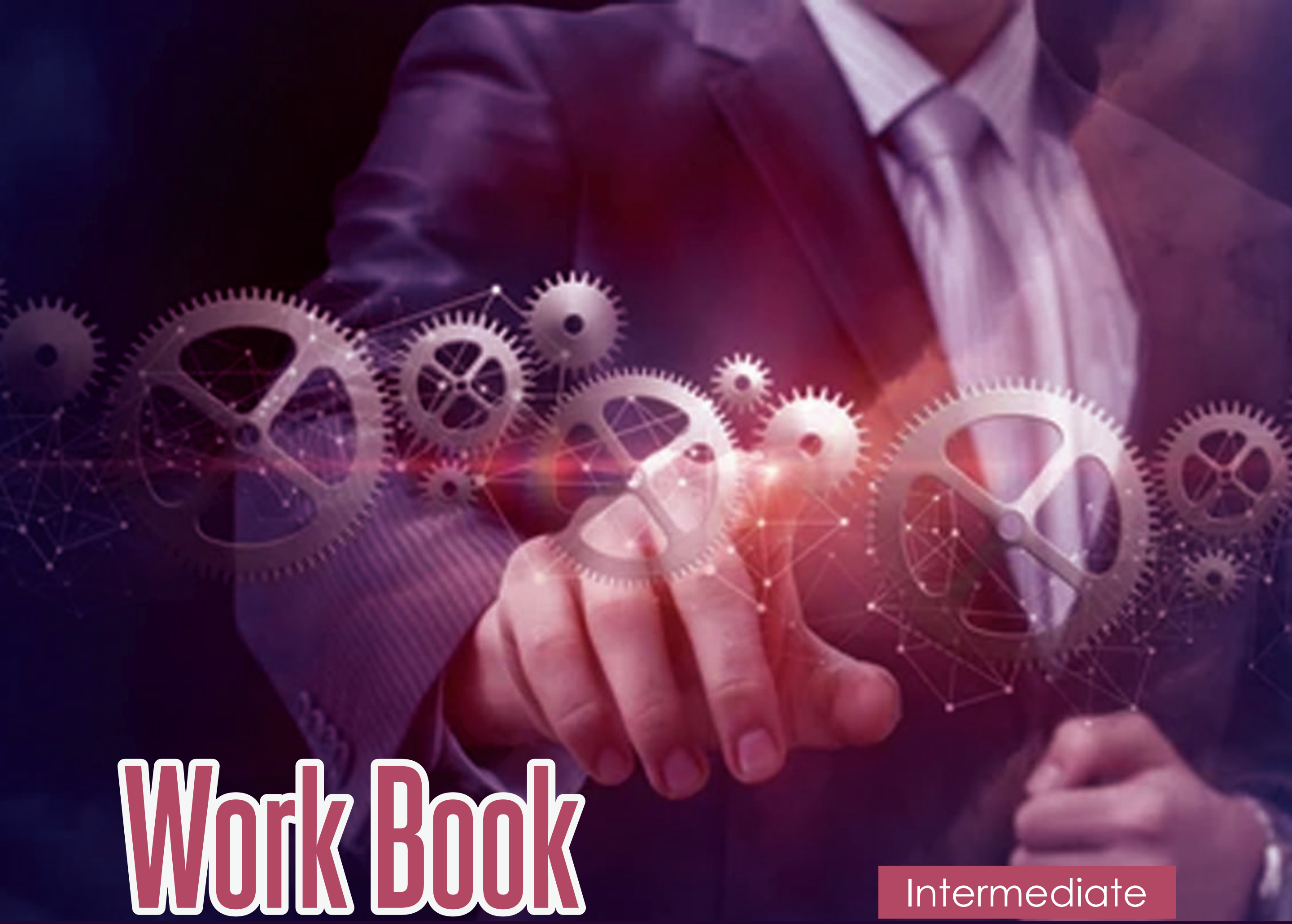

Work Book

Intermediate

Operations Management

Paper

9

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WORKBOOK

Operations Management and Strategic Management

INTERMEDIATE

Paper 9

SYLLABUS 2022

The Institute of Cost Accountants of India

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Preface

The landscape of professional education is undergoing a profound transformation, driven by the evolving demands of a globally integrated economy. In this dynamic environment, it is imperative to equip students not only with technical knowledge but also with the analytical skills and professional acumen essential for success.

Effective learning extends beyond theoretical understanding—it necessitates the development of strong conceptual foundations, critical thinking abilities, and disciplined study habits. These attributes are cultivated through continuous practice and engagement with thought-provoking academic material. To facilitate this process, the curriculum, instructional methods, and assessments must be designed to provide comprehensive, structured, and intellectually stimulating learning experiences.

