 200×6mt
300×7 mt 400×8 mt
MRC

2. Paasche's Index (P)

$$P = \left(\frac{\sum(P_1 q_1)}{\sum(P_0 q_1)}\right) \times 100$$

where:

- q_1 = Current year quantity

Note:

If prices or quantities change in the same ratio, then Laspeyres' Index will be equal to Paasche's Index.

A.M of L and P
Bowley

3. Bowley's Index (B)

- It is the Arithmetic Mean (AM) of Laspeyres' and Paasche's Index.

$$B = \frac{L + P}{2}$$

4. Fisher's Index (F)

GM of L and P

- It is the Geometric Mean (GM) of Laspeyres' and Paasche's Index.

$$F = \sqrt{L \times P}$$

- Alternatively, it can be written as:

$$F = \left(\frac{\sum(P_1 q_0)}{\sum(P_0 q_0)} \times \frac{\sum(P_1 q_1)}{\sum(P_0 q_1)}\right)^{\frac{1}{2}} \times 100$$

- Fisher's Index is considered an Ideal Index Number.

5. Marshall's Index (M)

$$M = \left(\frac{\sum P_1(q_0 + q_1)}{\sum P_0(q_0 + q_1)}\right) \times 100$$

- Marshall's Index is a good approximation to Fisher's Index. *

Q) Fisher's Index is based on:-

(a) Arithmetic Mean of Laspeyre and Paasche

(b) Geometric Mean of Laspeyre and Paasche

(c) Harmonic Mean of Laspeyre and Paasche

(d) Median of Laspeyre and Paasche.

Q) In Passche's index, weights are based on:

(a) Current year quantities

(b) Base year quantities

(c) Weighted average prices

(d) None of these

Q) In Laspeyre's Index Number, ____ are used as weights?

(a) Base year price

(b) Current year price

(c) Base year quantities

(d) Current year quantities

Q) Find the Paasche's index number for prices from the following data taking 1970 as the base year.

Commodity	1970 Price (P_0)	1970 Quantity (q_0)	1975 Price (P_1)	1975 Quantity (q_1)
A	1	6	3	5
B	3	5	8	5
C	4	8	10	6

(a) 261.36

(b) 265.48

(c) 274.32

(d) 282

$P = \left(\frac{\Sigma P_1 q_1}{\Sigma P_0 q_1}\right) \times 100$

3x5 mt 8x5 mt 10x6 mt

MRC

1x5 mt 3x5 mt 4x6 mt

$\frac{115}{44} \times 100$

$L = \frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times 100 = 260.37$

Q) Fisher's Ideal Index is obtained by:

(a) Arithmetic Mean of Laspeyre's & Paasche's index
(b) Geometric Mean of Laspeyre's & Paasche's index
(c) Sum of Laspeyre's & Paasche's index
(d) None of the above

Q) Bowley's index = 150, Laspeyer's index = 180, then Paasche's index = _____

(a) 120
(b) 30
(c) 165
(d) None of these

$B = \frac{L+P}{2}$

$150 = \frac{180+P}{2}$

$300 = 180 + P$

$P = 120$

Q) The weighted aggregative price index turnover for 2001 with 2000 as the base year using Fisher's Index Number is:

Commodity	Price (In ₹) 2000 (P_0)	Price (In ₹) 2001 (P_1)	Quantities 2000 (q_0)	Quantities 2001 (q_1)
A	10	12	20	22
B	8	8	16	18
C	5	6	10	11
D	4	4	7	8

(a) 12.26
(b) 112.25
(c) 112.32
(d) 126.01

$F = \sqrt{L \times P}$

$= \sqrt{\frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1}} \times 100$

$\Sigma P_1 q_0 = 12 \times 20 + 8 \times 16 + 6 \times 10 + 4 \times 7$ MRC

$\Sigma P_0 q_0 = 10 \times 20 + 8 \times 16 + 5 \times 10 + 4 \times 7$ MRC

$= \sqrt{\frac{456 \times 506}{406 \times 451}} \times 100$

Consumer Price Index (CPI)

- The **Consumer Price Index (CPI)** is also known as **the Cost of Living Index**.
- It measures the **average change in prices** paid by urban consumers for a **market basket of consumer goods and services** over time.
- The CPI is a key economic indicator used to track inflation.

Purpose of CPI:

- The primary purpose of the CPI is to **track changes in the cost of living.**
- It helps to estimate **the inflation rate**, which represents the **general rise in prices** over time.

Formula for CPI:

$$CPI = \frac{\sum(I \times W)}{\sum W}$$

where:

- I = **Group index**
- W = **Weight (importance of the item in the market basket)**

Deflated Value:

$$Deflated\ Value = \frac{Current\ Price}{Current\ CPI}$$

- **Deflation of value** occurs when retailers and service providers **reduce costs** by:
  - Selling **smaller packages**
  - Providing **smaller portions**
  - Offering **less quantity for the same price** while maintaining the same sticker price.

Q) Consumer price index is commonly known as

(a) Chain Based index

(b) Ideal index

(c) Wholesale price index

(d) Cost of living index

SRPI

Q) The index number for the year 2012 taking 2011 as the base year from the data given below by using the simple average of the price relative method is:

Commodity	A	B	C	D	E
Price in 2011 P_0	115	108	95	80	90
Price in 2012 P_1	125	117	108	95	95

(a) 112

(b) 117

(c) 120

(d) 111

$SRPI = \left(\frac{\sum \frac{P_1}{P_0}}{N}\right) \times 100$ $N = 5$

$\frac{125}{115}$ mt $\frac{117}{108}$ mt $\frac{108}{95}$ mt $\frac{95}{80}$ mt $\frac{95}{90}$ mt

MRC $\div 5 \times 100$

$= 111$

Splicing of Index Numbers

- Splicing is the process of constructing one continuous series from two different index number series using a **common base.**
- It is useful when a **new commodity is added** to the existing list of commodities.

105

A 1947 Pepper Turmeric clove indigo

Q) Which of the following statements is true?

(a) Paasche's Index Number is based on base year quantity

(b) Fisher's Index Number satisfies the circular test

(c) Arithmetic Mean is the most appropriate average for constructing the Index Number

(d) Splicing means constructing one continuous series from two different indices on the basis of a common base.

C

Shifted Price Index

$$Shifted\ Price\ Index = \left(\frac{Original\ Price\ Index}{Price\ Index\ of\ the\ year\ to\ which\ it\ has\ to\ be\ shifted}\right) \times 100$$

- The purpose of shifting a price index is to **recalibrate** it to a **different base year**.
- This adjustment is necessary when:
  - The original base year **becomes outdated.**
  - Comparison is required **across different index series** with different base years.

May 25

Chain Index Number (CIN)

$$CIN = \left(\frac{Link\ Relative\ of\ Current\ Year \times CIN\ of\ Previous\ Year}{100}\right)$$

- A **Chain Index** is different from a **fixed-base index**.
- It links **each period** to the **preceding one**, rather than a fixed base period.
- **Link Relative** is the **price relative**, meaning the **ratio of the price in the current year to the price in the previous year.**

Purchasing Power of Money

$$Purchasing\ Power\ of\ Money = \frac{1}{Price\ Index\ Number}$$

- Refers to the **quantity of goods and services** that can be bought with a **unit of currency.**
- **When prices increase (inflation)** → Purchasing power **falls.**
- **When prices decrease (deflation)** → Purchasing power **rises.**

Q) Purchasing power of money is

(a) Reciprocal of price index number

(b) Equal to price index number

(c) Unequal to price index number

(d) None of these

Real Wages

$$Real\ Wages = \left(\frac{Current\ Year\ Wages}{Current\ Year\ CPI}\right) \times Base\ Year\ CPI$$

- **Real wages** are **adjusted for inflation**, reflecting the **actual purchasing power** of wages.
- The formula accounts for the impact of **inflation** to compare **nominal wages** with base-year wages.

$1 - \left(\frac{1+\%\text{ in }P}{1+\%\text{ in }W}\right) \times 100$

Percentage Increase in Real Wages

$$Percentage\ Increase\ in\ Real\ Wages = \left[1 - \left(\frac{1 + \%\ Increase\ in\ Price}{1 + \%\ Increase\ in\ Wages}\right)\right] \times 100$$

- Percentage Increase in Price = Rate of inflation.
- Percentage Increase in Wages = Nominal increase in wages.
- This formula determines how much real wages have increased after adjusting for inflation.

Note:

- **The percentage increase in real wages will always be less than the difference between the percentage increase in wages and the percentage increase in price.**

Assume if not given | Base year index is 100

Q) The index number of prices at a place in the year 2008 is 225 with 2004 as the base year. Then there is:

(a) Average 125% increase in prices.
(b) Average 225% increase in prices.
(c) Average 100% increase in prices.
(d) None of the above.

2004 → 2008
100 → 225
+125%

Q) In the year 2005, the wholesale price index number is 286 with 1995 as the base year. How much have the prices increased in 2005 in comparison to 1995?

(a) 286%
(b) 386%
(c) 86%
(d) 186%

1995 → 2005
100 → 286
+186

Q) If with an increase of 10% in prices, the rise in wages is 20%, then the real wage has increased by:

(a) 20%

(b) 10%

(c) Less than 10%

(d) More than 10%

$$\%\text{ increase in real wages} = \left[1 - \left(\frac{1 + \%\text{ increase in } P}{1 + \%\text{ increase in wages}}\right)\right] \times 100$$

$$= \left[1 - \left(\frac{1.1}{1.2}\right)\right] 100 = 8.33\%$$

Q) Consumer price index number for the year 1977, was 313, with 1960 as the base year, and was 100 for the year 1960. The average monthly wages in 1977 of the workers into factory be ₹ 160, their real wages is:

(a) ₹ 48.40

(b) ₹ 51.12

(c) ₹ 40.30

(d) None of the above

1977	1960
313	100
160	x

$313x = 160 \times 100$

$x = 51.11$

Tests of Adequacy (Tests of Consistency)

Index numbers must satisfy certain mathematical tests to be considered **consistent** and **reliable**.

1) Unit Test

- An index number **passes the unit test** if it is **independent of the units of measurement**.
- Changing units (e.g., from kilograms to pounds or from dollars to euros) **should not** affect the index number.
- **The Simple Aggregative Price Index does not satisfy the Unit Test** because it is dependent on the units of the items being priced.

2) Time Reversal Test (TRT)

- This test checks whether an index number remains **consistent when the time order is reversed**.
- **Condition for satisfying TRT:**

$$P_{01} \times P_{10} = 1$$

P_0 P_1 1947 | P_1 P_0 2025

- **Fisher's Index and Marshall's Index satisfy the Time Reversal Test.**

3) Factor Reversal Test (FRT)

- This test checks whether the **value index** can be obtained by multiplying the **price index** and the **quantity index**.
- **Condition for satisfying FRT:**

$$P_{01} \times Q_{01} = V_{01}$$

where:

- P_{01} = **Price Index** (ratio of prices in year 1 to year 0).
- Q_{01} = **Quantity Index** (ratio of quantities in year 1 to year 0).
- V_{01} = **Value Index.**
- Mathematically:

$$V_{01} = \frac{\sum(P_1 q_1)}{\sum(P_0 q_0)}$$

- **Only Fisher's Index satisfies the Factor Reversal Test.**

0 1 2
1947 | 1993 | 2025

4) Circular Test

- This test ensures **consistency over multiple time periods.**
- **Condition for satisfying Circular Test:**

$$P_{01} \times P_{12} \times P_{20} = 1$$

- **Fisher's Index fails to satisfy the Circular Test.**
- The test is **satisfied by:**
  - The **Simple Geometric Mean of Price Relatives.**
  - The **Weighted Aggregative Index with Fixed Weights.**
- **The Circular Test is an extension of the Time Reversal Test (TRT).**
  - While TRT deals with the **reversal of two periods**, the Circular Test ensures consistency **across multiple periods in a cycle.**

Q) Fisher's Ideal Index does not satisfy:

(a) Time Reversal Test

(b) Factor Reversal Test

(c) Unit Test

(d) Circular Test

Q) $P_{01}Q_{01} = \frac{\sum P_1Q_1}{\sum P_0Q_0}$ which of the following tests satisfies the above?

(a) Time Reversal Test

(b) Factor Reversal Test

(c) Circular Test

(d) None of these

Q) Time reversal & factor reversal are:

(a) Quantity Index

(b) Ideal Index

(c) Price Index

(d) Test of Consistency

Q) The number of test of Adequacy is:

(a) 2

(b) 5

(c) 3

(d) 4

Finding Current Year Salary & Dearness Allowance (DA)

If the **Base Year Salary, Base Year CPI, and Current Year CPI** are given, the **Current Year Salary** is calculated as:

$$\boxed{Current\ Year\ Salary} = \left(\frac{Base\ Year\ Salary \times Current\ Year\ CPI}{Base\ Year\ CPI} \right)$$

Dearness Allowance (DA) Calculation:

$$\boxed{DA = Current\ Year\ Salary - Base\ Year\ Salary}$$

Note: If the **Base Year CPI is not given**, it is taken as **100**.

Q) In the year 2010 the monthly salary of a clerk was ₹ 24,000. The consumer price index was 140 in the year 2010, which rises to 224 in the year 2016. If he has to be rightly compensated, what additional monthly salary should be paid to him?

(a) ₹ 14,400

(b) ₹ 38,400

(c) ₹ 7,200

(d) None of these

2010	2016
140	224
24000	x

$x = \frac{24000 \times 224}{140}$

$x = 38400$

$DA = 38400 - 24000$

$= 14400$

Q) The monthly income of an employee was ₹ 8,000 in 2014. The consumer price index number was 160 in 2014, which rose to 200 in 2017. If he has to be rightly compensated, the additional dearness allowance to be paid to him in 2017 would be:

(a) ₹ 2,400

(b) ₹ 2,750

(c) ₹ 2,500

(d) None of these

2014	2017
160	200
8000	x

$x = 10000$

$DA = 10000 - 8000$

$= 2000$

5 marks

Chapter 11: Seating Arrangements

Definition

The process of organizing a group of people to sit in a pre-planned manner based on given conditions is called **Seating Arrangement**. In these types of questions, certain conditions or constraints are provided, and based on them, students need to arrange the individuals accordingly.

Types of Seating Arrangements

Seating arrangements can be broadly categorized into three types:

1. **Linear Arrangement** – People are seated in a straight line.
2. **Circular Arrangement** – People are seated in a circle or around a round table.
3. **Polygon Arrangement** – People are seated in a polygonal shape, such as a square, hexagon, etc.

General Instructions to Solve Seating Arrangement Questions

To efficiently solve seating arrangement problems, follow these steps:

1. **Review All Given Information**
   Carefully read and understand all details mentioned in the question.

2. **Identify Three Types of Information:**

A

- **Definite Information:** Statements that confirm exact placement.
  Example: "Person A is sitting at the left end of the bench."
- **Comparative Information:** Statements that describe a relative position but not an exact placement.
  Example: "Person B is sitting immediately to the right of C."

C B

- **Negative Information:** Statements that exclude possibilities but do not confirm exact positions.
  Example: "Person C is not sitting immediately to the right of D."

3. **Understanding Negative Information**
   - While negative information does not directly tell us a definite placement, it helps in eliminating possibilities.

Golden Rules for Arrangements

Start with Definite Information

- Always begin solving an arrangement problem with 100% confirmed information.
- This means placing elements (people, objects) only where their positions are explicitly stated, avoiding assumptions.

A

C

Understanding "And" vs. "Who"

- When reading a problem statement, the placement of words like **"and"** and **"who"** matters.

A is immediate right of C and immediate left of D

CAD

- Example:
  - **"A is right of B who is left of C"** → "Who" refers to B, meaning B is left of C, and A is right of B.
  - **"A is right of B and is left of C"** → A is both right of B and left of C, meaning A is between B and C.

A is at the left end of the bench. 2 people are between A and B who is at the right end of the Bench

A __ __ B

Left vs. Immediate Left

- In both **circular** and **linear** seating arrangements, do not assume the position of "left" unless specified.
- Example:
  - If a question states, **"X is to the left of Y"**, it does not mean X is **immediately** left of Y.
  - If a question specifically says **"X is immediately left of Y"**, only then is X placed directly next to Y.

Linear Arrangement

Notes

A _ _ B

or

B _ _ A

AB

A is sitting 2nd to the right of B

B _ A

Q) Five children are sitting in a row. S is sitting next to P but not T. K is sitting next to R, who is sitting on the extreme left, and T is not sitting next to K. Who is/are adjacent to S?

(a) K and P
(b) R and P
(c) Only P
(d) P and T

R K S P T

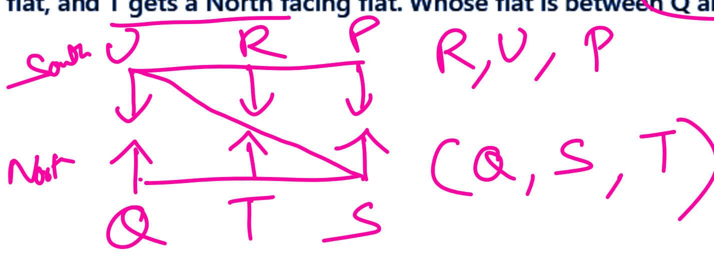
Q) Eight persons A, B, C, D, E, F, G, and H are sitting in a line. E sits second right to D. H sits fourth left to D. C and F are immediate neighbours, but C is not an immediate neighbour of A. G is not a neighbour of F. Only two persons sit between A and E. The persons on the left end and right end respectively are:

(a) G and E
(b) B and E
(c) H and E
(d) G and B

G H C F A D B E

Q) Six flats on a floor in two rows facing North and South are allotted to P, Q, R, S, T, and U. Q gets a North facing flat, and it is not next to S. S and U get diagonally opposite flats. R next to U gets a South facing flat, and T gets a North facing flat. Whose flat is between Q and S?

(a) T
(b) U
(c) R
(d) P



Q) Five persons are standing in a line. One of the two persons at the extreme ends is a professor and the other a businessman. An advocate is standing to the right of a student. An author is to the left of the businessman. The student is standing between the professor and the advocate. Counting from the left, the advocate is at which place?

(a) 1st

(b) 2nd

(c) 3rd

(d) 5th

Pro Studen Adv Author Businessm

1 2 3

~~1~~ 2 3

Q) Five girls are sitting on a bench to be photographed. Seema is to the left of Rani and to the right of Bindu. Mary is to the right of Rani. Reeta is between Rani and Mary. Who is sitting immediate right to Reeta?

(a) Seema

(b) Rani

(c) Bindu

(d) Mary

Bindu Seema Rani Reeta Mary

Q) In a line, P is sitting 13th from the left. Q is sitting 24th from the right and 3rd left from P. How many people are sitting in the line?

(a) 34

(b) 31

(c) 32

(d) 33

1 2 3 4 5 6 7 8 9 10 11 12 13

Q × × P

24 23 22 21 1

24 + 9 = 33

Q) Six persons A, B, C, D, E, and F are sitting in two rows with three persons in each row. Both rows are in front of each other. E is not at the end of any row, and D is second left to F. C is a neighbour of E and diagonally opposite to D. If B is a neighbour of F, who is in front of C, then who is sitting diagonally to F?

A E C

D B F

(a) C

(b) E

(c) A

(d) D

1 2 3 4 5 6 7 8

Q) Pran, Komal, Ravi, Shalu, Trilok, Urui, Baasu, and Volter are sitting in a row facing North

(i) Pran is fourth to the right of Trilok.

(ii) Volter is fourth to the left of Shalu.

(iii) Ravi and Urui, which are not at the ends, are neighbors of Komal and Trilok respectively.

(iv) Volter is immediate left of Pran, and Pran is the neighbor of Komal.

Identify who are sitting at the extreme ends.

(a) Pran and Volter

(b) Trilok and Urui

(c) Trilok and Shalu

(d) Shalu and Pran

Trilok Urui Baasu Volter Pran Komal Ravi Shalu

Q) A, B, C, D, and E are sitting on a bench. A is sitting next to B, C is sitting next to D, D is not sitting with E, who is on the left end of the bench. C is on the second position from the right. A is to the right of B and E. A and C are sitting together. In which position is A sitting?

(a) Between B and D

(b) Between B and C

(c) Between E and D

(d) Between C and E

E B A C D

~~E D C A~~

Q) There are four children P, Q, R, S sitting in a row. P occupies seat next to Q but not next to R. If R is not sitting next to S, who is occupying seat next to or adjacent to S?

(a) Q

(b) P

(c) P and Q

(d) None of these

S P Q R

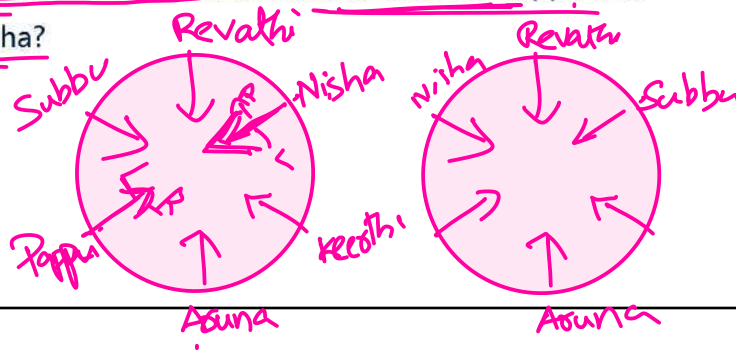
Q) Six girls are standing in such a way that they form a circle, facing the centre. Subbu is to the left of Pappu, Revathi is between Subbu and Nisha, Aruna is between Pappu and Keerthna. Who is to the right of Nisha?

(a) Ravathi

(b) Aruna

(c) Subbu

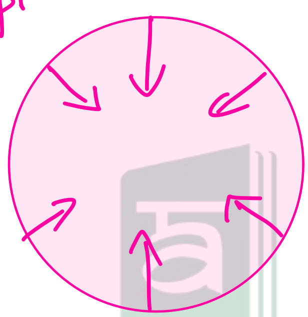
(d) Keerthana



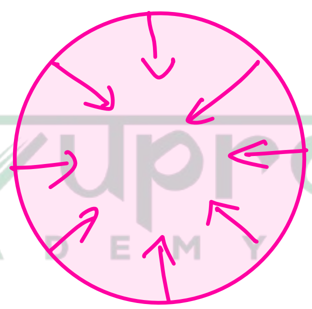
Circular Arrangement

Notes

6 people



8 people



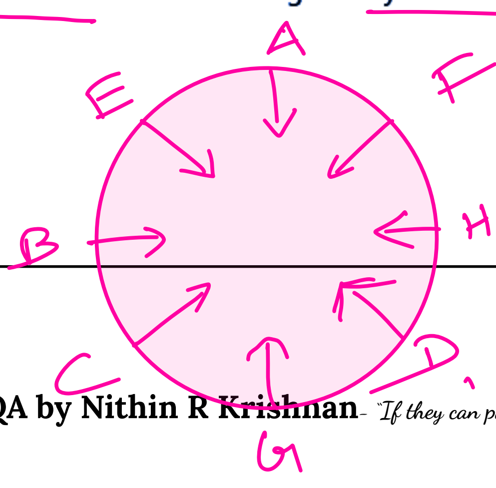
Q) A, B, C, D, E, F, G, H are sitting in a circle facing the center. D sits 3rd to the left of A. E sits to the immediate right of A. B is third to the left of D. G is second to the right of B. C is a neighbour of B. C is 3rd to the left of H. Who is sitting exactly in between F and E?

(a) C

(b) E

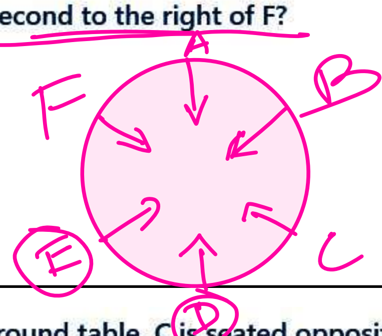
(c) H

(d) A



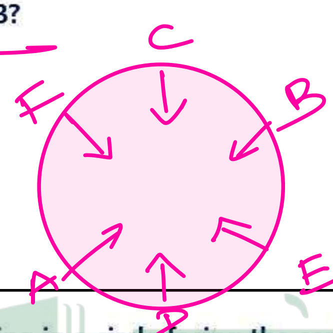
Q) Six friends - A, B, C, D, E, and F are sitting around a circular table facing towards the center of the circle. E is not sitting between B and A. A sits to the left of F, and C is fourth to the right of A. D is immediate right of E. Who sits second to the right of F?

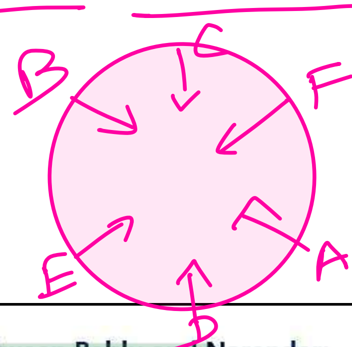
(a) C
(b) A
(c) D
(d) B



Q) A is seated between D and F at a round table. C is seated opposite to D. E is round adjust to D. Who sits opposite to B?

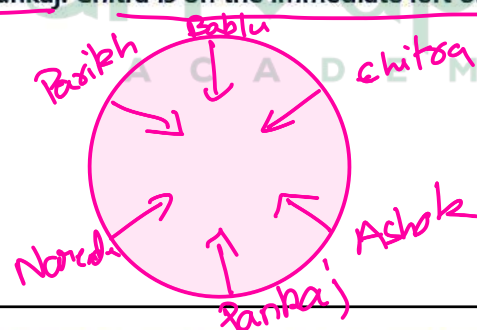
(a) A
(b) D
(c) C
(d) F





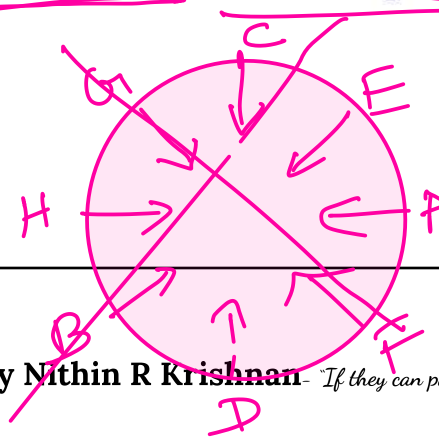
Q) Six persons are sitting in a circle facing the center. Parikh is between Bablu and Narender; Ashok is between Chitra and Pankaj. Chitra is on the immediate left of Bablu. Who is on the immediate right of Bablu?

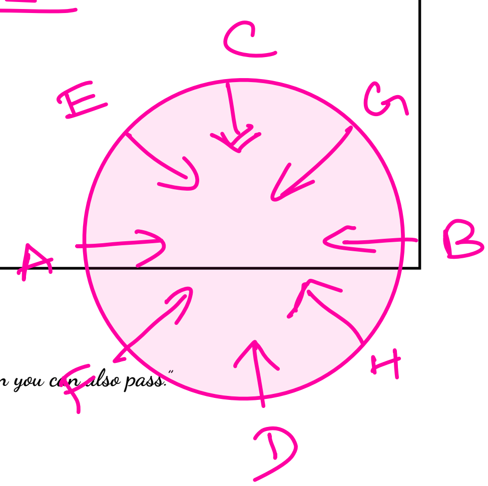
(a) Parikh
(b) Pankaj
(c) Narender
(d) Chitra



Q) Four Indian, A, B, C, and D and four Chinese E, F, G, and H are sitting in a circle around a table facing each other in a conference. No two Indians or Chinese are sitting side by side. C, who is sitting between G and E, is ~~being~~ Facing D. F is between D and A and facing G. H is to the left of B. Who is sitting left of A?

(a) E
(b) F
(c) G
(d) H





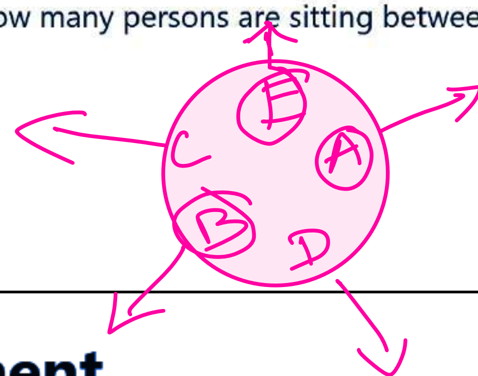
Q) Five people A, B, C, D, E are seated about a round table facing outside the center but not necessarily in the same order. A sits at the immediate right of E. C sits third to the left of D, who sits at the immediate right of A. How many persons are sitting between C & D?

(A) 1

(B) 2

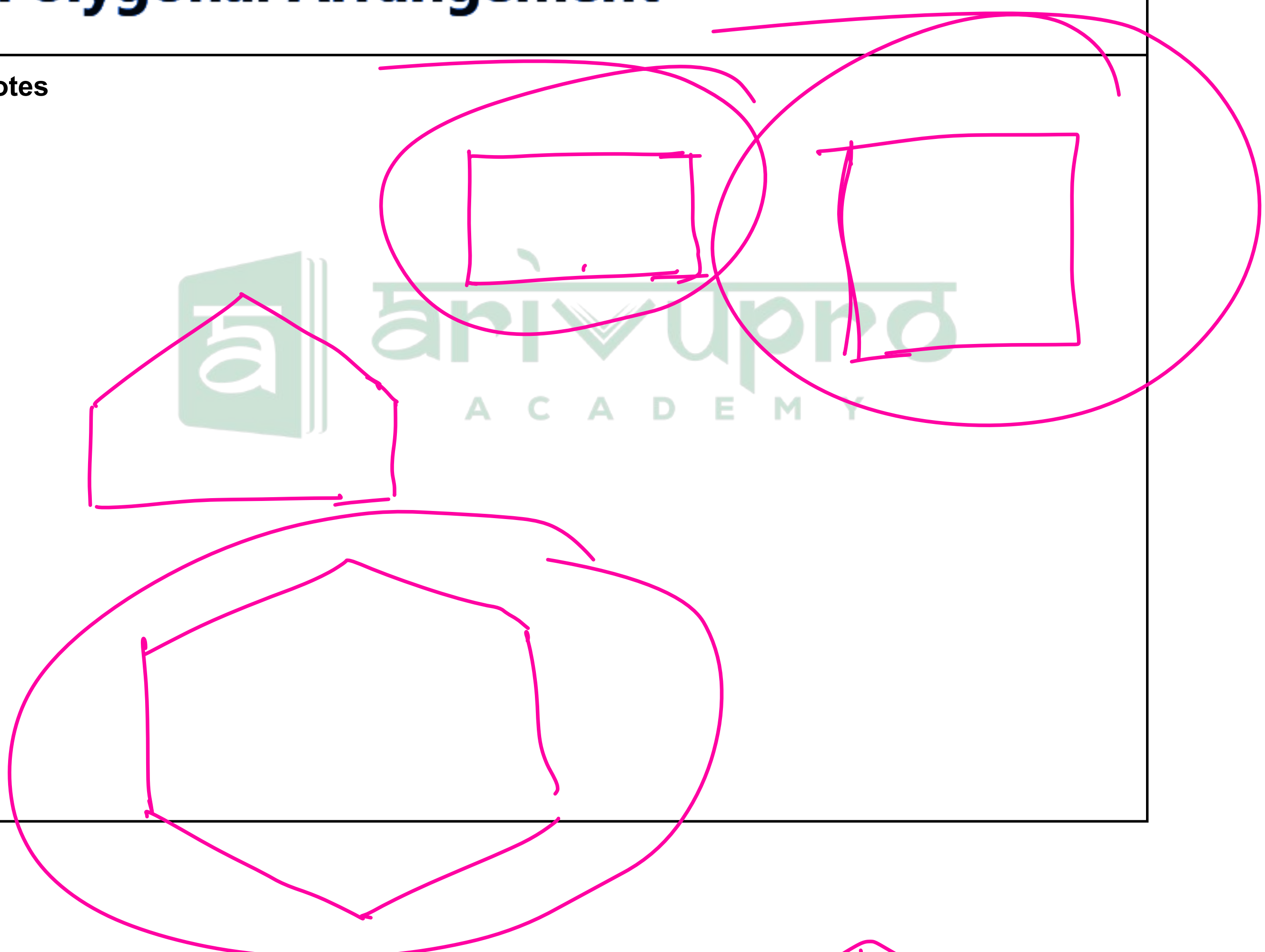
(C) 3

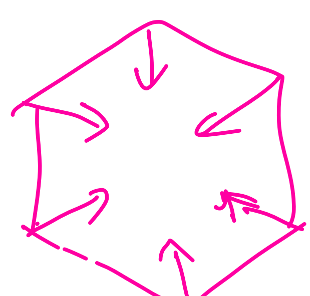
(D) 4



Polygonal Arrangement

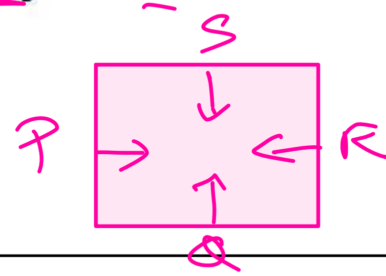
Notes





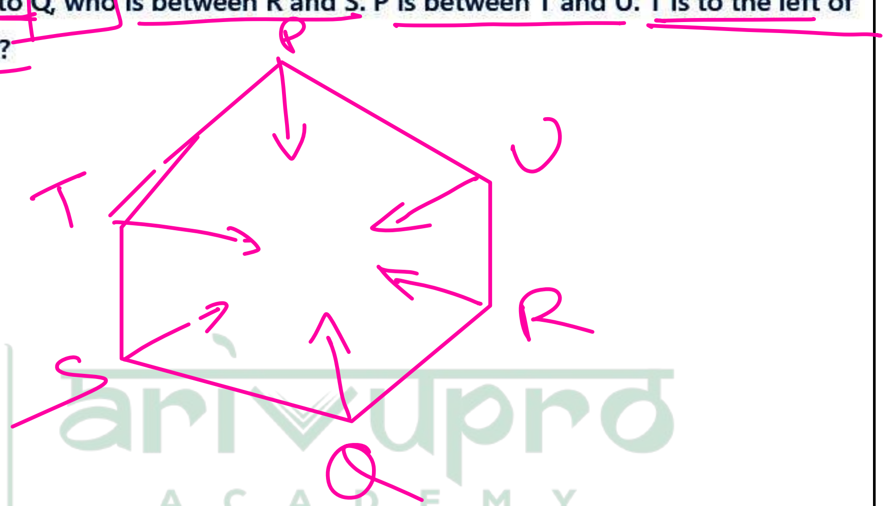
Q) P, Q, R, and S are playing a game of carrom. P, R and S, Q are partners. 'S' is to the right of 'R'. If 'R' is facing West, then 'Q' is facing which direction?

(a) South
(b) North
(c) East
(d) West



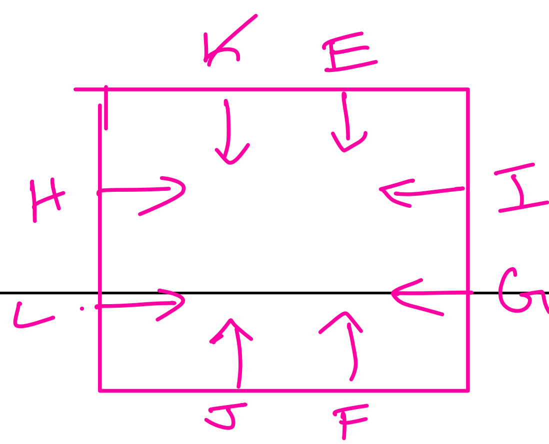
Q) If six persons are sitting at a hexagonal table, they are P, Q, R, S, T, and U, each facing the center. P is seated opposite to Q, who is between R and S. P is between T and U. T is to the left of S. Which of them is facing R?

(a) P
(b) Q
(c) U
(d) T



Q) Eight persons E, F, G, H, I, J, K, and L are seated around a square table, facing the table - two on each side. J is between L and F; G is between I and F; H, a lady member, is second to the left of J; F, a male member, is seated opposite to E, a lady member. There is a lady member between F and I. Who among the following is to the immediate left of F?

(a) G
(b) I
(c) J
(d) H



15 14
5 5

5

Ratio

A ratio is a comparison of the sizes of two or more quantities of the same kind by division. It denotes how many times one quantity is contained in another.

Definition

If a and b are two quantities of the same kind (in the same units), then the fraction $\frac{a}{b}$ is called the ratio of a to b. It is written as $a : b$. Thus, the ratio of a to b is $\frac{a}{b}$ or $a : b$.

Terms of the Ratio

The quantities a and b are called the terms of the ratio.

- a is called the first term or antecedent.
- b is called the second term or consequent.

Inverse Ratio

One ratio is the inverse of another if their product is 1. Thus, $a : b$ is the inverse of $b : a$ and vice-versa.

$a : b$ $\quad$ $b : a$

$2 : 3$ $\quad$ $3 : 2$

Definition

The inverse ratio of $a : b$ is $b : a$. It is obtained by taking the reciprocals of a and b.

Explanation

The inverse ratio of $a : b$ is:

$$\frac{1}{a} : \frac{1}{b} = b : a$$

$\cancel{a} : \cancel{b} : \cancel{c}$

$1 : 2 : 3$

$\boxed{bc : ac : ab}$

$2\times3 : 1\times3 : 1\times2$

$6 : 3 : 2$

Similarly, the inverse ratio of $a : b : c$ is:

$$\frac{1}{a} : \frac{1}{b} : \frac{1}{c} = bc : ac : ab$$

Q) Find the inverse ratio of 5 : 8

(a) 5 : 8

(b) 8 : 5

(c) 25 : 64

(d) 1 : 1

Q) Find the Inverse ratio of 5:8:9

(a) 72:45:40

8x9 : 5x9 : 5x8

:72 :

(b) 45:72:80

(c) 9:8:5

(d) 40:45:72

a : b

Q) The ratio of two quantities is 15 : 17. If the consequent of its inverse ratio is 15, then the antecedent is:

(a) 15

(b) √15

(c) 17

(d) 14

1 : 2 $1^2 : 2^2$

1. Duplicate Ratio

A duplicate ratio is obtained by squaring the terms of the original ratio. If the original ratio is $a : b$, the duplicate ratio is $a^2 : b^2$.

2. Triplicate Ratio

A triplicate ratio is obtained by cubing the terms of the original ratio. If the original ratio is $a : b$, the triplicate ratio is $a^3 : b^3$.

2 : 3

$2^3 : 3^3$

8 : 27

3. Sub-duplicate Ratio

$\sqrt{a} : \sqrt{b}$

A sub-duplicate ratio is obtained by taking the square root of the terms of the original ratio. If the original ratio is $a : b$, the sub-duplicate ratio is $\sqrt{a} : \sqrt{b}$.

4. Sub-triplicate Ratio

$\sqrt[3]{a} : \sqrt[3]{b}$

A sub-triplicate ratio is obtained by taking the cube root of the terms of the original ratio. If the original ratio is $a : b$, the sub-triplicate ratio is $\sqrt[3]{a} : \sqrt[3]{b}$.

5. Compound Ratio

A compound ratio is formed by multiplying the corresponding terms of two or more ratios. For example, the compound ratio of $a : b$ and $c : d$ is $(a \times c) : (b \times d)$.

Examples

- **Duplicate Ratio**: For the ratio **2 : 3**, the duplicate ratio is **4 : 9**.
- **Triplicate Ratio**: For the ratio **2 : 3**, the triplicate ratio is **8 : 27**.
- **Sub-duplicate Ratio**: For the ratio **4 : 9**, the sub-duplicate ratio is **2 : 3**.
- **Sub-triplicate Ratio**: For the ratio **8 : 27**, the sub-triplicate ratio is **2 : 3**.

  $2 \times 4 : 3 \times 5$
- **Compound Ratio**: For the ratios **2 : 3** and **4 : 5**, the compound ratio is **8 : 15**.
- **Inverse Ratio**: For the ratio **2 : 3**, the inverse ratio is **3 : 2**.
- **Continued Ratio**: For the quantities **2, 3, and 4**, the continued ratio is **2 : 3 : 4**.

  $2 : 3 : 4$

Q) Triplicate ratio of 4 : 5 is

(a) 125 : 64

(b) 16 : 25

(c) 64 : 125

(d) 120 : 46

$4^3 : 5^3$

$64 : 125$

Q) Find the sub-duplicate ratio of 49 : 64.

(a) 7 : 8

(b) 49 : 8

(c) 8 : 7

(d) 64 : 49

$\sqrt{49} : \sqrt{64}$

$7 : 8$

Q) Find the sub-triplicate ratio of 8 : 27.

(a) 2 : 3

(b) 3 : 2

(c) 4 : 9

(d) 8 : 27

$\sqrt[3]{8} : \sqrt[3]{27}$

$2 : 3$

Q) The ratio compounded of **4:5** and the sub-duplicate ratio of **a:9** is **8:15**, then 'a' is

(a) 2

(b) 3

(c) 4

(d) 5

$(\sqrt{a} : \sqrt{9})$

$\sqrt{a} : 3$

$\frac{\cancel{4} \times \sqrt{a}}{\cancel{5} \times 3} = \frac{\cancel{8}^{2}}{\cancel{15}}$

$\sqrt{a} = 2$

$a = 4$

Q) $\frac{3x-2}{5x+6}$ is the duplicate ratio of $\frac{2}{3}$. Then find the value of x:

(a) 2

(b) 6

(c) 5

(d) 9

Duplicate of $\frac{2}{3} = \frac{4}{9}$

$\frac{3x-2}{5x+6} = \frac{4}{9}$

$27x - 18 = 20x + 24$

$7x = 42$

$x = 6$

Continued Ratio

Find a:b:c:d

c:d = 9:5

Notes: a:b = 1 : 2

b:c = 3 : 4

a:b:c = 1×3 : 3×2 : 2×4

= 3 : 6 : 8

a:b = 1 : 2

b:c = 3 : 4

a:b:c = 3 : 6 : 8

c:d = 9 : 5

27 : 54 : 72 : 40

a:b = 3 : 4

b:c = 9 : 11

= 27 : 36 : 44

a:b:c

Continued Ratio Based Problems

Q) If **A : B = 4 : 7** and **B : C = 5 : 9**, then find **A : B : C**.

(a) **20 : 35 : 63**

(b) **4 : 7 : 9**

(c) **5 : 7 : 9**

(d) **20 : 28 : 45**

4 7

5 9

20 : 35 : 63

a b

b c

a:b = 4:3

b:c = 5:9

Q) If A : B = 3 : 4, B : C = 5 : 6, and C : D = 7 : 8, find A : B : C : D.