Building on the success of the previous editions, we are pleased to present the new edition of our 'Workbook' in an e-distributed format. This edition has been meticulously developed to enhance students' comprehension and application of key concepts. Each chapter is structured to offer a seamless learning experience and integrating practical illustrations in a phased manner to align with the evolving regulatory framework.

We are confident that this new edition will continue to serve as a valuable academic resource, empowering students to achieve their professional aspirations with confidence and competence.
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Operations Planning [Study Material - Module 2]

ILLUSTRATION 1:

From the following time series data of sale project, the sales for the next three years.

Year	2019	2020	2021	2022	2023	2024	2025
Sales (‘000 units)	160	180	184	166	188	198	184

Also find out the trend values for the years 2023, 2024 and 2025 and their deviations from the actual values.

Solution:

Year	**Time deviation from 2022 (X)**	**Sales (‘000 units) (Y)**	X^2	**XY**
2019	-3	160	9	-480
2020	- 2	180	4	-360
2021	- 1	184	1	-184
2022	0	166	0	0
2023	1	188	1	188
2024	2	198	4	396
2025	3	184	9	552
Total	$0 = \sum X$	$1{,}260 = \sum Y$	$28 = \sum X^2$	$112 = \sum XY$

Let the Trend equation be

Y = a + b.X

We can use the following short-cut method to find out the values of two constants:

$$a = \frac{\sum Y}{n} = \frac{1{,}260}{7} = 180, \text{ and } b = \frac{\sum XY}{\sum X^2} = \frac{112}{28} = 4$$

Therefore, the Trend equation is Y = 180 + 4.X

Projected sales for the next three years:

Year	Value of X	Projected sales ('000 units)
2026	4	180 + 4 × 4 = 196
2027	5	180 + 4 × 5 = 200
2028	6	180 + 4 × 6 = 204

Year	Value of X	Sales values ('000 units)		
		Trend Values	Actual values	Deviations
2023	1	180 + 4 × 1 = 184	188	-4
2024	2	180 + 4 × 2 = 188	198	-10
2025	3	180 + 4 × 3 = 192	184	+ 8

ILLUSTRATION 2:

An investigation into the demand for AC sets in 5 towns has resulted in the following data:

Population of the town (in lakh)	3	7	6	10	12
No. of AC demanded (in '000)	7	11	10	15	17

Fit a linear regression of Y on X and estimate the demand for CTV sets for two towns with a population of 13 lakhs, 15 lakhs and 21 lakhs.

Solution:

Population (X)	Demand (Y)	X^2	XY
3	7	9	21
7	11	49	77
6	10	36	60
10	15	100	150
12	17	144	204
38	60	338	512

Let the Trend equation be

$Y = a + b.X$

$XY = aX + b.X^2$

$\sum Y = 5a + b.$

$\sum XY = a. \ + b. \sum X^2$

Putting the values from the above table:

60 = 5a + 38b, i.e., 2,280 = 190a + 1,444b

512 = 38a + 338b, i.e., 2,560 = 190a + 1,690b

Subtracting the above two equations, we get

246b = 280, i.e., b = 1.138

$\therefore 60 = 5a + 38 \times 1.138, \quad a = \frac{60 - 43.244}{5} = 3.3512$

Therefore, the Trend equation is

Y = 3.3512 + 1.138X

Population (X)	Demand (Y) = 3.3512 + 1.138X
13	3.3512 + 1.138 × 13 = 18.146
15	3.3512 + 1.138 × 15 = 20.422
21	3.3512 + 1.138 × 21 = 27.250

ILLUSTRATION 3:

GOLDEN MOMENTS LIMITED is a producer of mobile sets. The following data is available from the recent estimates of the company:

Particulars	GOLD SET	GOLD PLUS SET
Estimated demand	50,000 sets	40,000 sets
Processing time per unit (hours)	24 minutes	48 minutes
Set up time per lot	5 hours	6 hours
Lot size	100 units	200 units

The factory works for two shifts per day, 8 hours per shift, and 300 days per year. Currently, the company operates 5 machines and desires a 20% capacity cushion.

(a) How many additional machines should be purchased to meet the upcoming year's demand without resorting to any short-term capacity solution?

(b) If the company makes double-shift operation, how many additional machines required to be purchased?

Solution:

(a) Total working hours available = 2 × 8 × 300 = 4,800

Less: Capacity cushion required (20%) 960

Effective working hours 3,840

Total hours required = [50,000 × (24 / 60) + (50,000 / 100) × 5] + [40,000 × (48 / 60) + (40,000 / 200) × 6] = 22,500 + 33,200 = 55,700.