(a) 105 : 140 : 168 : 192

(b) 15 : 20 : 24 : 28

(c) 30 : 40 : 48 : 56

(d) 21 : 28 : 35 : 40

3 4

5 6

a:b:c

15 : 20 : 24

c d

7 8

105 : 140 : 168 : 192

105 : 192

Q) P, Q, and R are three cities. The ratio of average temperature between P and Q is 11 : 12 and that between P and R is 9 : 8. The ratio between the average temperature of Q and R is:

(a) 22 : 27

1.222 (b) 27 : 22

(c) 32 : 33

(d) None

P : Q = 11 : 12

P : R = 9 : 8

Q : P = 12 : 11

P : R = 9 : 8

Q : P : R

108 : 99 : 88

Q : R

108 : 88 = 1.227

Q) X, Y, and Z together start a business. If X invests 3 times as much as Y invests and Y invests two-thirds of what Z invests, then the ratio of capitals of X, Y, and Z is:

(a) 3:9:2

(b) 6:3:2

(c) 3:6:2

(d) 6:2:3

$\frac{6}{2} = \frac{3}{1}$

X : Y : Z

X = 3Y

$\boxed{\frac{X}{Y} = \frac{3}{1}}$

$Y = \frac{2}{3}Z$

$\frac{Y}{Z} = \frac{2}{3}$

X : Y = 3 : 1

Y : Z = 2 : 3

X : Y : Z

6 : 2 : 3

An amount divided in Certain ratio

Q) Two numbers are in the ratio 7 : 8. If 3 is added to each of them, their ratio becomes 8 : 9. The numbers are:

(a) 14, 16

(b) 24, 27

(c) 21, 24

(d) 16, 18

$\frac{7}{8} = 0.875$

$\frac{21}{24} = 0.875$

$\frac{8}{9} = 0.8888$

24, 27 = 0.8888

Q) If the ratio of two numbers is 7:11. If 7 is added to each number, then the new ratio will be 2:3.
Then the numbers are:

(a) 49, 77 = 0.6363

(b) 42, 45

(c) 43, 42

(d) 39, 40

$\frac{7}{11} = 0.6363$

Q) The ages of two persons are in the ratio 5 : 7. Eighteen years ago, their ages were in the ratio 8 : 13. Their present ages (in years) are:

(a) 50, 70 = 0.7142

(b) 70, 50

(c) 40, 56

(d) None

$\frac{5}{7} = 0.7142$

18 years ago

$\frac{8}{13} = 0.6153$

32, 52

$\frac{32}{52} = 0.6153$

Q) The price of a scooter and moped are in the ratio 7 : 9. The price of the moped is ₹ 1,600 more than that of the scooter. Then the price of the moped is:

2 → 1600

1 → 800

(a) ₹ 7,200

(b) ₹ 5,600

(c) ₹ 800

(d) ₹ 700

7x is price of scooter

9x is the price of moped

$9x - 7x = 1600$

$2x = 1600$

$x = 800$

$9 \times 800 = 7200$

Q) The students of two classes are in the ratio 5 : 7. If 10 students left from each class, the remaining students are in the ratio 4 : 6. Then the number of students in each class is:

(a) 30, 40

(b) 25, 24

(c) 40, 60

(d) 50, 70

$5 \div 7 = 0.7142$

$\frac{5\cancel{0}}{7\cancel{0}}$

Q) Find three numbers in the ratio 1 : 2 : 3, so that the sum of their squares is equal to 504.

(a) 6, 12, 18

(b) 3, 6, 9

(c) 4, 8, 12

(d) 5, 10, 15

Option Hit

$6x = m^+$ $12x = m^+$ $18x = m^+$

MRC $= 504$

Q) The ratio of two numbers is 3 : 4. The difference of their squares is 28. The greater number is:

(a) 8

(b) 12

(c) ~~24~~

(d) 64

Let 8 be the greater no:

$4x = 8$

$x = 2$

smaller no: $= 6$

$8^2 - 6^2 = 28$

$1 \div 2 = 0.5$

Q) Find three numbers in the ratio 1:2:3 so that the sum of their cubes is equal to 972.

(a) 2, 4, 6

(b) 1, 2, 3

(c) 3, 6, 9

(d) none

Option Hits

$3 \div 6 = 0.5$

$3x$ (= 2 times)

$6x$ (= 2 times)

$9x$ (= 2 times)

$= 972$

Q) Eight people are planning to share equally the cost of a rental car. If one person withdraws from the arrangement and the others share equally the entire cost of the car, then the share of each of the remaining persons increased by:

(a) $\frac{1}{9}$
(b) $\frac{1}{8}$
(c) $\frac{1}{7}$
(d) $\frac{7}{8}$

$\frac{1}{7}$

Q) The salaries of **A, B, and C** are in the ratio **2:3:5.** If increments of **15%, 10%,** and **20%** are allowed respectively to their salaries, then what will be the new ratio of their salaries?

(a) 3:3:10
(b) 10:11:20
(c) 23:33:60
(d) Cannot be determined

A 2 $\xrightarrow{+15\%}$ 2.3
B 3 $\xrightarrow{+10\%}$ 3.3
C 5 $\xrightarrow{+20\%}$ 6

New salary 2.3 3.3 6

$\frac{2.3}{3.3} = 0.6964$

Q) If the salary of **P** is **35%** lower than that of **Q** and the salary of **R** is **40% higher** than that of **Q**, then the ratio of the salary of P & R will be:

(a) 13:28 = 0.464
(b) 7:12
(c) 9:20
(d) 11:25

Assume Q's salary is 100

P = 100 − 35% = 65

R = 100 + 40% = 140

P : R = 65 : 140 = 0.464

Q) Incomes of **R and S** are in the ratio $\boxed{7:9}$ and their expenditures are in the ratio $\boxed{4:5}$. Their total expenditure is equal to the income of R. What is the ratio of their savings?

(a) 23:36
(b) 28:41
(c) 31:43
(d) 35:46

	R	S
I/C	7×9	9×9 81
Exp	4×7	5×7

Total exp = 9×7 = 63

I/C 63 81

Exp 28. 35

Savin 35 : 46

Problems on Coins

Q) The ratio of the number of ₹5 coins and ₹10 coins is 8:15. If the value of ₹5 coins is ₹360, then the number of ₹10 coins will be:

(a) 72

(b) 120

(c) 135

(d) 185

	₹5	₹10
No. of coins	$8x$	$15x$
Values	$40x$	$150x$

$40x = 360$

$x = 9$

$15x = 15 \times 9$

$= 135$

Q) A bag contains 105 coins consisting of 50 paise and 25 paise coins. The ratio of the number of these coins is 4:3. The total value (in ₹) in the bag is:

(a) 43.25

(b) 41.25

(c) 39.25

(d) 35.25

	50ps	25ps
No. of coin	$4x$	$3x$
Value	$0.5 \times 4x = 2x$	$0.25 \times 3x = 0.75x$

Total value $= 2x + 0.75x$

$= 2.75x$

$4x + 3x = 105$

$x = \frac{105}{7} = 15$

$= 2.75 \times 15$

$= 41.25$

Problems on Mixture

Q) A dealer mixes rice costing **₹13.84 per kg** with rice costing **₹15.54 per kg** and sells the mixture at **₹17.60 per kg**. So, he earns a profit of **14.6%** on his sale price. The proportion in which he mixes the two qualities of rice is:

(a) **3 : 7** = 0.428

(b) **5 : 7**

(c) **7 : 9**

(d) **9 : 11**

A B

CP 13.84 15.54

Sp = 17.60

CP of mix = 17.60 − 14.6%

= 15.03

15.0304

0.5096 : 1.1904

$\frac{0.5096}{1.1904} = 0.428$

Q) The cost price of **Rice A** is **₹50 per kg** and that of **Rice B** is **₹70 per kg**. In what ratio should they be mixed so that the cost price of the mixture may be **₹65 per kg**?

(a) **1:3**

(b) **1:5**

(c) **2:3**

(d) **3:4**

Rice A Rice B

CP 50 70

Mix 65

5 15

5 : 15

1 : 3

Proportion

$\frac{a}{b} = \frac{c}{d}$

A proportion is defined as the equality of two ratios.

- If $a : b = c : d$, it is usually expressed as:

$ad = bc$

$$\frac{a}{b} = \frac{c}{d} \quad \text{or} \quad ad = bc \quad \text{or} \quad a : b :: c : d$$

- In this proportion, d is called the **fourth proportional** (or the highest proportional).

Key Points:

1. Terms of Proportion:

- The quantities a, b, c, and d are called the **terms of the proportion**.
- a is the first term, b is the second term, c is the third term, and d is the fourth term.
- The **first and fourth terms** are called **extremes** (or extreme terms).
- The **second and third terms** are called **means** (or middle terms).

2. Product of Extremes and Means:

- For a proportion $a : b :: c : d$,

$$\text{Product of extremes} = \text{Product of means}$$

i.e.,

$$ad = bc.$$

Continued Proportion

$\frac{a}{b} = \frac{b}{c}$

Definition:

- A proportion $a : b :: b : c$ is said to be a **continued proportion**.
- Here, c is the **third proportional** (or highest proportional) and b is the **mean proportional**.

$a : b = c : d$

Key Concepts:

1. **Continued Proportion:**
   - If $a : b :: b : c$, then:

$$\frac{a}{b} = \frac{b}{c} \quad \text{or} \quad b^2 = ac \quad \text{or} \quad b = \sqrt{ac}$$

2. Cross Product Rule:

- For three quantities a, b, and c in continued proportion:

$$a : b = b : c \quad \text{or} \quad \frac{a}{b} = \frac{b}{c} \quad \text{or} \quad b^2 = ac$$

- Here, b is the **mean proportional**, a is the **first proportional**, and c is the **third proportional**.

Properties of Proportion

1. Basic Properties

1. **Basic Proportion Property**
   - If $a : b = c : d$, then $ad = bc$.

$\frac{a}{b} = \frac{c}{d}$

$ad = bc$

2. Invertendo

- If $a : b = c : d$, then $b : a = d : c$.

$\frac{a}{b} = \frac{c}{d}$ $\quad \frac{b}{a} = \frac{d}{c}$

3. Alternendo

- If $a : b = c : d$, then $a : c = b : d$.

4. Componendo

- If $a : b = c : d$, then $a + b : b = c + d : d$.

$\frac{a}{b} = \frac{c}{d}$ $\frac{a+b}{b} = \frac{c+d}{d}$

5. Dividendo

- If $a : b = c : d$, then

$$a - b : b = c - d : d.$$

$\frac{a-b}{b} = \frac{c-d}{d}$

6. Componendo and Dividendo

- If $a : b = c : d$, then

$$a + b : a - b = c + d : c - d.$$

$\frac{a+b}{a-b} = \frac{c+d}{c-d}$

7. Addendo

- If $a : b = c : d = e : f = \ldots$, then each of these ratios is equal to

$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{a+c+e+}{b+d+f+\ldots}$

$$\frac{a + c + e + \ldots}{b + d + f + \ldots}.$$

8. Subtrahendo

- If $a : b = c : d = e : f = \ldots$, then each of these ratios is equal to

$$\frac{a - c - e - \ldots}{b - d - f - \ldots}.$$

Q) If $\frac{x}{y} = \frac{z}{w}$ implies $\frac{x}{z} = \frac{y}{w}$, then the process is called:

(a) Dividendo

(b) Addendo

(c) Alternendo

(d) Invertendo

Q) If $\frac{p}{q} = \frac{r}{s} = \frac{p+r}{q+s}$, the process is called:

(a) Addendo

(b) Subtrahendo

(c) Componendo

(d) Dividendo

Q) Which of the numbers are not in proportion?

(a) 6, 8, 5, 7

(b) 7, 3, 14, 6

(c) 18, 27, 12, 18

(d) 8, 6, 12, 9

$\frac{6}{8} = 0.75 \quad \frac{5}{7} = 0.7114$

Q) The mean proportion between 18 and 72 is:

a c

(a) 24

(b) 36

(c) 48

(d) 54

$b = ?$

$b^2 = ac$

$b = \sqrt{ac}$

$= \sqrt{18 \times 72} = 36$

Q) The 3rd proportion to 15 and 45 is:

a b c

(a) 90

(b) 135

(c) 105

(d) 75

$b^2 = ac$

$45^2 = 15 \times c$

$c = 135$

Q) If A : B = 2 : 5, then

$$\frac{(10A + 3B)}{(5A + 2B)}$$

$$\frac{10 \times 2 + 3 \times 5}{5 \times 2 + 2 \times 5}$$

$$= \frac{35}{20} = 1.75$$

is equal to:

(a) 7 : 4

(b) 7 : 3

(c) 6 : 5

(d) 7 : 9

Q) Find two numbers such that the mean proportional between them is 18 and the third proportional between them is 144:

(a) 9, 36

(b) 8, 32

(c) 7, 28

(d) 6, 24

$b = \sqrt{ac}$

$18 = \sqrt{9 \times 36}$

$= 18$

Q) The mean proportional between 24 and 54 is:

(a) 33

(b) 34

(c) 35

(d) 36

$b = \sqrt{24 \times 54}$

Q) If $x : y = 2 : 3$, then

$$\frac{5x + 2y}{3x - y} = \frac{5 \times 2 + 2 \times 3}{3 \times 2 - 3}$$

$$= \frac{16}{3}$$

(a) 19 : 3

(b) 16 : 3

(c) 7 : 2

(d) 7 : 3

Q) If $a : b = 3 : 7$, then

$$3a + 2b : 4a + 5b = ?$$

$$\frac{3\times3+2\times7}{4\times3+5\times7} = \frac{23}{47}$$

Options:

(a) 23 : 47

(b) 27 : 43

(c) 24 : 51

(d) 29 : 53

Q) If $a : b = 9 : 4$, then $\sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}} = ?$

$$\sqrt{\frac{9}{4}} + \sqrt{\frac{4}{9}}$$

$$= 2.1666$$

(a) $\frac{3}{2}$

(b) $\frac{2}{3}$

(c) $\frac{6}{13}$

(d) $\frac{13}{6}$ $= 2.1666$

a b

a b a c

Q) The ratio of the third proportion of 12, 30 to the mean proportion of 9, 25 is:

(a) 2 : 1

(b) 5 : 1

(c) 7 : 15

(d) 3 : 5

$b^2 = ac$

$\frac{30^2}{12} = c$

75

$b = \sqrt{9 \times 25}$

$= 15$

$75 : 15 = 5$

Q) If $x : y : z = 7 : 4 : 11$, then $\frac{x+y+z}{z}$ is:

(a) 2

(b) 3

(c) 4

(d) 5

$= \frac{7+4+11}{11}$

$= \frac{22}{11} = 2$

Indices (Exponents)

Definition:

- An index (plural: indices) or exponent is the power to which a number (the base) is raised.
- If a number a is multiplied by itself n times, it is written as a^n, where a is the base and n is the index or exponent.

4

3^7

3 3^1

Basic Concepts:

- **Base**: The number that is being multiplied.
- **Index (Exponent)**: The number of times the base is multiplied by itself.

Important Notes:

- Any base raised to the power of zero is defined to be **1**, i.e., $a^0 = 1$ (for $a \neq 0$).
- The nth root of a number a is written as $\sqrt[n]{a}$ and is equivalent to $a^{1/n}$.

Laws of Indices:

$3^4 \times 3^5 = 3^{4+5}$

1. **Product of Powers (Same Base):**

$$\boxed{a^m \times a^n} = a^{m+n}$$

   - The bases must be the same.

$\frac{3^5}{3^2} = 3^{5-2} = 3^3$

2. **Quotient of Powers (Same Base):**

$$\frac{a^m}{a^n} = a^{m-n}$$

$\frac{2}{2}$ $2^{1-1} = 2^0 = 1$

3. **Power of a Power:**

$(3^5)^9 = 3^{5\times9} = 3^{45}$

$$(a^m)^n = a^{m \cdot n}$$

$3^\infty = \infty$

4. **Power of a Product:**

$3^2 \times 3^5 = 3^7$

$$\boxed{(ab)^n = a^n \cdot b^n}$$

$(6)^4 = 3^4 \times 2^4$

5. **Power of a Quotient:**

$3^5 \times 3^4 = 3^9$

$$\boxed{\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}}$$

6. **Zero Exponent:**

$$\boxed{a^0 = 1} \quad (\text{for } a \neq 0)$$

$3^{-1} = \frac{1}{3}$

7. **Negative Exponent:**

$$a^{-m} = \frac{1}{a^m}$$

$(1+i)^{-n} = \frac{1}{(1+i)^n}$

8. **Fractional Exponent:**

$$\sqrt[m]{a} = a^{1/m}$$

$3^{1/n} = \sqrt[n]{3}$

$3^{1/2} = \sqrt[2]{3}$

$3^{1/3} = \sqrt[3]{3}$

9. **Equality of Exponents:**

- If $a^x = a^y$, then $x = y$.

Algebraic Identities

Key Identities:

1. $(a+b)^2 = a^2 + 2ab + b^2$
2. $(a-b)^2 = a^2 - 2ab + b^2$
3. $\boxed{a^2 - b^2 = (a+b)(a-b)}$
4. $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$
5. $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
6. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
7. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
8. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$(a+b)(a-b$

$(a+b)^3 = a^3 + b^3 + 3ab(a+b$

$(a-b)^3 =$

Special Note:

- If $a^{1/3} + b^{1/3} + c^{1/3} = 0$, then $(a+b+c)^3 = 27abc$.

Q) Simplify:

$$\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n}$$

$$\frac{\cancel{2^n}\left[1 + \frac{1}{2}\right]}{\cancel{2^n}\left[2 - 1\right]} = \frac{1.5}{1}$$

Put n = 1

$$\frac{2 + 1}{4 - 2} = \frac{3}{2}$$

(a) $\frac{1}{2}$

(b) $\frac{3}{2}$

(c) $\frac{2}{3}$

(d) $\frac{1}{3}$

Q) On simplification

$$\frac{1}{1+z^{a-b}+z^{a-c}}+\frac{1}{1+z^{b-c}+z^{b-a}}+\frac{1}{1+z^{c-a}+z^{c-b}}$$

reduces to:

(a) $\frac{1}{z^{2(a-b-c)}}$

(b) $\frac{1}{z^{(a-b-c)}}$

(c) 1

(d) 0

Q) If $4^x = 5^y = 20^z$, then z is equal to:

(a) xy

(b) $\frac{x+y}{xy}$

(c) $\frac{1}{xy}$

(d) $\frac{xy}{x+y}$

Assume $4^x = 5^y = 20^z = k$

$4^x = k$, $4 = k^{1/x}$ | $5 = k^{1/y}$ | $20 = k^{1/z}$

$4 \times 5 = k^{1/z}$

$k^{1/x} \times k^{1/y} = k^{1/z}$

$k^{\frac{1}{x}+\frac{1}{y}} = k^{1/z}$

$\frac{1}{x}+\frac{1}{y} = \frac{1}{z}$

$\frac{x+y}{xy} = \frac{1}{z}$

$z = \frac{xy}{x+y}$

Q) If $2^x \times 3^y \times 5^z = 360$, then what is the value of x, y, z?

(a) $3, 2, 1$

(b) $1, 2, 3$

(c) $2, 3, 1$

(d) $1, 3, 2$

option Hit

$2^3 \times 3^2 \times 5^1 = 8 \times 9 \times 5$

$= 360$

$= RHS$

Q) Find the value of x, if

$$x(x)^{1/5} = (x^{1/5})^x$$

(a) 3

(b) 4

(c) 2

(d) 6

$x^1 \times x^{1/5} = x^{x/5}$

$x^{1+\frac{1}{5}} = x^{\frac{x}{5}}$

$1 + 0.2 = \frac{x}{5}$

$1.2 = \frac{x}{5}$

$x = 6$

Q) The value of

$$\left(\frac{x^a}{x^b}\right)^{a^2+ab+b^2} \times \left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2} \times \left(\frac{x^c}{x^a}\right)^{c^2+ac+a^2}$$

is:

(a) 0

(b) 1

(c) 2

(d) none

Q) If $(25)^{150} = (25x)^{50}$, then the value of x will be:

(a) 5^3

(b) 5^4

(c) 5^2

(d) 5

$(25)^{150} = 25^{50} \times x^{50}$

$25^{100} = x^{50}$

$5^4 = 25^2$

$(5^2)^2 = (25)^2$

RHS $= (25^2)^{50}$

$= 25^{100} =$ LHS

Q) If $abc = 2$, then the value of

$$\frac{1}{1+a+2b^{-1}} + \frac{1}{1+\frac{1}{2}b+c^{-1}} + \frac{1}{1+c+a^{-1}}$$

is:

(a) 1

(b) 2

(c) 3

(d) $\frac{1}{2}$

arivupro ACADEMY

Q) If $a = \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}}$ and $b = \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}}$, then the value of $\frac{1}{a^2} + \frac{1}{b^2}$ is equal to:

(a) 480 $a = 21.9544$ $b = 0.045580$

(b) 482

(c) 484

(d) 486

$\log 35 = x$

$@ \quad 35 = a^x$

Logarithms

Definition:

- The logarithm of a number to a given base is the index or the power to which the base must be raised to produce the number, i.e., to make it equal to the given number.

Basic Concept:

- If there are three quantities indicated by a, x, and n, they are related as follows:

$$a^x = n$$

  - Where $n > 0$, $a > 0$, and $a \neq 1$.
  - Then x is said to be the logarithm of the number n to the base a.
  - Symbolically, it can be expressed as:

$$x = \log_a n$$

Types of Logarithms

Logarithms can be categorized into two main types:

1. Natural Logarithm

- A logarithm with base "e" (Euler's number) is called a **natural logarithm.**
- It is represented as **Log_ex** or **ln(x)**.
- The constant **e ≈ 2.7183.**

$e = 2.72$

- Used frequently in calculus, exponential growth, and scientific applications.

2. Common Logarithm

- A logarithm with base **10** is called a **common logarithm**.
- It is represented as **Log_{10}x** or simply **Log(x)** when no base is mentioned.
- If the base is not explicitly given, it is assumed to be **10** in most practical and commercial calculations.

Characteristic & Mantissa:

- The integral part of a common logarithm is called the **characteristic**.
- The non-negative decimal part is called the **mantissa**.

Antilogarithm:

- If x is said to be the logarithm of N to a given base, then N is said to be the antilogarithm of x to that base.

$$\text{If } \log_a N = x \Rightarrow N = \text{antilog } x$$

Additional Notes:

- $$\log_a n = x \Rightarrow a^x = n$$

Laws of Logarithms

$\log_{10} 5 + \log_{10} 30$
$= \log_{10} 5 \times 30$
$= \log_{10} 150$

1. **Product Rule:**

$$\log_a(xy) = \log_a x + \log_a y$$

- Where x and y are positive numbers.

2. **Quotient Rule:**

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$\log x^{10} = 10 \log x$

3. **Power Rule:**

$$\log_a(x^k) = k \log_a x$$

$\log x^{35} = 35 \log x$

4. **Change of Base Rule:**

$$\boxed{\log_b m = \frac{\log_a m}{\log_a b}}$$

$\log_{24} 35 = \frac{\log_a 35}{\log_a 24}$

5. Logarithm of 1:

$$\log_a 1 = 0$$

6. Logarithm of a Number to the Same Base:

- The logarithm of a number to the same base is equal to 1.

$$\log_a a = 1$$

- Examples:

$$\log_5 5 = 1$$
$$\log_{10} 10 = 1$$

$a^{\log_a x} = x$

7. Inverse Property:

$$\boxed{a^{\log_a x} = x}$$

8. Equality Property:

- If $\log x = \log y$, then $x = y$.

log

2^{64}

$\log 2^{64}$

* **9. Number of Digits in a Given Numeral:**

$$\text{Number of digits in a given numeral} = \log(\text{numeral})$$

Q) $7\log\left(\frac{16}{15}\right) + 5\log\left(\frac{25}{24}\right) + 3\log\left(\frac{81}{80}\right)$ is equal to:

(a) 0

(b) 1

(c) $\log 2$

(d) $\log 3$

$\log\left(\frac{16}{15}\right)^7 + \log\left(\frac{25}{24}\right)^5 + \log\left(\frac{81}{80}\right)^3$

$\log\left(\left(\frac{16}{15}\right)^7 \times \left(\frac{25}{24}\right)^5 \times \left(\frac{81}{80}\right)^3\right)$

$16 \div 15 \times (= 6 \text{ times})$

M^+

$25 \div 24 \times (= 4 \text{ times})$

$81 \div 80 \times = 2 \text{ times} \times MRC$

$1.27 \times$ MRC 2 times M^+

Q) If $\log_{10000} x = -\frac{1}{4}$, then x is given by:

(a) $\frac{1}{100}$

(b) $\frac{1}{10}$

(c) $\frac{1}{20}$

(d) None of these.

$x = (10000)^{-1/4}$

$= (10^4)^{-1/4}$

$= 10^{4 \times -\frac{1}{4}}$

$= \frac{1}{10}$

Q) If $\log_a(ab) = x$, then $\log_b(ab)$ is:

(a) $\frac{1}{x}$
(b) $\frac{x}{1+x}$
(c) $\frac{x}{x-1}$
(d) None of these

$\log_a ab = x$

$\boxed{\log_a a} + \log_a b = x$

$1 + \log_a b = x$

$\log_a b = x - 1$

$\boxed{\log_b a} = \frac{1}{x-1}$

$\log_b a + \log_b b = \log_b a + 1$

$= \frac{1}{x-1} + 1$

$= \frac{1 + x - 1}{x-1}$

$= \frac{x}{x-1}$

Q) Number of digits in the numeral for 2^{64}
(Given $\log 2 = 0.30103$)

(a) 18 digits
(b) 19 digits
(c) 20 digits
(d) 21 digits

$\log 2^{64} = 64 \times \log 2$

$= 64 \times 0.30103$

$= 19.265$

Q) The value of

$$\frac{\log_3 8}{\log_9 16 \cdot \log_4 10}$$

is:

(a) $3\log_{10} 2$

(b) $7\log_{10} 3$

(c) $3\log_e z$

(d) None

$$= \frac{\frac{\log 8}{\log 3}}{\frac{\log 16}{\log 9} \times \frac{\log 10}{\log 4}} = \frac{\frac{\log 2^3}{\log 3}}{\frac{\log 2^4}{\log 3^2} \times \frac{\log 10}{\log 2^2}} = \frac{\frac{3\log 2}{\cancel{\log 3}}}{\frac{4\cancel{\log 2} \times \log 10}{2\cancel{\log 3} \times \cancel{2} \times \cancel{\log 2}}}$$

$$= \frac{3(\log 2)}{(\log 10)}$$

$$= 3\log_{10} 2$$

Q) The value of

$$\log_5 3 \times \log_3 4 \times \log_2 5$$

is:

(a) 0

(b) 1

(c) 2

(d) $\frac{1}{2}$

$$\frac{\cancel{\log 3}}{\cancel{\log 5}} \times \frac{\log 4}{\cancel{\log 3}} \times \frac{\cancel{\log 5}}{\log 2}$$

$$= \frac{\log 2^2}{\log 2}$$

$$= 2\frac{\cancel{\log 2}}{\cancel{\log 2}}$$

Q) If $\log 2 = 0.3010$ and $\log 3 = 0.4771$, then the value of $\log 24$ is:

(a) 1.0791

(b) 1.7323

(c) 1.3801

(d) 1.8301

$$\log 24 = \log 3 \times 2^3$$
$$= \log 3 + \log 2^3$$
$$= 0.4771 + 3 \log 2$$
$$= 0.4771 + 3 \times 0.3010$$
$$= \underline{\underline{1.3801}}$$

Q) If $\log_3[\log_4(\log_2 x)] = 0$, then the value of x will be:

(a) 4

(b) 8

(c) 16

(d) 32

$$\log_4(\log_2 x) = 3^0$$
$$\log_4(\log_2 x) = 1$$
$$\log_2 x = 4$$
$$x = 2^4 = \underline{16}$$

Q) The value of

$$\log_5\left(1+\frac{1}{5}\right)+\log_5\left(1+\frac{1}{6}\right)+\cdots+\log_5\left(1+\frac{1}{624}\right)$$

is:

(a) 2

(b) 3

(c) 5

(d) 0

$\log \frac{6}{5} + \log \frac{7}{6} + \ldots + \log \frac{625}{624}$

$\log \frac{\cancel{6}}{5} \times \frac{\cancel{7}}{\cancel{6}} \times \frac{\cancel{8}}{\cancel{7}} \times \ldots \frac{\cancel{624}}{\cancel{623}} \times \frac{625}{\cancel{624}}$

$\log_5 \frac{625}{5}$

$\log_5 125 = \log_5 5^3$

$= 3$

Q) $\log_{0.01} 10,000 = ?$

(a) 2

(b) -2

(c) 4

(d) -4

$\log_{0.01} 10000 = x$

$10000 = (0.01)^x$

Q) $\log_a \sqrt{3} = \frac{1}{6}$, find the value of a

(a) 9

(b) 81

(c) 27

(d) 3

$\sqrt{3} = a^{1/6}$ Taking power of 6 on both si..

$(\sqrt{3})^6 = a$

$(3^{1/2})^6 = a$

$3^3 = a$

$a = 27$

Chapter 2: Equations

Equations

Definition of an Equation: A mathematical statement of equality.

- **Conditional Equation**: An equation that holds true for certain values of the variable involved. If the equality is true for a specific value of the variable, it is called a conditional equation and uses the equality sign '='.

  Example:

  $$\frac{x+2}{3}+\frac{x+3}{2}=3$$

  holds true only for $x=1$.

- **Identity**: An equation that holds true for all values of the variable involved.

  Example:

  $$\frac{x+2}{3}+\frac{x+3}{2}=\frac{5x+13}{6}$$

  This is an identity since it holds for all values of the variable x.

Solution or Root of the Equation: The value of the variable which satisfies an equation is called the solution or root of the equation.

Types of Equations:

- **Linear Equation (Simple Equation)**: An equation in which the highest power of the variable is 1. This is also called an equation of degree 1.
- **Simultaneous Linear Equations**: Two or more linear equations involving two or more variables.
- **Quadratic Equation**: An equation of degree 2 (highest power of the variable is 2).
- **Cubic Equation**: An equation of degree 3 (highest power of the variable is 3).

Simple Equation

A simple equation in one unknown x is in the form $\boxed{ax + b = 0}$.

Where a, b are known constants and $a \neq 0$.

Note: A simple equation has only one root.

$5x + 10 = 0$
$5x = -10$
$x = -2$

$\log_{10} 10$

Q) If $\log_{10} 5 + \log_{10}(5x + 1) = \log_{10}(x + 5) + 1$ then, the value of x = ____

(a) 7
(b) 3
(c) 5
(d) 10

$\cancel{\log_{10}} 5(5x+1) = \cancel{\log_{10}} 10(x+5)$
$25x + 5 = 10x + 50$
$15x = 45$
$x = 3$

Q) If |x - 2| + |x - 3| = 7 then, 'x' will be equal to

(a) 6
(b) -1
(c) 6 and -1
(d) None of the above.

Try option (a)

$|6-2| + |6-3|$
$= 4 + 3 = 7 = RHS$

$|-1-2| + |-1-3|$
$= 3 + 4 = 7$

Q) $\frac{2x+5}{10} + \frac{3x+10}{15} = 5$

(a) 10.58

(b) 9.58

(c) 9.5

(d) None

Option Hit using (b)

RHS = 5

LHS = $\frac{2\times9.58+5}{10} + \frac{3\times9.58+10}{15}$

$2\times9.58+5 \div 10$ M+ $3\times9.58+10 \div 15$ M+

MRC = 4.998 ≃ 5

Simultaneous Linear Equations in Two Unknowns

The general form of a linear equation in two unknowns x and y is $ax + by + c = 0$, where a, b are non-zero coefficients and c is a constant.

Two such equations, $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, form a pair of simultaneous equations in x and y.

A value for each unknown which satisfies both equations simultaneously will give the roots of the equations.

Solve the following simultaneous linear equations:

$$4x + 3y = 7$$

$$3x + 4y = 10$$

$x = \frac{4\times7 - 3\times10}{4\times4 - 3\times3}$

$= \frac{-2}{7}$

$y = \frac{4\times10 - 3\times7}{4\times4 - 3\times3}$

$= \frac{13}{7}$

Notes:

Q) A man sells 6 radios and 4 televisions for ₹18,480. If 14 radios and 2 televisions are sold for the same amount, what is the price of a television?

(a) ₹1,848

(b) ₹840 7x

(c) ₹1,680

(d) ₹3,360

42

3x

Price of radio x TV = y

$6x + \boxed{4y} = 18480$

$14x + \boxed{2y} = 18480$

2 | 6, 14
3, 7

$y = \frac{6 \times 18480 - 14 \times 18480}{6 \times 2 - 14 \times 4} = 3360$

Q) A man went to the Reserve Bank of India with ₹1,000. He asked the cashier to give him ₹5 and ₹10 notes only in return. The man got 175 notes in all. Find how many notes of ₹5 and ₹10 did he receive?

(a) (25, 150)

(b) (40, 110)

(c) (150, 25)

(d) None.

150 + 25 = 175

150 × 5 = 750

25 × 10 = 250

1000

Q) If the ratio of (5x - 3y) and (5y - 3x) is 3 : 4, then the value of x : y is:

(a) 27 : 29

(b) 29 : 27

(c) 3 : 4

(d) 4 : 3

$\frac{5x - 3y}{5y - 3x} = \frac{3}{4} = 0.75$

Try option (a)

$x = 27$ $y = 29$

$\frac{5 \times 27 - 3 \times 29}{5 \times 29 - 3 \times 27} = \frac{48}{64} = 0.75$

Q) The equation x + 5y = 33; $\frac{x+y}{x-y} = \frac{13}{3}$ has the solution (x, y) as:

Option Hi-

(a) ~~(4, 8)~~

(b) (8, 5)

(c) (4, 16)

(d) (16, 4)

$8 + 5 \times 5 = 8 + 25 = 33$

Q) If $\frac{3}{x+y} + \frac{2}{x-y} = -1$ and $\frac{1}{x+y} - \frac{1}{x-y} = \frac{4}{3}$, then (x, y) is:

$\rightarrow \frac{1}{1+2} - \frac{1}{1-2}$

$= \frac{1}{3} - (-1)$

$= \frac{1}{3} + 1$

$= \frac{4}{3}$

(a) ~~(2, 1)~~

(b) (1, 2)

(c) (-1, 2)

(d) (-2, 1)

Try option (a)

$\frac{3}{2+1} + \frac{2}{2-1} = 1 + 2 = 3$

Try option (b)

$\frac{3}{1+2} + \frac{2}{1-2} = 1 + \frac{2}{-1} = -1$

Q) A number consists of two digits such that the digit in one's place is thrice the digit in ten's place. If 36 be added then the digits are reversed. Find the number ________.

(a) 62

(b) 26 ✓

(c) 39

(d) None of these

$26 + 36 = 62$

Simultaneous Linear Equation in Three Variables

Definition

A system of simultaneous linear equations with three unknowns is a set of equations where each equation is linear and involves three variables. These systems aim to find the specific values of these variables that satisfy all the equations in the system simultaneously. Typically, such systems can be written in the form:

$$\begin{cases} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{cases}$$

Where:

- x, y, and z are the unknown variables.
- $a_1, a_2, a_3, b_1, b_2, b_3, c_1, c_2, c_3$ are the coefficients of the variables.
- d_1, d_2, d_3 are constants.

Q) $\frac{x}{4} = \frac{y}{3} = \frac{z}{2}$; $7x + 8y + 5z = 62$

$\frac{\text{Maths}}{40} = \frac{\text{LR}}{20} = \frac{\text{Stats}}{40}$

$\frac{25}{40} = \frac{10}{20} = \frac{30}{40}$

$7 \times 4 + 8 \times 3 + 5 \times 2$

$= 28 + 24 + 10$

$= 62$

(a) (4, 3, 2)

(b) (2, 3, 4)

(c) (3, 4, 2)

(d) (4, 2, 3)

Q) Solve the system of equations:

$$\frac{xy}{x+y} = 20, \quad \frac{yz}{y+z} = 40, \quad \frac{zx}{z+x} = 24$$

Find the values of (x, y, z).

(a) (120, 60, 30)

(b) (60, 30, 120)

(c) (30, 120, 60)

(d) (30, 60, 120)

x y z

Option Hit

$\frac{30 \times 60}{30+60} = \frac{1800}{90} = 20$

$\frac{60 \times 120}{180} = 40$

$\frac{30 \times 120}{15} = 24$

Quadratic Equation

An equation of the form $ax^2 + bx + c = 0$ where x is a variable and a, b, c are constants with $a \neq 0$ is called a quadratic equation or equation of the second degree.

- **Pure Quadratic Equation**: When $b = 0$, the equation is called a pure quadratic equation.
- **Affected Quadratic Equation**: When $b \neq 0$, the equation is called an affected quadratic.

General Solution:

The roots of the quadratic equation $ax^2 + bx + c = 0$ can be found using the quadratic formula:

0

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

zeros/roots/Solutions x

Sum and Product of the Roots:

Sum of root = $-\frac{\text{coef of } x}{\text{coef of } x^2}$

Product = $\frac{\text{const}}{\text{coef of } x^2}$

Let one root be α and the other root be β.

- **Sum of the Roots:**

$$\alpha+\beta=\frac{-b+\sqrt{b^2-4ac}}{2a}+\frac{-b-\sqrt{b^2-4ac}}{2a}=\frac{-2b}{2a}=\frac{-b}{a}$$

Thus, the sum of the roots is:

$$\alpha+\beta=-\frac{b}{a}$$

Which can be interpreted as:

$$\text{Sum of roots}=-\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

Product of the Roots:

$$\alpha\beta=\left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)\left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right)=\frac{c}{a}$$

So, the product of the roots is:

$$\boxed{\alpha\beta=\frac{c}{a}}$$

This can also be written as:

$$\text{Product of roots}=\frac{\text{constant term}}{\text{coefficient of } x^2}$$

How to Construct a Quadratic Equation

x² − (sum)x + product =

For the equation $ax^2 + bx + c = 0$, we have:

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

or

$$x^2 - \left(-\frac{b}{a}\right)x + \frac{c}{a} = 0$$

or

$$x^2 - (\text{Sum of the roots})x + (\text{Product of the roots}) = 0$$

Q) Given a quadratic equation $x^2 - 6x + 9 = 0$, what are the roots of the equation?

Options:

A) 3 and 3

B) 3 and -3

C) -3 and -3

D) 2 and 4

Sum = 6

Product = 9

3, 3

3+3 = 6

3×3 = 9

Q) Which of the following equations has the ~~sum of~~ the roots 3 and 5?

(A) $x^2 - 15x + 8 + 0$

(B) $x^2 - 8x + 15 = 0$

(C) $x^2 + 3x + 5 = 0$

(D) $x^2 + 8x - 15 = 0$

Sum = 3+5 = 8

Produ = 3×5 = 15

x² − (sum)x + pro = 0

x² − 8x + 15 = 0

Nature of the Roots for the Equation $ax^2 + bx + c = 0$

1. If $b^2 - 4ac = 0$:
   - The roots are real and equal.
2. If $b^2 - 4ac > 0$ and a perfect square:
   - The roots are real, distinct, and rational.
3. If $b^2 - 4ac > 0$ and not a perfect square:
   - The roots are real, distinct, and irrational.
4. If $b^2 - 4ac < 0$:
   - The roots are imaginary.

$\sqrt{b^2-4ac}$

$\sqrt{36}$
$=6$

Q) If $b^2 - 4ac$ is a perfect square but not equal to zero then the roots are:

(a) real and equal
(b) real, irrational and equal
(c) real, rational and unequal
(d) imaginary.

Q) Find the positive value of k for which the equations: $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$ will have real roots:

(a) 12
(b) 16
(c) 18
(d) 22

$b^2-4ac \geq 0$

$b^2-4ac=0$

$k^2-4\times1\times64=0$

$k^2=256$

$k=16$

Q) The quadratic equation $x^2 - 2kx + 16 = 0$ will have equal roots when the value of k is ______.

(a) ± 1
(b) ± 2
(c) ± 3
(d) ± 4

$b^2 - 4ac = 0$
$(-2k)^2 - 4 \times 16 = 0$
$4k^2 = 64$
$k^2 = 16$
$k = \pm 4$

Note

1. If one root of the quadratic equation is $P + \sqrt{q}$:
   - Then the other root will be $P - \sqrt{q}$.
2. If one root is the reciprocal of the other:
   - Then $\alpha\beta = 1$ (since $\alpha \cdot \frac{1}{\alpha} = 1$).

$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$
$-1 + \sqrt{3}$
$-1 - \sqrt{3}$
$3 + \sqrt{5}$
$3 - \sqrt{5}$

Q) If one root of an equation is $2 + \sqrt{5}$, then the quadratic equation is:

(a) $x^2 + 4x - 1 = 0$
(b) $x^2 - 4x - 1 = 0$
(c) $x^2 + 4x + 1 = 0$
(d) $x^2 - 4x + 1 = 0$

$\alpha = 2 + \sqrt{5}$ $\beta = 2 - \sqrt{5}$
$\alpha + \beta = \text{sum} = 4$
$\alpha\beta = (2+\sqrt{5})(2-\sqrt{5})$
$= 2^2 - (\sqrt{5})^2$
$= 4 - 5 = -1$

$x^2 - (\text{sum})x + \text{product} = 0$
$x^2 - 4x - 1 = 0$

Q) If one of the roots of the equation $x^2 + px + a$ is $\sqrt{3} + 2$, then the value of p and a is:

(a) -4, -1

(b) 4, -1

(c) -4, 1

(d) 4, 1

$\alpha = 2 + \sqrt{3}$ $\beta = 2 - \sqrt{3}$

Sum = 4 prod = 4 - 3 = 1

$x^2 - (-p)x + a = 0$

$-p = 4$

$p = -4$

$a = 1$

Q) Roots of the equation $3x^2 - 14x + k = 0$ will be reciprocal of each other if:

(a) $k = -3$

(b) $k = 0$

(c) $k = 3$

(d) $k = 14$

Product = 1 $\alpha\beta = 1$

2 , $\frac{1}{2}$

$2 \times \frac{1}{2} = 1$

$\frac{c}{a} = 1$

$\frac{k}{3} = 1$

$k = 3$

Q) One root of the equation: $x^2 - 2(5 + m)x + 3(7 + m) = 0$ is reciprocal of the other. Find the value of m.

(a) -7

(b) 7

(c) $\frac{1}{7}$

(d) $-\frac{1}{7}$

Product = 1

$\frac{c}{a} = 1$

$\frac{3(7+m)}{1} = 1$

$21 + 3m = 1$

$3m = -20$

$m = \frac{-20}{3}$

$= -6.666$

Q) If one root of the equation $x^2 - 3x + k = 0$ is 2, then value of k will be:

(a) -10

(b) 0

(c) 2

(d) 10

put $x = 2$

$2^2 - 3 \times 2 + k = 0$

$4 - 6 + k = 0$

$-2 + k = 0$

$k =$ ___

Q) If roots of equation $x^2 + x + r = 0$ are α and β and $\alpha^3 + \beta^3 = -6$. Find the value r?