No. of machines required = $\frac{55,700}{3,840}$ = 14.51 i.e., 15 machines

Number of additional machines to be bought = 15 – 5 = 10.

(b) No. of machines required = $\frac{55,700}{3,840 \times 2}$ = 7.25 i.e., 8 machines

Number of additional machines to be bought = 8 – 5 = 3.

ILLUSTRATION 4:

Department A of FULLMOON LIMITED works on 8 hours shift, 275 days a year. It uses a special machine to process four products, the details of which are given below:

Product	Processing hours per unit	Annual demand (units)
P	5	1,000
Q	8	1,250
R	4	800
S	3	1,500

(a) Compute the number of machines required to satisfy the above demand;

(b) Compute the surplus capacity.

Solution:

(a)

Product	Processing hours per unit	Annual demand (units)	Processing hours required
P	5	1,000	5 × 1,000 = 5,000
Q	8	1,250	8 × 1,250 = 10,000
R	4	800	4 × 800 = 3,200
S	3	1,500	3 × 1,500 = 4,500
Total			22,700

Total hours available per machine = 8 × 275 = 2,200

∴ Number of machines required = $\frac{22,700}{2,200}$ = 10.32 i.e., 11 machines.

(b) Total capacity of 11 machines = 11 × 2,200 =	24,200
Less: Machine-hours required	22,700
Surplus capacity (Processing hours)	1,500

ILLUSTRATION 5:

JANI-DUSHMAN LTD. is redesigning its production process so that it is capable to produce 640 chairs per day. Time estimates reveal that 72 hours of cutting work per day are required for the production of 640 chairs. 1 chair in every 20 is scrapped due to sub-standard quality which are sold as scrap by the company. The plant is expected to operate at 90% efficiency on a 2 shift of 8 hours each per day.

(a) Please compute the number of machines required to produce 640 standard quality chairs per day.

(b) If the cost of each machine is ₹50,000, find the total amount of investment in the machines.

(c) Compute the surplus capacity.

Solution:

(a) Scrap loss = $\frac{1}{20}$ × 100 = 5%.

Gross number of chairs to be produced = $\frac{640}{95\%}$ = 673.684, i.e. 674

Ideal number of machines required = $\frac{72}{16}$ = 4.5

Actual number of machines required = $\frac{4.5}{0.95 \times 0.90}$ = 5.26, i.e. 6.

(b) Total amount of investment = 6 × ₹ 50,000 = ₹ 3,00,000.

(c) Surplus capacity = (6 – 5.26) × 72 hours = 53.28 hours.

ILLUSTRATION 6:

The following information is available from the records of WHITE-TEETH LIMITED for the month of March 2025:

Capacity	10,000 Machine hours
Overhead costs	₹ 600,000
Machine hours worked	8,000

Machine hours constitute the main constraints on the physical capacity of the company.

Please compute the idle capacity costs.

Solution:

Overhead costs per MH = $\frac{₹\ 6,00,000}{10,000}$ = ₹. 60

Idle capacity = (10,000 – 8,000) MH = 2,000 MH

Therefore, Idle capacity costs = 2,000 × ₹. 60 = ₹. 120,000.

ILLUSTRATION 7:

The factory at Rourkela of GOOD INDUSTRY LTD has a design capacity of 7,20,000 tons of its product per day, effective capacity of 6,00,000 tons per day and an actual output of 5,60,000 tons of per day. Compute the efficiency of the plant and its utilisation.

Solution:

(a) Efficiency of the factory = $\frac{\text{Actual Output}}{\text{Effective Capacity}} \times 100 = \frac{5,60,000}{6,00,000} \times 100 = 93.33\%$

(b) Capacity utilisation = $\frac{\text{Actual Output}}{\text{Design Capacity}} = \times 100 = \frac{5,60,000}{7,20,000} \times 100 = 77.78\%$

ILLUSTRATION 8:

ORGANIC COMPOUND LIMITED buys and uses a component for production at ₹. 40 per piece. Annual requirement is 80,000 number. Carrying cost of inventory is 10% p.a. and the ordering cost is ₹. 400 per order. The purchase manager agrees that as the ordering cost is very high, it is advantageous to place a single order for the entire annual requirement. He also says that if we order 80,000 at a time, we get a 3% discount from the supplier. Evaluate this proposal and make your recommendation.