(a) $-\frac{5}{3}$

(b) $\frac{7}{3}$

(c) $-\frac{4}{3}$

(d) 1

$\alpha\beta = r$

Sum $= \alpha + \beta = -1$

$(\alpha+\beta)^3 = \alpha^3 + \beta^3 + 3\alpha\beta(\alpha+\beta)$

$(-1)^3 = -6 + 3 \times r(-1)$

$-1 = -6 - 3r$

$-1 + 6 = -3r$

$5 = -3r$

$r = \frac{-5}{3}$

Q) If the area and perimeter of a rectangle are 6000 cm^2 and 340 cm respectively, then the length of the rectangle is:

(a) 140 cm

(b) 120 cm

(c) 170 cm

(d) 200 cm

$lb = \text{area}$

$2(l+b) = \text{perime}$

$l(b) = 6000$

$\cancel{2}(l+b) = \cancel{340}$ 170

$b = \boxed{170 - l}$

$l(170 - l) = 6000$

option Hit

RHS = 6000

LHS

$l(170 - l)$

option b

$120(170 - 120)$

$= 6000 = RHS$

Q) If one root of the equation $px^2 + qx + r = 0$ is r, then the other root of the equation will be:

(a) $1/q$
(b) $1/r$
(c) $1/p$
(d) $\frac{1}{p+q}$

Product $= \frac{r}{p}$

$\alpha = r$ $\beta = ?$

$r \times \frac{1}{p}$

If two roots are in the ratio $\alpha : \beta$:

$\alpha\beta b^2 = (\alpha+\beta)^2 ac$

m : n

$mnb^2 = (m+n)^2 ac$

$$\alpha\beta b^2 = (\alpha+\beta)^2 ac$$

Q) The positive value of k for which the roots of the equation $12x^2 + kx + 5 = 0$ are in the ratio 3:2, is:

$m = 3$ $n = 2$

$mnb^2 = (m+n)^2 ac$

(a) $\frac{5}{12}$
(b) $\frac{12}{5}$
(c) $\frac{5\sqrt{10}}{2}$
(d) $5\sqrt{10}$ $= 15.81$

$3 \times 2 \times k^2 = (3+2)^2 \times 12 \times 5$

$6 \times k^2 = 25 \times 12 \times 5$

$k = \sqrt{250} = 15.81$

Q) If the ratio of the roots of the equation $4x^2 - 6x + p = 0$ is 1:2, then the value of p is:

(a) 1
(b) 2
(c) -2
(d) -1

$m = 1 \quad n = 2$

$mnb^2 = (m+n)^2 ac$

$1 \times 2 \times 36 = 9 \times 4 \times p$

$p = 2$

Q) If α and β are the roots of the equation $\boxed{x^2 + 7x + 12 = 0}$, then the equation whose roots $(\alpha+\beta)^2$ and $(\alpha-\beta)^2$ will be:

(a) $x^2 - 14x + 49 = 0$
(b) $x^2 - 24x + 144 = 0$
(c) $x^2 - 50x + 49 = 0$
(d) $x^2 - 19x + 144 = 0$

$x^2 + 7x + 12 = 0$

Sum $= -7$

Pro $= 12$

$\alpha = -3, \beta = -4$

$(\alpha+\beta)^2 = (-3-4)^2 = 49$

$(\alpha-\beta)^2 = (-3+4)^2 = 1$

Sum of root $= 49 + 1 = 50$

Product $= 49 \times 1 = 49$

Q) If $\alpha + \beta = -2$ and $\alpha\beta = -3$, then α and β are the roots of the equation, which is:

(a) $x^2 - 2x - 3 = 0$
(b) $x^2 + 2x - 3 = 0$
(c) $x^2 + 2x + 3 = 0$
(d) $x^2 - 2x + 3 = 0$

Q) If the difference between the roots of the equation $x^2 - kx + 8 = 0$ is 4, then the value of k is:

(a) 0

(b) ± 4

(c) $\pm 8\sqrt{3}$

(d) $\boxed{\pm 4\sqrt{3}}$ $= 6.9$

$(a+b)^2 = a^2+b^2+2ab$

$(a+b)^2 = (a-b)^2+4ab$

α, and β

$\alpha - \beta = 4$

$\alpha + \beta = k$

$\alpha\beta = 8$

$(\alpha+\beta)^2 = \alpha^2+\beta^2+2\alpha\beta$

$= \alpha^2+\beta^2-2\alpha\beta+2\alpha\beta+2\alpha\beta$

$= \boxed{\alpha^2+\beta^2-2\alpha\beta}+4\alpha\beta$

$(\alpha+\beta)^2 = (\alpha-\beta)^2+4\alpha\beta$

$k^2 = 4^2+4\times 8$

$= 16+32$

$= 48$

$k = \sqrt{48}$

$= 6.9$

Q) If α, β are the roots of the equation $x^2 + x + 5 = 0$ then $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$ is equal to:

(a) $\frac{16}{5}$

(b) 2

(c) 3

(d) $\frac{14}{5}$

$\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3+\beta^3}{\alpha\beta}$

$\alpha+\beta = -1$

$\alpha\beta = 5$

$(\alpha+\beta)^3 = \boxed{\alpha^3+\beta^3}+3\alpha\beta(\alpha+\beta)$

$\alpha^3+\beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

$= (-1)^3 - 3\times 5(-1)$

$= -1+15$

$= 14$

$= \frac{14}{5}$

Q) Find value of $x^2 - 10x + 1$ if $x = \frac{1}{5-2\sqrt{6}}$:

(a) 25

(b) 1

(c) 0

(d) 49

$10^2 - 10\times 10+1$

$= 100-100+1$

$= 1$

$x = 6\sqrt{}$

$\times 2$

change sign

$+5$

$\div =$

$x = 9.9 \approx 10$

Q) The value of

$$2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\dots}}} = x$$

$2+\frac{1}{x}=x$

$\frac{2x+1}{x}=x$

$2x+1=x^2$

$x^2-2x-1=0$

$\frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{+2\pm\sqrt{4+4}}{2}$

$\frac{2\pm\sqrt{8}}{2}$

$1\pm\sqrt{2}$

is:

(a) $1+\sqrt{2}$
(b) $2\pm\sqrt{5}$
(c) $2\pm\sqrt{3}$
(d) None.

Q) The value of

$$\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\dots}}}}$$

3×2

is:

(a) -3
(b) 2
(c) 3
(d) 4

Q) The value of

$$\sqrt{30+\sqrt{30+\sqrt{30+\sqrt{30+\dots}}}}$$

6×5

is:

(a) -7
(b) -8
(c) 6
(d) none

Cubic Equation

- **Definition:** A polynomial in x having degree 3 (highest power of x is 3).

General form:

$ax^3 + bx^2 + cx + d = 0$

Sum of the roots:

$\text{Sum of the roots} = -\frac{b}{a} = -\frac{\text{coe of } x^2}{\text{co of } x^3}$

Product of the roots:

$\text{Product of the roots} = -\frac{d}{a} = -\frac{\text{constant}}{\text{coe of } x^3}$

Q) The roots of the equation $y^3 + y^2 - y - 1 = 0$ are:

$1+1-1=1$

(a) ~~(1, 1, -1)~~

(b) $\boxed{(-1, -1, 1)}$

(c) (1, 1, 1)

(d) None of these

Sum $= -\frac{1}{1} = -1$

Product $= -\frac{(-1)}{1} = 1$

$-1-1+1=-1$

$-1 \times -1 \times 1 = 1$

Q) The roots of the cubic equation $x^3 - 7x + 6 = 0$ are:

(a) 1, 2, and 3

(b) 1, -2, and 3

(c) $\boxed{1, 2, \text{ and } -3}$

(d) 1, -2, and -3

Sum of roots = 0

Product = -6

$1+2-3=0$

$1 \times 2 \times (-3)$

$= -6$

Q) The roots of the cubic equation

$$x^3 + 7x^2 - 21x - 27 = 0$$

are:

Sum = −7

Prod = 27

(a) -1, 3, 9

(b) 1, -3, 9

(c) -1, 3, -9

(d) -1, -3, 9

$-1+3-9 = -7$

$-1 \times 3 \times -9 = 27$

Chapter 6: Sequence and Series – Arithmetic and Geometric Progressions

5

Difference Between Sequence and Series

Aspect	Sequence	Series
Definition	Ordered list of numbers.	Sum of the terms of a sequence.
Nature	List of terms.	Single numerical result (sum).
Example	$2, 4, 6, 8, \ldots$	$2 + 4 + 6 + 8 + \ldots$
Focus	On the arrangement of terms.	On the summation of terms.

Examples

1. **Arithmetic Sequence:** $1, 3, 5, 7, \ldots$
   - Series: $1 + 3 + 5 + 7 + \ldots$
2. **Geometric Sequence:** $2, 4, 8, 16, \ldots$
   - Series: $2 + 4 + 8 + 16 + \ldots$

Q) The nth terms of the series 3 + 7 + 13 + 21 + 31 + ... is

(a) ~~4n - 1~~

(b) ~~$n^2 + 2n$~~

(c) $n^2 + n + 1$

(d) $n^3 + 2$

$a_3 = 13$

12 -

option

Put n=3

$9+3+1=13$

Arithmetic Progression (AP):

- **Definition**: A sequence in which there exists a constant difference between consecutive terms.

$a_1\ a_2\ a_3\ \ldots\ldots a_n$

$d = a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \ldots = a_n - a_{n-1}$

Formula for the n-th Term:

$a_n = a + (n-1)d$

$$a_n = a_1 + (n-1)d$$

$a + (n-1)d$

$a_7 = a+6d$

$a_8 = a+7d$

$a_9 = a+8d$

$a_{1000} = a+999d$

$a_{200} = a+199d$

$(n-1)d$

Q) If in an A.P., Tn represents nth term. (a_n)

If $t_7 : t_{10} = 5:7$ then $t_8 : t_{11}$ = ______

(a) 13:16

(b) 17:23 = 0.739

(c) 14:17

(d) 15:19

$\frac{a_7}{a_{10}} = \frac{5}{7}$

$\frac{a+6d}{a+9d} = \frac{5}{7}$

$7a + 42d = 5a + 45d$

$2a = 3d$

$a = \frac{3}{2}d$

$\frac{a_8}{a_{11}} = \frac{a+7d}{a+10d}$

$= \frac{\frac{3}{2}d + 7d}{\frac{3}{2}d + 10d}$

$= \frac{d}{d}\left(\frac{1.5+7}{1.5+10}\right)$

$\frac{8.5}{11.5} = 0.739$

Q) The 4th term of an A.P. is three times the first and the 7th term exceeds twice the third term by 1. Find the first term 'a' and common difference 'd'.

(a) a = 3, d = 2

(b) a = 4, d = 3

(c) a = 5, d = 4

(d) a = 6, d = 5

$a_4 = 3a$

$a + 3d = 3a$

$3d = 2a$

option Hit

(a)

$3 \times 2 = 2 \times 3$

$6 = 6$

LHS = RHS

Q) If a, b, c are in Arithmetic Progression (A.P.), then the value of a - b + c is:

(a) a

(b) -b

(c) b

(d) c

Let $a=1$ $b=2$ $c=3$

$a-b+c = 1-2+3 = 2 = b$

Q) If the P^{th} term of an A.P. is 'q' and the q^{th} term is 'p', then its r^{th} term is:

(a) $\boxed{p + q - r}$

(b) $P + q + r$

(c) $p - q - r$

(d) $p - q$

Q) The value of K, for which the terms 7K + 3, 4K – 5, 2K + 10 are in A.P., is:

a_1 a_2 a_3

$a_2 - a_1 = a_3 - a_2$

(a) 13

(b) -13

(c) 23

(d) $\boxed{-23}$

$4k-5-(7k+3) = (2k+10)-(4k-5)$

$4k-5-7k-3 = 2k+10-4k+5$

$k = -23$

Q) The 20th term of arithmetic progression whose 6th term is 38 and 10th term is 66 is:

(a) 118

(b) ~~136~~

(c) 178

(d) 210

$a_6 = 38$

$a+5d = 38$

$a+35=38$

$a=3$

$a+9d=66$

$a+5d+4d=66$

$38+4d=66$

$4d=28$

$d=7$

$a_{20} = a+19d$

$= 3+19\times7 = 136$

Q) How many numbers between 74 and 25,556 are divisible by '5'?

(a) 5090

(b) 5097

(c) 5095

(d) 5075

75, 80 25555

a_n

$n = ?$ $a = 75$ $d = 5$

$a_n = a + (n-1)d$

$25555 = 75 + (n-1)5$

$n = 5097$

Q) Find the 17[th] term of an AP series if 15[th] and 21[st] terms are 30.5 and 39.5 respectively.

(a) 33.5

(b) 35.5

(c) 36.0

(d) 38.0

$a + 14d = 30.5$

$a + 20d = 39.5$

$a + 14d + 6d = 39.5$

$30.5 + 6d = 39.5$

$d = 1.5$

$a + 14 \times 1.5 = 30.5$

$a = 9.5$

$a_{17} = a + 16d$

$= 9.5 + 16 \times 1.5$

$= 33.5$

Sum of n Terms of an Arithmetic Progression (AP):

- **Formula**:

$$S_n = \frac{n}{2}(a + a_n)$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

- Where:
  - S_n: Sum of the first n terms.
  - n: Number of terms.
  - a: First term.
  - a_n: Last term (n-th term).

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Q) An Arithmetic progression has 13 terms whose sum is 143. The third term is 5, so the first term is:

(a) 4

(b) 7

(c) 9

(d) 2

$S_n = 143$

$n = 13$

$a_3 = 5$

$a + 2d = 5$

$\cancel{143}^{11} = \frac{\cancel{13}}{2}(2a + 12d)$

$\cancel{22}^{11} = \cancel{2}(a + 6d)$

$11 = a + 6d$

$11 = a + 2d + 4d$

$11 = 5 + 4d$

$4d = 6$

$d = \frac{6}{4} = 1.5$

$a + 2 \times 1.5 = 5$

$a = 2$

Q) The sum of all two-digit odd numbers is:

(a) 2475

(b) 2575

(c) 4950

(d) 5049

11, 13, ... 99

a $d=2$ a_n

$a_n = a+(n-1)d$

$99 = 11+(n-1)2$

$88 = (n-1)2$

$n = 45$

$S_n = \frac{n}{2}(a+a_n)$

$= \frac{45}{2}(11+99)$

$= 2475$

Q) If the sum of first 'n' terms of an A.P. is $6n^2 + 6n$, then the fourth term of the series is:

(a) 120

(b) 72

(c) 48

(d) 24

$a_4 = S_4 - S_3$

$= 120 - 72$

$= 48$

$S_4 = 6\times4^2+6\times4$

$= 120$

$S_3 = 6\times3^2+6\times3$

$= 72$

Q) If the sum of 'n' terms of an Arithmetic Progression (A.P.) is $3x^2 + 5x$ and its m^{th} term is 164, then the value of m is:

(a) 27

(b) 28

(c) 24

(d) 26

$a_m = S_m - S_{m-1}$

$164 = 3m^2+5m-(3(m-1)^2+5(m-1))$

$= 3m^2+5m-(3(m^2+1-2m)+5m-5)$

$= 3m^2+5m-(3m^2+3-6m+5m-5)$

$= \cancel{3m^2}+5m-\cancel{3m^2}+2+m$

$164 = 6m+2$

$162 = 6m$

$m = \frac{162}{6} = 27$

$3\times27^2+5\times27$

$-(3\times26^2+5\times26)$

Q) Find the value of 'x' for the following data:

1 + 7 + 13 + 19 + ... + x = 225

a_n

(a) 56

(b) 63

(c) 49

(d) 42

$S_n = 225$

$a=1$ $d=6$

31 37 43

+49

Q) If $S_n = n^2p$ and $S_m = m^2p$ ($m \neq n$) is the sum of an A.P., then S_p =

(a) p^2
(b) p^3
(c) $2p^3$
(d) p^4

Finding the Sum and Middle Term in an Arithmetic Progression (AP)

This section explains how to determine the **sum of n terms** when the **middle term** is given and vice versa.

1. Relation Between Sum and Middle Term

The sum of n terms (S_n) in an arithmetic progression can be directly related to the middle term:

1, 2, 3 Sum = 6

2 × 3 = 6

$$S_n = \text{Middle term} \times n$$

This means the total sum can be obtained by multiplying the middle term by the number of terms.

2. Finding the Middle Term

If the sum of the first n terms is given, the middle term can be calculated using:

$$\text{Middle term} = \frac{\text{Sum of n terms of an AP}}{n}$$

This formula helps find the central value of an arithmetic series when its total sum is known.

Q) If 8th term of an A.P. is 15, then sum of its 15 terms is:

(a) 15

(b) 0

(c) 225

(d) 225/2

8

$15 \times 15 = 225$

$a + 4d = 22$

$3a + 6d = 42$

Q) If the sum of 3 arithmetic means between "a" and 22 is 42 then 'a' = ______.

(a) 14

(b) 11

(c) 10

(d) 6

$a, a+d, a+2d, a+3d, a+4d$

$a_1 \quad a_2 \quad a_3 \quad a_4 \quad 22$

$\boxed{a+4d=22}$

$a+d+a+2d+a+3d = 42$

$3a+6d = 42$

$a=6 \quad d=4$

Q) Insert two arithmetic means between 68 and 260:

(a) 132, 196

(b) 130, 194

(c) ~~70, 258~~

(d) None of the above

68, 132, 196, 260

64 64 64

Q) Divide 69 into 3 parts which are in A.P. and are such that the product of first two parts is 460:

(a) 20, 23, 26

(b) ~~21, 23, 25~~

(c) ~~19, 23, 27~~

(d) ~~22, 23, 24~~

$a-d, a, a+d$

$20 \times 23 \times 26 = 460$

Divide 69 into 5 parts which are in AP

$a-2d, a-d, a, a+d, a+2d$

Geometric Progression (GP):

- **Definition:** A sequence where each term has a common ratio with the previous term.

General Form of GP:

a_n

$$a, ar, ar^2, ar^3, \ldots, \boxed{ar^{n-1}}$$

a

- **Where:**
    - a: First term
    - r: Common ratio
    - n: Number of terms
- **Common Ratio:**

$$r = \frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = \frac{a_5}{a_4} = \ldots = \frac{a_n}{a_{n-1}}$$

Formula for the n-th Term:

$$a_n = a \cdot r^{n-1}$$

Sum of n Terms in a Geometric Progression (GP):

$$S_n = a\left(\frac{r^n - 1}{r - 1}\right)$$

1. For $\boxed{r > 1}$:

$$S_n = a \cdot \frac{r^n - 1}{r - 1}$$

$$S_n = a\left(\frac{1 - r^n}{1 - r}\right)$$

2. For $\boxed{r < 1}$:

$$S_n = a \cdot \frac{1 - r^n}{1 - r}$$

Q) Find the number of terms of the series:

$25, 5, 1, \ldots, \frac{1}{3125}$

(a) 6

(b) 7

(c) 8

(d) 9

$a = 25$

$r = \frac{5}{25} = \frac{1}{5}$

$a_n = \frac{1}{3125}$

$a_n = a r^{n-1}$

$\frac{1}{3125} = 25 \times \left(\frac{1}{5}\right)^{n-1}$

Option Hit

RHS = 0.00032

LHS = $25 \times \left(\frac{1}{5}\right)^{n-1}$

Option a

$= 25 \times \left(\frac{1}{5}\right)^{5}$ ✗

Try opt c

$= 25 \times \left(\frac{1}{5}\right)^{7} = 0.00032 = \text{RHS}$

Q) In a geometric progression, the 3rd and 6th terms are respectively 1 and $-1/8$. The first term (a) and common ratio are respectively:

(a) ~~4 and~~ $\frac{1}{2}$

(b) 4 and $-\frac{1}{4}$

(c) 4 and $-\frac{1}{2}$

(d) 4 and $\frac{1}{4}$

$a r^2 = 1$

$a r^5 = -\frac{1}{8}$

Try option a

$4 \times \left(\frac{-1}{2}\right)^2 = 4 \times \frac{1}{4} = 1$

$4 \times \left(-\frac{1}{2}\right)^5 = -\frac{1}{8}$

Q) The sum of the first 20 terms of a G.P. is 1025 times the sum of the first 10 terms of the same G.P. Then the common ratio is:

(a) $\sqrt{2}$

(b) 2

(c) $2/\sqrt{2}$

(d) $1/2$

$S_{20} = 1025 \times S_{10}$

$\cancel{a} \times \frac{(r^{20} - 1)}{\cancel{r-1}} = 1025 \times \cancel{a} \times \frac{r^{10} - 1}{\cancel{(r-1)}}$

$(r^{20} - 1) = 1025 \times (r^{10} - 1)$

$(r^{10})^2 - 1^2 = 1025(r^{10} - 1)$

$\cancel{(r^{10} - 1)}(r^{10} + 1) = 1025\,\cancel{(r^{10} - 1)}$

$r^{10} = 1024$

$r = 2$

Q) In a G.P., the 5th term is 27 and the 8th term is 729. Find its 11th term:

(a) 729

(b) 6561

(c) 2187

(d) 19683

$a_5 = 27$

$a_8 = 729$

$a r^4 = 27$ — (1)

$a r^7 = 729$ — (2)

(2) ÷ (1)

$\frac{\cancel{a} r^7}{\cancel{a} r^4} = \frac{729}{27}$

$r^3 = 27$

$r = 3$

$a_{11} = 19683$

Finding the Product of n Terms Using the Middle Term in a Geometric Progression (GP)

This section explains how to determine the **product of n terms** in a geometric progression (GP) using its middle term.

The product of the first n terms of a geometric progression is given by:

$$\boxed{\text{Product of } n \text{ terms of GP}} = (\mathrm{G}_m)^n$$

where G_m is the middle term of the GP.

$(\text{Mid})^n$

Q) In a G.P., if the fourth term is '3', then the product of the first seven terms is:

(a) 3^5
(b) 3^7
(c) 3^6
(d) 3^8

$(3)^7 =$

Sum of Infinite Terms in Geometric Progression (GP)

1. **Formula:**
   $S_\infty = \frac{a}{1-r}$, where $r < 1$.

$S_\infty = \frac{a}{1-r}$

2. **Conditions:**
   - a: First term of the GP.
   - r: Common ratio (must satisfy $|r| < 1$).
3. **Important Note:**
   - S_∞ **is not defined** when $r > 1$.

Q) The sum of the infinite G.P. $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots$ is equal to:

(a) 1.95

(b) 1.5

(c) 1.75

(d) None of these

$r = \frac{1}{3}$ $a = 1$

$S_\infty = \frac{a}{1-r} = \frac{1}{1-\frac{1}{3}} =$

$1 \div 3$ change sign $+1$ $\div =$

Q) Sum up to infinity of the series:

$$\frac{1}{2} + \frac{1}{3^2} + \frac{1}{2^3} + \frac{1}{3^4} + \frac{1}{2^5} + \ldots$$

(a) $\frac{19}{24}$ $= 0.79$

(b) $\frac{24}{19}$

(c) $\frac{5}{24}$

(d) None

$\underbrace{\frac{1}{2} + \frac{1}{2^3} + \frac{1}{2^5} + \ldots}_{r=\frac{1}{4}\ a=\frac{1}{2}} + \underbrace{\frac{1}{3^2} + \frac{1}{3^4} + \ldots}_{a=\frac{1}{9}\ r=\frac{1}{9}}$

$S_\infty = \frac{\frac{1}{2}}{1-\frac{1}{4}} + \frac{\frac{1}{9}}{1-\frac{1}{9}} = 0.79$

Q) Find the sum to infinity of the following series:

$$1 - 1 + 1 - 1 + 1 - 1 + \ldots \infty$$

$a = 1$ $r = -1$

$S_\infty = \frac{a}{1-r} = \frac{1}{1-(-1)}$

$= \frac{1}{2}$

(a) 1

(b) ∞

(c) $\frac{1}{2}$

(d) Does not exist

Q) If $y = 1 + x + x^2 + \ldots \infty$, then $x =$:

(a) $\frac{y-1}{y}$

(b) $\frac{y+1}{y}$

(c) $\frac{y}{y-1}$

(d) $\frac{1}{y-1}$

$a = 1$ $r = x$

$y = S_\infty$

$= \frac{a}{1-r}$

$y = \frac{1}{1-x}$

$y(1-x) = 1$

$y - yx = 1$

$y - 1 = yx$

$x = \frac{y-1}{y}$

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Q) Three numbers in G.P. with their sum 130 and their product 27,000 are:

(a) 10, 30, 90

(b) 90, 30, 10

(c) (a) and (b) both

(d) 10, 20, 30

$10+30+90 = 130$

$90+30+10 = 130$

Q) If the sum and product of three numbers in G.P. are 7 and 8 respectively, then the 4th term of the series is:

(a) 6

(b) 4

(c) 8

(d) 16

$\frac{a}{r}, a, ar$

$1, 2, 4, 8$

$\frac{a}{\cancel{r}} \times a \times a\cancel{r} = 8$

$a^3 = 8$

$a = 2$

Sum of Infinite Terms with Combination of A.P. and G.P.

1. **Formula**:

$$\boxed{S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}}$$

2. **Components**:
   - a: First term.
   - r: Common ratio of G.P. (where $|r| < 1$).
   - d: Common difference of A.P.
3. **Purpose**:
   This formula calculates the sum of an infinite sequence where terms are generated by combining the progression rules of both arithmetic (A.P.) and geometric (G.P.) sequences.

Q) Sum of the series $1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \ldots \infty$ is:

+3 +3 +3

(a) $\frac{15}{36}$

(b) $\frac{35}{36}$

(c) $\frac{35}{16}$ =2.187

(d) $\frac{15}{16}$

$d = 3$ $r = \frac{1}{5} = 0.2$

$a = 1$

$S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$

$= \frac{1}{1-0.2} + \frac{3 \times 0.2}{(1-0.2)^2} = 2.187$

Notes on Combination of A.P., G.P., and H.P.

1. **Arithmetic Progression (A.P.):**

   For three terms a, b, c to be in A.P.:

$$\boxed{b = \frac{a+c}{2}}$$

   Example: $a, b, c = 1, 2, 3.$

   a b c

   $b = \frac{a+c}{2}$

2. **Geometric Progression (G.P.):**

   For three terms a, b, c to be in G.P.:

$$b = \sqrt{ac}$$

   Example: $a, b, c = 2, 4, 8.$

3. **Harmonic Progression (H.P.):**

   For three terms a, b, c to be in H.P.:

$$b = \frac{2ac}{a+c}$$

$HM = \frac{2ab}{a+b}$

a, b, c

$b = \frac{2ac}{a+c}$

Q) If x, y, z are the terms in $\boxed{\text{G.P.}}$ then the terms $x^2 + y^2, xy + yz, y^2 + z^2$ are in:

(a) A.P.

(b) G.P.

(c) H.P.

(d) None of these

Let $x = 1$ $y = 2$ $z = 4$

$1^2 + 2^2, 1\times2 + 2\times8, 2^2 + 4^2$

5, 10, 20

$10 = \sqrt{5\times20}$

$= \sqrt{100} = 10$

Q) If Geometric mean (G.M.) of a, b, c, d is 3, then the G.M. of $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}, \frac{1}{d}$ will be:

(a) $\frac{1}{3}$

(b) 3

(c) 81

(d) $\frac{1}{81}$

$(a \times b \times c \times d)^{1/4} = 3$

$a \times b \times c \times d = 81$

$\left(\frac{1}{a} \times \frac{1}{b} \times \frac{1}{c} \times \frac{1}{d}\right)^{1/4} = \left(\frac{1}{abcd}\right)^{1/4}$

$= \left(\frac{1}{81}\right)^{1/4}$

$= \frac{1}{3}$

Key Formulas for Sums of Natural Numbers

1. **Sum of First n Natural Numbers:**

$$\sum_{i=1}^{n} i = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

2. **Sum of Squares of First n Natural Numbers:**

$$\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

3. **Sum of Cubes of First n Natural Numbers:**

$$\sum_{i=1}^{n} i^3 = 1^3 + 2^3 + 3^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

4. **Sum of First n Even Natural Numbers:**

$$\sum_{i=1}^{n} 2i = 2 + 4 + 6 + \cdots = n(n+1)$$

5. **Sum of First n Odd Natural Numbers:**

$$\sum_{i=1}^{n} (2i-1) = 1 + 3 + 5 + \cdots = n^2$$

Q) Geometric Mean of $P, P^2, P^3, \ldots, P^n$ will be:

(a) $P^{1\cdot 1}$

(b) $P^{\frac{1\cdot n}{2}}$

(c) $P^{\frac{n(n-1)}{2}}$

(d) None of the above.

$(P^1 \times P^2 \times P^3 \times \ldots \ldots P^n)^{1/n}$

$(P^{1+2+3+\ldots\ldots n})^{1/n}$

$(P^{\frac{n(n+1)}{2} \times \frac{1}{n}}$

$= P^{\frac{n+1}{2}}$

SPECIAL SERIES

Q) The sum of n terms of the series $2 + 22 + 222 + \ldots$ is:

A) $\frac{2}{81}(10^{n+1} - 10) - \frac{2}{9}n$

B) $\frac{2n}{9} \div \frac{2}{81}(10^{n+1} - 10)$

C) $\frac{2}{81}(10^{n+1} - 10) + \frac{2}{9}n$

D) None

$S_2 = 24$

Put $n = 2$ in option and check which option gives 24

option A is the ans

Q) The sum to m terms of the series $1 + 11 + 111 + \ldots$ up to m terms, is equal to:

(a) $\frac{1}{81}(10^m - 9m - 10)$

(b) $\frac{1}{27}(10^m - 9m - 10)$

(c) $10^m - 9m - 10$

(d) None of these

12

Put $n = 2$

Chapter 12: Blood Relations

(5 marks)

Definition of Blood Relations:

- Blood relations refer to people who are connected by birth rather than marriage. These include direct family members like parents, grandparents, children, and siblings, as well as extended relatives like aunts, uncles, and cousins.

Remember the relations as given below:

(i) Basic Relations

- Children of Same Parents → Siblings
- One's Husband or Wife → Spouse
- Relatives on Mother's Side → Maternal
- Relatives on Father's Side → Paternal

(ii) Extended Family Relations

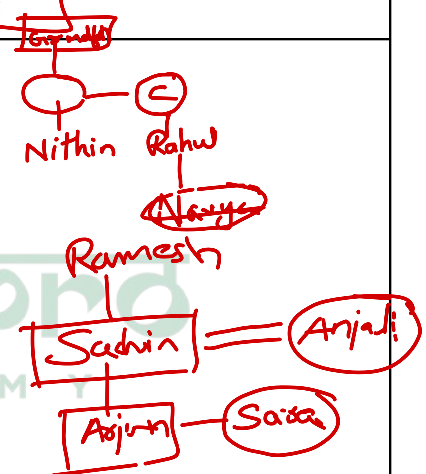
- Father's Father → Paternal Grandfather
- Father's Mother → Paternal Grandmother
- Mother's Father → Maternal Grandfather
- Mother's Mother → Maternal Grandmother
- Father's Brother → Paternal Uncle
- Father's Sister → Paternal Aunt
- Mother's Brother → Maternal Uncle
- Mother's Sister → Maternal Aunt
- Uncle's or Aunt's Child → Cousin
- Sibling's Son → Nephew
- Sibling's Daughter → Niece
- Son's or Daughter's Child → Grandchild

(iii) Additional Relations

1. Grandfather's Son → Father or Uncle
2. Grandmother's Son → Father or Uncle
3. Grandfather's Only Son → Father
4. Grandmother's Only Son → Father
5. Mother's or Father's Mother → Grandmother

6. Son's Wife → Daughter-in-Law
7. Daughter's Husband → Son-in-Law
8. Husband's or Wife's Sister → Sister-in-Law
9. Brother's Son → Nephew
10. Brother's Daughter → Niece
11. Uncle's or Aunt's Son or Daughter → Cousin
12. Sister's Husband → Brother-in-Law
13. Brother's Wife → Sister-in-Law
14. Grandson's or Granddaughter's Daughter → Great Granddaughter

Notes



TYPE 1

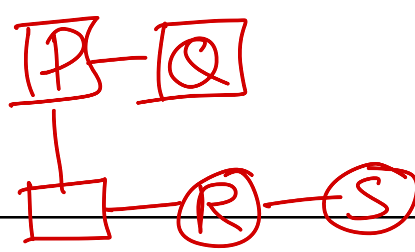
Q) P and Q are brothers, R and S are sisters. P's son is R's brother. How is Q related to R?

(a) Uncle

(b) Brother

(c) Father

(d) Grandfather



Q) Six persons are seen together in a group. They are A, B, C, D, E, and F.

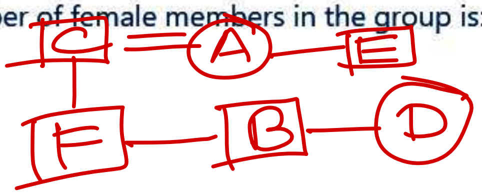
B is the brother of D, but D is not the brother of B.

F is the brother of B, C, and A are married together.

F is the son of C, but C is not the mother of F.

E is the brother of A.

The number of female members in the group is:



(a) 1

(b) 2

(c) 3

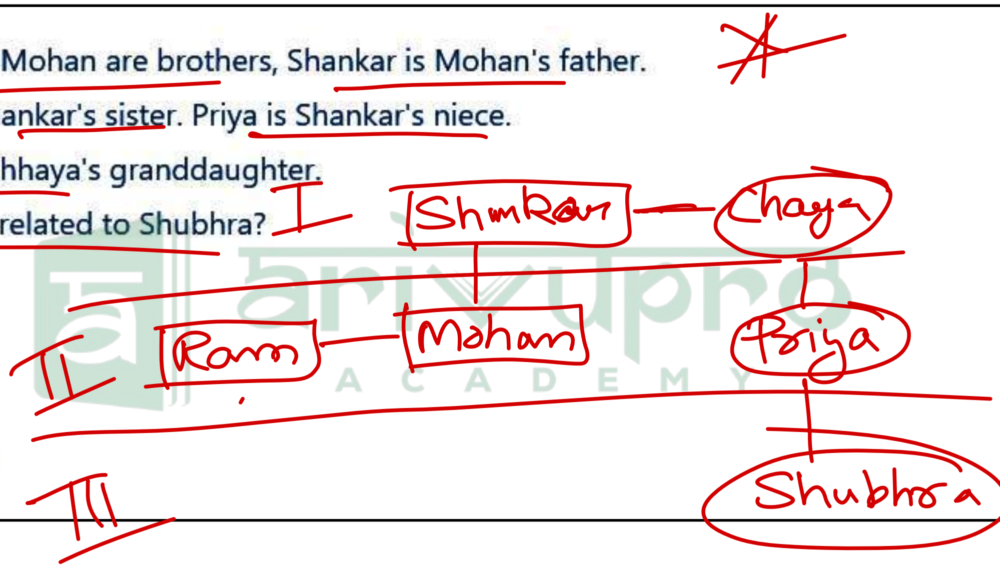
(d) 4

Q) Ram and Mohan are brothers, Shankar is Mohan's father.

Chhaya is Shankar's sister. Priya is Shankar's niece.

Shubhra is Chhaya's granddaughter.

How is Ram related to Shubhra?



(a) Brother

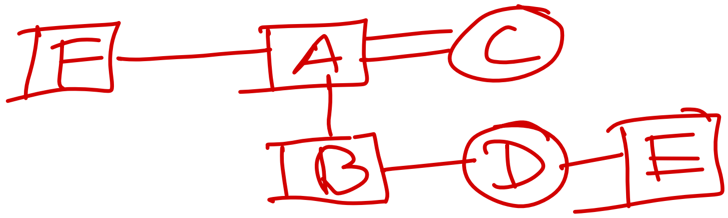
(b) Uncle

(c) Cousin

(d) Nephew

Q) A, B, C, D, E, and F are members of a family. B is the son of A, but A is not the mother of B. A and C are a married couple. F is the brother of A. D is the sister of B, and E is the son of C.
How many male members are there in the family?

(a) 1
(b) 2
(c) 3
(d) 4



Q) How is F related to B?

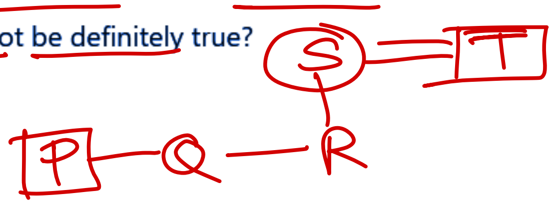
(a) Uncle
(b) Daughter
(c) Son
(d) Niece

Q) How many children does A have?

(a) 3
(b) 2
(c) 4
(d) 1

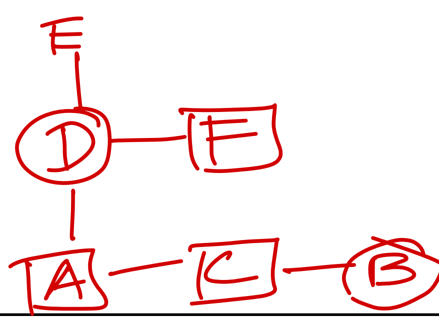
Q) P is the brother of Q and R, S is the mother of R. T is the father of P.
Which of the following statements cannot be definitely true?

(a) S is the mother of P
(b) P is the son of S
(c) T is the husband of S
(d) Q is the son of T



Q) D is the daughter of E. A is the son of D. C is a brother of A, and B is the sister of A. F is the brother of D.
How is F related to B?

(a) Father-in-law
(b) Uncle
(c) Brother
(d) Mother-in-law



Q) A is B's sister. C is B's mother. D is C's father. E is D's mother.

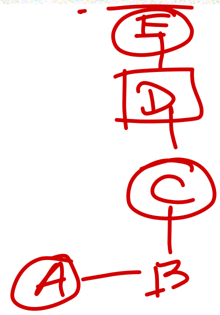
Then how is A related to D?

(a) Grandfather

(b) Grandmother

(c) Daughter

(d) Granddaughter



Q) Suresh's sister is the wife of Ram. Ram is Rani's brother. Ram's father is Madhur. Sheetal is Ram's grandmother. Rema is Sheetal's daughter-in-law. Rohit is Rani's brother's son.

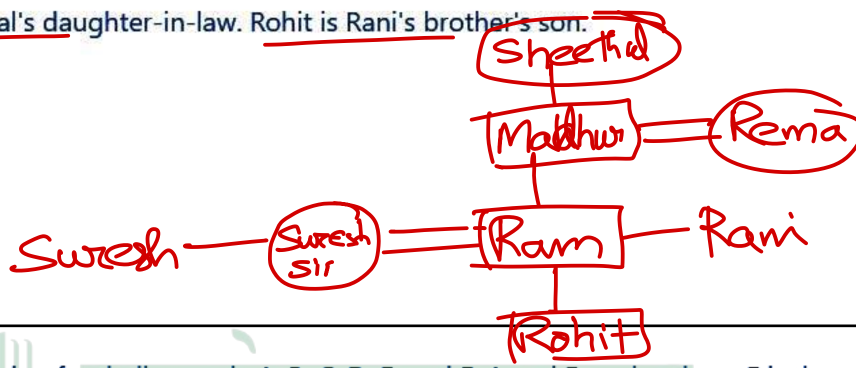
Who is Rohit to Suresh?

(a) Brother-in-law

(b) Son

(c) Brother

(d) Nephew



Q) There are six children playing football, namely A, B, C, D, E, and F. A and E are brothers. F is the sister of E. C is the only son of A's uncle. B and D are daughters of the brother of C's father.

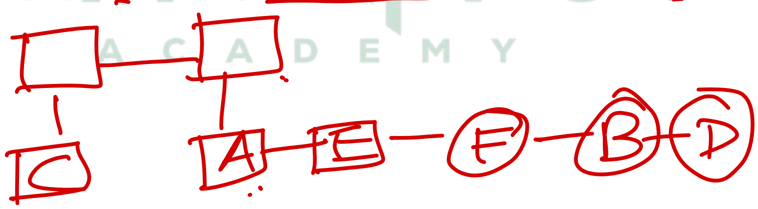
How is D related to A?

(a) Uncle

(b) Cousin

(c) Niece

(d) Sister



Q) In a joint family, there are a father, mother, 3 married sons, and one unmarried daughter. Out of the sons, two have 2 daughters each, and one has a son only.

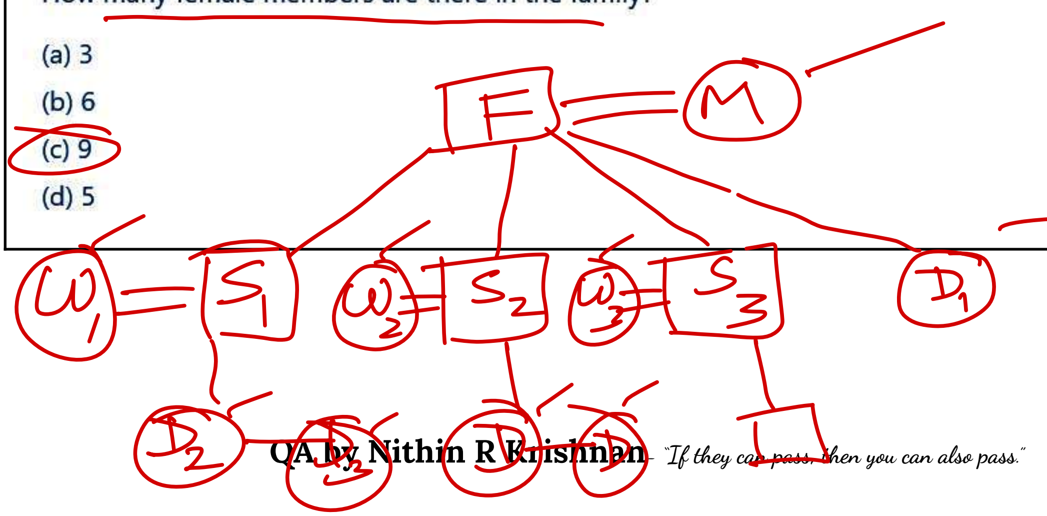
How many female members are there in the family?

(a) 3

(b) 6

(c) 9

(d) 5



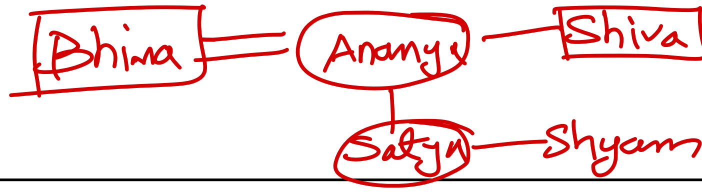
Q) Ananya is the mother of Satya, and Shyam is the son of Bhima. Shiva is the brother of Ananya. If Satya is the sister of Shyam, how is Bhima related to Shiva?

(a) Son

(b) Cousin

(c) Brother-in-law

(d) Son-in-law



Q) Based on the statements given below, find out who is the uncle of P?

(i) K and J are brothers.

(ii) K's sister is M.

(iii) P and N are siblings.

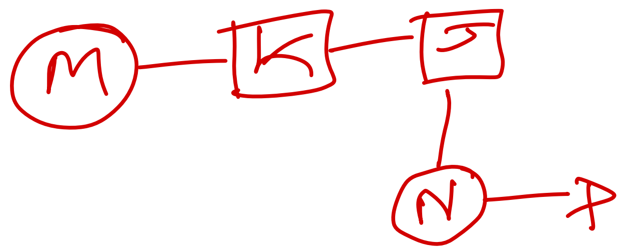
(iv) N is the daughter of J.

(a) K

(b) J

(c) N

(d) M



Q) In a family, there are six members: A, B, C, D, E, and F.

A and B are a married couple, with A being the male member.

D is the only son of C and is the brother of A.

E is the sister of D.

B is the daughter-in-law of F, whose husband is deceased.

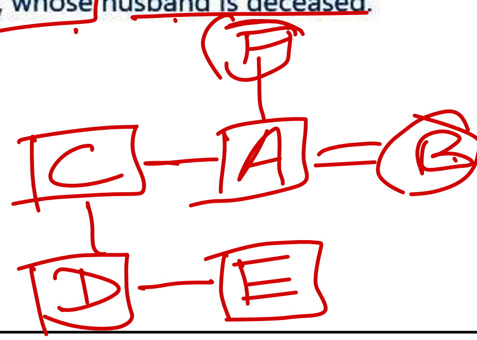
How is C related to B?

(a) Brother

(b) Nephew

(c) Brother-in-law

(d) Sister-in-law



Q) P, Q, R, S, T, U are six members of a family in which there are two married couples.
T, a teacher, is married to a doctor who is the mother of R and U.
Q, the lawyer, is married to P. P has one son and one grandson.
Of the two married ladies, one is a housewife.
There is also one student and one male engineer in the family.
Which of the following is true about the granddaughter of the family?