Solution:

$$EOQ = \sqrt{\frac{2 \times A \times B}{C}} = \sqrt{\frac{2 \times 80,000 \times 400}{4}} = 4,000 \text{ Units}$$

(a) Ordering quantity	4,000	80,000
(b) No. of orders [80,000 ÷ OQ]	20	1
(c) Price per unit (₹.)	40	38.80
(d) Carrying cost per unit per annum [c × 10%]	4	3.88
(e) Total ordering costs [(b) × ₹. 400]	8,000	400

(f) Total carrying costs [$\frac{OQ}{2}$ × (d)]	8,000	1,55,200
(g) Purchase price [80,000 × Price per unit]	32,00,000	31,04,000
(h) Total inventory costs [e + f + g] (₹.)	32,16,000	32,59,600

Recommendation: The Purchase Manager's proposal should not be accepted. The optimum ordering quantity is 4,000 units.

ILLUSTRATION 9:

A company planning to manufacture a household cooking range has to decide on the location of the plant. Three locations are being considered viz., Patna, Ranchi, and Dhanbad. The fixed costs of the three locations are estimated to be ₹30 lakh, ₹50 lakh, and ₹25 lakh per annum respectively. The variable costs are ₹300, ₹200 and ₹350 per unit respectively. The expected sales price of the cooking range is ₹700 per unit.

Find out:

(i) The range of annual production/sales volume for which each location is most suitable and

(ii) Which one of the three locations is the best location at a production/sales volume of 18,000 units?

Solution:

$$\text{Cost Indifference point (CIP)} = \frac{\text{Change in Fixed Costs}}{\text{Change in Variable Cost Per Unit}}$$

(a) Patna V. Ranchi: $\text{CIP} = \frac{50,00,000 - 30,00,000}{300 - 200} = 20,000$ Units.

This indicates that above 20,000 units, Ranchi is cheaper and below 20,000 units, Patna is cheaper.

(b) Patna V. Dhanbad: $\text{CIP} = \frac{30,00,000 - 25,00,000}{350 - 300} = 10,000$ Units.

This indicates that above 10,000 units, Patna is cheaper and below 10,000 units, Dhanbad is cheaper.

(c) Ranchi V. Dhanbad: $\text{CIP} = \frac{50,00,000 - 25,00,000}{350 - 200} = 16,667$ Units.

This indicates that above 16,667 units, Ranchi is cheaper and below 16,667 units, Dhanbad is cheaper.

Comments: Combining the above three observations, we can recommend that below 10,000 units, Dhanbad is cheaper. Above 10,000 units but below 20,000 units, Patna is cheaper and above 20,000 units, Ranchi is cheaper.

ILLUSTRATION 10:

The production schedule for a certain part call for manufacturing approximately740 units per week. Five operations are to be performed on five different machines. The time requirements for these operations, adjusted for delays, labour efficiency and expected scrap are as follows:

Operation No.	Required time per unit (hour)
1	0.095
2	0.200
3	0.197
4	0.098
5	0.100

(a) Please calculate the number of machines required to perform each operation if each machine is scheduled to work for 40 hours per week.

(b) Compute the surplus capacity of each operation.

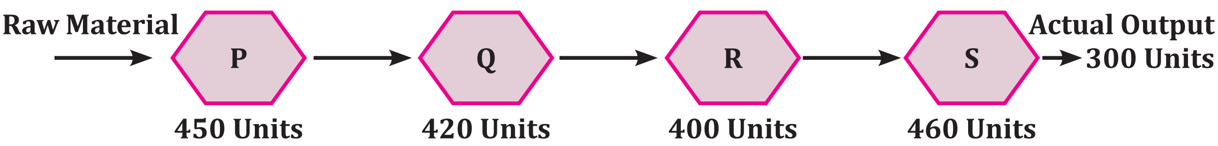
Solution:

Computation of required number of machines & surplus hours

Operation No.	(a) Number of machines required	(b) Surplus capacity (hours)
1	$\frac{740 \times 0.095}{40} = 1.76$ i.e., 2	(2 – 1.76) × 40 = 9.6
2	$\frac{740 \times 0.200}{40} = 3.7$ i.e., 4	(4 – 3.7) × 40 = 12
3	$\frac{740 \times 0.197}{40} = 3.64$ i.e., 4	(4 – 3.64) × 40 = 14.4
4	$\frac{740 \times 0.098}{40} = 1.813$ i.e., 2	(2 – 1.813) × 40 = 7.48
5	$\frac{740 \times 0.100}{40} = 1.85$ i.e., 2	(2 – 1.85) × 40 = 6

ILLUSTRATION 11:

A firm has four work centres, P, Q, R & S, in series with individual capacities in units per day shown in the figure below.



(a) Identify the bottle neck centre.

(b) What is the system capacity?

(c) What is the system efficiency?

Solution:

(a) The bottle neck centre is the work centre having the minimum capacity. Hence, work centre 'R' is the bottleneck centre.