Q = P

(a) She is a lawyer

(b) She is an engineer

(c) She is a student

(d) She is a doctor

Q Lawyer = P Housewife

T Teacher = Doctor

R son — U student

TYPE 2(Pointing at a Person)

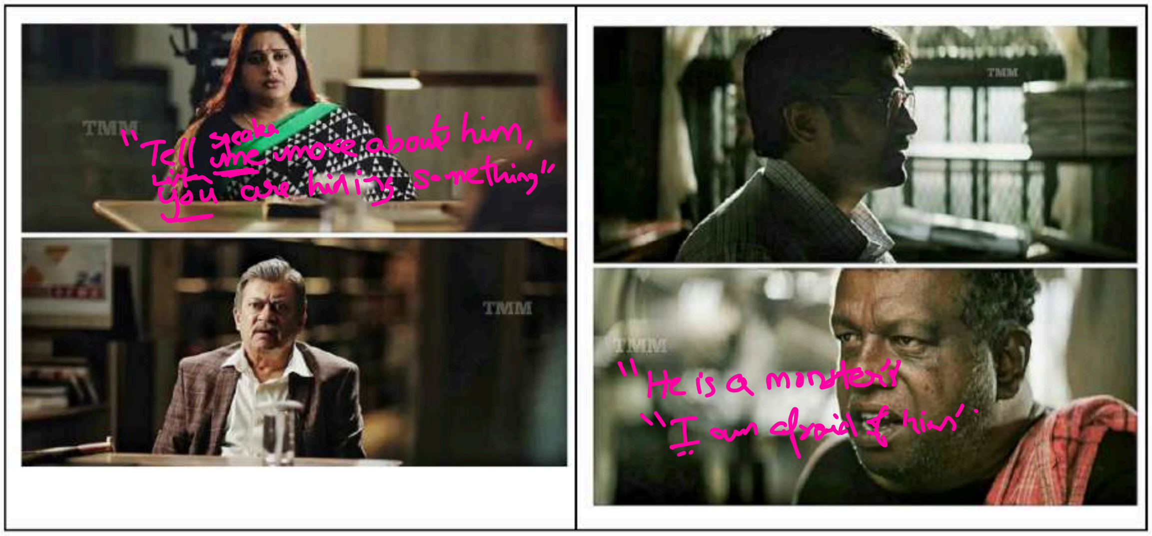
Things You Should Know About Pointing at a Person or Introducing in a Conversation

Notes:

1. The person who is introducing: Speaker
2. The person who is being introduced: Reference Person
3. The person who is listening: Listener

Rules for Pronoun Usage:

- Statements within quotes: " "
  - **I/mine/myself**: Refers to the **speaker.**
  - **He/she/her/his/this girl/this boy**: Refers to the **reference person.**
  - **You/your**: Refers to the **listener.**
  - If listener information is not provided, **you/your** refers to the **reference person.**
- **Statements without quotes:**
  - **He/his/she/her**: Refers to the **speaker.**



Q) Pointing to a man (Ref) in a photograph, a woman said (Speaks), "The father of his brother (man) is the only son of my (women) grandfather."

How is the woman related to the man in the photograph?

(a) Mother

(b) Aunty

(c) Daughter

(d) Sister

Grandfather . I

Father II

Woman — man — Brother III

Q) Vinod introduces Vishal as the son of the only brother of his father's wife.

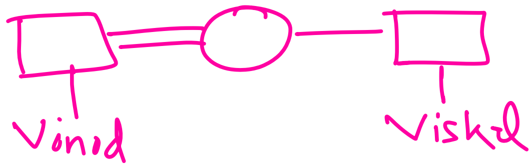
How is Vinod related to Vishal?

(a) Cousin

(b) Brother

(c) Son

(d) Uncle



Q) Suresh introduces a man as "he is the son of the woman who is the mother of the husband of my mother."

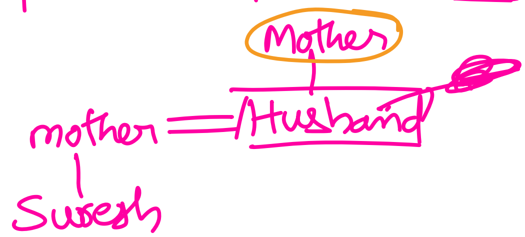
How is Suresh related to the man?

(a) Brother-in-law

(b) Son

(c) Brother

(d) Nephew



Q) Pointing to a woman in a picture, Sumit said, "She is the mother of my son's wife's daughter."

How is the lady related to Sumit?

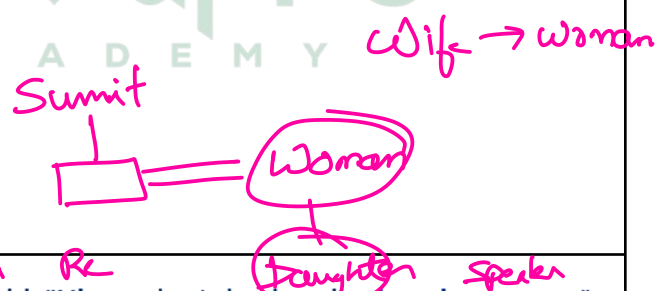
(a) Uncle

(b) Cousin

(c) Daughter

(d) None

Daughter in Law



Q) Pointing to a man in a photograph, a man said, "His mother's husband's sister is my aunt."

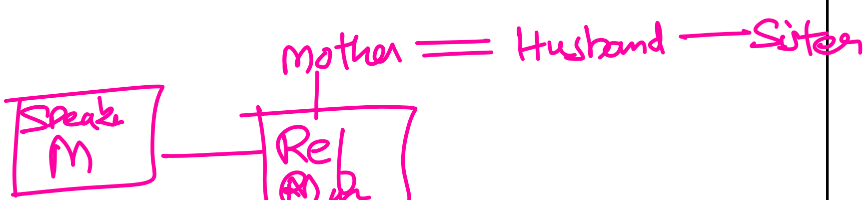
What is the relation between that man and him?

(a) Son

(b) Uncle

(c) Nephew

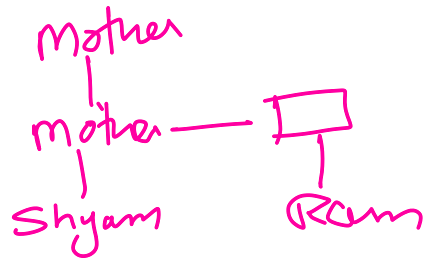
(d) Brother



Speaker Listener Shyam's mom's

Q) Shyam's mother said to Shyam, "My mother has a son whose son is Ram."
Shyam is related to Ram as:

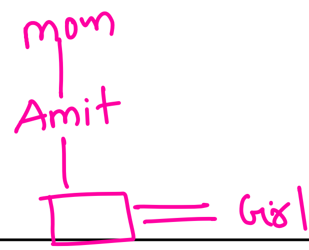
(a) Uncle
(b) Cousin
(c) Nephew
(d) Grandfather



Speaker Rel Amit's

Q) Amit said, "This girl is the wife of the grandson of my mother."
How is Amit related to the girl?

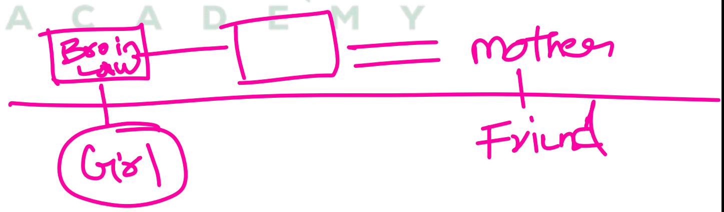
(a) Father-in-law
(b) Grandson
(c) Father
(d) Son



Speak List

Q) R told M as, "The girl I met at the beach was the youngest daughter of the brother-in-law of my friend's mother."
How is the girl related to R's friend?

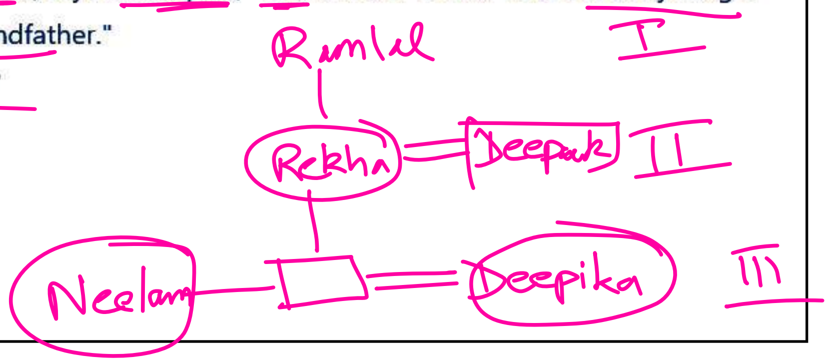
(a) Cousin
(b) Daughter
(c) Niece
(d) Aunt



Speaker Listen Deepika's

Q) Neelam, who is Deepak's daughter, says to Deepika, "Your mother-in-law Rekha is the younger daughter of Ramlal, who is ~~the~~ my grandfather."
How is Neelam related to Deepika?

(a) Cousin
(b) Niece
(c) Sister-in-law
(d) Aunt



→Speaker

P Q P

Q) When Mr. P saw Mr. Q, he recalled, "He is the son of the father of my daughter's mother." Who is Mr. Q to Mr. P?

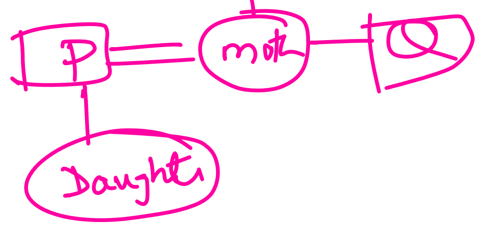
Father

(a) Brother

(b) Cousin

(c) Nephew

(d) Brother-in-law



TYPE 3

Q) If P + Q means P is the mother of Q;

P ÷ Q means P is the father of Q;

P – Q means P is the sister of Q;

+ → mother

÷ → father

– → Sister

Then which of the following relationships shows that M is the daughter of R?

(a) R ÷ M + N

(b) R + N ÷ M

(c) R – M ÷ N

(d) None of the above

Q) If:

- A $ B means A is the father of B,
- A # B means A is the daughter of B,
- A @ B means A is the sister of B,

$ — father

daughter

@ Sister

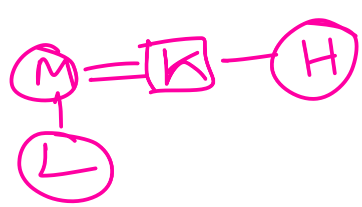
Then how is K related to M in the expression H @ K $ L # M?

(a) Husband

(b) Uncle

(c) Father

(d) Grandson



Chapter 9: Number Series, Coding and Decoding and Odd Man Out

Coding and Decoding Techniques

Definition:

- **Coding** is a method of transmitting a message using a system of **words, letters, figures, or symbols** to hide its meaning from a third person.
- **Decoding** is the process of converting the coded message back into its original form.
- **Objective:** The test of Coding-Decoding is designed to judge a candidate's logical and analytical abilities.

Type 1 - Letter to Digit Coding

Each specific alphabet of a word is assigned to a specific digit.

Q) In a certain code, RIPPLE is written as 613382 and LIFE is written as 8192. How is PILLER written in that code?

(a) 318826
(b) 318286
(c) 618826
(d) 338816

R I P P L E
6 1 3 3 8 2
L I F E
8 1 9 2
3 1 8 8 2 6

Q) If LOSE is coded as 1357 and GAIN is coded as 2468, what does figure 82146 stand for?

(a) NGLAI
(b) NGLIA
(c) ~~GNLIA~~
(d) ~~GNLA~~

N 1

Q) If MOUSE is coded as 34651 and KEY is coded as 217, then how will YES be coded?

(a) 715
(b) 517
(c) 175
(d) 571

7 1

E J O T Y
5 10 15 20 25

Type 2 - Based on Sum of Forward or Backward Positions

In this type of coding, the sum of the forward or backward positions of the letters in a word is calculated to derive a corresponding numeric code.

27 − (Forward position)
27 − 1 = 26

27 − 13 = 14
27 − 12 = 15

Alphabet Position Table (Forward & Backward)

Reverse Pos	26	25	24	23	22	21	20	19	18	17	16	15	14
Alphabet	A	B	C	D	E	F	G	H	I	J	K	L	M
Forward Pos	1	2	3	4	5	6	7	8	9	10	11	12	13
Reverse Pos	13	12	11	10	9	8	7	6	5	4	3	2	1
Alphabet	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Forward Pos	14	15	16	17	18	19	20	21	22	23	24	25	26

1. Forward Position of Letters in English Alphabet:

- The forward position refers to the numeric position of a letter in the alphabet starting from 1 (A=1, B=2, ..., Z=26).
- **Mnemonic for Key Letters:**
  - **EJOTY:**
    - E = 5
    - J = 10
    - O = 15
    - T = 20
    - Y = 25

arivupro ACADEMY

TAC = 20+1+3 = 24

CAT = 3+1+20 = 24

DOG = 4+15+7 = 26

27×3 − 26 = 81 − 26 = 55

2. Backward Position of Letters:

- The backward position is calculated as:
  Backward position = 27 - Forward position
  - Example:
    - Forward position of **I** = **9**
    - Backward position = 27 - 9 = **18**
- To calculate the **sum of backward positions** for a word:
  Formula:
  (27 × number of letters) - sum of forward positions

Q) If FRAME is coded as 0618011305, then ARISE is coded as:

(a) 0118091905
(b) 0119091805
(c) 0118190905
(d) 0118091805

06

01 18 09 19 05

Q) In a certain code language, "EXAM" is coded as 39 and "PAPER" is coded as 51. Then "PASS" is coded as:

(a) 47
(b) 51
(c) 50
(d) 52

EXAM = 5+24+1+13 = 43 − 4 = 39

PAPER = 16+1+16+5+18 = 56 − 5 = 51

PASS = 16+1+19+19 = 55 − 4 = 51

Q) In a certain coded language, the word "AND" is assigned the value 62. Following the same coding pattern, what would be the code for the word "NAND"?

Options:
(a) 75
(b) 33
(c) 78
(d) None

AND = 62

1+14+4 = 19

Backward sum = 3×27 − 19 = 62

NAND = 14+19 = 33

4×27 − 33 = 75

Type 3 - Letter to Letter Coding

This coding technique involves transforming one letter into another based on specific rules. The methods used include:

1. **Forward Coding (+ or -):**
   - The position of a letter is shifted forward or backward by adding or subtracting a specific value.
2. **Reverse Coding (+ or -):**
   - The alphabet is considered in reverse order (Z=1, Y=2, ..., A=26), and a specific value is added or subtracted.
3. **Word Divided into Parts with Reverse Coding for Both Halves:**
   - A word is divided into two parts, and reverse coding is applied separately to each part.

Q) In a certain code, MADRAS is coded as NBESBT. How is DELHI coded in that code?

(a) EMMJI
(b) JIFEM
(c) EFMIJ
(d) CDKGH

M A D R A S
+1 +1 +1 +1 +1 +1
N B E S B T

D E L H I
+1 +1
E F

Q) If HONEY is coded as JQPGA, which word is coded as VCTIGVU?

(a) CARPETS
(b) TRAPETS
(c) TARGETS
(d) UMBRELU

H O N E Y
+2 +2 +2 +2 +2
J Q P G A

T A
-2 -2
V C T I G V U

Q) In a certain code, MENTION is written as LNEITNO. How is PRESENT coded?

(a) QFSFTUM
(b) ONESERP
(c) QRESTNO
(d) OERESTN

M E N T I O N
-1
L N E I T N O

PRESENT
O E R

27 − 14 = 13 7N

Q) In a certain code language, BEAT is written as YVZG. What will be the code for MILD?

(a) ONRW

(b) NOWR

(c) ONWR

(d) NROW

B E A T
Y V Z G

13th 4
M I L D
N W

Q) If in a certain code "THANKS" is written as "SKNTHA," then how is "STUPID" written?

(a) DIPUTS

(b) DISPUT

(c) DIPUST

(d) DIPSTU

T H A N K S
S K N T H A

S T U P I D
D I P S T U

Q) In a certain code, MENTION is written as LNEITNO. How is PRESENT written in that code?

(a) NTSEREO

(b) OERESTN

(c) ERESTNO

(d) ROESTNE

Q) What comes at the last place in R, U, X, A, D, ...?

(a) E

(b) F

(c) G

(d) H

+3 +3 +3 +3 +3
R U X A D G

Q) Find the missing term: P 3 C, R 5 F, T 8 I, V 12 L, ? X O

+2 +2 +2 +2
+3 +3 +3 +3

(a) Y 17 O

(b) X 17 M

(c) X 17 O

(d) X 16 O

3 5 8 12 17
2 3 4 +5

Q) If "HEALTH" is written as "IFBMUL," then how will "NORTH" be written in that code?

(a) OPSUL

(b) GSQNM

(c) FRPML

(d) IUPSO

HEALTH → IFBMUL (+1, +1, +1, +4)

NORTH → O P

Q) Which of the following is the odd one?

(a) CEHL (+2 +3 +4)

(b) KMPT (+2 +3 +4)

(c) OQTX (+2 +3 +4)

(d) NPSV (+2 +3 +3)

Q) If SUMMER is coded as RUNNER, the code for WINTER will be:

(a) SUITER

(b) VIOUER

(c) WALKER

(d) SUFFER

SUMMER → RUNNER

WINTER → I ER

Type 4 - Word to Word / Word to Character / Word to Digit Coding

This type of coding assigns a specific tag, label, or identifier to a word, which may not be related to its position or numerical value. The coding follows a set pattern or logic unique to the system but does not necessarily involve calculations based on positions in the alphabet.

Q) In a certain code, '256' means 'you are good,' '637' means 'we are bad,' and '358' means 'good and bad.' Which of the following represents 'and' in that code?

(a) 2

(b) 5

(c) 8

(d) 3

5 → good

3 → Bad

8 → and

Q) In a certain code language:

- 'dugo hui mul zo' stands for 'work is very hard,'
- 'hui dugo ba ki' stands for 'Bingo is very smart,'
- 'nano mul dugo' stands for 'cake is hard,' and
- 'mul ki gu' stands for 'smart and hard.'

is very → Hui Dugo
Smart → ki

Which of the following words stands for "Bingo"?

Options:

(a) Jalu

(b) Dugo

(c) Ki

(d) Ba

Q) If in a certain code '493' means 'Friendship difficult challenge', '961' means 'Struggle difficult Exam', and '178' means 'Exam believable subject', then which digit is used for believable?

1 → Exam

(a) 7 or 8

(b) 7 or 9

(c) 8

(d) 8 or 1

Letter Repeating Series

A **Letter Repeating Series** is a type of **alphabet series** where certain letters appear repeatedly in a structured pattern. These series often involve **fixed repetitions, alternating repetitions, or progressive changes** in repetition.

Notes

12 4×3 | 3×4

15 5×3 | 3×5

16 4×4

18 6×3 or 3×6

ab aba ba ab

a b / aba b / a ba b / ab

(a) abbba

(b) abbab

(c) baabb

(d) bbaba

abbab

Q) b aa b ba a bb a ab b aab

(A) aaabb

(B) babab

(C) bbaab

(D) bbbaa

bbaab

Q) In the following letter-series some letters are missing. The missing letters are given in the proper sequence as one of the alternatives. Find the correct alternative.

ab_abcab_abc_bca_c

ab c abc ab c abc a bca b c

(a) abac

(b) bcac

(c) ccab

(d) bbac

ccab

Types of Series and Identification:

1. **Natural Numbers:**
   Series: 1, 2, 3, 4, 5, ...
2. **Even Numbers:**
   Series: 2, 4, 6, 8, 10, ...
3. **Odd Numbers:**
   Series: 1, 3, 5, 7, 9, ...
4. **Prime Numbers:**
   Series: 2, 3, 5, 7, 11, 13, 17, ...
5. **Squares of First n Natural Numbers:**
   Series: 1, 4, 9, 16, 25, 36, 49, ...
6. **Squares of Odd Natural Numbers:**
   Series: 1, 9, 25, 49, 81, ...
7. **Squares of Even Natural Numbers:**
   Series: 4, 16, 36, 64, 100, ...
8. **Cubes of First n Natural Numbers:**
   Series: 1, 8, 27, 64, 125, 216, ...
9. **Cubes of Odd Natural Numbers:**
   Series: 1, 27, 125, 343, 729, ...
10. **Cubes of Even Natural Numbers:**
    Series: 8, 64, 216, 512, 1000, ...
11. **Combination of Squares and Cubes (Alternating):**
    Series: 1, 4, 8, 9, 27, 16, 64, ...
12. **Arithmetic Progression (Common Difference):**
    Example Series: 1, 3, 5, 7, 9, ...
13. **Geometric Progression (Common Ratio):**
    Example Series (Ratio = 4): 4, 16, 64, 256, ...

14. **Plus and Minus Series:**

Example: 1, 4, 2, 5, 3, 6, 4, ...

15. **Plus and X Series:**

Example: 1, 2, 2, 4, 8, 11, 33, ...

16. **Fibonacci Series:**

Series: 1, 2, 3, 5, 8, 13, 21, 34, ... 55. . . ..

Huge increament
power or multiplication
is involve.

Q) Complete the series: 4, 16, 64, 256, 1024

×4 ×4 ×

(a) 32

(b) 48

(c) 64

(d) 46

Q) Find the next number in the series: 7, 23, 47, 119, 167

3^2-2 5^2-2 7^2-2 11^2-2 13^2-2 17^2-2

prime no:

(a) 211

(b) 223

(c) 287

(d) 319

Q) Complete the series: 4, 16, 36, 64, 100, 144

2^2 4^2 6^2 8^2 10^2 12^2

(a) 144

(b) 121

(c) 49

(d) 120

Q) Find out the next term: 6, 13, 28, 59, ? 114

7 15 31 55

8 16 24

(a) 122

(b) 114

(c) 113

(d) 112

6×2 = 12+1 = 13

13×2 = 26+2 = 28

28×2 = 56+3 = 59

59×2 + 4 = 122

Q) Find the missing value in:

$\frac{3}{8}, \frac{8}{19}, \frac{18}{41}, ?, \frac{78}{173}$

×2=40

38+40

(a) $\frac{37}{84}$

(b) $\frac{40}{87}$

(c) $\frac{39}{86}$

(d) $\frac{38}{85}$

8 19 41

×2+3 ×2+2 ×2+2

85

Q) Complete the series: 7, 26, 63, 124, 215, 342, _____

2^3-1 3^3-1 4^3-1 5^3-1 6^3-1 7^3-1

8^3-1

(a) 511

(b) 672

(c) 508

(d) 556

Q) Complete the series: 4, 6, 9, 13.5, _______, 30.375

×1.5 ×1.5 ×1.5 ×1.5

(a) 40.50

(b) 20.25

(c) 40.75

(d) 60.25

Q) Find the odd man out in the sequence: 8, $\cancel{25}$ 27, 64, 125, 216

2^3 4^3 5^3 6^3

(a) 25

(b) 64

(c) 125

(d) 216

Q) Which of the following is the odd one: 6, 9, 15, 21, 24, ~~26~~, 30

(a) 30

(b) 24

(c) ~~26~~

(d) 9

26 is not a multiple of 3

Q) Find the odd one out: 1, 5, 14, 30, ~~49~~, 55, 91

$1+2^2$ $+3^2$ $+4^2$ $+5^2$

(a) 49

(b) 30

(c) 55

(d) 91

Q) Find the odd man out of the following series: 15, 21, 63, ~~81~~, 69

(a) 15

(b) 21

(c) 63

(d) 81

81 is a perfect sq

Q) Which of the following is the odd one: 4, 12, $\boxed{44}$, 176, 890

(a) 4

(b) 12

(c) 44

(d) 176

$4\times2+4$

$12\times3+6 = 42$

$42\times4+8 = 176$

$176\times5+10 = 890$

Q) The wrong term in the series 225, 196, 169, 144, 121, 100, $\boxed{77}$, 64 is:

15^2 14^2 13^2 12^2 11^2 10^2 8^2

81

(a) 121

(b) ~~77~~

(c) 100

(d) 169

$$\frac{\cancel{dx}}{\cancel{dx}} = 1 \qquad \frac{dx^1}{dx} = 1 \times x^{1-1} = 1 \times x^0 = 1 \qquad \frac{\cancel{5}}{\cancel{5}} = 1$$

$$\frac{d}{dx} x^n = n x^{n-1}$$

$$\frac{d}{dx} x^{11} = 11 x^{10} \qquad \frac{\cancel{5}}{\cancel{5}} =$$

$$\frac{d}{dx} x^{1000} = 1000\, x^{999} \qquad 5^{1-1} = 5^0 = 1$$

$$\frac{d}{dx} x^{2} = 2x^{2-1} = 2x$$

$$\frac{d}{dx}\sqrt{x} = \frac{d}{dx} x^{1/2} = \frac{1}{2} \times x^{\frac{1}{2}-1}$$
$$= \frac{1}{2} x^{-1/2}$$
$$= \frac{1}{2\sqrt{x}}$$

$$\boxed{\frac{d}{dx} e^x = e^x}$$

$$\frac{d}{dx} a^x = a^x \log a$$

$$\frac{d}{dx} \log x = \frac{1}{x} \qquad \frac{d}{dx} \log 11 = 0 \qquad \frac{d}{dx}\, \text{constant} = 0$$

$$\frac{d}{dx}\left(x^5 + e^x + 10^x + \log x\right)$$

$$5x^4 + e^x + 10^x \log 10 + \frac{1}{x}$$

Chapter 8: Basic Applications of Differential and Integral Calculus in Business and Economics

Basic Rules of Differentiation

1. Sum and Difference Rule:

$$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x)$$

- The derivative of a sum (or difference) of two functions is the sum (or difference) of their derivatives.

2. Scalar Multiple Rule:

$$\frac{d}{dx}[\alpha f(x)] = \alpha \frac{d}{dx}f(x)$$

- Multiplying a function by a constant scales the derivative by the same constant.

3. Constant Rule:

$$\frac{d}{dx}(c) = 0$$

- The derivative of any constant is zero.

4. Power Rule:

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

- The derivative of x^n involves multiplying by the power and reducing the power by 1.

5. **Exponential Function with a Constant Base:**

$$\frac{d}{dx}(a^x) = a^x \cdot \ln(a)$$

- Where a is a constant.

6. **Exponential Function with Base e:**

$$\frac{d}{dx}(e^x) = e^x$$

7. **Logarithmic Function:**

$$\boxed{\frac{d}{dx}(\ln(x))} = \frac{1}{x}$$

8. **Square Root Function:**

$$\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$$

9. **Logarithmic Function with Base a:**

$$\boxed{\frac{d}{dx}(\log_a(x))} = \frac{1}{x \cdot \ln(a)} \qquad \frac{1}{x \log a}$$

10. **Variable as Exponent:**

$$\boxed{\frac{d}{dx}(x^x) = x^x(1 + \ln(x))}$$

Sum Rule

The **Sum Rule** in differentiation allows us to find the derivative of the sum or difference of two functions by differentiating each function separately and then combining the results.

Formula:

$$\frac{d}{dx}(u \pm v) = \frac{d(u)}{dx} \pm \frac{d(v)}{dx}$$

Explanation:

- u and v are functions of x.
- The "+" or "−" sign in the original expression is retained in the derivative.

Example:

If $u = x^2$ and $v = 3x$, then:

$$\frac{d}{dx}(u+v) = \frac{d}{dx}(x^2) + \frac{d}{dx}(3x)$$

$$= 2x + 3$$

Q) If $y = x(x-1)(x-2)$, then $\frac{dy}{dx}$ is:

Options:

(a) $-6x$

(b) $3x^2 - 6x + 2$

(c) $6x + 4$

(d) $3x^2 - 6x$

$x(x^2 - 2x - x + 2)$

$x(x^2 - 3x + 2)$

$y = x^3 - 3x^2 + 2x$

$\frac{dy}{dx} = \frac{d}{dx}(x^3 - 3x^2 + 2x)$

$= 3x^2 - 3\times 2x + 2$

$= 3x^2 - 6x + 2$

$\frac{d}{dx} -3x^2$

$-3\frac{d}{dx}x^2$

Q) If $u = 3t^4 + 5t^3 + 2t^2 + t + 4$, then the value of $\frac{du}{dt}$ at $t = -1$ is:

Options:

(a) 0

(b) 1

(c) 2

(d) 5

$\frac{du}{dt} = 3\times 4t^3 + 5\times 3t^2 + 2\times 2t + 1 + 0$

$= 12t^3 + 15t^2 + 4t + 1$

$\left.\frac{du}{dt}\right|_{\text{at } t=-1} = 12\times(-1)^3 + 15\times(-1)^2 + 4\times(-1) + 1$

$= -12 + 15 - 4 + 1$

$= 16 - 16$

$= 0$

$\frac{d}{dx}\log(x) = \frac{1}{x} \times 1$

$\frac{d}{dx}e^{x} = e^{x} \times \frac{dx}{dx}$ 1

$\frac{d}{dx}e^{5x} = e^{5x} \times \frac{d}{dx}5x$

$= e^{5x} \times 5$

Product Rule

The **Product Rule** in differentiation is used when finding the derivative of the product of two functions.

Formula:

$$\frac{d}{dx}(uv) = u\frac{d(v)}{dx} + v\frac{d(u)}{dx}$$

Alternate Notation:

$$\boxed{\frac{d}{dx}(I \times II)} = I\frac{d(II)}{dx} + II\frac{d(I)}{dx}$$

Q) If $y = \log x^x$, then $\frac{dy}{dx} =$:

Options:

(a) $\log(ex)$

(b) $\log(e/x)$

(c) $\log(x/e)$

(d) 1

$y = x \log x$ (I = x, II = $\log x$)

$\frac{dy}{dx} = \frac{d}{dx} x \log x$

$= x \times \frac{d}{dx}\log x + \log x \times \frac{d}{dx} x$

$= x \times \frac{1}{x} + \log x$

$= 1 + \log x$

$= \log_e e + \log_e x = \log_e(ex)$

Quotient Rule

The **Quotient Rule** in differentiation is used when finding the derivative of the quotient of two functions.

Formula:

$$\frac{dy}{dx} = \frac{v\frac{d(u)}{dx} - u\frac{d(v)}{dx}}{v^2}$$

$\frac{d}{dx}\frac{\text{Num}}{\text{Denom}} = \frac{\text{Deno} \times \frac{d}{dx}\text{Num} - \text{Num} \times \frac{d}{dx}\text{Deno}}{(\text{Deno})^2}$

eg: $\frac{d}{dx}\frac{x}{e^x} = \frac{e^x \times \frac{dx}{dx} - x \times \frac{d}{dx}e^x}{(e^x)^2}$

$= \frac{e^x \times 1 - x e^x}{(e^x)^2}$

Q) If $y = \frac{x^4}{e^x}$, then $\frac{dy}{dx}$ is equal to:

Options:

(a) $\frac{x^3(4-x)}{(e^x)^2}$

(b) $\frac{x^3(4-x)}{e^x}$

(c) $\frac{x^2(4-x)}{e^x}$

(d) $\frac{x^3(4x-1)}{e^x}$

$$\frac{d}{dx}\frac{x^4}{e^x} = \frac{e^x \times 4x^3 - x^4 \times e^x}{(e^x)^2}$$

$$= \frac{\cancel{e^x} x^3 (4-x)}{(e^x)^2}$$

$$= \frac{x^3(4-x)}{e^x}$$

Shortcut Rule for Rational Functions

This is a shortcut for differentiating functions of the form:

$$y = \frac{af(x)+b}{cf(x)+d}$$

$$\frac{dy}{dx} = \frac{(ad-bc)\cdot f'(x)}{[cf(x)+d]^2}$$

$f'(x)$

$\frac{df(x)}{dx} = \frac{d}{dx}x$

$\frac{x+1}{x-1}$

$\boxed{f(x)} = x$

$a = 1$

$b = 1$

$c = 1$

$d = -1$

Q) If $y = \frac{x+1}{x-1}$, find $\frac{dy}{dx}$:

Options:

(a) $\frac{-2}{(x-1)^2}$

(b) $\frac{2}{(x-1)^2}$

(c) $\frac{1}{(x-1)^2}$

(d) $\frac{-1}{(x-1)^2}$

$$\frac{(-1-1)\times 1}{(x-1)^2}$$

$$= \frac{-2}{(x-1)^2}$$

Q) If $y = \frac{3\log x+6}{4\log x-10}$, find $\frac{dy}{dx}$:

$a = 3 \quad b = 6 \quad c = 4 \quad d = -10$

Options:

(a) $\frac{-54}{x(4\log x-10)^2}$

(b) $\frac{-54}{x(3\log x+6)^2}$

(c) $\frac{54}{x(4\log x-10)^2}$

(d) $\frac{-36}{x(4\log x-10)^2}$

$$\frac{(-30-24)\frac{1}{x}}{(4\log x-10)^2} = \frac{-54}{x(4\log x-10)^2}$$

Chain Rule

The **Chain Rule** is used for finding the derivative of a composite function (a function within another function).

Formula:

For a composite function $Y = [f(g(x))]$, the derivative is:

$$\frac{dY}{dx} = f'(g(x)) \cdot g'(x)$$

$$\frac{de^{5x}}{dx} = e^{5x} \times \frac{d5x}{dx} = 5e^{5x}$$

$$\frac{de^{x^2}}{dx} = e^{x^2} \times \frac{dx^2}{dx} = e^{x^2} \times 2x = 2xe^{x^2}$$

$$\frac{d}{dx}\log(1000x) = \frac{1}{1000x} \times \frac{d1000x}{dx} = \frac{1000}{1000x} = 1/x$$

Q) Find $\frac{dy}{dx}$ if $y = \log(\log x)$:

Options:

(a) $\frac{1}{x \cdot \log x}$

(b) $\frac{1}{x(\log x)^2}$

(c) $\frac{\log x}{x}$

(d) $\frac{1}{x}$

$$\frac{1}{\log x} \times \frac{d\log x}{dx} = \frac{1}{x\log x}$$

$f'(x) = \frac{d}{dx} 3e^{x^4} = 3\frac{d}{dx} e^{x^4} = 3 \times e^{x^4} \times 4x^3 = 12x^3 e^{x^4}$

$f'(0) = 0 \quad f(0) = 3 \times e^0 = 3$

Q) If $f(x) = 3e^{x^4}$, then $f'(x) - 4x^3 f(x) + \frac{1}{3} f(0) - f'(0)$ is equal to:

Options:

(a) 0

(b) e^{x^2}

(c) 1

(d) -1

$= 12x^3 e^{x^4} - 4x^3 \times 3e^{x^4} + \frac{1}{3} \times 3 - 0$

$= 12x^3 e^{x^4} - 12x^3 e^{x^4} + 1$

$= 1$

$\boxed{\text{Parametric Differentiation}}$

Spoiler

In **parametric differentiation**, both variables (x and y) are expressed in terms of a third variable, called the **parameter** (e.g., t).

If:

$$x = f(t) \quad \text{and} \quad y = g(t),$$

then the derivative $\frac{dy}{dx}$ is calculated as:

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \quad (\text{provided } \tfrac{dx}{dt} \neq 0).$$

Q) If $x = 2t + 5$ and $y = t^2 - 5$, then $\frac{dy}{dx} = ?$

Options:

(a) t

(b) $-\frac{1}{t}$

(c) $\frac{1}{t}$

(d) 0

$\frac{dy}{dt} = \frac{d}{dt}(t^2 - 5)$

$\frac{dy}{dt} = 2t$ —— (1)

$\frac{dx}{dt} = \frac{d}{dt}(2t+5) = 2$ —— (2)

eq (1) ÷ (2)

$\frac{\frac{dy}{\cancel{dt}}}{\frac{dx}{\cancel{dt}}} = \frac{\cancel{2}t}{\cancel{2}}$

$\frac{dy}{dx} = t$

Q) If $x = at^2$ and $y = 2at$, then $\frac{dy}{dx} = ?$

Options:

(a) $\frac{1}{t}$

(b) $-\frac{1}{t}$

(c) t

(d) None of the above

$\frac{dy}{dt} = \frac{d(2a)t}{dt} = 2a \times \frac{dt}{dt} = 2a$ — (1)

$\frac{dx}{dt} = \frac{d}{dt} at^2 = a\frac{d}{dt}t^2 = 2at$ — (2)

(1) ÷ (2)

$\frac{dy}{dx} = \frac{\cancel{2a}}{\cancel{2a}t} = \frac{1}{t}$

Q) If $x = ct$ and $y = \frac{c}{t}$, then $\boxed{\frac{dy}{dx}}$ is equal to:

Options:

(a) $\frac{1}{t}$

(b) ~~$t \cdot e'$~~

(c) $-\frac{1}{t^2}$

(d) None of these

$\frac{dy}{dt} = c \times \frac{d t^{-1}}{dt} = c \times -1 \times t^{-2}$

$\frac{dy}{dx} = -\frac{c}{t^2}$ — (1)

$\frac{dx}{dt} = c$ — (2)

(1) ÷ (2)

$\frac{dy}{dx} = \frac{-1}{t^2}$

Implicit Functions

Implicit functions are those where x and y are not explicitly separated (e.g., x and y are on the same side of the equation).

$x^2y + y^2x = 0$ Find $\frac{dy}{dx}$

Shortcut Formula:

$$\boxed{\frac{dy}{dx} = -\frac{\text{Differentiate with respect to } x\text{, treating } y \text{ as constant}}{\text{Differentiate with respect to } y\text{, treating } x \text{ as constant}}}$$

$= -\frac{\frac{d}{dx}(x^2y + y^2x)}{\frac{d}{dy}(x^2y + y^2x)}$

$= -\frac{(y \times 2x + y^2 \times 1)}{x^2 \times 1 + x \times 2y}$

$= -\frac{(2xy + y^2)}{2xy + x^2}$

Q) If $e^{xy} - 4xy = 4$, find $\boxed{\frac{dy}{dx}}$:

Options:

(a) ~~$\frac{y}{x}$~~

(b) $-\frac{y}{x}$

(c) $\frac{x}{y}$

(d) $-\frac{x}{y}$

$-\frac{\left(\frac{d}{dx}e^{xy} - \frac{d}{dx}4xy\right)}{\frac{d}{dy}e^{xy} - \frac{d}{dy}4xy}$

$= -\frac{(ye^{xy} - 4y)}{xe^{xy} - 4x}$

$= -\frac{[y\cancel{(e^{xy} - 4)}]}{x\cancel{(e^{xy} - 4)}}$

$= -y/x$

If a function is given in the form:

$$x^m \cdot y^n = (x+y)^{m+n},$$

then the derivative is simplified as:

$$\frac{dy}{dx} = \frac{y}{x}.$$

Q) If $x^p y^q = (x+y)^{p+q}$, then $\frac{dy}{dx}$ is equal to:

Options:

(a) $\frac{q}{p}$

(b) $\frac{x}{y}$

(c) $\frac{y}{x}$

(d) $\frac{p}{q}$

Q) If $x^5 + y^5 - 5xy = 0$, then $\frac{dy}{dx}$ is

Options:

(a) $\frac{y+x^4}{x+y^4}$

(b) $\frac{y-x^4}{y^4-x}$

(c) $\frac{x-y^4}{x^4-y}$

(d) $\frac{x+y^4}{x^4+y}$

$$\frac{dy}{dx} = -\frac{\left[\frac{d}{dx} \text{ taking } y \text{ const}\right]}{\frac{d}{dy} \text{ taking } x \text{ const}}$$

$$= -\frac{\left[\frac{d}{dx}x^5 + \frac{d}{dx}y^5 - \frac{d}{dx}5xy\right]}{\frac{d}{dy}x^5 + \frac{d}{dy}y^5 - \frac{d}{dy}5xy}$$

$$= -\frac{[5x^4 + 0 - 5y]}{0 + 5y^4 - 5x}$$

$$= \frac{y - x^4}{y^4 - x}$$

Logarithmic Differentiation

Logarithmic Differentiation is used for differentiating functions where the variable appears as both the base and the exponent or in complex products/quotients.

Shortcut Formula:

$$\frac{dy}{dx} = y\left[\frac{P}{B} \cdot \frac{d(B)}{dx} + \ln(B) \cdot \frac{d(P)}{dx}\right]$$

- P: Power (exponent).
- B: Base.
- $\ln(B)$: Natural logarithm of the base.

$y = x^x$

Q) Differentiate x^x with respect to x:

Options:

(a) $x^x(1 + \log x)$

(b) $\frac{y}{x}$

(c) $-\frac{y}{x}$

(d) $y + x^x \log x$

$$\frac{dy}{dx} = x^x\left[\frac{x}{x} \times \frac{dx}{dx} + \log x \times \frac{dx}{dx}\right]$$

$$= x^x(1 + \log x)$$

$P = \log x \quad B = \sqrt{x}$

Q) If $y = (\sqrt{x})^{\log x}$, find $\frac{dy}{dx}$:

Options:

(a) $y \cdot \frac{\log x}{2x}$

(b) $y \cdot \frac{2\log x}{x}$

(c) $y \cdot \frac{\log x}{x}$

(d) $y \cdot \frac{\log x}{4x}$

$$\frac{dy}{dx} = y\left[\frac{\log x}{\sqrt{x}} \times \frac{d\sqrt{x}}{dx} + \log\sqrt{x} \times \frac{d\log x}{dx}\right]$$

$$= y\left[\frac{\log x}{\sqrt{x}} \times \frac{1}{2\sqrt{x}} + \log x^{1/2} \times \frac{1}{x}\right]$$

$$= y\left[\frac{\log x}{2x} + \frac{1 \times \log x}{2x}\right]$$

$$= y\,\cancel{\frac{2\log x}{2x}} = y\,\frac{\log x}{x}$$

Successive Differentiation (Higher Order Derivatives)

Successive Differentiation refers to the process of repeatedly differentiating a function to find its higher-order derivatives.

Explanation:

- The **first derivative** ($f'(x)$) represents the rate of change or slope of the function.
- The **second derivative** ($f''(x)$) represents the rate of change of the first derivative (often related to acceleration or concavity).
- The **nth derivative** ($f^{(n)}(x)$) is obtained by differentiating the function n times.

Notation:

1. First derivative: $\frac{dy}{dx}$ or y'
2. Second derivative: $\frac{d^2y}{dx^2}$ or y''
3. Third derivative: $\frac{d^3y}{dx^3}$ or y'''
4. n-th derivative: $\frac{d^ny}{dx^n}$ or $y^{(n)}$

$$\frac{dy}{dx} = 3x^2$$

$$\frac{d^2y}{dx^2} = 3 \times \frac{d}{dx} x^2 = 6x$$

$$\frac{d^3y}{dx^3} = \frac{d}{dx} 6x = 6$$

Example:

For $y = x^3$:

1. First derivative (y'):

$$\frac{dy}{dx} = 3x^2$$

2. Second derivative (y''):

$$\frac{d^2y}{dx^2} = 6x$$

3. Third derivative (y'''):

$$\frac{d^3y}{dx^3} = 6$$

Q) Find the second derivative of $y = \sqrt{x+1}$

$\frac{d}{dx}\sqrt{x+1} = \frac{1}{2\sqrt{x+1}}$

Options:

(a) $\frac{1}{2}(x+1)^{-\frac{3}{2}}$

(b) $-\frac{1}{4}(x+1)^{-\frac{3}{2}}$

(c) $\frac{1}{4}(x+1)^{-\frac{1}{2}}$

(d) None of these

$\frac{dy}{dx} = \frac{1}{2\sqrt{x+1}}$

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{1}{2}\frac{d}{dx}(x+1)^{-1/2}$

$= \frac{1}{2} \times \frac{-1}{2} \times (x+1)^{-\frac{1}{2}-1}$

$= -\frac{1}{4}(x+1)^{-3/2}$

Derivatives of Infinite Series

When dealing with a function that represents an **infinite power series**, like:

$$y = f(x)^{f(x)^{f(x)^{\cdots}}}$$

$y = f(x)^{f(x)^{f(x)^{f(x)\cdots}}}$

there's a **shortcut formula** to differentiate it.