(b) System capacity is the maximum units that are possible to produce in the system as a whole. Hence, system capacity is the capacity of the bottle neck centre i.e., 400 units.

(c) System efficiency = $\frac{\text{Actual Output}}{\text{System Capacity}} \times 100 = \frac{300}{400} \times 100 = 75\%$ (approx.)

ILLUSTRATION 12:

A metal processing firm desires to install sufficient number of automatic moulders to produce 2,00,000 good parts every year. The firm operates 2,000 hours per annum but the Moulding equipment is part of a production line which will be used only 60% of the time and its output is about 3% defective. A moulding operation takes 90 seconds per part. Although the temperature adjustments and maintenance downtime the moulders are about 80% efficient.

How many moulders are required to satisfy the production demand of 2,00,000 good parts per annum ?

Solution:

Required system capacity = $\frac{\text{Actual Output}}{\text{System Capacity}} = \frac{2,00,000}{97\%}$ = 2,06,186 parts per annum,

Total Moulding hours needed to produce gross output of 2,06,186 units $= \frac{2,06,186 \times 90}{60 \times 60}$

= 5,154.65.

Capacity utilization per Moulding equipment = 2,000 × 60% × 80% = 960 hours.

Therefore, number of Moulding machines needed = $\frac{5,154.65}{960}$ = 5.40 i.e., 6 machines.

2 Application of Operation Research - Production Planning and Control [Study Material - Module 4]

ILLUSTRATION 1:

The time study of a work operation yields a cycle time of 10 minutes. The analyst rated the worker observed at 80%. The firm uses a 15% allowance factor. Please compute the normal time and standard time for this operation.

Solution:

Average Cycle Time (ACT) = 10 Minutes

Performance rating (PR) = 80%

Therefore, Normal time (NT) = ACT × PR = 10 Minutes × 80% = 8 Minutes

$$\text{Standard time} = \frac{\text{Normal Time}}{(1 - \text{Allowance Fraction})} = \frac{8}{(1 - 0.15)} = 9.41 \text{ Minutes}$$

ILLUSTRATION 2:

A work measurement study was conducted in a company for 80 hours and the following results were obtained:

- Units produced : 3200
- Idle time : 15%
- Performance rating : 120%
- Allowance time : 12% of standard time

Compute the standard time for the task.

Solution:

Computation of Standard time:

Total time (80 × 60) Less: Idle time 15%	4,800 Minutes 720 Minutes
Effective time	4,080 Minutes

Units produced	3,200 units
ACT	1.275 Minutes
Performance Rating	120%
Normal time (ACT × PR)	1.53 Minutes
Standard time (ST) = NT/(1 – AF)	1.53/(1 – 0.12) = 1.74 Minutes

ILLUSTRATION 3:

A time study of a shop worker revealed the actual times shown below. The analyst rated the worker at 90% rating factor and the company allows the following per 8-hour day:

Worker	Machine time (minute / cycle)	Total
2.30	0.80	3.10
1.80	0.80	2.60
2.00	0.80	2.80
2.20	0.80	3.00
1.90	0.80	2.70
10.20*	0.80	11.00
2.20	0.80	3.00
1.80	0.80	2.60

Personal time : 20 minutes and delay time : 30 minutes.

*Unusual non-recurring situation.

Please compute the standard time.

Solution:

Step 1: Computation of normal time (NT)

	ACT	PR	NT = ACT × PR
Average Cycle Time (ACT):			
• Worker: (2.30 + 1.80 + 2.00 + 2.20 + 1.90 + 2.20 + 1.80) / 7	2.0286	90%	1.8257
• Machine	0.8000	100%	0.8000
Total Normal Time			**2.6257**

Allowance Fraction (AF) = (20 + 30) / (8 × 60) = 0.1042

Standard Time (ST) = NT / (1 – AF) = 2.6257 / (1 – 0.1042) = 2.93 Minutes (Approx.)

ILLUSTRATION 4:

A time study was conducted for a worker in shipping department in a factory. The observations (in minutes) are as under:

Elements	A	B	C	D	Performance rating
P: Obtain the case	0.15	0.25	0.20	0.17	90%
Q: Place 3 dozen bottles in case	1.56	*	1.80	1.75	105%
R: Set case aside	0.20	0.10	0.10	0.15	95%

* Dropped the bottle.

Please compute the following:

(i) Normal time;

(ii) Standard time if the allowance factor is 10%.

Solution:

Elements	ACT	ACT	PR	NT = ACT × PR
P	(0.15+0.25+0.20+0.17) /4	0.1925	90%	0.1733
Q	(1.56+1.80+1.75) / 3	1.7033	105%	1.7885
R	(0.20+0.10+0.10+0.15) / 4	0.1375	95%	0.1306
Total				2.0924

Standard Time (ST) = NT / (1 – AF) = 2.0924 / (1 – 0.10) = 2.32 Minutes

ILLUSTRATION 5:

MR. SUNIL GAVASKAR has recently constructed his house and after completing the construction he discovered that 100 sq. ft. of plywood scrap and 80 sq. ft. of pine scrap are in usable form for the construction of tables and book cases. It takes 16 sq. ft. of plywood and 8 sq. ft. of white pine to make a table and 12 sq. ft. of plywood and 16 sq. ft. of white pine to make a book case. By selling the finished products to the nearest furniture store, he can realize a profit of ₹25 on each table and 20 on each book case.

Formulate the L.P.P. and find out the optimum solution. [**Ans.:** 4 tables, 3 book cases; Z = ₹160]

Solution:

	Tables (x_1)	Book cases (x_2)	Capacity
Plywood scrap (sq. ft.)	16	12	100
Pine scrap (sq. ft.)	8	16	80
Profit per unit (₹.)	25	20	

Let x_1 number of Tables and x_2 number of Book cases should be made to make maximum amount of profit.

Let Z = Total amount of profit

The formulation of LPP is as follows:

Maximise $Z = 25x_1 + 20x_2$

Subject to —

$16x_1 + 12x_2 \le 100$

$8x_1 + 16x_2 \le 80$

$x_1, x_2 \ge 0$

Applying Simplex Method or Graphical Method, the optimum solution will be as follows: -

Number of tables = 4 and number of book cases = 3; Maximum amount of profit = ₹. 160.

ILLUSTRATION 6:

A company produces three products P, Q and R from three raw materials A, B and C. One unit of product P requires 2 units of A and 3 units of B. One unit of product Q requires 2 units of B and 5 units of C and one unit of product R requires 3 units of A, 2 units of B and 4 units of C. The company has 8 units of material A, 10 units of material B and 15 units of material C available to it. Profits per unit of products P, Q and R are ₹. 3, ₹. 5 and ₹. 4 respectively. Formulate the question mathematically to maximize the profit.

ILLUSTRATION 7:

SOURAV GANGULY & CO. is manufacturing two products A & B. The manufactured time required to make them, the profit and the capacity available at each work center is given as follows:

Products	Machining (hours)	Fabrication (hours)	Assembly (hours)	Profit per unit (₹)
A B	1 2	5 4	3 1	80 100
Capacity (hours)	720	1800	990	

Please determine the product mix that will maximize the profit by using—

(a) Simplex Method: and (b) Graphical Method. [**Ans.**: A = 120 units, B = 300 units & Z = ₹39,600]

Solution:

	A (x_1)	B (x_2)	Capacity
Machine hours	1	2	720
Fabrication hours	5	4	1,800
Assembly hours	3	1	990
Profit per unit (₹.)	80	100	

Let x_1 number of product A and x_2 number of product B should be made to make maximum amount of profit.

Let Z = Total amount of profit

The formulation of LPP is as follows:

Maximise $Z = 80x_1 + 100x_2$

Subject to —

$1x_1 + 2x_2 \leq 720$

$5x_1 + 4x_2 \leq 1,800$

$3x_1 + 1x_2 \leq 990$

$x_1, x_2 \geq 0$

Applying Simplex Method or Graphical Method, the optimum solution will be as follows: -

Product A = 120 and Product B = 300; Maximum amount of profit = ₹. 39,600.

ILLUSTRATION 8:

MR. SHARAD POWER, a scrap metal dealer has received an order from a customer for at least 2,000 kg of scrap metal. The customer requires that at least 1,000 kg of the shipment of the metal must be of high-quality copper that can be melted down and used to produce copper tubings. Furthermore, the customer will not accept delivery of the order if it contains more than 175 kg. of metal that he thinks unfit for commercial use, i.e. metal that contains excessive amount of impurities and cannot be melted down and refined profitably.

The dealer can purchase scrap metal from two different suppliers in unlimited quantities with the following percentage (by weight) of high-quality copper and unfit scrap.

	Supplier A	Supplier B
Copper	25%	75%
Unfit copper	5%	10%

The cost ₹ per kg of metal purchased from supplier A and supplier B are ₹10 and ₹40 respectively. Please determine the optimal quantities of metal for the dealer to purchase from each of the two suppliers.