Shortcut Formula:

$$\frac{dy}{dx} = \frac{y^2}{1 - y\ln f(x)} \cdot \frac{f'(x)}{f(x)}$$

$= \frac{y^2}{1 - y\log f(x)} \times \frac{f'(x)}{f(x)}$

Q) If $Y = x^{x^{x^{\cdots}}}$ (infinite power tower), then $\frac{dY}{dx}$:

~~Options:~~

(a) $\frac{y^2}{x(1-y\log x)}$

(b) $\frac{y}{x(1-y\log x)}$

(c) $\cancel{\frac{y^2}{x(1+y\log x)}}$

(d) $\cancel{\frac{y}{x(1+y\log x)}}$

$$\frac{y^2}{(1-y\log x)} \times \frac{1}{x}$$

$$= \frac{y^2}{x(1-y\log x)}$$

Application of Derivatives: Slope of the Tangent (Gradient to the Curve)

$\frac{dy}{dx}$

Key Idea:

The **derivative** of a function at a specific point gives the **slope of the tangent** to the curve at that point.

Formula:

If the equation of the curve is $y = f(x)$, then:

$$\text{Slope of the tangent} = \boxed{\left.\frac{dy}{dx}\right|_{x=x_1}}$$

where x_1 is the point of interest.

Equation of the Tangent

Once we know the slope of the tangent at a specific point, the **equation of the tangent** to the curve at that point can be found using the **point-slope form** of a straight line.

Formula:

$\frac{dy}{dx}$

$$\boxed{y - y_1 = m(x - x_1)}$$

Where:

- (x_1, y_1): The point of tangency on the curve.
- m: The slope of the tangent, given by $\frac{dy}{dx}\Big|_{x=x_1}$

When the tangent to a curve is parallel to the x-axis, its slope is zero. In such cases:

$$\boxed{\frac{dy}{dx} = 0}$$

Q) The equation of the tangent to the curve $f = x^3 - 2x + 3$ at the point $(2, 7)$ is:

$m = \frac{dy}{dx} = \frac{d}{dx} x^3 - 2x + 3 = \boxed{3x^2 - 2}$

$\frac{dy}{dx}\Big|_{x=2} = 3 \times 4 - 2$
$= 10$

Options:

(a) $y = 2x - 13$

(b) ~~$y = 10x$~~

(c) $y = 10x - 13$

(d) $y = 10$

$y - y_1 = m(x - x_1)$
$y - 7 = 10(x - 2)$
$y - 7 = 10x - 20$

$y = 10x - 13$

Q) For a given curve $y = 2 - x^2$, when 'x' increases at the rate of 3 units/s, then the slope of the curve will:

$\frac{dy}{dx} = \frac{d}{dx} 2 - x^2$
$= -2x$
$= -2 \times 3 = -6$

Options:

(a) Increase at 6 units/s

(b) Increase at 3 units/s

(c) Decrease at 6 units/s

(d) Decrease at 3 units/s

Maximum and Minimum Value of a Function

To determine whether a function $y = f(x)$ has a maximum or minimum value at a given point, we use the first and second derivatives.

Steps:

1. **Find the First Derivative:**
   - Compute $\frac{dy}{dx}$ and set it equal to zero:
   $$\frac{dy}{dx} = 0$$
   - Solve for x to find the critical points.
2. **Find the Second Derivative:**
   - Compute $\frac{d^2y}{dx^2}$.
   - Substitute the critical points (values of x) into $\frac{d^2y}{dx^2}$ to determine the nature of the critical points.

$x^3 - 2x^2 + 3x + 1 = 0$

$3x^2 - 4x + 3 = 0$

Find x value

$6x - 4$

Conditions:

- If $\frac{d^2y}{dx^2} < 0$, the function has a **maximum** value at that point.
- If $\frac{d^2y}{dx^2} > 0$, the function has a **minimum** value at that point.
- If $\frac{d^2y}{dx^2} = 0$, the test is inconclusive, and the point may be a saddle point (neither maximum nor minimum).

Q) If $y = 2x^3 - 15x^2 + 36x + 15$, then find the value of x at which the function will have a minimum value. Also, find the minimum value of the function.

Options:

(a) $x = 3$, Minimum value = 42

(b) $x = 2$, Minimum value = 30

(c) $x = 1$, Minimum value = 15

(d) $x = 4$, Minimum value = 50

① $\frac{dy}{dx} = 6x^2 - 30x + 36 = 0$

$x^2 - 5x + 6 = 0$

$x = 3, 2$

② $\frac{d^2y}{dx^2} = \frac{d(6x^2 - 30x + 36)}{dx}$

$= 12x - 30$

Put $x = 3$

$\frac{d^2y}{dx^2} = 36 - 30 = 6$

Put $x = 2$

$\frac{d^2y}{dx^2} = -6$

Q) In a market, there are 30 shops to allocate to people. If they allocate x shops, then their monthly expenses, in rupees, is given by $p(x) = -8x^2 + 400x - 1000$. Then the number of shops they should allocate to minimize the expenses is:

$p'(x) = \frac{d}{dx}(-8x^2 + 400x - 1000)$

$= -16x + 400 = 0$

$16x = 400$

$x = 25$

Options:

(a) 0

(b) 30

(c) 25

(d) 10

Application of Derivatives in Cost and Revenue

Key Definitions

1. **Average Cost (AC):**

$$\text{Avg Cost} = \frac{C(x)}{x}$$

Where $C(x)$ is the cost function, and x is the number of units of production.

2. **Marginal Cost (MC):**

$$\text{Marginal Cost} = \frac{d}{dx}[C(x)]$$

It represents the rate of change of total cost with respect to the number of units produced.

3. **Average Revenue (AR):**

$$\text{Avg Revenue} = \frac{R(x)}{x}$$

Where $R(x)$ is the revenue function.

4. **Marginal Revenue (MR):**

$$\text{Marginal Revenue} = \frac{d}{dx}[R(x)]$$

It represents the rate of change of total revenue with respect to the number of units sold.

Notes

1. **Revenue Function:**

$$\boxed{R(x) = \text{Demand Function} \times x}$$

2. **Profit Maximization:**

- Profit is maximized at the equilibrium point under perfect competition when:

$$\text{Marginal Revenue} = \text{Marginal Cost}.$$

3. **Break-Even Point:**

- Profit is zero when:

$$\text{Total Revenue} = \text{Total Cost}.$$

Q) The cost function is given by

$$C(x) = 125 + 500x - x^2 + \frac{x^3}{3}, \quad 0 \le x \le 100$$

and the demand function for the items is given by

$$p(x) = 1500 - x$$

Find the marginal profit when 18 items are sold.

$R(x) = (1500 - x)x$

$= 1500x - x^2$

$P_{ro}(x) = R(x) - C(x) = 1500x - \cancel{x^2} - 125 - 500x + \cancel{x^2} - \frac{x^3}{3}$

$P(x) = 1000x - 125 - x^3/3$

$\frac{d}{dx}P(x) = 1000 - \cancel{\frac{1}{3}\times 3}x^2$

$\frac{d}{dx}(P(18)) = 1000 - x^2$

$= 1000 - 18^2$

$= 676$

(a) 751

(b) 571

(c) 676

(d) 875

Q) The speed of a train at a distance x (from the starting point) is given by $3x^2 - 5x + 4$. What is the rate of change (of distance) at $x = 1$?

Options:

(a) -1

(b) 0

(c) 1

(d) 2

$6x - 5$

$= 1$

Integrals

1. $\int e^x \, dx = e^x + C$
2. $\int e^{ax} \, dx = \frac{e^{ax}}{a} + C$ (where $a \neq 0$)
3. $\int \frac{1}{x} \, dx = \ln x + C$
4. $\int a^x \, dx = \frac{a^x}{\ln a} + C$ (where $a > 0$)
5. $\int 1 \, dx = x + C$
6. $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$ (for $n \neq -1$)

$\frac{e^{ax}}{a}$

$\int x^2 = \boxed{\frac{x^3}{3} + c}$

$\frac{d}{dx}\frac{x^3}{3} + \frac{d}{dx}c$

$\frac{1}{3}\times 3x^2 + 0$

$= x^2$

$\int dx = x + c$

$\int \frac{1}{x} dx = \log x$

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Problems on Sum Rule in Integration

The Sum Rule for Integration states:

$$\int (f(x) + g(x)) \, dx = \int f(x) \, dx + \int g(x) \, dx$$

$\int \frac{1}{x} + x^3 + e^{10x}$

$= \log x + \frac{x^4}{4} + \frac{e^{10x}}{10} + c$

Q) Evaluate the integral:

$$\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) dx$$

$\int (x^{1/2} + x^{-1/2}) . dx$

$\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{x^{\frac{-1}{2}+1}}{\frac{-1}{2}+1}$

$= \frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2}$

$\frac{2x^{3/2}}{3} + \frac{2x^{1/2}}{1}$

$2x^{1/2}\left(\frac{x}{3} + 1\right) + C$

Options:
(a) $2x^{1/2}\left(\frac{1}{3}x - 1\right)$
(b) $2x^{1/2}\left(\frac{1}{3}x + 1\right)$
(c) $2\left(\frac{1}{3}x + x^{1/2}\right)$
(d) None of these

Q) Evaluate the integral:

$\frac{d}{dx} 3^x = 3^x \times \log 3$

$$\int (3x^2 - 2e^{2x} + 5 - 3^x) dx$$

$3 \times \frac{x^3}{3} - \cancel{2} \times \frac{e^{2x}}{\cancel{2}} + 5x - \frac{3^x}{\log 3} + C$

Options:
(a) $x^3 - e^{2x} + 5x - \frac{3^x}{\ln 3} + C$
(b) $x^3 - 2e^{2x} + 5x - \frac{3^x}{\ln 3} + C$
(c) $x^3 - e^{2x} + 5x - 3^x + C$
(d) $x^3 + e^{2x} + 5x - \frac{3^x}{\ln 3} + C$

Q) $\int 2^{3x} \cdot 3^{2x} \cdot 5^x \, dx =$

$\int a^x . dx$

$\int 8^x \times 9^x \times 5^x . dx$

$\int (360)^x . dx$

$\frac{360^x}{\log 360} + C$

Options:
(a) $\frac{2^{3x} \cdot 3^{2x} \cdot 5^x}{\log(720)} + C$
(b) $\frac{2^{3x} \cdot 3^{2x} \cdot 5^x}{\log(360)} + C$
(c) $\frac{2^{3x} \cdot 3^{2x} \cdot 5^x}{\log(180)} + C$
(d) $\frac{2^{3x} \cdot 3^{2x} \cdot 5^x}{\log(90)} + C$

Q) The equation of the curve which passes through the point (1, 2) and has the slope $3x - 4$ at any point (x, y) is:

Options:

(a) $2y = 3x^2 - 8x + 9$

(b) $y = 6x^2 - 8x + 9$

(c) $y = x^2 - 8x + 9$

(d) $2y = 3x^2 - 8x + c$

$\frac{dy}{dx} = 3x - 4$

$dy = (3x - 4)dx$

$y = \int (3x - 4) . dx$

$y = \frac{3 \times x^2}{2} - 4x + c$

$2 = \frac{3}{2} - 4 + c \quad c = 4.5$

$y = \frac{3}{2}x^2 - 4x + 4.5$

$2y = 3x^2 - 8x + 9$

Definite Integrals

A **definite integral** is an integral with specific limits of integration, usually expressed as:

$$\int_a^b f(x)\, dx$$

$\int_2^5 x^2 . dx$

$= \left[\frac{x^3}{3}\right]_2^5$

$= \frac{5^3}{3} - \frac{2^3}{3} = \frac{125 - 8}{3}$

$= 39$

- a and b are the lower and upper limits, respectively.
- The result represents the **net area under the curve** $f(x)$ from $x = a$ to $x = b$.

Key Properties of Definite Integrals

1. Linearity:

$$\int_a^b (f(x) + g(x))\, dx = \int_a^b f(x)\, dx + \int_a^b g(x)\, dx$$

$$\int_a^b c \cdot f(x)\, dx = c \cdot \int_a^b f(x)\, dx$$

2. **Reversing Limits:**

$$\int_a^b f(x)\, dx = -\int_b^a f(x)\, dx$$

3. **Zero Width Interval:**

$$\int_a^a f(x)\, dx = 0$$

4. **Additivity:**

$$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$$

Steps to Evaluate Definite Integrals

1. **Find the Indefinite Integral:** Compute $F(x)$, the antiderivative of $f(x)$
2. **Apply Limits:** Use the Fundamental Theorem of Calculus:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Q) Solve: $\int_1^1 (e^x - e^{-x})dx$

Options:

(a) 0

(b) 1

(c) 12

(d) None of the above

Q) Evaluate:

$$\int_1^2 7x^6\,dx$$

$$\left[\cancel{7} \times \frac{x^7}{\cancel{7}}\right]_1^2$$

$$2^7 - 1^7$$

$$128 - 1$$

$$= \underline{\underline{127}}$$

Options:

(a) 126

(b) 127

(c) 128

(d) 129

Q) Find the area under the curve $f(x) = x^2 + 5x + 2$ with the limits 0 to 1?

$\int f(x).dx = \int_0^1 (x^2+5x+2)dx$

Options:

(a) 3.833

(b) 4.388

(c) 4.833

(d) 3.338

$= \left[\frac{x^3}{3} + \frac{5x^2}{2} + 2x\right]_0^1$

$= \frac{1}{3} + \frac{5}{2} + 2 = 4.833$

Q) The value of $\int_{-2}^{2} f(x)\,dx$, where $f(x) = 1 + x$, $x \le 0$, $f(x) = 1 - 2x$, $x \ge 0$, is:

$\int_{-2}^{2} f(x).dx = \int_{-2}^{0} (1+x).dx + \int_0^2 1-2x$

Options:

(a) 20

(b) -2

(c) -4

(d) 0

$= \left[x + \frac{x^2}{2}\right]_{-2}^{0} + \left(x - \frac{2x^2}{2}\right)_0^2$

$= 0 + 0 - \left[-2 + \frac{4}{2}\right] + \left(2 - 4\right]$

$= -\left[\cancel{-2+2}\right] + 2 - 4 = -2$

Property 5: Symmetry in Definite Integrals

$$\int_a^b f(x)\,dx = \int_a^b f((a+b) - x)\,dx$$

Special Definite Integral Formula

$$\int_a^b \frac{x^n}{x^n + (a+b-x)^n}\,dx = \boxed{\frac{b-a}{2}}$$

Q) The value of $n = \frac{1}{2}$ $a + b = 2$

$$\int_0^2 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{2-x}}\,dx \text{ is:}$$

$\frac{b-a}{2} = \frac{2-0}{2}$

a) 0

b) 3

c) 2

$= 1$

d) 1

Integration by Substitution

Integration by substitution is a technique used to simplify integrals by changing the variable of integration. It is based on the **chain rule** in reverse.

Evaluate:

$$\int xe^{x^2}dx$$

Solution:

1. Substitution: Let $u = x^2$, so $du = 2xdx$, or $\frac{du}{2} = xdx$.
2. Rewrite the integral:

$$\int xe^{x^2}dx = \int e^u \frac{du}{2}$$

$$= \frac{1}{2}\int e^u du$$

3. Integrate:

$$\frac{1}{2}\int e^u du = \frac{1}{2}e^u + C$$

4. Back-substitute $u = x^2$:

$$\frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$

Final Answer:

$$\int xe^{x^2}dx = \frac{1}{2}e^{x^2} + C$$

Q) Evaluate:

$$\int_1^2 \frac{2x}{1+x^2}dx$$

Assume

$1+x^2 = t$

Different

$2x = \frac{dt}{dx}$

$2x\,dx = dt$

$$\int_1^2 \frac{dt}{t}$$

$\log t$

$\left[\log(1+x^2)\right]_1^2 = \log[5] - \log 2$

$= \log \frac{5}{2}$

(a) $\log_e \frac{5}{2}$

(b) $\log_e 5 - \log_e 2 + 1$

(c) $\log_e \frac{2}{5}$

(d) None of these

Q) Evaluate $\int \frac{1}{x(1+\log x)^2} dx$

Assume

$1 + \log x = t$

$\frac{1}{x} = \frac{dt}{dx}$

$\frac{1}{x} dx = dt$

Options:

(a) $-\frac{1}{2(1+\log x)^2} + C$

(b) $\frac{1}{(1+\log x)} + C$

(c) $-\frac{1}{(1+\log x)} + C$

(d) None of these

$\int \frac{dt}{t^2}$

$\int t^{-2} \cdot dt = \frac{t^{-2+1}}{-2+1}$

$= \frac{t^{-1}}{-1} = \frac{-1}{t}$

Important Formula

$$\int e^x (f(x) + f'(x))\, dx = e^x f(x) + C$$

Q) $\int_1^2 e^x \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$ **is:**

$\left[e^x \times \frac{1}{x}\right]_1^2$

Options:

(a) $e\left(\frac{e}{2} - 1\right)$

(b) $e(e-1)$

(c) a

(d) $e^2(e - i)$

$\left[e^2 \times \frac{1}{2} - e\right]$

$e\left(\frac{e}{2} - 1\right)$

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Integration by Parts

Integration by parts is a method used to integrate products of two functions. The formula is derived from the product rule for differentiation.

Formula:

$$\int u\, v\, dx = u \int v\, dx - \int \left(\frac{du}{dx} \int v\, dx\right) dx$$

L A E

ILATE Rule:

The priority for selecting u (the part to differentiate) is:

1. I: Inverse Trigonometric Functions (e.g., $\arctan x, \arcsin x$).
2. L: Logarithmic Functions (e.g., $\ln x$).
3. A: Algebraic Functions (e.g., $x^2, x, \sqrt{x}$).
4. T: Trigonometric Functions (e.g., $\sin x, \cos x$).
5. E: Exponential Functions (e.g., e^x, a^x).

$\int x \log x \, dx$

$\int \log x \times x \, dx$

Shortcut:

For repeated integration by parts, the solution can be written as:

$$\int uv\,dx = uv_1 - u'v_2 + u''v_3 - \dots$$

Terms in the Formula:

1. u: The function chosen for differentiation (based on the ILATE rule).
2. u', u'': Successive derivatives of u.
3. v_1: The first integral of v.
4. v_2: The integral of v_1.
5. v_3: The integral of v_2, and so on.

How to Use:

1. Differentiate u repeatedly until it becomes zero or simplifies the integral.
2. Compute successive integrals of v.
3. Substitute into the formula:
   - Alternate the signs (+, -, +, ...) for each term.
4. Stop when u becomes zero or the integral is fully solved.

Q) $\int_0^1 xe^x \, dx$ is equal to:

Options:

(a) 0

(b) 2

(c) 1

(d) 3

$xe^x - 1 \times e^x + 0$

$= \left[xe^x - e^x\right]_0^1$

$= \left[1 \times \cancel{e^1} - \cancel{e^1}\right] - \left[0 - e^0\right]$

$= 0 - (-e^0) = 1$

Integration by Partial Fractions

Integration by partial fractions is a technique used to integrate rational functions by breaking them into simpler fractions.

Type 1: Linear Factors in the Denominator

When the denominator can be factored into **distinct linear factors**, the function is decomposed as:

$$\frac{P(x)}{Q(x)} = \frac{A}{(x-a)} + \frac{B}{(x-b)} + \dots$$

Q) Evaluate the integral:

$$\int \frac{6x+4}{(x-2)(x-3)} dx = \int \frac{A}{x-2} + \frac{B}{x-3}$$

$A \log(x-2) + B \log x - 3$

a) $22\log(x-3) - 16\log(x-2)$

b) $11\log(x-3) - 8(x-2)$

c) $22\log(x-3) - 16\log(x-2)$

d) $22\log(x-3) + 16\log(x-2)$

To find value of A
Put $x = 2$

$\frac{6 \times 2 + 4}{2-3} = -16$

Put $x = 3$

$\frac{6 \times 3 + 4}{3-2} = \frac{22}{1}$

Type 2: Repeated Linear Factors

When the denominator has repeated linear factors, the rational function is decomposed as:

$$\frac{P(x)}{Q(x)} = \frac{A}{(x-a)} + \frac{B}{(x-a)^2} + \dots$$

Q) Evaluate the integral:

$$\int \frac{3x+5}{(x-4)(x-5)^2}\,dx$$

Options:

a) $17\log\left(\frac{x-5}{x-4}\right) - \frac{20}{x-5} + C$
b) $15\log\left(\frac{x-5}{x-4}\right) - \frac{25}{x-5} + C$
c) $17\log(x-5) + \frac{20}{x-4} + C$
d) $10\log\left(\frac{x-4}{x-5}\right) + \frac{5}{x-5} + C$

Q) Evaluate the integral:

$$\int_0^1 \frac{1-x}{1+x}\,dx = \int \frac{1-x+1-1}{1+x}$$

$$= \int \frac{2-x-1}{1+x}$$

$$= \int \frac{2-(1+x)}{(1+x)}.dx = \int \frac{2}{1+x} - \int 1.dx = \left[2\log(1+x) - x\right]_0^1 - \left[2\log 1 - 0\right]$$

$$= 2\log 2 - 1$$

$$= 2\log 2 - 1$$

a) $2\log 2 - 1$
b) $4\log 2 - 1$
c) $2\log 2$
d) None of these

Q) Evaluate $\int \frac{x-1}{x+3}\,dx$:

Options: a) $x - 4\ln(x+3) + C$
b) $x + 4\ln(x+3) + C$
c) $x - \ln(x+3) + C$
d) $x - 4(\ln(x+3)) + C$

$$\int \frac{x-1+3-3}{x+3}$$

$$\int \frac{x+3-4}{x+3}$$

$$\int \frac{x+3}{x+3}.dx - \int \frac{4}{x+3}$$

$$x - 4\log(x+3)$$

Standard Integrals for Common Forms

1. $$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \log\left(x+\sqrt{x^2-a^2}\right)+C$$
2. $$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \log\left(x+\sqrt{x^2+a^2}\right)+C$$
3. $$\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log\left(x+\sqrt{x^2+a^2}\right)+C$$
4. $$\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left(x+\sqrt{x^2-a^2}\right)+C$$
5. $$\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\log\left(\frac{x-a}{x+a}\right)+C$$
6. $$\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\log\left(\frac{a+x}{a-x}\right)+C$$

Chapter 3: Linear Inequalities

Definition of an Inequality

An **inequality** is a statement that compares two real numbers or two algebraic expressions using the symbols $<$, $>$, $\leq$, $\geq$. For example:

- $3<5$ or $7>5$ (numerical inequalities).
- $x<5$, $y\geq 2$, $x\geq 3$, $y\leq 4$ (inequalities involving variables, sometimes called literal inequalities).

Inequalities in One Variable

A one-variable inequality has the form $ax+b<0$, $ax+b>0$, $ax+b\leq 0$, or $ax+b\geq 0$, where a and b are constants.
Its **solution set** is a range (interval) on the real number line.

Key Property of Multiplication by a Negative Number

- If $a>0$ and $b>c$, then multiplying both sides by a preserves the inequality: $ab>ac$.
- If $a<0$ and $b>c$, then multiplying both sides by a **reverses** the inequality: $ab<ac$.

Inequalities in Two Variables

An inequality in two variables x and y often appears in the form:

$$ax + by < c, \quad ax + by > c, \quad ax + by \leq c, \quad ax + by \geq c,$$

where a, b, and c are real constants.

The **solution set** (also called the **solution region** or **solution space**) is the set of all points (x, y) in the plane that satisfy the inequality.

Linear Equations vs. Linear Inequalities

A **linear equation** in two variables has the form:

$$ax + by + c = 0.$$

A **linear inequality** in two variables replaces " $=$ " with one of the inequality symbols:

$$ax + by + c < 0, \quad ax + by + c > 0, \quad ax + by + c \leq 0, \quad ax + by + c \geq 0$$

Boundary Lines and Graphical Representation

Boundary Line: For a linear inequality $ax + by + c < 0$, > 0, ≤ 0, or ≥ 0, the line

$$ax + by + c = 0$$

is called the **boundary line**.

Half-Planes: This line divides the plane into two half-planes:

- One side satisfies $ax + by + c > 0$.
- The other side satisfies $ax + by + c < 0$.

Including/Excluding the Boundary:

- For strict inequalities ($<$ or $>$), the boundary line itself is **not** part of the solution.
- For non-strict inequalities ($\leq$ or $\geq$), the boundary line **is** included in the solution region.

Special Case: If $ax + by = 0$ (i.e., $c = 0$), the line passes through the origin $(0, 0)$.

Examples

1. **Numerical Inequality:**
   $7 > 5$ (True statement comparing two numbers.)
2. **One-Variable Inequality:**
   $x + 2 < 5$
   $\implies x < 3.$
   The solution is all real x less than 3.
3. **Two-Variable Inequality:**
   $2x + 3y \leq 6.$
   - Boundary line: $2x + 3y = 6.$
   - To determine which side to shade, pick a test point (e.g., the origin $(0, 0)$): $2(0) + 3(0) = 0 \leq 6$ is true, so the side containing $(0, 0)$ is part of the solution region.
   - Include the boundary line since the inequality is $\leq$.

Graphical Representation of Linear Inequalities

Understanding the Graphical Solution of Linear Inequalities

- A **line** divides the Cartesian plane into two parts. Each part is called a **half-plane**.
- A **vertical line** divides the plane into **left and right half-planes.**
- A **non-vertical line** divides the plane into **upper and lower half-planes.**

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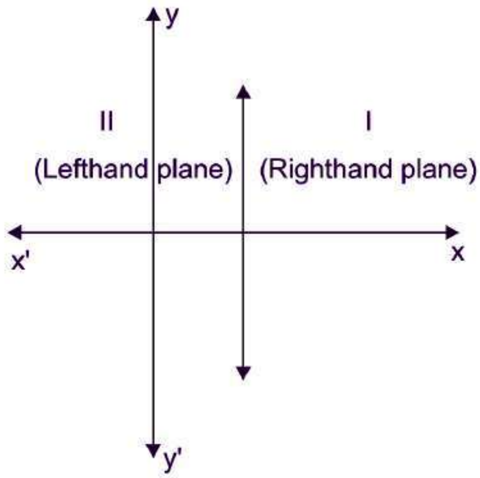
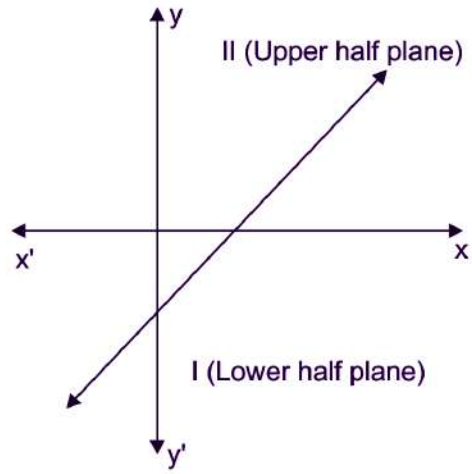




Steps to Graph a Given Inequality

1. **Plot the Boundary Line:**
   - Convert the given inequality into an equation by replacing the inequality sign ($<$, $>$, $\leq$, $\geq$) with $=$.
   - The resulting equation represents a straight line.
   - If the inequality is **strict** ($<$ or $>$), use a **dashed** boundary line (since points on the line are not included).
   - If the inequality is **non-strict** ($\leq$ or $\geq$), use a **solid** boundary line (since points on the line are included).
2. **Choose a Test Point:**
   - Pick a test point (often the origin $(0, 0)$ if it is not on the line).
   - Substitute this point into the original inequality.
   - If the inequality holds true, shade the **half-plane** containing the test point.
   - If the inequality does not hold true, shade the **other half-plane**.

Using Arrows to Indicate the Solution Region

- **Arrows** are used to indicate which half-plane satisfies the inequality.
- If the **coefficient of y is positive**:
  - For **less than** ($<$ or $\leq$), arrows should be **below the line**.
  - For **greater than** ($>$ or $\geq$), arrows should be **above the line**.
- If the **coefficient of y is negative**:
  - For **less than** ($<$ or $\leq$), arrows should be **above the line**.
  - For **greater than** ($>$ or $\geq$), arrows should be **below the line**.

x	0	10
y	-10	0

$x - y = 10$

Q) The following graph is represented by:

$x + y = 10$

(10,0)

(-10)

(a) $x > 0$

(b) $x \geq 0$

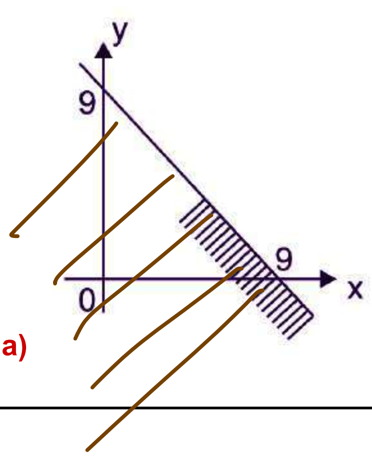
(c) $x < 0$

(d) $x \leq 0$

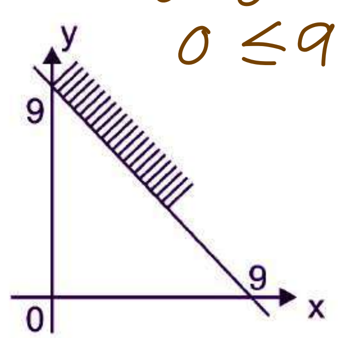
Q) The graph to express the inequality $x + y \leq 9$ is:

$0 + 0$

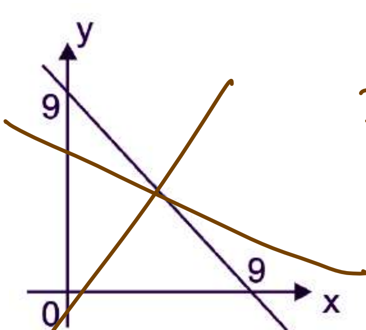
$0 \leq 9$



a)



b)



c)

$x + y \geq 9$

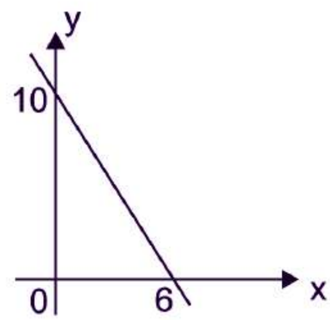
$0 + 0 \geq 9$

$0 > 9$

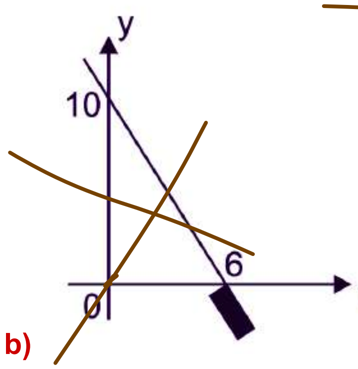
d) None

Q) The graph to express the inequality $5x + 3y \geq 30$ is:

$0 + 0 \geq 30$



a)



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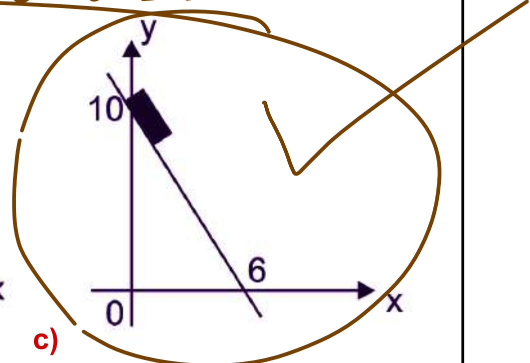
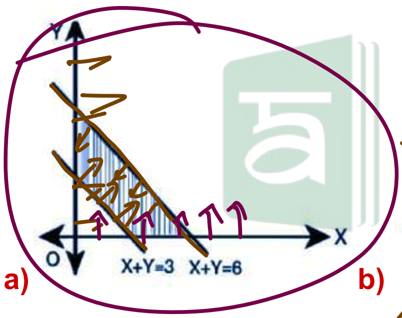


c)

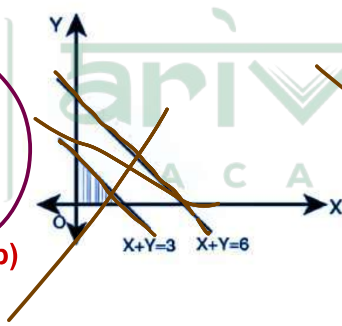
d)None

Q) The common region of $x + y \leq 6$; $x + y \geq 3$; $x \geq 0$; $y \geq 0$

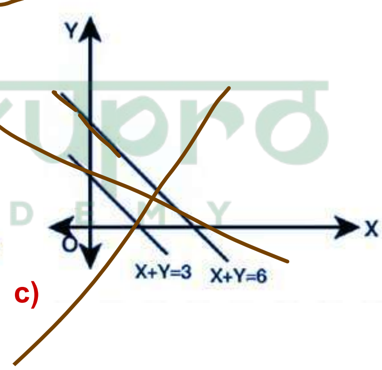
$0 < 6$



a)



b)



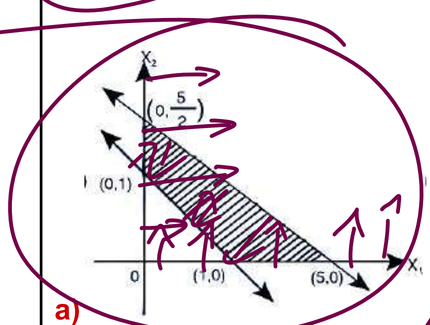
c)

d)None

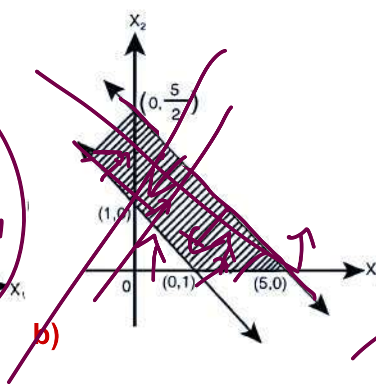
Q) The common region by the inequalities $x_1 + 2x_2 \leq 5$, $x_1 + x_2 \geq 1$, $x_1 \geq 0$, $x_2 \geq 0$ is given as shaded portion in:

$0 + 0 \leq 5$

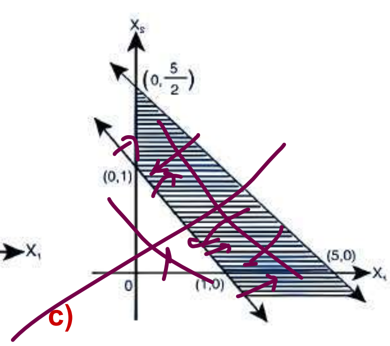
$0 + 0 > 1$



a)



b)



c)

d)None

Q) The common regions by the inequalities $4x + 3y \leq 60$; $y \geq 2x$; $x \geq 3$; $x \geq 0$; and $y \geq 0$ is:

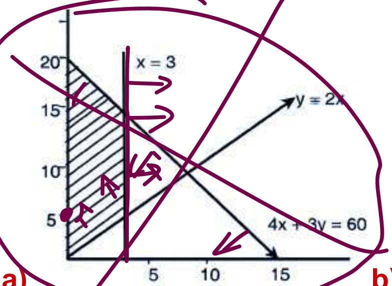
$x=0$ $y=5$

$0 \quad 0 < 60$

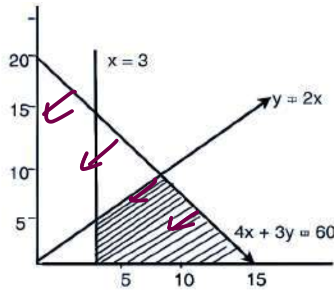
$y - 2x \geq 0$

$5 - 2\times 0 \geq 0$

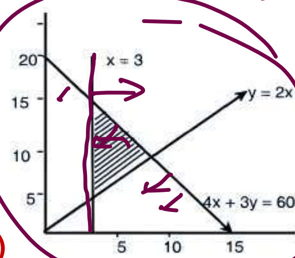
$5 > 0$



a)



b)



c)

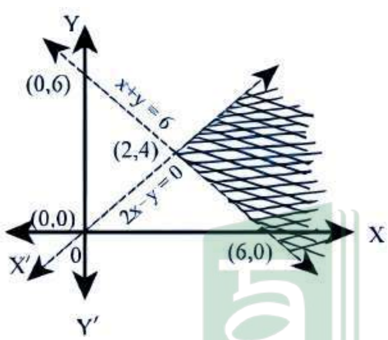
$0 - 10 \geq 0$

$-10 > 0$

d)None

Q) The shaded region represents:

$x \geq 3$



(a) $x + y > 6$, $2x - y > 0$

(b) $x + y < 6$, $2x - y > 0$

(c) $x + y > 6$, $2x - y < 0$

(d) None of these

Solving Problems on Linear Inequalities

Key Rule for Multiplication with a Negative Number

- When an inequality is **multiplied or divided by a negative number**, the inequality sign reverses.

- That is:
  - $<$ changes to $>$
  - $>$ changes to $<$
  - $\leq$ changes to $\geq$
  - $\geq$ changes to $\leq$

Q) The solution of the inequality $\frac{(5-2x)}{3} \leq \frac{x}{6} - 5$ is:

(a) $x \geq 8$
(b) ~~$x \leq 8$~~
(c) ~~$x = 8$~~
(d) None of these.

$\frac{5-18}{3} \leq \frac{9}{6} - 5$

$4.33 \leq -3.5$

Q) The solution of the inequality $8x + 6 < 12x + 14$ is:

(a) ~~$(-2, 2)$~~
(b) ~~$(0, -2)$~~
(c) $(2, \infty)$
(d) $(-2, \infty)$

$-16+6 < -24+14$

$-10 < -10$

$24+6 < 36+14$

$30 < 50$

$8x+6 < 12x+14$

$6-14 < 12x-8x$

$-8 < 4x$

$-2 < x$

$\boxed{x > -2}$

Q) Solve for x of the inequalities $2 \leq \frac{3x-2}{5} \leq 4$ where $x \in \mathbb{N}$:

(a) {5, 6, 7}
(b) ~~{3, 4, 5, 6}~~
(c) {4, 5, 6}
(d) None

$2 \leq \frac{3x-2}{5} \leq 4$

$10 \leq 3x-2 \leq 20$

$12 \leq 3x \leq 22$

$\div 3$

$4 \leq x \leq 7.33$

WORD PROBLEMS

Key Guidelines for Setting Up Inequalities

- For time-related constraints, use "$\leq$" (less than or equal to).

- **For work-related constraints**, use "$\geq$" (greater than or equal to).

Q) On average, an experienced person does 7 units of work while a fresh one does 5 units of work daily, but the employer has to maintain an output of at least 35 units of work per day. The situation can be expressed as:

(a) $7x + 5y < 35$
(b) $7x + 5y \leq 35$
(c) $7x + 5y > 35$
(d) $7x + 5y \geq 35$

$7x + 5y \geq 35$

Q) On average, an experienced person does 5 units of work while a fresh person does 3 units of work daily, but the employer has to maintain an output of at least 30 units of work per day. The situation can be expressed as:

(a) $5x + 3y \leq 30$
(b) $5x + 3y \geq 30$
(c) $5x + 3y > 30$
(d) $5x + 3y = 30$

≥ 30

$\leq$

Q) A dietician wishes to mix together two kinds of food so that the vitamins content of the mixture is at least 9 units of vitamin A, 7 units of vitamin B, 10 units of vitamin C, and 12 units of vitamin D. The vitamin content per kg of each food is shown in the table. Assuming x units of food I is to be mixed with y units of food II, the situation can be expressed as:

	A	B	C	D
Food I	2 x	1	1	2
Food II	1 y	1	2	3

(a) $2x + y \leq 9, x + y \leq 7, x + 2y \leq 10, 2x + 3y \leq 12, x > 0, y > 0$

(b) $\cancel{2x + y \geq 30, x + y} \leq 7, x + 2y \geq 10, 2x + 3y \geq 12, x > 0, y > 0$

(c) $\cancel{2x + y \geq 9, x + y \geq 7, x + 2y \leq 10, 2x + 3y \leq 12, x \geq 0, y \geq 0}$

(d) $2x + y \geq 9, x + y \geq 7, x + 2y \geq 10, 2x + 3y \geq 12, x \geq 0, y \geq 0$

Q) An employer recruits experienced (x) and fresh workmen (y) under the condition that he cannot employ more than 11 people. x and y can be related by the inequality:

(a) $x + y \neq 11$

(b) $x + y \leq 11, x \geq 0, y \geq 0$

(c) $x + y \geq 11, x \geq 0, y \geq 0$

(d) None of these

Q) The union forbids the employer to employ less than two experienced persons (x) for each fresh person (y). This situation can be expressed as:

(a) $x \leq y/2$

(b) $y \leq x/2$

(c) $y \geq x/2$

(d) None of these.

$\frac{x}{2} \geq y$

Demand

Forbid

Problems on Boundary Points

Definition of Boundary Points

- A **boundary point** is a point that lies on the dividing line or curve of an inequality.
- The boundary separates the solution region from the non-solution region.

Q) On solving the inequalities $5x + y \leq 100$, $x + y \leq 60$, $x \geq 0$, $y \geq 0$, we get ~~the following solution:~~ $5 \times 60 + 0 \leq 10$ (a) $(0, 0), (20, 0), (10, 50), (0, 60)$ (b) ~~$(0, 0), (60, 0), (10, 50), (0, 60)$~~ (c) ~~$(0, 0), (20, 0), (0, 100), (10, 50)$~~ (d) None of these
$0 + 0 \geq 18$ Q) On solving the inequalities $6x + y \geq 18$, $x + 4y \geq 12$, $2x + y \geq 10$, we get (a) $(0, 18), (12, 0), (4, 2), (7, 6)$ $0 + 3 \geq 18$ (b) ~~$(3, 0), (0, 3)$~~, $(4, 2), (7, 6)$ (c) ~~$(5, 0), (0, 10), (4, 2), (7, 6)$~~ (d) ~~$(0, 18), (12, 0), (4, 2), (0, 0), (7, 6)$~~
Q) On solving the inequalities $2x + 5y \leq 20$, $3x + 2y \leq 12$, $x \geq 0$, $y \geq 0$, we get the following situation: (a) $(0, 0), (0, 4), (4, 0)$ and $(20/11, 36/11)$ $= \frac{2 \times 20}{11} + \frac{5 \times 36}{11}$ (b) ~~$(0, 0), (10, 0), (0, 6)$ and $(20/11, 36/11)$~~ $= \frac{40}{11} + \frac{180}{11}$ (c) $(0, 0), (0, 4), (4, 0)$ and $(2, 3)$ (d) ~~$(0, 0), (10, 0), (0, 6)$ and $(2, 3)$~~ $= \frac{220}{11}$

$= 20$

Chapter 5: Basic Concepts of Permutations and Combinations

Chapter 5 - Permutations and Combinations

Key Definitions:

1. **Permutation:** Arrangement of items.
2. **Combination:** Selection of items.

Factorial of n (n!):

- **Definition:** Product of the first *n* natural numbers.
- **Formula:**

$$n! = n \times (n-1) \times (n-2) \cdots \times 1$$

1! = 1
2! = 2×1
3! = 3×2×1
4! = 4×3×2×1
0! = 1

Values of Factorials:

- 1! = 1
- 2! = 2
- 3! = 6
- 4! = 24
- 5! = 120
- 6! = 720
- 7! = 5040
- 8! = 40,320

Q) The value of N in $\frac{1}{7!} + \frac{1}{8!} = \frac{N}{9!}$ is:

(a) 81
(b) 78
(c) 89
(d) 64

$\frac{1}{5040} + \frac{1}{40320} = \frac{N}{9 \times 40320}$

N = 80.999

Permutation Formula:

- Arrangement of n items, taken r at a time:

$$\boxed{{}^nP_r} = \frac{n!}{(n-r)!}$$

$${}^5P_2 = \frac{5!}{3!} = 5\times4$$

$${}^5P_5 = 5\times4\times3\times2\times1$$

$${}^5P_3 = \frac{5!}{2!} = 5\times4\times3$$

Properties of Permutations:

1. Full arrangement (all items):

$$\boxed{{}^nP_n = n!}$$

2. Zero selection:

$$\boxed{{}^nP_0 = 1}$$

$${}^{100}P_2 = \frac{100!}{98!} = \frac{\cancel{98!}\times99\times100}{\cancel{98!}} = 100\times99$$

Q) Find 'n' if ${}^nP_2 = 72$:

(a) 12

(b) 36

(c) 24

(d) 9

Option Hit

use (d)

$${}^9P_2 = 9\times8 = 72 = RHS$$

Q) If ${}^nP_{13} : {}^{n+1}P_{12} = 3 : 4$, then the value of 'n' will be:

(a) 13

(b) 15

(c) 18

(d) 31

$$\frac{{}^nP_{13}}{{}^{n+1}P_{12}} = 0.75$$

Option Hit using (b)

$$\frac{{}^{15}P_{13}}{{}^{16}P_{12}} = \frac{\cancel{15}\times\cancel{14}\times\cancel{13}\times\cancel{12}\times\cancel{11}\times\cancel{10}\times\cancel{9}\times\cancel{8}\times\cancel{7}\times\cancel{6}\times\cancel{5}\times4_{\times3}}{16\times\cancel{15}\times\cancel{14}\times\cancel{13}\times\cancel{12}\times\cancel{11}\times\cancel{10}\times\cancel{9}\times\cancel{8}\times\cancel{7}\times\cancel{6}\times\cancel{5}}$$

$$= \frac{12}{16} = 0.75$$

Linear Permutation

NITHIN

$\frac{6!}{2!2!} = \frac{720}{2\times2} = 180$

Type – 1: Permutation of Letters Without Condition

This refers to the arrangement of all letters in a given word, considering possible repetitions of letters.

Formula for Permutation with Repeated Letters

If a word contains repeated letters, the total number of unique arrangements is calculated using the formula:

$$\frac{n!}{n_1! \times n_2! \times n_3! \times \ldots}$$

Where:

- $n!$ = Factorial of the total number of letters.
- $n_1, n_2, n_3, \ldots$ = The number of times each letter is repeated.

Special Case: No Repeated Letters

- If no letter is repeated, the number of unique arrangements simplifies to:

$$n!$$

ISHA 4!

- This means all letters can be freely rearranged without any repetition constraints.

Q) The number of words which can be formed by the letters of the word 'ALLAHABAD' is:

(a) 7560
(b) 3780
(c) 30240
(d) 15120

$\frac{9!}{2!4!} = \frac{9\times40320}{2\times24}$

Q) The number of ways in which 4 persons can occupy $\boxed{\text{9 vacant}}$ seats is:

(a) 6048
(b) 3024
(c) 1512
(d) 4536

${}^9P_4 = 9\times8\times7\times6 = 3024$

Q) The number of numbers between 1,000 and 10,000, which can be formed by the digits 1, 2, 3, 4, 5, 6 without repetition is:

(a) 720

(b) 180

(c) 360

(d) 540

6 × 5 × 4 × 3

360

Type 2: Permutation with Conditions

This type of permutation involves additional conditions related to the arrangement of letters or digits.

Key Points:

- The **condition must be satisfied first** before applying permutation rules.
- The word **"or"** in a problem translates to **addition (+)** in calculations.
- The word **"and"** in a problem translates to **multiplication (×)** in calculations.

Q) If a man travels in a train from place A to B in 10 ways, then by how many ways can he come back by another train?

(a) 94

(b) 110

(c) 90

(d) 99

10 × 9

= 90

Q) How many 3-digit odd numbers can be formed using the digits 5, 6, 7, 8, 9, if the digits can be repeated?

(a) 55

(b) 75

(c) 65

(d) 85

5 × 5 × 3

= 75

If Even

5 × 5 × 2

= 50

EA 1

Q) The number of different ways the letters of the word "DETAIL" can be arranged in such a way that the vowels can occupy only the odd positions is:

(a) 32
(b) 36
(c) 48
(d) 60

$3 \times 3 \times 2 \times 2 \times 1 \times 1$

Q) The number of words that can be formed using the letters of the word "PETROL" such that the words do not have "P" in the first position is:

(a) 720
(b) 120
(c) 600
(d) 540

$5 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 600$

Type 3: Permutations with Specific Pairing Conditions

1. **When two specific items must always stay together:**
   - Treat the two items as a single unit (block).
   - The total number of arrangements is:

$$\boxed{(n-1)! \times 2!}$$

$(n-1)! \times 2!$

- $(n-1)!$: Arrange the block and the remaining items.
- $2!$: Arrange the two items within the block.

Note: If three specific items must always stay together, treat them as one block, and the formula becomes:

$$(n-2)! \times 3!$$

$3! \times (n-2)!$

Never together: Total arrangements − Always together arrangements

$4! \times (n-3)!$

$5! \times (n-4)!$

$100! \times (n-99)!$

$n=10$

Q) The number of ways of arranging 6 boys and 4 girls in a row so that all 4 girls are together is:

(a) $6! \cdot 4!$
(b) $2 \cdot (7! \cdot 4!)$
(c) $7! \cdot 4!$
(d) $2 \cdot (6! \cdot 4!)$

$4! \times (10-3)!$

$4! \times 7!$

Q) How many permutations can be formed from the letters of the word "DRAUGHT", if both vowels may not be separated?

(a) 720
(b) 1,440
(c) 140
(d) 1,000

$2! \times (7-1)!$

$2! \times 6! = 2 \times 720$

$= 1440$

Q) The number of words from the letters of the word "BHARAT", in which B and H will never come together, is:

(a) 360
(b) 240
(c) 120
(d) None of the above

Total arrangement − Arrangem in which B and H comes toget

$\frac{6!}{2!} - 2! \times 5!$

$= 360 - 240 = 120$

- **Shortcut for boys and girls occupying alternate positions (unequal in number):**
  Total arrangement = (smaller number)! × (larger number)!
- **Shortcut for boys and girls occupying alternate positions (equal in number):**
  Total arrangement = 2 × boys! × girls!
- **Shortcut for no two boys being together:**
  Total arrangement = girls! × ${}^{(girls+1)}P_{boys}$
- **Shortcut for no two boys being together:**
  Total arrangement = girls! × ${}^{(girls+1)}P_{boys}$
- **Shortcut for no two girls being together:**
  Total arrangement = boys! × ${}^{(boys+1)}P_{girls}$

Q) The number of ways 4 boys and 3 girls can be seated in a row so that they are alternate is:

(a) 12

(b) 288

(c) 144

(d) 256

$4! \times 3! = 24 \times 6$
$= 144$

Q) Six boys and five girls are to be seated for a photograph in a row such that no two girls sit together and no two boys sit together. Find the number of ways in which this can be done.

(a) 74,200

(b) 96,900

(c) 45,990

(d) 86,400

$6! \times 5! = 720 \times 120$
$= 86400$

Type 4: Different Ways to Answer Questions

This type of problem deals with counting the number of ways to respond to multiple-choice questions, including the possibility of leaving them unanswered.

1. **Total number of ways to answer 'n' questions:**
   - Each question has two choices: answer it or leave it unanswered.
   - Formula:

$$2^n$$

   where n is the number of questions.

2. **Ways to answer at least one question out of 'n' questions:**
   - This is the total ways to answer all questions minus the case where no question is answered.
   - Formula:

$$\boxed{2^n - 1}$$

Q) There are 12 questions to be answered in Yes or No. How many ways can these be answered?

(a) 1024

(b) 2048

(c) 4096

(d) None

2^{12} $2\times$ = 11 times

Q) An examination paper with 10 questions consists of 6 questions in mathematics and 4 questions in the statistics part. At least one question from each part is to be attempted. In how many ways can this be done?

(a) 1024

(b) 945

(c) 1005

(d) 1022

$(2^6-1)\times(2^4-1)$

$63\times15 = 945$

Q) There are 5 questions, each having four options. Then, in how many different ways can we answer the questions?

(a) 20

(b) 120

(c) 1024

(d) 60

4^5

Q.) 100

A B C D Not ans.

$\boxed{5^{100}}$

Circular Permutation (Without Condition)

1. **Linear Arrangement:**
   - When arranging n items in a straight line (row), the number of arrangements is:

$$n!$$

2. **Circular Arrangement:**
   - In a circle, the starting point is fixed because rotating the arrangement does not create a new order.
   - Therefore, the number of arrangements is reduced by 1 factorial (to account for the fixed starting point):

$$\boxed{(n-1)!}$$

Key Idea:

- **Linear Arrangement** counts all possible orders.
- **Circular Arrangement** avoids overcounting identical rotations by fixing one item as the reference point.

Circular Permutation with Condition

In circular permutations, objects are arranged in a circle rather than in a linear order. The number of ways to arrange **n** objects in a circle depends on specific conditions.

1. **Shortcut for no two girls being together in a circular manner**
   Total arrangement = $(\text{boys} - 1)! \times {}^{\text{boys}}P_{\text{girls}}$
2. **Shortcut for no two boys being together**
   Total arrangement = $(\text{girls} - 1)! \times {}^{\text{girls}}P_{\text{boys}}$

Q) The number of ways 5 boys and 5 girls can be seated at a round table, so no two boys are adjacent, is:

(a) 2,550
(b) 2,880
(c) 625
(d) 2,476

(Girls − 1)! Girls P Boys

4! × ${}^5P_5$

4! × 5!
24 × 120 = 2880

Q) In how many ways can 6 women and 6 men be seated at a circular table so that no two women are adjacent?

A) 86400
B) 72000
C) 43200
D) 94600

5! × 6!

= 120 × 720

EXTRA

1. Number of Ways of Arranging 'r' Things Taken from 'n' Things, Such That One Particular Thing Is Always Included

- When one specific item is always included, it occupies one position. The remaining $r-1$ items are arranged from the $n-1$ remaining items.

Formula:

$$r \cdot {}_{(n-1)}P_{(r-1)}$$

$\boxed{r \times {}^{n-1}P_{r-1}}$

- r: The position occupied by the specific item.
- ${}_{(n-1)}P_{(r-1)}$: Permutations of $r-1$ items from $n-1$ items.

2. Number of Ways of Arranging 'r' Things Taken from 'n' Things, Such That One Particular Thing Is Always Excluded

- When one specific item is excluded, r items are arranged from the remaining $n-1$ items.

Formula:

$$\boxed{{}_{(n-1)}P_r}$$

${}^{n-1}P_r$

Q) **Find the number of arrangements of 5 things taken out of 12 things, in which one particular thing must always be included.**

$5 \times {}^{11}P_4 = 5 \times 11 \times 10 \times 9 \times 8$

(a) 39,000

(b) 37,600

(c) 39,600

(d) 36,000

Q) Find the number of arrangements of 4 things taken out of 10 things such that 1 particular thing is always excluded.

${}^{9}P_4 = 9 \times 8 \times 7 \times 6$

A) 3024

B) 4032

C) 5040

D) 2520

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Combination (Selection of 'r' Things from 'n' Things)

A combination is used when the order of selection does **not** matter. It gives the number of ways to choose r items from n items.

Formula:

$$^nC_r = \frac{^nP_r}{r!}$$

$$^nC_r = \frac{^nP_r}{r!} = \frac{n!}{(n-r)!\,r!}$$

Where:

- $^nP_r = \frac{n!}{(n-r)!}$: The number of permutations of r items from n items.
- $r!$: Accounts for the fact that the order of selection doesn't matter.

If:

$$\boxed{^nC_r = ^nC_s}$$

Then:

$$r + s = n$$

$$^nC_0 = \frac{n!}{\cancel{n!} \times 0!} = 1$$

Basic Properties of Combination:

1. $^nC_0 = 1$: Selecting 0 items from n items is always 1.
2. $^nC_1 = n$: Selecting 1 item from n items can be done in n ways.
3. $^nC_n = 1$: Selecting all n items from n is always 1.
4. $^nC_r = ^nC_{n-r}$: Symmetry property—selecting r items is the same as excluding r items.
5. $^nC_r + ^nC_{r-1} = ^{n+1}C_r$: Pascal's identity.

$^nC_r = ^nC_{n-r}$

$^5C_3 = ^5C_2$

$^7C_5 = ^7C_2$

Q) If $^{18}C_r = ^{18}C_{r+2}$, find the value of $^{18}C_5$:

(a) 55

(b) 50

(c) 56

(d) None of these

$$^{18}C_5 = \frac{^{18}P_5}{5!} = \frac{18 \times 17 \times 16 \times 15 \times 14}{120} = 8568$$

Q) $^{15}C_3 + ^{15}C_{13}$ is equal to:

(a) $^{16}C_3$

(b) $^{30}C_{16}$

(c) $^{15}C_6$

(d) $^{15}C_{15}$

$^{15}C_3 + ^{15}C_2 = {}^{15+1}C_3$

$= {}^{16}C_3$

Q) A fruity basket contains 7 apples, 6 bananas, and 4 mangoes. How many selections of 3 fruits can be made so that all 3 are apples?

(a) 35 ways

(b) 120 ways

(c) 165 ways

(d) 70 ways

$^{7}C_3 = \frac{7 \times 6 \times 5}{3!}$

$= 35$

Q) A business house wishes to simultaneously elevate two of its six branch heads. In how many ways can these elevations take place?

(a) 12

(b) 3

(c) 6

(d) 15

$^{6}C_2 = \frac{6 \times 5}{2!}$

$= 15$

Applications in Geometry and Other Scenarios:

1. Number of straight lines formed using n points, where m are collinear:

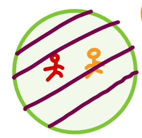
$$\boxed{{}^nC_2 - {}^mC_2 + 1}$$

2. Number of triangles formed using n points, where m are collinear:

$$ {}^nC_3 - {}^mC_3 $$

3. Number of matches played by n teams (one match per pair):

$$ {}^nC_2 $$



$^nC_2$

- If each team plays two matches against every other team:

$$\boxed{2 \cdot {}^nC_2}$$

4. Number of handshakes among n people:

$$ {}^nC_2 $$

5. Number of quadrilaterals (or parallelograms) formed by n parallel lines intersecting m parallel lines:

$^2C_2 \times {}^2C_2 = 1 \times 1 = 1$

$$ {}^nC_2 \times {}^mC_2 $$

6. Number of diagonals in a polygon with n sides:

$$\boxed{{}^nC_2 - n}$$

7. Number of chords in a circle formed by n points on its circumference:

$$ {}^nC_2 $$

Q) Six points are on a circle. The number of quadrilaterals that can be formed are:

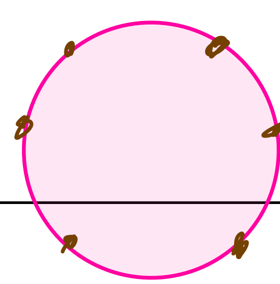
(a) 30

(b) 360

(c) 15

(d) None of the above

$^6C_4 = {}^6C_2$



$\frac{6 \times 5}{2}$

$= 15$

Q) A polygon has 44 diagonals. Then the number of its sides are:

(a) 8
(b) 9
(c) 10
(d) 11

$^nC_2 - n = 44$

option Hit using d

$^{11}C_2 - 11 = \frac{11 \times 10}{2} - 11$
$= 55 - 11$
$= 44$

Q) How many parallelograms can be formed from a set of 6 parallel lines intersecting another set of 4 parallel lines?

(a) 80
(b) 70
(c) 90
(d) 100

$^6C_2 \times {^4C_2} = 15 \times 6$
$= 90$

Q) Number of ways of shaking hands in a group of 10 persons, shaking hands with each other, are:

(a) 45
(b) 54
(c) 90
(d) 10

$^{10}C_2 = \frac{10 \times 9}{2} = 45$

Problems With Conditions

Q) How many ways a team of 11 players can be made out of 15 players if one particular player is not to be selected in the team?

(a) 364
(b) 728
(c) 1,001
(d) 1,234

$^{14}C_{11} = {^{14}C_3}$

$= \frac{14 \times 13 \times 12}{6}$

$= 364$

Q) Out of 4 gents and 6 ladies, a committee is to be formed. Find the number of ways the committee can be formed such that it comprises at least 2 gents, and the number of ladies should at least be double the number of gents.

(a) 94
(b) 132
(c) 136
(d) 104

2M and 4L or 2M and 5L or 2M and 6L or 3M and 6L

$^4C_2 \times {}^6C_4 + {}^4C_2 \times {}^6C_5 + {}^4C_2 \times {}^6C_6 + {}^4C_3 \times {}^6C_6$

$6 \times 15 + 6 \times 6 + 6 \times 1 + 4$

$= 136$

Q) In how many ways can a selection of 6 out of 4 teachers and 8 students be done so as to include at least two teachers?

(a) 220
(b) 672
(c) 596
(d) 968

Case I: 2T and 4S or Case II: 3T and 3S or Case III: 4T and 2S

$^4C_2 \times {}^8C_4 + {}^4C_3 \times {}^8C_3 + {}^4C_4 \times {}^8C_2$

$6 \times \frac{8 \times 7 \times 6 \times 5}{24} + 4 \times \frac{8 \times 7 \times 6}{6} + 1 \times \frac{8 \times 7}{2} = 672$

Q) From a group of 8 men and 4 women, 4 persons are to be selected to form a committee so that at least 2 women are there on the committee. In how many ways can it be done?

(a) 168
(b) 201
(c) 202
(d) 220

2W and 2M or 3W and 1M or 4W and 0M

$^4C_2 \times {}^8C_2 + {}^4C_3 \times {}^8C_1 + {}^4C_4 \times {}^8C_0$

$6 \times 28 + 4 \times 8 + 1 \times 1$

$= 201$

Q) Eight chairs are numbered from 1 to 8. Two women and three men are to be seated by allowing one chair for each. First, the women choose the chairs from the chairs numbered 1 to 4, and then men select the chairs from the remaining. The number of possible arrangements is:

(a) 120
(b) 288
(c) 32
(d) 1440

Two women are selecting chair

$^4C_2$ and $2!$ $= {}^4C_2 \times 2$
$= 6 \times 2 = 12$

Men selecting chair $^6C_3 \times 3! = \frac{6 \times 5 \times 4}{6} \times 6 = 120$

$12 \times 120 = 1440$

Q) In the next World Cup, there will be 12 teams, divided equally into two groups. Teams of each group will play matches against one another. From each group, 3 top teams will qualify for the next round. In this round, each team will play against each other. Four top teams of this round will qualify for the semi-final round, where each team will play against the others once. Two top teams of this round will go to the final round, where they will play the best of three matches. The minimum number of matches in the next World Cup will be:

(a) 56
(b) 53
(c) 37
(d) 43

Round 1 $^6C_2 + {}^6C_2 = 30$
Round 2 $^6C_2 = 15$
Semifinal $^4C_2 = 6$
Final $= 2$
$30 + 15 + 6 + 2 = 53$

Chapter 7: Sets,Relations,functions,limits & continuity

Sets

- **Definition**: A set is a collection of well-defined elements.

$A = \{a, b, c, d \ldots x, y, z\}$
$B = \{a, e, i, o, u\}$

Forms of Sets

1. Roster Form:
   - Elements are listed explicitly within curly brackets {}.
   - Example: $A = \{a, e, i, o, u\}$

$C = \{b, c, d \ldots x, y, z\}$

2. Set Builder Form:
   - Describes the set using a property or rule that its elements satisfy.
   - Example: $A = \{x : x \text{ is a vowel}\}$

$A = \{x : x \text{ is a alphabet}\}$
$A = \{a, b, c, d, \ldots x, y, z\}$
$n(A) = 26$

Cardinal Number of a Set

- **Definition**: The number of elements in a set.
- **Notation**: If A is a set, $n(A)$ denotes its cardinal number.

Null Set

- **Definition**: An empty set is called a null set. It contains no elements.
- **Notation**:
  $A = \{\}$ or $A = \phi$
- **Cardinal Number**: The cardinal number of a null set is zero.

Complement of a Set

- **Definition**: If A is a set, then A^1 represents the complement of A.
- **Formula**:
  $A^1 = U - A$ (Elements in the universal set U but not in A)
- **Example**:
  $U = \{1, 2, 3, 4, 5, 6\}, A = \{2, 4, 6\}$
  $A^1 = U - A = \{1, 3, 5\}$

Types of Sets

Equal Sets

- **Definition**: Two sets A and B are said to be equal if they contain the same and equal number of elements.
- **Example**:
  $A = \{a, b, c\}$
  $B = \{a, b, c\}$

Equivalent Sets

- **Definition**: Two sets A and B are equivalent if they have the same number of elements.
  $n(A) = n(B)$
- **Example**:
  $A = \{1, 2, 3\}$
  $B = \{x, y, z\}$

Note: All equal sets are equivalent, but not all equivalent sets are equal.

Q) The set of cubes of natural numbers is:

(a) Null set

(b) A finite set

(c) An infinite set ✓

(d) Singleton set

Operations on Two Sets

1. Union Operation

- **Definition**: The union of two sets A and B is the set of all elements that belong to A, B, or both.
- **Notation**: $A \cup B$

$A \cup B = \{2, 4, 6, 8, 10, 12\}$

- **Example**:
  $A = \{2, 4, 6, 8\}$, $B = \{6, 8, 10, 12\}$
  $A \cup B = \{2, 4, 6, 8, 10, 12\}$

2. Intersection Operation

- **Definition**: The intersection of two sets A and B is the set of all elements that are common to both A and B.
- **Notation**: $A \cap B$
- **Example**:
  $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$
  $\boxed{A \cap B} = \{3, 4\}$

$A = \{1, 2, 3, 4\}$
$B = \{2, 3, 5, 6\}$
$A - B = \{1, 4\}$
$B - A = \{5, 6\}$

3. Difference of Sets (A - B)

- **Definition**: The difference $A - B$ is the set of elements that belong to A but not to B. Similarly, $B - A$ is the set of elements in B but not in A.
- **Example**:
  $A = \{1, 3, 5, 7\}$, $B = \{3, 5, 8\}$
  - $A - B = \{1, 7\}$
  - $B - A = \{8\}$

At least / either / or $\Rightarrow \cup$

and $\Rightarrow \cap$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Formulas for Union of Sets

1. **For Any Two Sets A and B:**

   The number of elements in the union of A and B is given by:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$n(\text{only } A) = n(A) - n(A \cap B)$

$n(\text{only } B) = n(B) - n(A \cap B)$

   - **Example:**

     $A = \{1, 2, 3, 4\}, B = \{3, 4, 5, 6\}$

     $n(A) = 4, n(B) = 4, n(A \cap B) = 2$

     $n(A \cup B) = 4 + 4 - 2 = 6$

2. **For Any Three Sets $A, B,$ and C:**

   The number of elements in the union of $A, B,$ and C is given by:

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

   - **Example:**

     $A = \{1, 2, 3\}, B = \{2, 3, 4\}, C = \{3, 4, 5\}$

     $n(A) = 3, n(B) = 3, n(C) = 3$

     $n(A \cap B) = 2, n(B \cap C) = 2, n(A \cap C) = 1, n(A \cap B \cap C) = 1$

     $n(A \cup B \cup C) = 3 + 3 + 3 - 2 - 2 - 1 + 1 = 5$

$n(\text{only } A) = n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$

$n(\text{only } B) = n(B) - n(B \cap C) - n(A \cap B) + n(A \cap B \cap C)$

For Any Two Sets A and B:

1. At least one of A or B

$$\boxed{n(A \cup B)} = n(A) + n(B) - n(A \cap B)$$

2. Only A

$$n(A - B) = n(A) - n(A \cap B)$$

3. Only B

$$n(B - A) = n(B) - n(A \cap B)$$

4. Exactly one of A or B

$$\boxed{n(A - B) + n(B - A)} = n(A) + n(B) - 2n(A \cap B)$$

5. Neither A nor B

$$= n(U) - n(A \cup B)$$

A - B   B-A

For Any Three Sets A, B, C:

1. At least one of A, B, C

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

2. Only A

$$n(A - (B \cup C)) = n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$$

3. Only B

$$n(B - (A \cup C)) = n(B) - n(A \cap B) - n(B \cap C) + n(A \cap B \cap C)$$

4. Only C

$$n(C - (A \cup B)) = n(C) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

5. Exactly one of A, B, C

$$n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(B \cap C) - 2n(A \cap C) + 3n(A \cap B \cap C)$$

6. Neither A, B, nor C

$$= n(U) - n(A \cup B \cup C)$$

De Morgan's Laws

1. Complement of Union:

$(A \cup B)' = A' \cap B'$

$$(A \cup B)^c = A^c \cap B^c$$

$(A \cap B)' = A' \cup B'$

- Example:
  $A = \{1, 2\}, B = \{2, 3\}, U = \{1, 2, 3, 4\}$
  $A^c = \{3, 4\}, B^c = \{1, 4\}$
  $(A \cup B)^c = A^c \cap B^c = \{4\}$

2. Complement of Intersection:

$$(A \cap B)^c = A^c \cup B^c$$

- Example:
  $A = \{1, 2\}, B = \{2, 3\}, U = \{1, 2, 3, 4\}$
  $A^c = \{3, 4\}, B^c = \{1, 4\}$
  $(A \cap B)^c = A^c \cup B^c = \{1, 3, 4\}$

Additional Notes

$(A \cup B)' = U - A \cup B$

- **Complement of Union:**

$$(A \cup B)^c = U - (A \cup B)$$

- **Formula for Complement of Union Count:**

$$n((A \cup B)^c) = n(U) - n(A \cup B)$$

- **Usage:** To calculate elements that belong to neither A nor B.

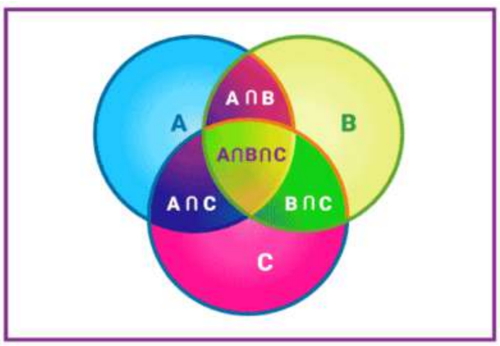
Problems Using Venn Diagram

Understanding Venn Diagrams in Problem Solving

- A **Venn diagram** is a visual tool used to represent relationships between different sets or groups.
- It is particularly useful when solving problems that involve counting elements in overlapping or distinct categories.

When to Use Venn Diagrams?

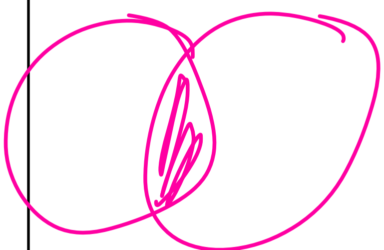
- Venn diagrams are **most effective** when it is required to find the number of elements related to **only one specific event** or **multiple overlapping events.**



Q) There are 40 students, 30 of them passed in English, 25 of them passed in Maths, and 15 of them passed in both. Assuming that every student has passed at least in one subject, how many students passed in English only but not in Maths?

(a) 15
(b) 20
(c) 10
(d) 25

$$n(E\,only) = n(E) - n(E \cap M)$$



$$= 30 - 15$$

$$= 15$$

Q) Of the 200 candidates who were interviewed for a position at a call center, 100 had a two-wheeler, 70 had a credit card, and 140 had a mobile phone.

- 40 of them had both a two-wheeler and a credit card.
- 30 had both a credit card and a mobile phone.
- 60 had both a two-wheeler and a mobile phone.
- 10 had all three.

How many candidates had none of the three?

(a) 0
(b) 20
(c) 10
(d) 18

$$\text{None of these} = n(\text{Total}) - n(A \cup B \cup C)$$

$$= 200 - [n(A)+n(B)+n(C) - n(A\cap B) - n(A \cap C) - n(B \cap C) + n(A\cap B \cap C)]$$

$$= 200 - (100 + 70 + 140 - 40 - 30 - 60 + 10)$$

$$= 200 - 190$$

$$= 10$$

Subset

$A = \{1, 2, 3, 4\}$

$B = \{1, 2\}$

- **Definition**:
  A set A is said to be a subset of set B if all elements of A are also elements of B.
  Denoted as $A \subseteq B$.

$A = \{1, 2\}$

$B = \{1, 2, 3\}$

- **Example**:
  $A = \{x, y\}, B = \{w, x, y, z\}$
  Here, $A \subseteq B$, as all elements of A are present in B.
- **Note**:
  - Any set is a subset of itself.
  - The empty set ϕ is a subset of every set.

1. **Number of Subsets**:
   - For a set with n elements, the number of subsets is:

$$2^n$$

2. **Number of Proper Subsets**:
   - For a set with n elements, the number of proper subsets is:

$$2^n - 1$$

3. **Power Set**:
   - **Definition**: The power set of a set A is the set of all subsets of A, including the empty set and A itself.

ϕ

   - **Example**:
     For $A = \{x, y\}$,
     Power set of $A = \{\{\}, \{x\}, \{y\}, \{x, y\}\}$

Q) The number of proper subsets of the set $\{3, 4, 5, 6, 7\}$ is:

(a) 32
(b) 31
(c) 30
(d) 25

$2^n - 1$

$2^5 - 1 = 31$

Q) The number of subsets of the set formed by the unique letters in the word "Allahabad" is:

(a) 128

(b) 16

(c) 32

(d) 64

A L H B D

$2^n = 2^5 = 32$

Q) If $A = \{1, 2, 3, 4, 5, 7, 8, 9\}$ and $B = \{2, 4, 6, 7, 9\}$, then how many $\boxed{\text{proper}}$ subsets of $A \cap B$ can be created?

$n = 4$

(a) 16

(b) 15

(c) 32

(d) 31

$2^n - 1$

$= 2^4 - 1$

$= 15$

Q) $A \cap E'$ is equal to (E is a superset of A)

a) E

b) φ

c) A

d) none

$U = \{1, 2, 3, 4\}$

$E = \{1, 2, 3\}$

$A = \{1, 2\}$

$E' = \{4\}$

Q) $A \cap \varphi =$

a) A

b) φ

c) E

d) none

$A = \{1, 2, 3\}$

$\phi = \{\}$

$A \cap \phi = \{\}$

Q) $A \cup A'$ is equal to (E is a superset of A)

a) A

b) E

c) φ

d) none

$U = E = \{1, 2, 3, 4\}$

$A = \{1, 2\}$

$A' = \{3, 4\}$

$A \cup A' = \{1, 2, 3, 4\}$

Q) I is a set of Isosceles triangles and E is the set of Equilateral triangles. Then

a) $I \subset N$

b) $E \subset I$

c) $E = I$

d) none

Equilat → All side are equal

Isosceles → 2 sides

$I = \{I_1, I_2, I_3, I_4, E_1, E_2, E_3 \ldots\}$

$E \subset I$

$E = \{E_1, E_2, E_3, E_4, -$

Cartesian Product of Sets

- **Definition:**
  The Cartesian product of two sets A and B is the set of all possible ordered pairs, where the first element is from A and the second element is from B.
  Denoted as $A \times B$.

A

$B \times A = \{(1,x)\ (1,y)\ (2,x)\ (2,y)\}$

Example 1:
If $A = \{x, y\}$ and $B = \{1, 2\}$,

$$\boxed{A \times B} = \{(x, 1), (x, 2), (y, 1), (y, 2)\}$$

Example 2:
If $A = \{a, b, c\}$ and $B = \{p, q\}$,

$$A \times B = \{(a, p), (a, q), (b, p), (b, q), (c, p), (c, q)\}$$

$$B \times A = \{(p, a), (p, b), (p, c), (q, a), (q, b), (q, c)\}$$

Note:

$4 = 2 \times 2$

- **The cardinal number of $\boxed{A \times B}$ is given by:**

$$n(A \times B) = \boxed{n(A)} \times \boxed{n(B)}$$

Q) If $A = \{1, 2, 3, 4, 5\}$, $B = \{2, 4\}$, and $C = \{1, 3, 5\}$, then $(A - C) \times B$ is:

~~(a) $\{(2, 2), (2, 4), (4, 2), (4, 4), (5, 2), (5, 4)\}$~~
~~(b) $\{(1, 2), (1, 4), (3, 2), (3, 4), (5, 2), (5, 4)\}$~~
(c) $\{(2, 2), (4, 2), (4, 4), (4, 5)\}$
(d) $\{(2, 2), (2, 4), (4, 2), (4, 4)\}$

$\{2,4\} \times \{2,4\}$

$(2,2)(2,4)(4,2),(4,4)$

Q) If $A = \{2, 3\}$, $B = \{4, 5\}$, $C = \{5, 6\}$, then $A \times (B \cap C) = \_\_\_$

(a) $\{(5, 2), (5, 3)\}$
(b) $\{(2, 5), (3, 5)\}$
(c) $\{(2, 4), (3, 5)\}$
(d) $\{(3, 5), (2, 6)\}$

$\{2, 3\} \times \{5\}$
$\{(2, 5)\ (3, 5)\}$

Q) If $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $B = \{2, 4, 6, 8\}$, the cardinal number of $A - B$ is:

(a) 4
(b) 3
(c) 9
(d) 7

Relations

- **Definition:**
  A relation is a subset of the Cartesian product of two sets A and B. It consists of ordered pairs (a, b), where $a \in A$ and $b \in B$.

Example 1:
If $A = \{1, 2\}$ and $B = \{3, 4\}$,
Cartesian Product:
$\boxed{A \times B} = \{(1, 3), (1, 4), (2, 3), (2, 4)\}$ 3
Relation $R = \{(1, 3), (2, 4)\}$

Example 2:

If $A = \{x, y\}$ and $B = \{5, 6, 7\}$,

Cartesian Product:

$A \times B = \{(x,5),(x,6),(x,7),(y,5),(y,6),(y,7)\}$

Relation $R = \{(x,5),(y,7)\}$

The number of relations from A to B is given by:

$$2^{n(A)\times n(B)}$$

$2^{n(A)\times n(B)}$

Q) If $A = \{1, 2\}$ and $B = \{3, 4\}$, determine the number of relations from A to B:

(a) 3

(b) 16

(c) 5

(d) 6

$2^{2\times 2} = 2^4$

$= 16$

Types of Relations

$A = \{1, 2\}$

$R = \{(1,1)(2,2)\}$

$R = \{(1,2)\}$

1. Reflexive Relation:
   - **Definition**: A relation R on a set A is reflexive if every element $a \in A$ satisfies $(a, a) \in R$.
2. Symmetric Relation:
   - **Definition**: A relation R on a set A is symmetric if, for any $(a, b) \in R$, it implies $(b, a) \in R$.

(a, b) then (b, a)

$(1,2)(2,1)$

3. Transitive Relation:
   - **Definition**: A relation R on a set A is transitive if, for any $(a, b) \in R$ and $(b, c) \in R$, it implies $(a, c) \in R$.

$(1,2)(2,1)$ $(1,1)$

$(2,3)$ $(3,4)$ $(2,4)$

Additional Types of Relations

1. Equivalence Relation

- **Definition:**
  A relation R on a set A is called an equivalence relation if it satisfies the following three properties:
  1. **Reflexive:** $(a, a) \in R$ for all $a \in A$.
  2. **Symmetric:** If $(a, b) \in R$, then $(b, a) \in R$.
  3. **Transitive:** If $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$.
- **Example:**
  Let $A = \{1, 2, 3\}$, and define $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$.
  Here:
  - R is **reflexive** because $(1, 1), (2, 2), (3, 3) \in R$.
  - R is **symmetric** because $(1, 2) \in R$ implies $(2, 1) \in R$.
  - R is **transitive** because $(1, 2) \in R$ and $(2, 1) \in R$ imply $(1, 1) \in R$.
    Therefore, R is an equivalence relation.

2. Partial Order Relation

- **Definition:**
  A relation R on a set A is called a partial order relation if it satisfies the following three properties:
  1. **Reflexive:** $(a, a) \in R$ for all $a \in A$.
  2. **Antisymmetric:** If $(a, b) \in R$ and $(b, a) \in R$, then $a = b$.
  3. **Transitive:** If $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$.

Q) Let $A = \{1, 2, 3\}$, then the relation $R = \{(1, 1), (2, 3), (2, 2), (3, 3), (1, 2)\}$ is:

~~(a) Symmetric~~

~~(b) Transitive~~

(c) Reflexive

(d) Equivalence

Q) If $S = \{1, 2, 3\}$, then the relation $\{(1, 1), (2, 2), (1, 2), (2, 1)\}$ is symmetric and:

(a) Reflexive but not transitive

(b) Reflexive as well as transitive

(c) Transitive but not reflexive

(d) Neither transitive nor reflexive

$(1,2)(2,1)\ (1,1)$

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Q) On the set of lines, being perpendicular is a ______ relation.

~~(a) Reflexive~~

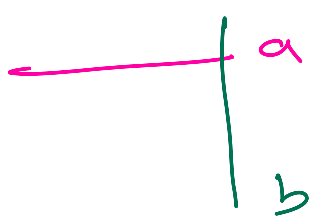
(b) Symmetric

(c) Transitive

(d) None of these

(a,a)

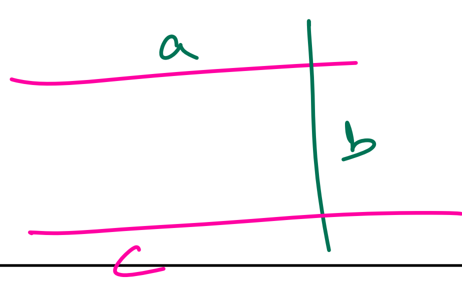
$a \perp a$



$a \perp b$

$b \perp a$

$(a,b)(b,a)$



Function

(a, b)

Domain Co-domain/Range/y-v.

- **Definition:**
  A relation R is said to be **a function** if:
  1. Each element of the domain is mapped to exactly one element in the range (i.e., no domain element is repeated).
  2. Every domain element must have an image in the range.

- **Example 1 (Not a Function):**
  $R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\}$
  Here, domain elements 1 and 2 are repeated, so R is **not a function.**

- **Example 2 (A Function):**
  $R = \{(1, 2), (2, 3), (3, 4), (4, 1)\}$
  Here, no domain element is repeated, so R is a **function.**

Past year

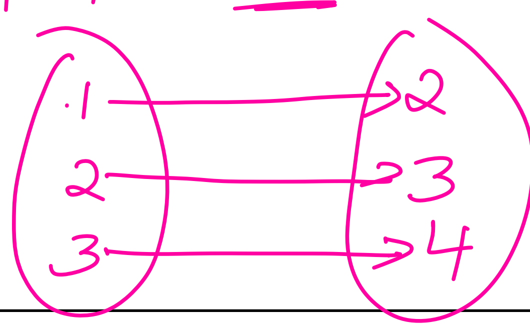
- **Key Notes:**
  1. All functions are relations, but not all relations are functions.
  2. The **first element** in each ordered pair is the **domain element.**
  3. The **second element** in each ordered pair is the **range element.**

Types of Functions

1. **One-to-One Function (Injective):**
   - **Definition:** A function is one-to-one if every element of the domain maps to a distinct element in the codomain.
   - **Example:** $f(x) = \boxed{x + 1}$ where $f : \{1, 2, 3\} \to \{2, 3, 4\}$.

y=

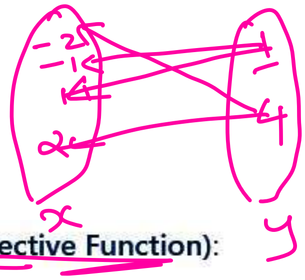
x+1



2. **Onto Function (Surjective):**

- **Definition:** A function is onto if every element of the codomain has at least one pre-image in the domain.
- **Note:** If a function is **not onto**, it is called an "into function."
- **Example:** $f(x) = x^2$, where $f : \{-2, -1, 1, 2\} \rightarrow \{1, 4\}$.

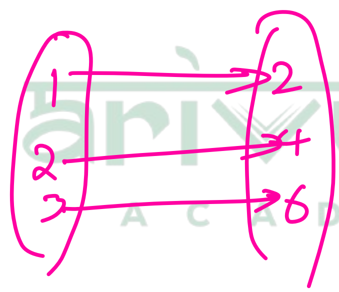
y=



into function

3. **One-to-One and Onto (Bijective Function):**

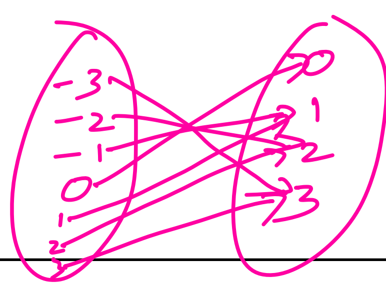
- **Definition:** A function is bijective if it is both one-to-one and onto.
- **Example:** $f(x) = 2x$ where $f : \{1, 2, 3\} \rightarrow \{2, 4, 6\}$.



4. **Many-to-One Function:**

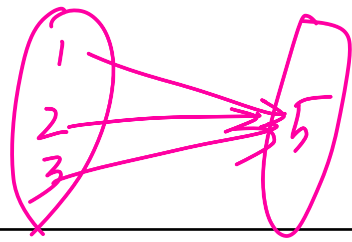
- **Definition:** A function is many-to-one if two or more elements in the domain map to the same element in the codomain.

$f(x) = |x|$, where $f : \{-3, -2, -1, 0, 1, 2, 3\} \rightarrow \{0, 1, 2, 3\}$.



5. **Constant Function:**

- **Definition:** A function is constant if all elements of the domain map to the same element in the codomain.
- **Example:** $f(x) = 5$, where $f : \{1, 2, 3\} \to \{5\}$.
- **Note:** All constant functions are many-to-one, but not all many-to-one functions are constant.

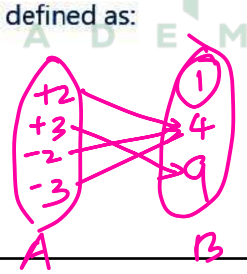


Q) Identify the function from the following:

(a) $\{(1, 1), (1, 2), (1, 3)\}$
(b) $\{(1, 1), (2, 1), (2, 3)\}$
(c) $\{(1, 2), (2, 2), (3, 2), (4, 2)\}$
(d) None of these

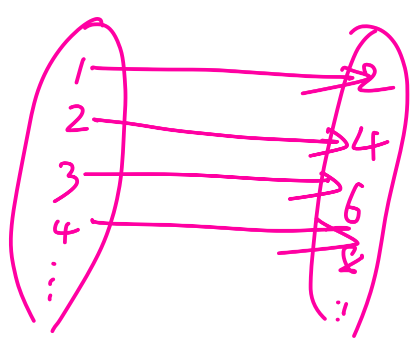
Q) If $A = \{\pm 2, \pm 3\}$, $B = \{1, 4, 9\}$, and $F = \{(2, 4), (-2, 4), (3, 9), (-3, 4)\}$, then F is defined as:

(a) One-to-one function from A into B.
(b) One-to-one function from A onto B.
(c) Many-to-one function from A onto B.
(d) Many-to-one function from A into B.



Q) Let $\mathbb{N}$ be the set of all natural numbers, and E be the set of all even natural numbers. Then the function $f : \mathbb{N} \to E$ defined as $f(x) = 2x$, $x \in \mathbb{N}$, is:

(a) One-one into
(b) One-one onto
(c) Many-one into
(d) Many-one onto



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Q) If $\mathbb{R}$ is the set of all real numbers, then the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 2^x$ is:

(a) One-one onto

(b) One-one into

(c) Many-one into

(d) Many-one onto

Given the function $f(x) = 2x + 3$, then the value of $f(2x) - 2f(x) + 3$ will be:

(a) 3

(b) 2

(c) 1

(d) 0

$f(2x) = 2(2x) + 3$
$= 4x + 3$

$4x + 3 - 2(2x+3) + 3$
$\cancel{4x} + \cancel{3} - \cancel{4x} - \cancel{6} + \cancel{3}$

Q) If $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x + 1$,
and $g : \mathbb{R} \to \mathbb{R}$, $g(x) = x^2 + 1$,
then $f \circ g(-2)$ equals to:

(a) 6

(b) 5

(c) -2

(d) None

$f(g(x)) = g(x) + 1$
$= x^2 + 1 + 1$
$f(g(-2)) = x^2 + 2$
$= (-2)^2 + 2$
$= 6$

Q) $f(x) = 3 + x$, for $-3 < x < 0$, and $\boxed{f(x) = 3 - 2x}$, for $0 < x < 3$.