[**Ans.**: 2,500 kg from Supplier A and 500 kg from Supplier B. Z = ₹45,000]

[Hints: Maximise Z (Total costs) = $x_1 + 4x_2$

Subject to

$0.25x_1 + 0.75x_2 \geq 1{,}000$

$0.05x_1 + 0.10x_2 \leq 175$

$x_1 + x_2 \geq 2{,}000$]

ILLUSTRATION 9:

A pension fund manager is considering investing in two shares A and B. It is estimated that:

(i) Share A will earn a dividend of 12% per annum and share B 4% per annum.

(ii) Growth in the market value in one year of share A will be 10 paise per ₹.1 invested and in B paise per ₹.1 invested.

He requires investing the minimum total sum which will give: Dividend income of at least ₹600 per annum and growth in one year of at least ₹1,000 on the initial investment.

You are required to State the mathematical formulation of the problem which will facilitate computation of the minimum sum to be invested to meet the manager's objective.

Solution:

Let the manager should invest ₹x_1 in share A and ₹x_2 in share B

	Share		**Minimum Income (₹)**
	Share A	**Share B**	
Dividend	12%	4%	600
Growth	0.10	0.40	1,000

Formulation:

Minimise Z = $x_1 + x_2$

Subject to:

$0.12\ x_1 + 0.04\ x_2 = 600$

$0.10\ x_1 + 0.40\ x_2 = 1{,}000$

$x_1 + x_2 \geq 0.$

ILLUSTRATION 10:

SURESH RAINA & CO. has two grades of inspectors—Grade 1 & Grade 2 to undertake quality control inspection. At least 3,500 pieces must be inspected in an 8 hour day. Grade 1 inspector can check 50 pieces in an hour with an accuracy of 95% whereas Grade 2 inspectors can check 25 pieces with an accuracy of 90%.

The daily wages of Grade 1 inspector are ₹6 per hour while those of Grade 2 are ₹5 per hour. Any error made by an inspector cost ₹4 to the company. If there are in all 20 Grade 1 inspectors and 25 Grade 2 inspectors in the company, formulate the linear programming problem to determine the optimal assignment of inspectors that minimises the daily inspection costs.

Solution:

Let x_1 be the number of Grade 1 inspectors and x_2 be the number of Grade 2 inspectors.

Z = Daily inspection costs.

∴ Cost per hour for Grade 1 inspectors = ₹. (6 + 50 × 5% × 4) = ₹. 16

For Grade 2 inspectors = ₹. (5 + 25 × 10% × 4) = ₹. 15

The LPP formulation is as follows:

Minimise $Z = 8 \times (16x_1 + 15x_2)$

Subject to:

$50 \times 8x_1 + 25 \times 8x_2 \geq 3{,}500$

$x_1 \leq 20$

$x_2 \leq 25$

$x_1, x_2 \geq 0$

ILLUSTRATION 11:

From the following transportation cost matrix, find the initial feasible solution using North-West corner Rule:

WAREHOUSES

FACTORIES	W_1	W_2	W_3	W_4	Capacity
F_1	47	59	55	57	150
F_2	44	54	52	59	270
F_3	49	64	59	61	370
F_4	51	63	54	60	230
Requirement	210	330	260	220	

Solution:

North-West Corner Rule

WAREHOUSES

FACTORIES	W_1	W_2	W_3	W_4	Capacity
F_1	47 (150)	59	55	57	150
F_2	44 (60)	54 (210)	52	59	~~270~~ 210
F_3	49	64 (120)	59 (250)	61	~~370~~ 250
F_4	51	63	54 (10)	60 (220)	~~230~~ 220
Requirement	~~210~~ 60	~~330~~ 120	~~260~~ 10	220	

Total transportation costs = ₹. (47×150 + 44×60 + 54×210 + 64×120 + 59×250 + 54×10 + 60×220

= ₹. 57,200

ILLUSTRATION 12:

The Transportation problem of CO. RAJ & RAJ is given as follows:

WAREHOUSES

FACTORIES	W_1	W_2	W_3	W_4	Capacity
F_1	21	16	25	13	11
F_2	17	18	14	23	13
F_3	32	27	18	41	19
Requirement	6	10	12	15	

Please find out the Basic Feasible solution and total transportation cost under—

(a) North-West Corner Rule;

(b) Least Cost Method;

Solution:

(a) North-West Cornet Rule

WAREHOUSES

FACTORIES	W_1	W_2	W_3	W_4	Capacity
F_1	21 (6)	16 (5)	25	13	11
F_2	17	18 (5)	14 (8)	23	13
F_3	32	27	18 (4)	41 (1)	19
Requirement	6	10	12	15	