Then the value of $f(2)$ will be:

(a) -1

(b) 1

(c) 3

(d) 5

$f(2) = 3 - 2 \times 2$

$= -1$

Q) If $f(x) = \frac{x}{x-1}$, then $\frac{f(x/y)}{f(y/x)} =$ ______

(a) x/y

(b) y/x

(c) $-x/y$

(d) $-y/x$

$\boxed{f(x/y)} = \frac{x/y}{\frac{x}{y} - 1} = \frac{\frac{x}{\cancel{y}}}{\frac{x-y}{\cancel{y}}}$

$= \frac{x}{x-y}$ —— (1)

$f(y/x) = \frac{\frac{y}{x}}{\frac{y}{x} - 1} = \frac{\frac{y}{\cancel{x}}}{\frac{y-x}{\cancel{x}}} = \frac{y}{y-x}$ —— (2)

eq 1 ÷ 2

$\frac{x}{x-y} \div \frac{y}{y-x}$

$\frac{x}{-\cancel{(y-x)}} \times \frac{\cancel{y-x}}{y}$

$= -\frac{x}{y}$

Inverse of a Function

Definition:

- If $f(x)$ is a function, then $f^{-1}(x)$ is called the **inverse** of $f(x)$.
- The inverse of a function is obtained by interchanging the domain and codomain elements in the ordered pairs.

Example:

Let $f(x) = \{(1,2),(2,3),(4,5),(3,6)\}$.

- To find $f^{-1}(x)$, interchange the elements in each pair:
  $f^{-1}(x) = \{(2,1),(3,2),(5,4),(6,3)\}$.

Finding $f^{-1}(x)$ When $f(x)$ is Given as an Expression:

To find $f^{-1}(x)$ when $f(x)$ is given as a mathematical expression:

1. **Step 1**: Assume $y = f(x)$.
2. **Step 2**: Solve for x in terms of y.
3. **Step 3**: Replace y with x to get $f^{-1}(x)$.

Example:

If $f(x) = \boxed{2x+3,}$

1. Let $y = 2x + 3$.
2. Solve for x: $x = \frac{y-3}{2}$.
3. Replace y with x: $f^{-1}(x) = \frac{x-3}{2}$.

(1) Assume $f(x) = y$

$y = 2x + 3$

$y - 3 = 2x$

$x = \frac{y-3}{2}$

Replace y with x and x with $f^{-1}(x)$

$\boxed{f^{-1}(x) = \frac{x-3}{2}}$

Q) If $f(x) = \frac{x+1}{x}$, find $f^{-1}(x)$:

(a) $\frac{1}{x-1}$

(b) $\frac{1}{y-1}$

(c) $\frac{1}{y} - 1$

(d) x

$y = \frac{x+1}{x}$

$xy = x+1$

$xy - x = 1$

$x(y-1) = 1$

$x = \frac{1}{y-1}$

$f^{-1}(x) = \frac{1}{x-1}$

$f(y) = \frac{y+1}{y}$. Find $f^{-1}(y)$

Let $f(y) = x$

$x = \frac{y+1}{y}$

$yx = y+1$

$yx - y = 1$

$y(x-1) = 1$

$y = \frac{1}{x-1}$

$f^{-1}(y) = \frac{1}{y-1}$

Q) If $u(x) = \frac{1}{1-x}$, then $u^{-1}(x)$ is:

(a) $\frac{1}{x-1}$

(b) $1 - x$

(c) $1 - \frac{1}{x}$

(d) $\frac{1}{x} - 1$

$y = \frac{1}{1-x}$

$y - yx = 1$

$y - 1 = yx$

$x = \frac{y-1}{y}$

$u^{-1}(x) = \frac{x-1}{x}$

$= \frac{x}{x} - \frac{1}{x}$

$= 1 - 1/x$

Q) Given:

$X = \{x, y, w, z\}, Y = \{1, 2, 3, 4\}$
$H = \{(x, 1), (y, 2), (y, 3), (z, 4), (x, 4)\}$

Options:

(a) H is a function from X to Y

(b) H is not a function from X to Y

(c) H is a relation from Y to X

(d) None of the above

Limits

Definition:

The limit of a function $f(x)$ at a point $x = a$ is the value that $f(x)$ approaches as x gets arbitrarily close to a.

It is denoted as:

$$\lim_{x \to a} f(x)$$

Left-Hand Limit (LHL):

The value that $f(x)$ approaches as x approaches a from the left ($x < a$).

$$\lim_{x \to a^-} f(x)$$

Right-Hand Limit (RHL):

The value that $f(x)$ approaches as x approaches a from the right ($x > a$).

$$\lim_{x \to a^+} f(x)$$

Existence of a Limit:

A limit exists at $x = a$ if and only if:

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = L$$

where L is the common value.

$\lim_{x \to 2} x^2 = 2^2 = 4$

$\lim_{x \to 5} x^2 = 25$

$\lim_{x \to 3} (x^2 + x + 4) = 3^2 + 3 + 4 = 16$

Properties of Limits:

1. **Sum Rule:**

$$\lim_{x \to a}[f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$$

2. **Product Rule:**

$$\lim_{x \to a}[f(x) \cdot g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$$

3. **Quotient Rule:**

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}, \text{ if } \lim_{x \to a} g(x) \neq 0$$

4. **Constant Rule:**

$$\lim_{x \to a} c = c$$

$\lim_{x \to 4} \frac{(x-4)}{x^2-16} = \frac{\cancel{x-4}}{\cancel{(x-4)}(x+4)}$

$\lim_{x \to 4} \frac{1}{x+4} = \frac{1}{8}$

Constant Multiplication Rule:

If C is a constant, then:

$$\lim_{x \to a}[C \cdot f(x)] = C \cdot \lim_{x \to a} f(x)$$

$\lim_{x \to \infty} \frac{1}{x} = 0$

$\lim_{x \to \infty} \frac{x^2+2x+1}{x^2}$

Important Limits

1. Polynomial Limit:

$$\boxed{\lim_{x \to a} \frac{x^n - a^n}{x - a} = \boxed{n \cdot a^{n-1}}}$$

$$\lim_{x \to 0} \frac{5^x - 1}{x}$$

2. Exponential Limit:

$$\lim_{x \to 0} \frac{a^x - 1}{x} = \ln a \quad (\text{where } a > 0)$$

$$\log_e 5$$

3. Natural Exponential Limit:

$$\boxed{\lim_{x \to 0} \frac{e^x - 1}{x} = 1}$$

4. Logarithmic Limit:

$$\lim_{x \to 0} \frac{\ln(1 + x)}{x} = 1$$

$$3 \times \boxed{\lim_{x \to 0} \frac{\ln(1 + 3x)}{3x}}$$

$$3 \times 1 = 3$$

5. Limit Definition of e (Infinity Form):

$$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e$$

Expansion Limit:

$$\lim_{x \to 0} \frac{(1 + x)^n - 1}{x} = n$$

Q) Evaluate $\lim_{x \to 2} \frac{x^2+x+2}{x^3+1}$

(a) $\frac{8}{9}$
(b) $\frac{9}{8}$
(c) $\frac{2}{3}$
(d) 1

$\frac{2^2+2+2}{2^3+1} = \frac{8}{9}$

Q) Evaluate $\lim_{x \to 4} \sqrt[3]{5x+7}$.

(a) 3
(b) 4
(c) 5
(d) 2

$\lim_{x \to 4} \sqrt[3]{5x+7}$

$\sqrt[3]{5 \times 4 + 7}$

$= \sqrt[3]{27}$

$= 3$

Type II: Factorization

- When both the numerator and denominator approach 0 ($\frac{0}{0}$ form), factorize and cancel common terms:

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{p(a)}{q(a)}$$

- Example:

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = \lim_{x \to 1} \frac{(x - 1)(x + 1)}{x - 1} = \lim_{x \to 1}(x + 1) = 2$$

Q) Evaluate $\lim_{x \to -3} \frac{x^2+7x+12}{x+3}$

(a) 1

(b) 0

(c) -1

(d) 12

$$\lim_{x \to -3} \frac{\cancel{(x+3)}(x+4)}{\cancel{x+3}}$$

$= -3+4$

$= 1$

$$\frac{\frac{d}{dx}\, x^2+7x+12}{\frac{d}{dx}\, x+3}$$

$$\lim_{x \to -3} \frac{2x+7}{1}$$

$= 2(-3)+7$

$= 1$

Type III: Infinity Limits

- Divide the numerator and denominator by the highest power of x:

$$\lim_{x \to \infty} \frac{f(x)}{g(x)}$$

- Example:

$$\lim_{x \to \infty} \frac{3x^2 + 5}{2x^2 + 7} = \frac{3}{2}$$

$5\frac{x^2}{x^2}$

Q) Evaluate $\lim_{x\to\infty} \frac{2x^2+7x+5}{4x^2+3x+1}$, which is equal to l, where l is:

(a) $-\frac{1}{2}$

(b) $\frac{1}{2}$

(c) 2

(d) None

$$\lim_{x\to\infty} \frac{\cancel{x^2}\left(2+\frac{7}{x}+\frac{5}{x^2}\right)}{\cancel{x^2}\left(4+\frac{3}{x}+\frac{1}{x^2}\right)}$$

Apply limi

$$\frac{2+\cancel{\frac{7}{\infty}}+\cancel{\frac{5}{\infty}}}{4+\cancel{\frac{3}{\infty}}+\cancel{\frac{1}{\infty}}}$$

$$\frac{2}{4} = \frac{1}{2}$$

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Q) Evaluate $\lim_{n\to\infty} \left((2^n-2)(2^n+1)^{-1}\right)$:

$2^\infty = \infty$

(a) $\frac{1}{4}$

(b) $\frac{1}{2}$

(c) 1

(d) None

$$\lim_{n\to\infty} \frac{2^n-2}{2^n+1}$$

$$\lim_{n\to\infty} \frac{\cancel{2^n}\left(1-\frac{2}{2^n}\right)}{\cancel{2^n}\left(1+\frac{1}{2^n}\right)}$$

$$\frac{1-\cancel{\frac{2}{\infty}}}{1+\cancel{\frac{1}{\infty}}}$$

$= 1$

Type IV: Standard Limits

- Exponential Limit:

$$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$$

- Logarithmic Limit:

$$\lim_{x \to 0} \frac{\ln(1+x)}{x} = 1$$

- Power Limit:

$$\lim_{x \to a} \frac{x^n - a^n}{x - a} = n \cdot a^{n-1}$$

Q) Evaluate $\lim_{x \to 0} \frac{\log(1+3x)}{x}$ which is:

(a) 1

(b) 3

(c) -3

(d) None

$3 \lim_{x \to 0} \frac{\log(1+3x)}{3x}$

$= 3$

Q) Evaluate $\lim_{x \to 0} \frac{3^x - 1}{x}$, which is equal to:

(a) $10^3 \cdot \log_{10} 3$

(b) $\log_{10} e$

(c) $\log_e 3$

(d) None

log 3

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Type V: Using L'Hôpital's Rule

- If the limit is in indeterminate form ($\frac{0}{0}$ or $\frac{\infty}{\infty}$):

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

- Example:

Example 2: $\lim_{x \to \infty} \frac{x}{e^x}$

1. Evaluate the limit directly:

$$\frac{x}{e^x} \to \frac{\infty}{\infty} \text{ as } x \to \infty$$

2. Differentiate numerator and denominator:

$$f'(x) = 1, \quad g'(x) = e^x$$

3. Apply L'Hôpital's Rule:

$$\lim_{x \to \infty} \frac{x}{e^x} = \lim_{x \to \infty} \frac{1}{e^x} = 0$$

When to Use L'Hôpital's Rule:

- The rule is applicable **only** for indeterminate forms ($\frac{0}{0}$ or $\frac{\infty}{\infty}$).

Continuity

Definition:

The term "continuous" in mathematics means a function progresses without interruption or abrupt changes. A function $f(x)$ is said to be **continuous at a point** $x = a$ if the following conditions are satisfied:

1. **Function is Defined:**
   $f(a)$ must exist.
2. **Left-Hand Limit Equals Right-Hand Limit:**

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$$

3. **The Value of the Limit Equals the Function Value:**

$$\lim_{x \to a} f(x) = f(a)$$

Explanation of Limits:

- **Left-Hand Limit (LHL):**

$$LHL = \lim_{x \to a^-} f(x)$$

  This is the value $f(x)$ approaches as x approaches a from the left ($x < a$).

- **Right-Hand Limit (RHL):**

$$RHL = \lim_{x \to a^+} f(x)$$

  This is the value $f(x)$ approaches as x approaches a from the right ($x > a$).

If $LHL = RHL = f(a)$, the function is continuous at $x = a$.

Important Notes on Continuity:

1. **Continuity and Limit:**
   - For a function to be continuous, the limit must exist, but the existence of a limit alone is not sufficient for continuity.
   - Example: $f(x) = \frac{|x|}{x}$ at $x = 0$ has a limit but is not continuous.
2. **Operations on Continuous Functions:**
   - The **sum**, **difference**, **product**, and **quotient** (if the denominator is non-zero) of continuous functions are always continuous.
3. **Polynomials:**
   - All polynomials are continuous everywhere.
4. **Rational Functions:**
   - A function of the form $\frac{f(x)}{g(x)}$, where $f(x)$ and $g(x)$ are polynomials, is continuous everywhere except at points where $g(x) = 0$ (as the function becomes undefined).

Points of Discontinuity

Definition:

A **point of discontinuity** is a value $x = a$ in the domain of a function $f(x)$ where the function is not continuous.

Removable Discontinuity:

- Occurs when the limit exists at $x = a$, but $f(a)$ is either undefined or not equal to the limit.
- Example:

$$f(x) = \frac{x^2 - 1}{x - 1}, \quad x \neq 1$$

  Here, $\lim_{x \to 1} f(x) = 2$, but $f(1)$ is undefined, so $x = 1$ is a removable discontinuity.

Q) Find the points of discontinuity of the function $f(x) = \frac{x^2+2x+5}{x^2-3x+2}$.

$x^2-3x+2=0$

Sum=3 product-2

2,1

(a) 1

(b) 2

(c) (a) and (b) both

(d) None of these

Q) The function $f(x) = \frac{x^2-9}{x-3}$ is undefined at $x=3$. What value must be assigned to $f(3)$ if $f(x)$ is to be continuous at $x=3$?

(a) 6

(b) -6

(c) Cannot be decided

(d) None

$f(x) = \frac{x^2-9}{x-3}$

$= \frac{(x+3)\cancel{(x-3)}}{\cancel{x-3}}$

$f(3) = x+3$

$= 3+3 = 6$

Q) A function $f(x)$ is defined by $f(x) = (x-2)+1$ over all real values of x. Now $f(x)$ is:

(a) Continuous at $x = 2$

(b) Discontinuous at $x = 2$

(c) Undefined at $x = 2$

(d) None of these

$(x-2)+1$
$= x-1$

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Chapter 15: Probability

Probability is the **chance of occurrence of an event.**

If A is an event, the probability of occurrence of A is given by:

$$\boxed{P(A) = \frac{n(A)}{n(S)}}$$

$P(A) = \frac{n(A)}{n(S)}$

Where:

- $n(A)$: Number of favorable outcomes.
- $n(S)$: Total number of possible outcomes.

$S = \{a, b, c, \ldots x, y, z\}$
$n(S) = 26$
$A = \{a, e, i, o, u\}$

$\frac{5}{26}$

Classification of Probability:

Kadapa

1. **Subjective Probability**:
   - Based on one's experience and observation.
2. **Objective Probability**:
   - Based on mathematical facts.

$\frac{1}{2}$

Key Properties of Probability

1. **Range of Probability**:
   - Probability of an event always lies between 0 and 1 (both inclusive):

$$0 \leq P(A) \leq 1$$

2. **Special Cases**:
   - If $P(A) = 0$: The event is **impossible** or **improbable**.
   - If $P(A) = 1$: The event is a **sure event**.
   - If $0 < P(A) < 1$: The event is **possible or probable**.
3. **Complementary Events**:
   - The sum of the probabilities of an event and its non-occurrence equals 1:

$$P(A) + P(A') = 1$$

Where A' is the complement of A, meaning the event does not occur.

$P(\text{☺}) + P(\text{☹}) = 1$

$P(A) + P(A)' = 1$

$P(A)' = 1 - P(A)$

4. **Simple Event**:
   - An event that produces only one outcome is called a **simple event**.

     Example: Getting a 2 when rolling a die.
5. **Compound Event**:
   - An event that produces more than one outcome is called a **compound (or composite) event**.

     Example: Getting a multiple of 2 when rolling a die, where the outcomes are $2, 4, 6$.

Sample Space (S):

The **sample space** is the set of all possible outcomes of an experiment.

Sample Space for Tossing a Coin:

H or T

1. **When a coin is tossed once:**

   Sample space:

   $$S = \{H, T\}$$

   Number of outcomes:

   $$n(S) = 2$$

2. **When a coin is tossed twice:**

   Sample space:

   $$S = \{HH, HT, TH, TT\}$$

   Number of outcomes:

   $$n(S) = 4$$

3. **When a coin is tossed thrice:**

   Sample space:

   HHH, HHT, HTH, THH

   $$S = \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$$

   TTT, TTH, HTT, THT

   Number of outcomes:

   $$n(S) = 8$$

General Rule:

For n tosses of a coin:

$$n(S) = 2^n$$

Where n is the number of tosses.

Note:

The **probability** of getting heads and tails alternatively on tossing a coin n times is given by:

$$P = \frac{2}{2^n}$$

Sample Space for Tossing/Rolling a Die

a) When Tossed Once:

- Sample space:

$$S = \{1, 2, 3, 4, 5, 6\}$$

- Total number of outcomes:

$$\boxed{n(S) = 6}$$

b) When Tossed Twice:

- Sample space:

$$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), \ldots, (6,6)\}$$

This includes all ordered pairs of outcomes where each die can show any number between 1 and 6.

(1,1) (1,2) . . . . (1,6)
(2,1) 2,6
(3,1) 3,6
(4,1) . . . . . (4,6)
(5,1) . . . . . (5,6)
(6,1) . . . . . (6,6)

- Total number of outcomes:

$$n(S) = 6 \times 6 = 36$$

$n(S)$ $6^2 = 36$

Note:

For tossing a die n times, the total number of outcomes is given by:

$$\boxed{n(S) = 6^n}$$

52

$\frac{26}{52}$

Sample Space for a Deck of Cards

26 Red 26 BC

Total number of cards:

$$n(S) = 52$$

13 13 13 13

K K K K
Q Q Q Q
J J J J
A A A A
2 2 2 2
3 3 3 3
4 4 4 4
5 5 5 5
6 6 6 6
7 7 7 7
8 8 8 8
9 9 9 9
10 10 10 10

Face = 12

36 number

Classification of Cards:

1. **Red Cards** (26 cards):
   - **Hearts**: 13 cards
   - **Diamonds**: 13 cards
2. **Black Cards** (26 cards):
   - **Spades**: 13 cards
   - **Clubs**: 13 cards

Each Suit Consists of 13 Cards:

- **Number Cards**: 9 cards (2 to 10)
- **King**: 1 card
- **Queen**: 1 card
- **Jack**: 1 card
- **Ace**: 1 card

Number of Face Cards:

- Face cards: **King, Queen, Jack**
- Total face cards per suit = 3
- Total face cards across all suits:

$$4 \times 3 = 12$$

Q) In a pack of playing cards with two jokers, the probability of getting the king of spades is:

(a) 4/13

(b) 4/52

(c) 1/52

(d) 1/54

$n(S) = 52 + 2 = 54$

$n(A) = 1$

$P(A) = \frac{1}{54}$

$\frac{1}{36}$ $\frac{4}{36}$

Q) If x be the sum of two numbers obtained when two dice are thrown simultaneously, then $P(x \geq 7)$ is:

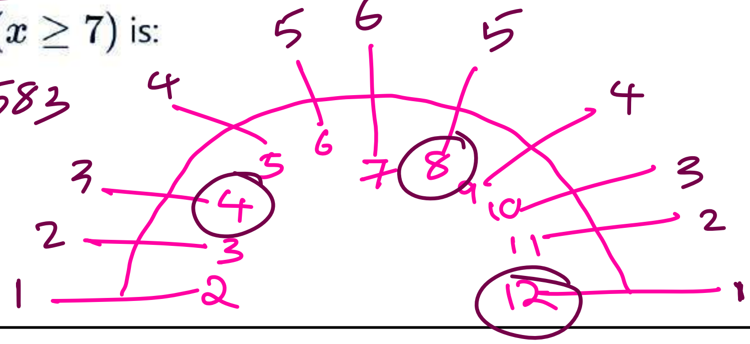
(a) $\frac{5}{12}$

(b) $\frac{7}{12}$ 0.583

(c) $\frac{11}{15}$

(d) $\frac{3}{8}$

$\frac{21}{36} = 0.583$



Q) An unbiased die is thrown twice. The probability of the sum of numbers obtained on the two faces being divisible by 4 is:

(a) $\frac{7}{36}$

(b) $\frac{1}{3}$

(c) $\frac{11}{36}$

(d) $\frac{1}{4}$

$\frac{3+5+1}{36} = \frac{9}{36}$

Q) Three coins are tossed together. The probability of getting exactly two heads is:

(a) $\frac{5}{8}$

(b) $\frac{3}{8}$

(c) $\frac{1}{8}$

(d) None

$n(S)=8$ HHT, HTH, THH

$\frac{3}{8}$

Q) If a coin is tossed 5 times, then the probability of getting tail and head alternately is:

(a) $\frac{1}{8}$

(b) $\frac{1}{16}$

(c) $\frac{1}{32}$

(d) $\frac{1}{64}$

$\frac{2}{2^n} = \frac{2}{2^5}$

$= \frac{1}{16}$

Q) Two different dice are thrown simultaneously. Then the probability that the sum of two numbers appearing on the top of the dice is 9 is:

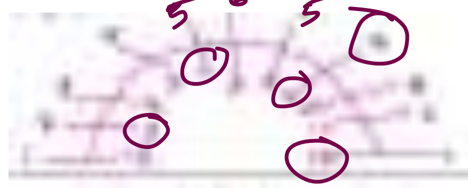
(a) $\frac{8}{9}$

(b) $\frac{1}{9}$

(c) $\frac{7}{9}$

(d) None of the above

$\frac{4}{36} = \frac{1}{9}$



Q) When 2 fair dice are thrown, what is the probability of getting the sum which is a multiple of 3?

(a) $\frac{4}{36}$

(b) $\frac{13}{36}$

(c) $\frac{2}{36}$

(d) $\frac{12}{36}$

$\frac{2+5+4+1}{36} = \frac{12}{36}$

Q) When two coins are tossed simultaneously, the probability of getting at least one tail is:

(a) 1

(b) 0.75

(c) 0.5

(d) 0.25

$n(S) = 2^2 = 4$

(T, H) (H, T) (T, T)

$\frac{3}{4} = 0.75$

Q) Two perfect dice are rolled. What is the probability that one appears at least in one of the dice?

(a) $\frac{7}{36}$

(b) $\frac{11}{36}$

(c) $\frac{9}{36}$

(d) $\frac{15}{36}$

(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)
(2, 1) (3, 1) (4, 1) (5, 1) (6, 1)

$\frac{11}{36}$

Key Concepts for Two Events A and B

1. Probability of $A \cup B$ (At least one of A or B occurs):

$$\boxed{P(A \cup B)} = P(A) + P(B) - P(A \cap B)$$

- $P(A \cup B)$: Probability of A or B.
- $P(A \cap B)$: Probability of A and B.

Note: This is also known as the **Addition Theorem of Probability.**

2. Mutually Exclusive Events:

- Two events A and B are **mutually exclusive** if they have no common outcomes:

$$\boxed{P(A \cap B) = 0}$$

- Probability simplifies to:

$$\boxed{P(A \cup B) = P(A) + P(B)}$$

3. Exhaustive Events:

- Two events A and B are **exhaustive** if their union includes all possible outcomes:

$$n(A \cup B) = n(S)$$

$$\boxed{P(A \cup B) = 1}$$

4. Mutually Exclusive and Exhaustive Events:

If A and B are both mutually exclusive and exhaustive:

- $P(A \cup B) = 1$
- $P(A \cap B) = 0$
- Formula simplifies to:

$$\boxed{1 = P(A) + P(B)}$$

$P(A \cap B) = 0$
$P(A \cup B) = 1$

5. Conditional Probability:

$P(A|B)$

If A and B are two events:

$P(A \cap B)$

1. **Probability of A given B (Event B has already occurred):**

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

2. **Probability of B given A (Event A has already occurred):**

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$P(B|A) =$

Where:

- $P(A|B)$: Conditional probability of A given B.
- $P(B|A)$: Conditional probability of B given A.

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Q) One card is drawn from a pack of 52. What is the probability that it is a king or a queen?

(a) $\frac{11}{13}$

(b) $\frac{2}{13}$

(c) $\frac{1}{13}$

(d) None of these

$P(K \cup Q) = P(K) + P(Q)$

$= \frac{4}{52} + \frac{4}{52}$

$= \frac{8}{52} =$

Q) A number is selected from the first 30 natural numbers. What is the probability that it would be divisible by 3 or 8?

(a) 0.2

(b) 0.4

(c) 0.6

(d) 0.8

$P\left(\frac{Div}{3} \cup \frac{div}{8}\right) = P\left(\frac{Div}{3}\right) + P\left(\frac{Div}{8}\right) - P\left(\frac{Div}{3} \cap \frac{Div}{8}\right)$

$= \frac{10}{30} + \frac{3}{30} - \frac{1}{30}$

$= \frac{12}{30}$

Q) A card is drawn out of a standard pack of 52 cards. What is the probability of drawing a king or a red card?

(a) $\frac{1}{4}$

(b) $\frac{4}{13}$

(c) $\frac{7}{13}$

(d) $\frac{1}{2}$

$P(K \cup R) = P(K) + P(R) - P(K \cap R)$

$= \frac{4}{52} + \frac{26}{52} - \frac{2}{52}$

$= \frac{28}{52}$

Q) The theorem of compound probability states that for any two events A and B:

(a) $P(A \cap B) = P(A) \times P(B|A)$

(b) $P(A \cup B) = P(A) \times P(B|A)$

(c) $P(A \cap B) = P(A) \times P(B)$

(d) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(B|A) = \frac{P(A \cap B)}{P(A)}$

$P(A) \times P(B|A) = P(A \cap B)$

Q) The sum of all probabilities of mutually exclusive and exhaustive events is equal to:

(a) 0

(b) $\frac{1}{2}$

(c) $\frac{1}{4}$

(d) 1

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Q) If in a class 60% of the students study both mathematics and science, and 90% of the students study science, then the probability of a student studying mathematics given that he/she is already studying science is:

(a) $\frac{1}{4}$

(b) $\frac{2}{3}$

(c) 1

(d) $\frac{1}{2}$

A B

$P(A \cap B) = 0.6$ $P(B) = 0.9$

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

$= \frac{0.6}{0.9} = 0.666$

Independent event
A and B
$P(A \cap B) = P(A) \times P(B)$

De Morgan's Theorem

1. $P(A' \cap B') = P(A \cup B)'$
2. $P(A' \cup B') = P(A \cap B)'$

$P(A' \cap B') = P(A \cup B)'$
$P(A' \cup B') = P(A \cap B)'$

Note:

The probability of neither A nor B occurring can be calculated using:

$$P(A \cup B)' = 1 - P(A \cup B)$$

Q) If A and B are two independent events and $P(A \cup B) = \frac{2}{5}$, $P(B) = \frac{1}{3}$ Find $P(A)$.

(a) $\frac{2}{9}$
(b) $-\frac{1}{3}$
(c) $\frac{2}{10}$
(d) $\frac{1}{10}$

$P(A \cup B) = P(A) + P(B) - P(A) \times P(B)$
$\frac{2}{5} = P(A)\left(1 - \frac{1}{3}\right) + \frac{1}{3}$
$\frac{2}{5} = P(A) \times \frac{2}{3} + \frac{1}{3}$

$\frac{2}{5} - \frac{1}{3} = \frac{2}{3} \times P(A)$
$2 \div 5\ m^+$
$1 \div 3\ m^-$
MRC $\times 3 \div 2$
0.1

Q) $P(A) = \frac{2}{3}$, $P(B) = \frac{3}{5}$, $P(A \cup B) = \frac{5}{6}$. Find $P(B|A)$.

(a) $\frac{11}{20}$
(b) $\frac{13}{20}$
(c) $\frac{13}{18}$
(d) None

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$
$= \frac{2}{3} + \frac{3}{5} - \frac{5}{6}$
$2 \div 3\ m^+\ 3 \div 5\ m^+\ 5 \div 6\ m^-$
MRC

$\frac{P(A \cap B)}{P(A)} = \frac{13}{20}$
$\frac{0.4333}{0.66666} =$

Q) If $P(A|B) = P(A)$ then A and B are:

(a) Mutually exclusive events
(b) Dependent events
(c) Independent events
(d) Composite events

$P(A) = \frac{P(A \cap B)}{P(B)}$
$P(A) \times P(B) = P(A \cap B)$

Q) For any two events A_1, A_2, let $P(A_1) = \frac{2}{3}$, $P(A_2) = \frac{3}{8}$, and $P(A_1 \cap A_2) = \frac{1}{4}$. Then A_1, A_2 are:

$P(A_1) \times P(A_2) = \frac{2}{3} \times \frac{3}{8} = \frac{1}{4}$

$= P(A_1 \cap A_2)$

~~(a) Mutually exclusive but not independent events~~

~~(b) Mutually exclusive and independent events~~

(c) Independent but not mutually exclusive

(d) None of these

Q) The probability of a girl getting a scholarship is 0.6, and the same probability for a boy is 0.8. Find the probability that at least one of the categories gets a scholarship:

$P(G) = 0.6 \quad P(G)' = 0.4 \quad P(B) = 0.8 \quad P(B)' = 0.2$

(a) 0.32

(b) 0.44

(c) 0.92

(d) None of the above

Case I — G gets and Boy Doesn't; or Case II — G Doesn't and Boy Does; or Case III — G Gets and Boy Gets

$P(G) \times P(B)' + P(G)' \times P(B) + P(G) \times P(B)$

$0.6 \times 0.2 + 0.4 \times 0.8 + 0.6 \times 0.8 = 0.92$

Q) Let A and B be two events in a sample space S such that $P(A) = \frac{1}{2}$, $P(B) = \frac{5}{8}$, $P(A \cup B) = \frac{3}{4}$. Find $P(\overline{A} \cap B)$:

(a) $\frac{3}{4}$

(b) $\frac{1}{4}$

(c) $\frac{3}{16}$

(d) None of these

Q) If $P(A) = 0.45$, $P(B) = 0.35$, and $P(A \cap B) = 0.25$, then $P(A|B) = ?$

(a) 1.4

(b) 1.8

(c) 0.714

(d) 0.556

Q) If for two mutually exclusive events A and B, $P(A \cup B) = \frac{2}{3}$ and $P(A) = \frac{2}{5}$, then what is the value of $P(B)$?

(a) $\frac{4}{15}$

(b) $\frac{4}{9}$

(c) $\frac{5}{9}$

(d) $\frac{7}{15}$

Q) For any two events A and B:

(a) $P(A - B) = P(A) - P(B)$

(b) $P(A - B) = P(A) - P(A \cap B)$

(c) $P(A - B) = P(B) - P(A \cap B)$

(d) $P(B - A) = P(B) + P(A \cap B)$

Q) If A speaks 75% of the truth and B speaks 60% of the truth, in what percentage are both of them likely to contradict each other in narrating the same questions?

$P(A) = 0.75$ $P(A)' = 0.25$ $P(B) = 0.6$ $P(B)' = 0.4$

(a) 0.60

(b) 0.45

(c) 0.65

(d) 0.35

$P(A) \times P(B)' + P(B) \times P(A)'$

$0.75 \times 0.4 + 0.6 \times 0.25$

$= 0.45$

Q) The probability of a cricket team winning a match at Kanpur is $\frac{2}{5}$, and losing a match at Delhi is $\frac{1}{7}$. What is the probability of the team winning at least one match?

$P(K) = \frac{2}{5}$ $P(K)' = \frac{3}{5}$ $P(D) = \frac{6}{7}$ $P(D)' = \frac{1}{7}$

(a) $\frac{3}{35}$

(b) $\frac{32}{35}$

(c) $\frac{18}{35}$

(d) $\frac{17}{35}$

$P(K) \times P(D)' + P(D) \times P(K)' + P(D) \times P(K)$

$\frac{2}{5} \times \frac{1}{7} + \frac{6}{7} \times \frac{3}{5} + \frac{6}{7} \times \frac{2}{5}$

$\frac{2 + 18 + 12}{35} = \frac{32}{35}$

P(atleast one) + P(winning zero) = 1

P(atleast one) = 1 − P(0) = 1 − P(K)' × P(D)' = 1 − $\frac{3}{5} \times \frac{1}{7}$

= 1 − $\frac{3}{35}$ = $\frac{32}{35}$

Q) Arun and Tarun appear for an interview for two vacancies. The probability of Arun's selection is $\frac{1}{3}$, and that of Tarun's selection is $\frac{1}{5}$. Find the probability that only one of them will be selected: $P(A) = \frac{1}{3}$ $P(A)' = \frac{2}{3}$ $P(T) = \frac{1}{5}$ $P(T)' = \frac{4}{5}$

(a) $\frac{2}{5}$
(b) $\frac{4}{5}$
(c) $\frac{6}{5}$
(d) $\frac{8}{5}$

$P(A) \times P(T)' + P(A)' \times P(T)$

$\frac{1}{3} \times \frac{4}{5} + \frac{2}{3} \times \frac{1}{5} = \frac{6}{15}$

Q) The probabilities of a person getting qualified for 2 different entrance exams are $\frac{1}{3}$ & $\frac{3}{4}$. Find the probability that he would get qualified in one of the 2 exams.

(a) $\frac{2}{3}$
(b) $\frac{5}{6}$
(c) $\frac{1}{2}$
(d) $\frac{3}{4}$

arivupro ACADEMY

Sample Space for Leap Year and Non-Leap Year Problems

$$7\overline{)365}\,52$$
35
15
14
1

Non-Leap Year:

- Total Days: 365
- Odd Days: 1
- Sample Space: `{Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}`
- n(S): 7
- Key Notes:
  - $P(A \cap B) = 0$, since there are no overlapping days for non-leap years.

52
$$7\overline{)366}$$
364
2

Leap Year:

- **Total Days**: 366
- **Odd Days**: 2
- **Sample Space**: `{Sunday-Monday, Monday-Tuesday, Tuesday-Wednesday, Wednesday-Thursday, Thursday-Friday, Friday-Saturday, Saturday-Sunday}`
- **n(S)**: 7
- **Key Notes**:
  - $P(A \cap B) = 0$ if the given days are non-successive.
  - $P(A \cap B) = \frac{1}{7}$ if the days are successive.

Q) The probability that a leap year has 53 Mondays is:

(a) $\frac{1}{7}$

(b) $\frac{2}{3}$

(c) $\frac{2}{7}$

(d) $\frac{3}{5}$

$$\frac{2}{7}$$

Q) What is the probability of getting 53 Sundays or 53 Wednesdays in a non-leap year?

(a) $\frac{2}{7}$
(b) $\frac{3}{7}$
(c) $\frac{1}{7}$
(d) $\frac{4}{7}$

$\frac{1}{7} + \frac{1}{7} = \frac{2}{7}$

Q) What is the probability of getting 53 Fridays or 53 Saturdays in a leap year?

(a) $\frac{2}{7}$
(b) $\frac{3}{7}$
(c) $\frac{4}{7}$
(d) $\frac{5}{7}$

$\frac{2}{7} + \frac{2}{7} - \frac{1}{7} = \frac{3}{7}$

For Any Three Events $A, B, \& C$

1. Probability that at least one event occurs:

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

This is the general addition rule for three events.

2. If $A, B, \& C$ are mutually exclusive:

$$P(A \cap B) = 0, \quad P(B \cap C) = 0, \quad P(A \cap C) = 0, \quad P(A \cap B \cap C) = 0$$

$$\boxed{P(A \cup B \cup C) = P(A) + P(B) + P(C)}$$

3. If $A, B, \& C$ are exhaustive:

If $A, B, \& C$ cover the entire sample space:

$$\boxed{P(A \cup B \cup C) = 1}$$

Mutu exclu and exhaust. $1 = P(A) + P(B) + P(C)$

4. If $A, B, \& C$ are mutually exclusive and exhaustive:

$$P(A) + P(B) + P(C) = 1$$

5. If $A, B, \& C$ are equally likely:

$$\boxed{P(A) = P(B) = P(C) = K}$$

Where K is a constant.

P(A) P(B) P(C)

Q) There are 3 events A, B, C having probabilities $\frac{1}{2}, \frac{1}{3}$, and $\frac{3}{4}$. If the probabilities of occurrence of $A\&B$, $B\&C$, $A\&C$, and $A\&B\&C$ are $\frac{1}{3}, \frac{1}{5}, \frac{1}{6}$, and $\frac{1}{15}$, respectively, then find the probability of occurrence of at least one event.

$P(A \cap B)$ $\quad P(B \cap C)$ $\quad P(A \cap C)$ $\quad P(A \cap B \cap C)$

(a) 0.95
(b) 0.85
(c) 0.90
(d) 1.00

$P(A \cup B \cup C)$

Random Variable (x)

- A random variable is a function that assigns a numerical value to every outcome of a random experiment.
- Example:
  - Tossing a coin:
    - Assign $x = 0$ for heads and $x = 1$ for tails.
  - Rolling a die:
    - x can take values $1, 2, 3, 4, 5, 6$, corresponding to the outcomes of the die.

x	0	1
$P(x)$	1/2	1/2

$E(x) = \bar{x} = \sum x\, p(x) = 1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + \dots 6 \times \frac{1}{6}$

x	1	2	3	4	5	6
$P(x)$	1/6	1/6	1/6	1/6	1/6	1/6

TTH
THT
HTT
HHT
HTH
THH

x	0	1	2
$P(x)$	1/4	2/4	1/4

x	0	1	2	3
$P(x)$	1/8	3/8	3/8	1/8

$= 1$

Probability Distribution

- A **probability distribution** lists all possible values of a random variable along with their probabilities.
- Key Property:

$$\sum P(x) = 1$$

This means that the probabilities of all possible outcomes add up to 1.

- Example:
  - Rolling a fair die:
    - Probability of each outcome (1 to 6): $P(x) = \frac{1}{6}$.
    - Total: $P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1$.

Mathematical Expectation of x (Mean)

- **Definition:** The **expected value** or **mean** of a random variable is the weighted average of all possible values of x, weighted by their probabilities.
- Formulas:

$\sum x \cdot p(x)$

  - $\boxed{E(x)} = \sum x \cdot P(x)$
    - Example:
      - For a die roll:

$$E(x) = 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + \cdots + 6 \cdot \frac{1}{6} = \frac{1+2+3+4+5+6}{6} = 3.5$$

- $E(x^2) = \boxed{\sum x^2 \cdot P(x)}$
  - This is used to compute **variance**.

$1x = x\ p(x)\ M^{+}$
$2x = x \frac{1}{6}\ M^{+}$
$3x = x \frac{1}{6}\ M^{+}$ ... $6x = x \frac{1}{6} M^{+}$

Variance of x

- **Definition:** Variance measures the spread or dispersion of the random variable around its mean. It tells us how far the values of x are likely to be from the expected value $E(x)$.
- **Formula:**

$$V(x) = E(x^2) - [E(x)]^2$$

Why Are These Concepts Useful?

- **Random variables** are used to model outcomes of experiments in real life.
- **Probability distributions** allow us to predict the likelihood of outcomes.
- **Expected value** helps us determine the "average" or central value of a process (e.g., average profit in business).
- **Variance** quantifies uncertainty or risk (e.g., in stock prices or insurance).

Detailed Explanation of Properties of $E(x)$:

1. $E(K) = K$:
   - This property states that if a random variable X is a constant value K, then its expected value $E(X)$ is simply K.
   - This makes sense because, in such a case, there is no variability or randomness, and the constant K itself is the average or expected value.
2. $E(x+y) = E(x) + E(y)$:
   - The expected value of the sum of two independent random variables x and y is equal to the sum of their individual expected values.
   - This is a linear property of expectation and is applicable irrespective of whether x and y are dependent or independent.

3. $E(XY) = E(X) \cdot E(Y)$ **(For Independent Variables):**

- When two random variables X and Y are independent, the expected value of their product is the product of their individual expected values.
- However, this equality holds only when X and Y are statistically independent.

4. $E(a + bX) = a + bE(X)$:

$= E(a) + E(bX)$

$= a + E(b) \times E(X)$

$= a + bE(X)$

- This property shows how expectation interacts with linear transformations of a random variable.
- Here:
  - a is a constant added to X, which directly gets added to the expected value.
  - b is a scaling factor that multiplies X, which directly scales the expected value by b.

Q) A random variable X has the following probability distribution:

X	0	1	2	3
$P(X)$	0	$2K$	$3K$	K

Then $P(X < 3)$ would be:

$\sum p(x) = 1$

$2K + 3K + K = 1$

$6K = 1$

$K = \frac{1}{6}$

(a) $\frac{1}{6}$

(b) $\frac{1}{3}$

(c) $\frac{2}{3}$

(d) $\frac{5}{6}$

$\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

Q) $E(XY)$ is also known as:

(a) $E(X) + E(Y)$

(b) $E(X)E(Y)$

(c) $E(X) - E(Y)$

(d) $\frac{E(X)}{E(Y)}$

Q) $E(13x + 9) =$

(a) $13x$

(b) $13E(x)$

(c) $13E(x) + 9$

(d) 9

$13E(x) + 9$

5 6 5

Q) Two unbiased dice are thrown. The expected value of the sum of numbers on the upper side is:

(a) ~~3.5~~

(b) 7

(c) 12

(d) 6

x	2	3	4	5	6	7	8	9	10	11	12
$p(x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

$$E(x) = \Sigma x\,p(x) = \frac{1}{36}\left\{2\times1+3\times2+4\times3+5\times4+6\times5+7\times6+8\times5+9\times4+10\times3+11\times2+12\times1\right\}$$

Q) A player tosses two fair coins. He wins ₹5 if 2 heads appear, ₹2 if one head appears, and ₹1 if no head occurs. Find his expected amount of winning.

(a) 2.5

(b) 3.5

(c) 4.5

(d) 5.5

Amount	1	2	5
x	0	1	2
$p(x)$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$

$$E(x) = \Sigma \text{Amount} \times p(x)$$

$$= 1\times\frac{1}{4} + 2\times\frac{2}{4} + 5\times\frac{1}{4}$$

$$= 2.5$$

1	-0.25
C	W
$\frac{1}{4}$	$\frac{3}{4}$

$1\times\frac{1}{4} + (-0.25)\times\frac{3}{4}$

$\frac{1}{4} + \frac{-0.75}{4}$

$\frac{-0.25}{4}$

$= 0.062$

Q) Find the expected value of the following probability distribution:

x	-20	-10	30	75	80
p(x)	3/20	1/5	1/2	1/10	1/20

(a) 20.5

(b) 21.5

(c) 22.5

(d) 24.5

$E(x) = 20 \times \frac{3}{20}\ M^-$

$10 \times \frac{1}{5}\ M^-$

$30 \times \frac{1}{2}\ M^+$

$75 \times \frac{1}{10}\ M^+$

$80 \times \frac{1}{20}\ M^+$

MRC

Q) The probability distribution of the demand for a commodity is given below:

Demand (x)	5	6	7	8	9	10
Probability P(x)	0.05	0.10	0.30	0.40	0.10	0.05

The expected value of demand will be:

(a) 7.55

(b) 7.85

(c) 1.25

(d) 8.35

$5 \times 0.05\ M^+$ $6 \times 0.1\ M^+$ $7 \times 0.3\ M^+$

$8 \times 0.4\ M^+$ $9 \times 0.1\ M^+$ $10 \times 0.05\ M^+$

MRC

Q) Assume that the probability for rain on a day is 0.4. An umbrella salesman can earn ₹ 400 per day in case of rain on that day and will lose ₹ 100 per day if there is no rain. The expected earnings (in ₹) per day of the salesman is:

(a) 400

(b) 200

(c) 100

(d) 0

	Rain	Not Rain
x	400	−100
$P(x)$	0.4	0.6

$\sum x\,p(x) = 400 \times 0.4\ M^+$

$100 \times 0.6\ M^-$

MRC

Q) The probability distribution of a random variable x is given below:

x	1	2	4	5	6
$P(x)$	0.15	0.25	0.20	0.30	0.10

What is the standard deviation of x?

(a) 1.49

(b) 1.56

(c) 1.69

(d) 1.72

Variance $= E(x^2) - (E(x))^2$

$E(x) = \Sigma x\, p(x)$
$= 3.55$

$E(x^2) = \Sigma x^2\, p(x)$

1x = x 0.15 M+ 2x = x 0.25 M+
4x = x 0.2 M+ 5x = x 0.3 M+ 6x = x 0.1 M+
MRC
= 15.45

Variance $= 15.45 - (3.55)^2$
$= 2.875$

$SD = \sqrt{2.875} = 1.69$

Bayes' Theorem Explained:

Bayes' Theorem helps us calculate the probability of an event E_i, given that another event A has already occurred. This is particularly useful when dealing with conditional probabilities.

The Formula:

$$P(E_i/A) = \frac{P(A/E_i) \cdot P(E_i)}{\sum_{i=1}^{n} P(A/E_i) \cdot P(E_i)}$$

- $P(E_i/A)$: The probability of event E_i occurring, given that A has occurred.
- $P(A/E_i)$: The probability of A, given that E_i has occurred.
- $P(E_i)$: The prior probability of event E_i.
- $\sum_{i=1}^{n} P(A/E_i) \cdot P(E_i)$: The sum of all probabilities where A is conditioned on each E_i, weighted by the prior probabilities of the E_is.

How It Works:

1. **Mutually Exclusive Events:**
   - Events $E_1, E_2, ..., E_n$ must be mutually exclusive, meaning no two events can occur simultaneously.
   - A is the observed event.
2. **Finding $P(E_i/A)$:**
   - The theorem updates our belief about the likelihood of E_i after observing A.
3. **Total Probability Rule:**
   - The denominator $\sum_{i=1}^{n} P(A/E_i) \cdot P(E_i)$ ensures that all possible events contributing to A are considered.

Example Application:

Imagine you're diagnosing a disease based on symptoms. E_i could represent different diseases, and A represents the observed symptom. Bayes' Theorem helps calculate the probability of each disease given the symptom.

This theorem is fundamental in probability theory and is widely applied in areas like medical diagnosis, machine learning, and decision-making under uncertainty.

Q) There are 3 bags:

Bag 1 consists of 3 red and 4 white balls.

Bag 2 consists of 4 red and 5 white balls.

Bag 3 consists of 5 red and 2 white balls.

If one ball is drawn at random, what is the probability that the ball is selected from Bag 2 given that the selected ball is red?

(a) 0.28

(b) 0.252

(c) 0.112

(d) 0.184

Q) According to Bayes' theorem,

$$P(E_k/A) = \frac{P(E_k)P(A/E_k)}{\sum_{i=1}^{n} P(E_i)P(A/E_i)}$$

here:

(a) $\boxed{E_1, E_2, \ldots \text{ are mutually exclusive}}$

(b) $P(E/A_1), P(E/A_2), \ldots$ are equal to 1

(c) $P(A_1/E), P(A_2/E), \ldots$ are equal to 1

(d) A and E_1 are disjoint sets

Odds in Favor of an Event

- If the odds in favor of an event are given as a:b:
  - a represents the number of favorable outcomes.
  - b represents the number of outcomes against the event.
  - The total number of outcomes is $a + b = n(S)$, where $n(S)$ is the total sample space.

A B C D

$1 : 3 \} \quad \frac{1}{1+3} = \frac{1}{4}$

$\frac{3}{3+1} \{ 3 : 1 \} = \frac{1}{3+1} = \frac{1}{4}$

$= \frac{3}{4}$

Odds Against an Event

- If the odds against an event are given as **a:b**:
  - **b** represents the number of favorable outcomes.
  - **a** represents the number of outcomes against the event.
  - Again, the total number of outcomes is $a + b = n(S)$.

Q) The odds in favor of A solving a problem are $5 : 7$ and the odds against B solving the same problem are $9 : 6$. What is the probability that if both of them try, the problem will be solved?

(a) $\frac{117}{180}$
(b) $\frac{181}{200}$
(c) $\frac{147}{180}$
(d) $\frac{119}{180}$

$P(A) = \frac{5}{12}$ $P(A)' = \frac{7}{12}$ $P(B) = \frac{6}{15}$ $P(B)' = \frac{9}{15}$

$P(A) \times P(B)' + P(B) \times P(A)' + P(A) \times P(B)$

$\frac{5}{12} \times \frac{9}{15} + \frac{6}{15} \times \frac{7}{12} + \frac{5}{12} \times \frac{6}{15} = \frac{117}{180}$

Q) The odds against A solving a certain problem are $\boxed{4 : 3}$, and the odds in favor of B solving the same problem are $7 : 5$. What is the probability that the problem will be solved if they both try?

(a) $\frac{15}{21}$
(b) $\frac{16}{21}$
(c) $\frac{17}{21}$
(d) $\frac{13}{21}$

$P(A) = \frac{3}{7}$ $P(A)' = \frac{4}{7}$ $P(B) = \frac{7}{12}$ $P(B)' = \frac{5}{12}$

$1 - P(A)' \times P(B)' = 1 - \frac{4}{7} \times \frac{5}{12}$

$= 1 - \frac{20}{84}$

$= \frac{64}{84}$

Q) The odds that a book will be favorably received by 3 independent reviewers are $5 : 2$, $4 : 3$, and $3 : 4$ respectively. What is the probability that out of 3 reviewers, a majority will be favorable?

A, B, C

(a) $\frac{209}{343}$
(b) $\frac{209}{434}$
(c) $\frac{209}{443}$
(d) $\frac{209}{350}$

$P(A) = \frac{5}{7}$ $P(A)' = \frac{2}{7}$ $P(B) = \frac{4}{7}$ $P(B)' = \frac{3}{7}$

$P(C) = \frac{3}{7}$ $P(C)' = \frac{4}{7}$

$P(A) \times P(B) \times P(C)' + P(A) \times P(B)' \times P(C) + P(A)' \times P(B) \times P(C) + P(A) \times P(B) \times P(C)$

$\frac{5}{7} \times \frac{4}{7} \times \frac{4}{7} + \frac{5}{7} \times \frac{3}{7} \times \frac{3}{7} + \frac{2}{7} \times \frac{4}{7} \times \frac{3}{7} + \frac{5}{7} \times \frac{3}{7} \times \frac{4}{7}$

Q) If $p : q$ are the odds in favor of an event, then the probability of that event is:

(a) $\frac{p}{q}$
(b) $\frac{p}{p+q}$
(c) $\frac{q}{p+q}$
(d) $\frac{q}{p}$

$\boxed{\frac{p}{p+q}}$

Extra Questions With the Concept of Combinations

3

Q) A bag contains 12 balls of which 3 are red. Five balls are drawn at random. Find the probability that in 5 balls, 3 are red.

(a) $\frac{3}{132}$
(b) $\frac{5}{396}$
(c) $\frac{1}{36}$
(d) $\frac{1}{22}$

$P(A) = \frac{n(A)}{n(S)}$

$n(S) = {}^{12}C_5 = \frac{12 \times 11 \times 10 \times 9 \times 8}{120} = 792$

$n(A) = {}^3C_3 \times {}^9C_2$

$P(A) = \frac{36}{792} = 1/22$

Q) In a bag, there were 5 white, 3 red, and 2 black balls. Three balls are drawn at a time. What is the probability that the three balls drawn are white?

(a) $\frac{1}{12}$
(b) $\frac{1}{24}$
(c) $\frac{1}{120}$
(d) None of these

$n(A) = {}^5C_3$

$n(S) = {}^{10}C_3$

$P(A) = \frac{10}{120} = 1/12$

Q) Four married couples have gathered in a room. Two persons are selected at random amongst them. Find the probability that the selected persons are a gentleman and a lady but not a couple.

(a) $\frac{1}{7}$
(b) $\frac{3}{7}$
(c) $\frac{1}{8}$
(d) $\frac{3}{8}$

4 G & 4 L are present

$\frac{{}^4C_1 \times {}^3C_1}{{}^8C_2} = \frac{4 \times 3}{28} = \frac{12}{28}$

* DIY

Q) A team of 5 is to be selected from 8 boys and 3 girls. Find the probability that it includes two particular girls.

(a) $\frac{2}{30}$
(b) $\frac{1}{5}$
(c) $\frac{2}{11}$
(d) $\frac{8}{9}$

$n(s) = {}^{14}C_4$

Q) From 6 positive and 8 negative numbers, 4 numbers are chosen at random without replacement and are then multiplied. The probability that the product of the chosen numbers will be a positive number is:

Case I Or Case II or Case III

2P and 2N 4P 4N

$\frac{8 \times 7 \times 6 \times 5}{24}$

(a) $\frac{409}{1001}$
(b) $\frac{70}{1001}$
(c) $\frac{505}{1001}$
(d) $\frac{420}{1001}$

$$\frac{{}^6C_2 \times {}^8C_2 + {}^6C_4 + {}^8C_4}{{}^{14}C_4} = \frac{15 \times 28 + 15 + 70}{1001} = \frac{505}{1001}$$

Chapter 16: Theoretical Distributions

Theoretical Probability Distribution:

1. **Definition:**
   - A probability distribution distributes the total probability (1) across different mass points (discrete random variable) or class intervals (continuous random variable). This is known as a **Theoretical Probability Distribution**, existing conceptually.
2. **Applications:**
   - Useful for **short-term projections** and predictions about the future.

3. **Characteristics:**
   - Exhibits properties of observed distributions, including measures such as:
     - **Mean**
     - **Median**
     - **Mode**
     - **Standard Deviation**
4. **Types of Probability Distribution:**
   - **Discrete:** Deals with countable outcomes.
     - Examples:
       - **Binomial Distribution (BD)**
       - **Poisson Distribution (PD)**
   - **Continuous:** Deals with outcomes over a continuum.
     - Example:
       - **Normal Distribution**
5. **Important Note:**
   - A **parameter** is a characteristic of a population.
     - Example: **Population Mean** is a parameter.

PD
Discrete
Continuous
Binomial
Biparametric
n, p
no: of trials
p → probability of success in a single trial
Poisson
one parameter
Normal Distribution
(Biparametric)

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Key Features of Binomial Distribution:

1. **Based on Bernoulli Trials:**
   - A Bernoulli trial is a random experiment with exactly two possible outcomes: "success" and "failure."
2. **Discrete Probability Distribution:**
   - The distribution is for discrete random variables.
3. **Biparametric Distribution:**
   - Two parameters define it:
     - n: Number of trials
     - p: Probability of success in a single trial

Probability Mass Function (PMF): The probability of exactly x successes in n trials is

$$\boxed{P(x) = nC_x p^x q^{n-x}}$$

$P(x) = \boxed{{}^nC_x} p^x q^{n-x}$

Where:

- nC_x (combination formula) is:

$$nC_x = \frac{n!}{x!(n-x)!}$$

- x: Number of successes
- $q = \boxed{1 - p}$: Probability of failure

Properties of the Binomial Distribution:

1. Mean of a Binomial Distribution:

$n \times p$

- The **mean** of a binomial distribution is given by:

$$\boxed{\text{Mean} = np}$$

variance = npq

- n: Number of trials.
- p: Probability of success in each trial.

$E(x) = \text{mean}$

$np =$

- This represents the expected number of successes in n trials.

2. Standard Deviation of a Binomial Distribution:

- The **standard deviation** measures the spread of the distribution and is given by:

$$\boxed{\sigma = \sqrt{npq}}$$

- $q = 1 - p$: Probability of failure.
- The square root ensures that the measure remains in the same units as the random variable X.

3. Variance of a Binomial Distribution:

- The **variance** quantifies the dispersion of the distribution and is given by:

$$\text{Variance} = npq$$

  - Variance is the square of the standard deviation: $(\sqrt{npq})^2 = npq$.

- Key observations:
  - The **mean** is always greater than the **variance** for a binomial distribution.
  - Variance is maximum when $p = q = 0.5$ (equal likelihood of success and failure).

4. Mode of a Binomial Distribution:

- The **mode** is the most frequent value in the distribution.
- Conditions for the mode:
  - If $(n+1)p$ is a **non-integer**, the mode is the **integral part** of $(n+1)p$.
  - If $(n+1)p$ is an **integer**, the binomial distribution is **bimodal**, with modes:

$$\text{Modes} = (n+1)p \text{ and } (n+1)p - 1$$

$(n+1)p = 4$

4, 3

$(n+1)p = 20$, 20, 19

4, 3

$(n+1)p = 13$

mode = 13, 12

$(n+1)p = \boxed{4}.5$

4

$(n+1)p = 3.5$

Mode = 3

$(n+1)p = 100$

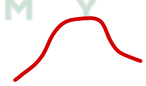
100, 99

$(n+1)p = 1000.5$

1000

5. Symmetry and Skewness:

- If $p = q = 0.5$: The distribution is symmetric.
- If $p < 0.5$: The distribution is positively skewed.
- If $p > 0.5$: The distribution is negatively skewed.



6. Applications of Binomial Distribution:

- **Conditions for Applicability:**
  - Trials are independent.
  - Each trial has two possible outcomes: success (p) or failure (q).

Additive Property of Binomial Distribution:

- If $X \sim B(n_1, p)$ and $Y \sim B(n_2, p)$, then:

$$(X + Y) \sim B(n_1 + n_2, p)$$

- This property allows combining independent binomial random variables with the same probability of success p.

Examples of Binomial Experiments:

1. Tossing a coin n times and counting the number of heads (successes).
2. Rolling a die n times and counting the occurrences of a specific number.
3. Testing light bulbs and counting how many fail.

Notation:

If X is a binomial random variable, it is denoted as:

$$\boxed{X \sim B(n, p)}$$

$1 - \frac{1}{4} = \frac{3}{4}$

Where n is the number of trials and p is the probability of success.

Key Observations: $X \sim B(6, \frac{1}{4})$ npq $= 6 \times \frac{1}{4} \times \frac{3}{4} = 1.125$

- When $p = q$, the binomial distribution is perfectly symmetric.
- The binomial distribution approaches the normal distribution as n increases, provided p is not too close to 0 or 1.

Usage of Binomial Distribution:

- Used to calculate probabilities in binary outcomes.
- Helps in quality control processes, such as defect analysis.
- Widely used in fields like biology (e.g., Mendelian genetics), marketing, and finance.

The third central moment for a binomial distribution $X \sim B(n, p)$ (where n is the number of trials and p is the probability of success) is given by:

$$\mu_3 = npq(1 - 2p)$$

mean variance

Q) For binomial distribution $E(X) = 2$, $V(X) = \frac{4}{3}$. Find the value of n.

(a) 3

(b) 4

(c) 5

(d) 6

$\boxed{np} = 2$ $\boxed{np}q = \frac{4}{3}$ $p = 1 - \frac{2}{3}$

$2 \times q = \frac{4}{3}$ $p = \frac{1}{3}$

$np = 2$ $q = \frac{2}{3}$

$n \times \frac{1}{3} = 2$

$n = 6$

Q) What are the parameters of binomial distribution?

(a) n

(b) p

(c) Both n and p

(d) None of these

$P(x) = {}^nC_x p^x q^{n-x}$

$n = 6$

Q) For a Binomial distribution $B(6, p)$, $P(X = 2) = 9P(X = 4)$, then p is:

(a) $\frac{1}{2}$

(b) $\frac{1}{3}$

(c) $\frac{10}{13}$

(d) $\frac{1}{4}$

$\cancel{{}^6C_2} \times \cancel{p^2} \times q^4 = 9 \times \cancel{{}^6C_4} \times p^4 \times q^2$

$\frac{q^2}{p^2} = 9$ $\frac{1-p}{p} = 3$

$\boxed{\left(\frac{1-p}{p}\right)^2 = 9}$ $1 - p = 3p$

$1 = 4p$

$p = \frac{1}{4}$

Q) If for a Binomial distribution $B(n, p)$, the mean = 6 and variance = 2, then p is:

(a) $\frac{2}{3}$

(b) $\frac{1}{3}$

(c) $\frac{3}{5}$

(d) $\frac{1}{4}$

$\boxed{np} = 6$ $npq = 2$

$6 \times q = 2$

$q = \frac{2}{6} = \frac{1}{3}$

$p = 1 - q = 1 - \frac{1}{3} = \frac{2}{3}$

Q) For binomial distribution:

(a) Variance < Mean

(b) Variance = Mean

(c) Variance > Mean

(d) None of the above

$(n+1)p$

Q) The mode of the Binomial Distribution for which the mean is 4 and variance is 3 is equal to?

(a) 4

(b) 4.25

(c) 4.5

(d) 4.1

$np = 4$ $\quad npq = 3$

$4 \times q = 3$

$q = \frac{3}{4}$

$p = 1 - q$

$= 1 - \frac{3}{4} = \frac{1}{4}$

$n \times \frac{1}{4} = 4$

$n = 16$

$(n+1)p = \frac{17}{4}$

$= 4.25$

Mode = 4

Q) If x is a binomial variable with parameters 15 and $\frac{1}{3}$, then the value of the mode of the distribution:

(a) 5

(b) 5 and 6

(c) 5.50

(d) 6

$n = 15 \quad p = \frac{1}{3}$

$(n+1)p = \frac{16}{3} = 5.33$

Mode = 5

Q) Find the mode when $n = 15$ and $p = \frac{1}{4}$ in binomial distribution:

(a) 4

(b) 4 and 3

(c) 4.2

(d) 3.75

$(n+1)p = \frac{16}{4} = 4$

Mode = 4, 3

$^nC_x p^x q^{n-x}$

Q) In a binomial distribution with 5 independent trials, the probabilities of 2 and 3 successes are 0.4362 and 0.2181, respectively. Parameter p of the binomial distribution is:

(a) $\frac{3}{4}$

(b) $\frac{1}{3}$

(c) $\frac{2}{3}$

(d) $\frac{1}{4}$

$P(x=2) = 0.4362$

$^5C_2 \times p^2 q^3 = 0.4362$ — (1)

$P(x=3) = 0.2181$

$^5C_3 \times p^3 q^2 = 0.2181$ — (2)

eq (1) ÷ (2)

$\frac{\cancel{^5C_2} \times \cancel{p^2} \times \cancel{q^3}}{\cancel{^5C_3}\, p^{\cancel{3}}\, \cancel{q^2}} = \frac{0.4362}{0.2181}$

$\frac{q}{p} = 2$

$\frac{1-p}{p} = 2$

$1 - p = 2p$

$1 = 3p$

$p = 1/3$

$np = 4$ $\quad npq = \frac{4}{3}$ $\quad 4 \times q = \frac{4}{3}$ $\quad q = \frac{1}{3}$ $\quad p = \frac{2}{3}$ $\quad \frac{2}{3} \times n = 4$ $\quad n = 6$

Q) Mean and variance of a binomial distribution are 4 and $\frac{4}{3}$, respectively. Then $P(X \geq 1)$ will be:

(a) $\frac{728}{729}$

(b) $\frac{1}{729}$

(c) $\frac{723}{729}$

(d) None of the above.

$P(x \geq 1) = 1 - P(x < 1)$

$= 1 - P(x = 0)$

$= 1 - {}^6C_0 \left(\frac{2}{3}\right)^0 \times \left(\frac{1}{3}\right)^6$

$= 1 - \frac{1}{729} = \frac{728}{729}$

$n = 6$ $\quad x = 2$

Q) If six coins are tossed simultaneously, the probability of obtaining exactly two heads is:

(a) $\frac{1}{64}$

(b) $\frac{63}{64}$

(c) $\frac{15}{64} = 0.234375$

(d) None of these

$P(x=2) = {}^nC_2\, p^2 q^{n-2}$

$= {}^6C_2 \times \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^4$

$= 15 \times (0.5)^6 = 0.234375$

Q) If x is a binomial variable with parameters n and p, then x can assume:

(a) Any value between 0 and n

(b) Any value between 0 and n, both inclusive

(c) Any whole number between 0 and n, both inclusive

(d) Any number between 0 and infinity

Q) For a Binomial distribution, if variance = (Mean)^2, then the values of n and p will be:

(a) 1 and $\frac{1}{2}$

(b) 2 and $\frac{1}{2}$

(c) 3 and $\frac{1}{2}$

(d) 1 and 1

$\cancel{n}\cancel{p}q = n^{\cancel{2}}p^{\cancel{2}}$

$1 - p = np$

Option Hit

option (a)

$LHS = 1 - \frac{1}{2} = \frac{1}{2}$

$RHS = 1 \times \frac{1}{2} = \frac{1}{2}$

$LHS = RHS$

p = 0.2 q = 0.8 n = 6

Q) The incidence of skin diseases in a chemical plant occurs in such a way that its workers have a 20% chance of suffering from it. What is the probability that 6 workers, 4 or more, will have skin diseases? $P(x \geq 4)$

(a) 0.1696

(b) 0.01696

(c) 0.1643

(d) 0.01643

$P(x \geq 4) = P(x=4) + P(x=5) + P(x=6)$

$= {}^6C_4 \times (0.2)^4 \times (0.8)^2 + {}^6C_5 \times (0.2)^5 \times (0.8)^1 + {}^6C_6 \times (0.2)^6 \times (0.8)^0$

$= 15 \times (0.2)^4 \times (0.8)^2 + 6 \times (0.2)^5 \times (0.8) + 1 \times (0.2)^6$

0.2 × = 3 times) M+
× 0.8 × 0.8 × 15

0.2 × = 4 times) M+
× 0.8 × 6

0.2 × = 5 times) M+

MRC

Poisson Distribution (PD)

1. **Definition:**
   - Poisson distribution is a discrete probability distribution used to model the number of events occurring in a fixed interval of time or space, given that these events happen with a known constant mean rate and independently of each other.

2. **Key Parameters:**

   m → mean / variance

   SD = $\sqrt{m}$

   λ
   - m: The mean of the distribution, which is also the variance.

3. **Characteristics:**
   - It is used when $n \to \infty$ and $p \to 0$ while np remains finite.
   - Uniparametric distribution: The parameter is m.

4. **Formulas:**
   - Mean $= m$
   - Variance $= m$
   - Standard Deviation $= \sqrt{m}$

$m = 4$
Modes = 4, 3
$m = 10$
Mode = 10, 9

$m = 7.5$
Mode = 7
$M = 2.36$
Mode = 2

5. **Mode:**
   - If m is a non-integer: The mode is the integral part of m.
   - If m is an integer: The distribution is bimodal, and the modes are m and $m - 1$.

6. **Probability Mass Function (PMF):**

$\frac{e^{-m} \times m^x}{x!}$

$$P(x) = \frac{e^{-m} m^x}{x!} \quad \text{for } x = 0, 1, 2, \ldots, \infty$$

   where:
   - $e = 2.72$ (base of the natural logarithm),
   - x: Poisson variate.

7. **Interpretation:**

$x \sim P(\sqrt{2})$
$m = \sqrt{2}$

   - $x \sim P(m)$: x follows a Poisson distribution with parameter m.

Additive Property of Poisson Distribution

Additive Property:

If $X \sim P(m_1)$ and $Y \sim P(m_2)$,
then $(X + Y) \sim P(m_1 + m_2)$.

This property holds because the sum of two independent Poisson random variables with parameters m_1 and m_2 also follows a Poisson distribution with parameter equal to the sum of the individual means $m_1 + m_2$.

Applications of Poisson Distribution

1. **When to Use:**
   - The Poisson distribution is applicable when:
     - The total number of events or trials is very large.
     - The probability of success for each trial is very small.
   - It models rare events occurring over a fixed interval of time or space.

2. Examples: PP
   - **Printing Errors**: Distribution of the number of printing mistakes per page in a large book.
   - **Road Accidents**: Distribution of the number of road accidents on a busy road per minute.

Skewness:

- **Note**: Poisson's distribution is always positively skewed, meaning the tail on the right side is longer or fatter than on the left side.

Q) In a Poisson distribution, $P(X = 0) = P(X = 2)$. Find $E(X)$: $\to m$

(a) $\sqrt{2}$

(b) 2

(c) -1

(d) 0

$\frac{e^{-m} m^{x}}{x!}$

$\frac{\cancel{e^{-m}} \times m^0}{0!} = \frac{\cancel{e^{-m}} \times m^2}{2!}$

$2 = m^2$

$m = \sqrt{2}$

Q) If the standard deviation of a Poisson distribution is 2 then its:

(a) Mode is 2

(b) Mode is 4

(c) Modes are 3 and 4

(d) Modes are 4 and 5

$\sqrt{m} = 2$

$m = 4$

Mode = 4, 3

Q) For Poisson Distribution:

(a) Mean and Standard Deviations are equal

(b) Mean and Variance are equal

(c) Standard Deviation and Variance are equal

(d) Both (a) and (b) are correct

$m = 30$

Q) If a random variable x follows a Poisson distribution such that $\boxed{E(x) = 30}$, then the variance of the distribution is:

(a) 7

(b) 5

(c) 30

(d) 20

Q) It is a Poisson variate such that $P(X = 1) = 0.7$, $P(X = 2) = 0.3$, then $P(X = 0)$ is:

(a) $e^{6/7}$

(b) $e^{-6/7}$

(c) $e^{-2/3}$

(d) $e^{-1/3}$

$\frac{e^{-m} m^1}{1!} = 0.7$ —(1) $\quad e^{-m} \times m^2 = 0.6$ —(2)

eq (2) ÷ 1

$\frac{\cancel{e^{-m}} \times m^2}{\cancel{e^{-m}} \times m} = \frac{0.6}{0.7}$ $\quad m = \frac{6}{7}$

$P(x=0) = \frac{e^{-m} m^0}{0!} = e^{-6/7}$

Q) The average number of advertisements per page appearing in a newspaper is 3. What is the probability that in a particular page zero advertisements are there?

$m = 3$

(a) e^{-3}

(b) e^{0}

(c) e^{+3}

(d) e^{-1}

$P(x=0) = \frac{e^{-m} \times m^0}{0!} = e^{-3}$

Q) If, for a Poisson distributed random variable X, the probability for X taking value 2 is 3 times the probability for X taking value 4, then the variance of X is:

(a) 4

(b) 3

(c) 2

(d) 5

$P(x=2) = 3 \times P(x=4)$

$\frac{\cancel{e^{-m}} \times m^2}{2!} = \cancel{3} \times \frac{\cancel{e^{-m}} \times m^4}{\cancel{24}\ 8\ 4}$ $\quad m^2$

$m^2 = 4 \quad m = 2$

Q) A renowned hospital usually admits 200 patients every day. One percent of patients, on average, require special room facilities. On one particular morning, it was found that only one special room is available. What is the probability that more than 3 patients would require special room facilities?

(a) 0.1428

(b) 0.1732

(c) 0.2235

(d) 0.3450

$m = 200 \times 1\% = 2$

$P(x > 3) = 1 - P(x \leq 3)$

$= 1 - [P(x=3) + P(x=2) + P(x=1) + P(x=0)]$

$= 1 - \left[\frac{e^{-2}\, 2^3}{3!} + \frac{e^{-2} \times 2^2}{2!} + \frac{e^{-2} \times 2^1}{1!} + e^{-2}\right]$

$= 1 - e^{-2}\left(\frac{8}{6} + \frac{4}{2} + 2 + 1\right)$

8÷6 +2 +2 +1 m+

2.72 × =
÷ =
× MRC
change sign
+1

Normal Distribution

Definition:

The **Normal Distribution** is a continuous probability distribution widely used to model natural phenomena like heights, weights, test scores, or any variable following a bell-shaped curve.

Key Features:

1. **Parameters:**
   - μ **(mean)**: The central value or average of the distribution.
   - σ^2 **(variance)**: The measure of spread or variability. It determines the width of the bell curve.
2. **Probability Density Function (PDF):** The formula for the normal distribution is:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

$-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2$

$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e$

   - x: Random variable.
   - μ: Mean of the distribution.
   - σ: Standard deviation.

The PDF defines the probability of x taking a particular value within the range $-\infty < x < \infty$.

3. **Symmetry:**
   - The curve is perfectly symmetrical around the mean (μ).
   - The left and right sides of the distribution are mirror images.
4. **Skewness:**
   - A normal distribution has **zero skewness**, meaning it is not tilted to the left or right.

5. **Equality of Measures:** Symmetry
   - For a normal distribution: **Mean = Median = Mode.**

6. **Shape Independence:** μ, σ
   - The shape of the distribution remains consistent regardless of the values of μ and σ. The mean shifts the center, while the variance scales the spread.
7. **Unimodal:**
   - The distribution is unimodal, meaning it has a single peak at the mean.

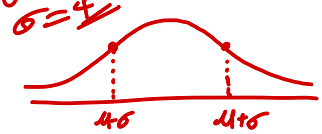
1. Points of Inflection:

6, 14.

- The **points of inflection** occur at $\mu - \sigma$ and $\mu + \sigma$, where:
  - μ: Mean
  - σ: Standard deviation

$10 - \sigma = 6$
$\sigma = 4$

$\mu - \sigma = 6$
$\mu + \sigma = 14$



- At these points, the curve transitions from concave to convex or vice versa.

$2\mu = 20$
$\mu = 10$

2. Quartile and Mean Deviations:

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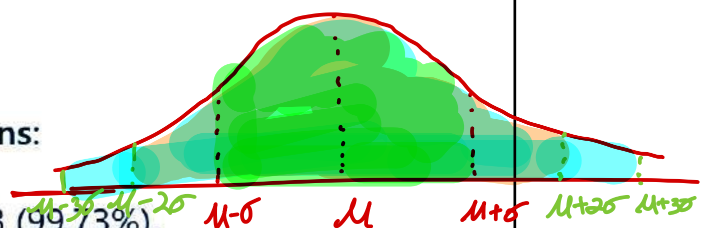
- **Quartiles (Q1 and Q3):**
  - $Q1 = \mu - 0.6745\sigma$
  - $Q3 = \mu + 0.6745\sigma$
  - The median is exactly in the middle of $Q1$ and $Q3$.

QD
2SD = 2.5MD = 3QD

- **Quartile Deviation (QD):**
  - $QD = \frac{Q3 - Q1}{2} = 0.6745\sigma$
- **Mean Deviation (MD):**
  - $MD = 0.8\sigma$

3. Area Under the Normal Curve:

- The total area under the curve is 1, representing 100% of the probability
- The area is symmetric around the mean (μ):
  - Area from $-\infty$ to 0 = 0.5.
  - Area from 0 to $+\infty$ = 0.5.
- **Specific areas between standard deviations:**



  - Between $\mu - 3\sigma$ and $\mu + 3\sigma$: 0.9973 (99.73%).
  - Between $\mu - 2\sigma$ and $\mu + 2\sigma$: 0.9546 (95.46%).
  - Between $\mu - \sigma$ and $\mu + \sigma$: 0.6828 (68.28%).

Additive Property:

If $X \sim N(\mu_1, \sigma_1^2)$ and $Y \sim N(\mu_2, \sigma_2^2)$, then:

- $(X + Y) \sim N\left[(\mu_1 + \mu_2), (\sigma_1^2 + \sigma_2^2)\right]$
- The standard deviation of $(X + Y)$ is:

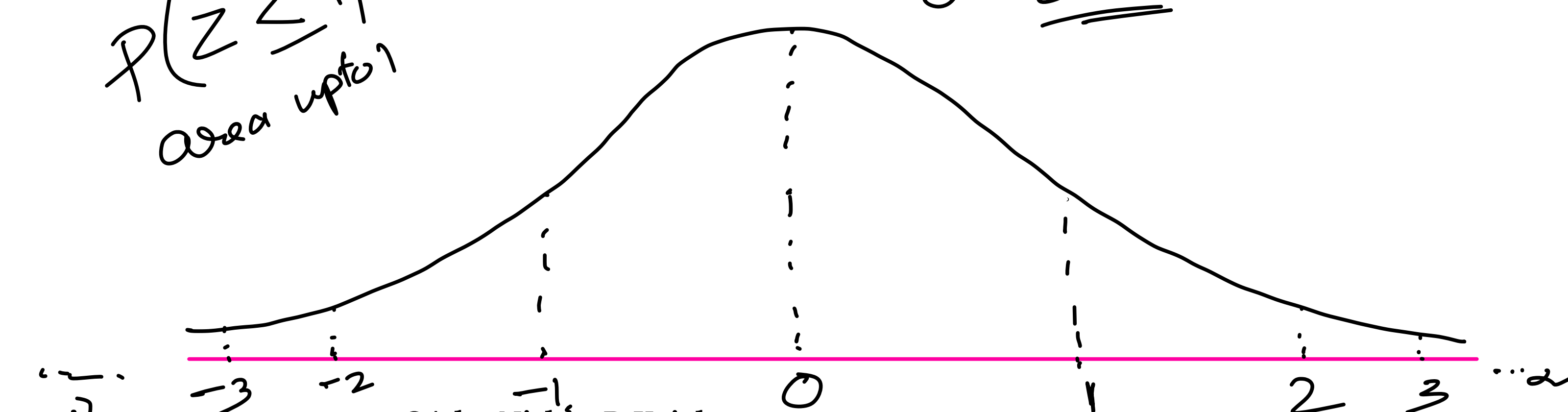
$$\sqrt{\sigma_1^2 + \sigma_2^2}$$

Area-Related Problems:

$P(Z \leq 1)$

area upto 1

0.6828



$\phi(2) - \phi(1)$

$\frac{\mu+\sigma-\mu}{\sigma}$

1

$P(x<3)$

Area-related problems in a normal distribution involve calculating probabilities that correspond to certain intervals of the variable X. These probabilities are found using the **cumulative distribution function (CDF)** of the standard normal distribution, denoted by $\phi(K)$.

(i) For Single Value: $P(X<a)$

- **Conversion to Standard Normal Form:** The normal variable X is converted into the standard normal variable Z using:

$$Z=\frac{X-\mu}{\sigma}$$

$Z=\frac{x-\mu}{\sigma}$

$\frac{3-\mu}{\sigma}$

  where:

  - μ is the mean of X,
  - σ is the standard deviation of X.

- **Probability Expression:**

$$P(X<a)=P\left(Z<\frac{a-\mu}{\sigma}\right)=\phi(K)$$

area upto K

  where $K=\frac{a-\mu}{\sigma}$.

- **Interpretation:** $\phi(K)$ gives the area under the normal curve from $-\infty$ to K.

(ii) For a Range: $P(a<X<b)$

- **Conversion to Standard Normal Form:** The range $a<X<b$ is converted to the standard normal variable Z:

$$P(a<X<b)=P(K_1<Z<K_2)$$

  where:

  - $K_1=\frac{a-\mu}{\sigma}$,
  - $K_2=\frac{b-\mu}{\sigma}$.

$P(a<x<b)$

$P\left(\underbrace{\frac{a-\mu}{\sigma}}_{K_1}<Z<\underbrace{\frac{b-\mu}{\sigma}}_{K_2}\right)$

$\phi(K_2)-\phi(K_1)$

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- **Probability Expression:**

$$P(a < X < b) = \phi(K_2) - \phi(K_1)$$

Here, $\phi(K_2)$ is the area under the curve up to K_2, and $\phi(K_1)$ is the area up to K_1

(iii) Special Area Results:

The following probabilities are standard results for a normal distribution:

- Total area under the curve: 1
- Area to the left of the mean ($X = \mu$): 0.5

$\boxed{\phi(-2) = 1 - \phi(2)}$

1. Symmetry of the normal curve:
   - $\boxed{\phi(-K)} = 1 - \phi(K)$
   - This property allows calculation of areas for negative K.

$\phi(-3) = 1 - \phi(3)$

2. Boundary Area Formulae:
   - Area from $-\infty$ to 0: 0.5,
   - Area from 0 to K: $\phi(K) - 0.5$ (for $K > 0$).

The standard normal distribution is a normal distribution that has been standardized. This means:

1. The **mean** (μ) is 0.
2. The **standard deviation** (σ) is 1.

Q) Shape of Normal Distribution Curve:

(a) Depends on its parameters

(b) Does not depend on its parameters

(c) Either (a) or (b)

(d) Neither (a) nor (b)

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Q) The variance of the $\boxed{\text{standard normal distribution}}$ is:

(a) 1

(b) μ

(c) σ^2

(d) 0

$1^2 = 1$

Q) The area under the Normal curve is:

(a) 1

(b) 0

(c) 0.5

(d) -1

Q) For a normal distribution $N(\mu, \sigma^2)$, $P(\mu - 3\sigma < x < \mu + 3\sigma)$ is equal to:

(a) 0.9973

(b) 0.9546

(c) 0.9899

(d) 0.9788

Q) If the inflexion points of a Normal Distribution are 6 and 14, find its Standard Deviation:

(a) 4

(b) 6

(c) 10

(d) 12

$\mu - \sigma = 6$

$\mu + \sigma = 14$

$2\mu = 20$

$\mu = 10$

$10 - 6 = \sigma$

$= 4$

Q) In a Normal Distribution:

(a) The first and second quartiles are equidistant from the median

(b) The second and third quartiles are equidistant from the median

(c) The first and third quartiles are equidistant from the mean $=$ Median $= Q_2$

(d) None of the above

Q_1 Q_2 Q_3

Q) Which of the following is **not** a characteristic of a normal probability distribution?

(a) Mean of the normally distributed population lies at the center of its normal curve.

(b) It is multi-modal.

(c) The mean, median, and mode are equal.

(d) It is a symmetric curve.

Q) The wages of workers of a factory follow:

(a) Binomial distribution

(b) Poisson distribution

(c) Normal distribution

(d) Chi-square distribution

Q) The normal curve is:

(a) Positively skewed

(b) Negatively skewed

(c) Symmetrical

(d) All these

Q) If the area of the standard normal curve between $z = 0$ to $z = 1$ is 0.3412, then the value of $\phi(1)$ is:

(a) 0.5000

(b) 0.8413

(c) -0.5000

(d) 1

$\phi(1)$ area upto 1

= Area upto 0 + area from 0 to 1

$= 0.5 + 0.3412$

$= 0.8412$

Q) If for a normal distribution $Q_1 = 54.52$ and $Q_3 = 78.86$, then the median of the distribution is:

(a) 12.17

(b) 39.43

(c) 66.69

(d) None of these

$M = Q_2 = \frac{Q_3 + Q_1}{2}$

$= 66.69$

Q) What is the mean of X having the following density function?

$\mu =$

$$f(x) = \boxed{\frac{1}{4\sqrt{2\pi}}} e^{-\frac{(x-10)^2}{32}}, \ -\infty < x < \infty$$

$$P(x) = \boxed{\frac{1}{\sigma\sqrt{2\pi}}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

(a) 10

(b) 4

(c) 40

(d) None of the above

$$\frac{1}{4\cancel{\sqrt{2\pi}}} = \frac{1}{\sigma\cancel{\sqrt{2\pi}}} \qquad \sigma = 4$$

Q) What is the SD and mean of x if $f(x) = \boxed{\frac{\sqrt{2}}{\sqrt{\pi}}} e^{2(x-3)^2}, -\infty < x < \infty$?

$$f(x) = \boxed{\frac{1}{\sigma\sqrt{2\pi}}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

(a) 3, $\frac{1}{2}$

(b) 3, $\frac{1}{4}$

(c) 2, $\frac{1}{2}$

(d) 2, $\sqrt{2}$

$$\frac{\sqrt{2}}{\cancel{\sqrt{\pi}}} = \frac{1}{\sigma\sqrt{2}\times\cancel{\sqrt{\pi}}}$$

$$\sigma = \frac{1}{\sqrt{2}\times\sqrt{2}} = \frac{1}{2} \qquad \mu = 3$$

$\mu = 50 \quad \sigma = 15$

Q) For a certain type of mobile, the length of time between charges of the battery is normally distributed with a mean of 50 hours and a standard deviation of 15 hours. A person owns one of these mobiles and wants to know the probability that the length of time will be between 50 and 70 hours is (given φ(1.33) = 0.9082, φ(0) = 0.5)?

$$P(50 < x < 70)$$

(a) -0.4082

(b) 0.5

(c) 0.4082

(d) -0.5

$$P\left(\frac{50-\mu}{\sigma} < Z < \frac{70-\mu}{\sigma}\right)$$

$$P(0 < Z < 1.33)$$

$$= \phi(1.33) - \phi(0) = 0.9082 - 0.5 = 0.4082$$

Q) Let X be a normal distribution with mean 2.5 and variance 1. If $P[a < X < 2.5] = 0.4772$ and the cumulative normal probability value at 2 is 0.9772, then $a = ?$

$\mu = 2.5 \quad \sigma = 1$

(a) 0.5

(b) 3

(c) -3.5

(d) -4.5

$$P(a < x < 2.5) = 0.4772$$

$$P\left(\frac{a-2.5}{1} < Z < \frac{2.5-2.5}{1}\right) = 0.4772$$

$$P(a-2.5 < Z < 0) = 0.4772$$

option Hit

$$P(0.5 - 2.5 < Z < 0)$$

$P(-2 < Z < 0) = \phi(0) - \phi(-2)$
$= \phi(0) - (1 - \phi(2))$
$= 0.5 - 1 + 0.9772$
$= 0.4772$
$= RHS$

Central Moments by Distribution

1. First Central Moment (μ_1)

- Represents the **mean deviation.**
- For symmetric distributions:
  - Binomial: $\mu_1 = 0$ (No mean deviation from itself).
  - Poisson: $\mu_1 = 0$ (Mean deviation is zero).
  - Normal: $\mu_1 = 0$ (Mean deviation is zero).

2. Second Central Moment (μ_2)

- Represents the **variance.**
- For each distribution:
  - **Binomial:** $\mu_2 = npq$, where n is the number of trials, p is the probability of success, and $q = 1 - p$ is the probability of failure.
  - **Poisson:** $\mu_2 = m$, where m is the mean (in Poisson, mean = variance).
  - **Normal:** $\mu_2 = \sigma^2$, where σ^2 is the variance.

3. Third Central Moment (μ_3)

- Represents **skewness,** which describes asymmetry in the distribution.
- For each distribution:
  - **Binomial:** $\mu_3 = npq(q - p)$. Indicates the direction of skewness based on p and q.

    $npq(1-2p)$
  - **Poisson:** $\mu_3 = m$. The distribution is positively skewed for $m > 0$.
  - **Normal:** $\mu_3 = 0$. Normal distribution is symmetric.