Total Transportation costs = ₹. 1,095

(b) Least Cost Method

WAREHOUSES

FACTORIES	W_1	W_2	W_3	W_4	Capacity
F_1	21	16	25	13 (1)	11
F_2	17 (1)	18	14 (1)	23	~~13~~ 1
F_3	32 (5)	27 (1)	18	41 (4)	~~19~~ 9
Requirement	~~6~~ 5	10	12	~~15~~ 4	

Total Transportation costs = ₹. 922

ILLUSTRATION 13:

The Transportation problem of MORNING WALK LTD. is given as follows:

WAREHOUSES

FACTORIES	W_1	W_2	W_3	W_4	Capacity
F_1	21	16	25	13	11
F_2	17	18	14	23	13
F_3	32	27	18	41	19
Requirement	6	10	12	15	

Using Vogel's Approximation Method (VAM), please find out the—

(a) Basic Feasible solution;

(b) Optimum solution; and

(c) Total transportation cost

Solution:

(a)

WAREHOUSES

FACTORIES	W_1	W_2	W_3	W_4	Capacity
F_1	21	16	25	13 (1)	11
F_2	17 (6)	18 (3)	14 5	23 (4)	13
F_3	32	27 (7)	18 (1)	41	19
Requirement	6	10	12	15	

(b) Optimality Test [Stepping Stone Method]

Square evaluation of the unoccupied cells:

F_1W_1 = 21 – 13 + 23 – 17 = + 14

F_1W_2 = 16 – 13 + 23 – 18 = + 8

F_1W_3 = 25 – 13 + 23 – 18 + 27 – 18 = + 26

F_2W_3 = 14 – 18 + 27 – 18 = + 5

F_3W_1 = 32 – 27 + 18 – 17 = + 6

F_3W_4 = 41 – 23 + 18 – 27 = + 9

As all the square evaluations are non-negative, the above solution is optimal solution.

(C) Total Transportation costs = ₹. 796

ILLUSTRATION 14:

DELICIOUS FOOD LTD has 3 plants P1, P2 & P3 each producing 50, 100 and 150 units of a similar products. There are 5 Warehouses—W1, W2, W3, W4 & W5 having demand of 100,70, 50, 40 & 40 units respectively. The cost of transporting a unit from various plants to the warehouses differs as given by the following cost matrix:

	Warehouses				
Plants	W_1	W_2	W_3	W_4	W_5
P_1	20	28	32	55	70
P_2	48	36	40	44	25
P_3	35	55	22	45	48

Using North-West Corner Rule & Stepping Stone method, please determine the optimum transportation schedule.

Solution:

Step 1: North-West Corner Rule:

	Warehouses					
Plants	**W_1**	**W_2**	**W_3**	**W_4**	**W_5**	**Supply**
P_1	20 (50)	28	32	55	70	50
P_2	48 (50)	36 (50)	40	44	25	100
P_3	35	55 (20)	22 (50)	45 (40)	48 (40)	150
Demand	100	70	50	40	40	

Step 2: After applying the Stepping Stone Method, the optimum solution is as follows:

	Warehouses				
Plants	**W_1**	**W_2**	**W_3**	**W_4**	**W_5**
P_1	20 (40)	28 (10)	32	55	70
P_2	48	36 (60)	40	44	25 (40)
P_3	35 (60)	55	22 (50)	45 (40)	48

Total Transportation costs = ₹. (20×40 + 28×10 + 36×60 + 25×40 + 35×60 + 22×50 + 45×40)

= ₹. 9,240

ILLUSTRATION 15:

SOUTH INDIAN DISH LTD has 3 factories F1, F2 & F3 each producing 100, 200 and 300 units of a similar products. There are 5 Warehouses—W1, W2, W3, W4 & W5 having demand of 200,140, 100, 80 & 80 units respectively. The cost of transporting a unit from various plants to the warehouses differs as given by the following cost matrix:

	Warehouses				
Plants	**W_1**	**W_2**	**W_3**	**W_4**	**W_5**
F_1	40	56	64	110	140
F_2	96	72	80	88	50
F_3	70	110	44	90	96

Find out the minimum transportation costs using VAM.

Solution:

Using Vogel's Approximation Method, the allocation is as follows:

	Warehouses					
Plants	**W_1**	**W_2**	**W_3**	**W_4**	**W_5**	**Supply**
F_1	40 (80)	56 (20)	64	110	140	100
F_2	96	72 (120)	80	88	50 (80)	200
F_3	70 (120)	110	44 (100)	90 (80)	96	300
Demand	200	140	100	80	80	

The application of Stepping stone method gives the following result:

$F_1W_3 = 64 - 44 + 70 - 40 = + 54$

$F_1W_4 = 110 - 90 + 70 - 40 = + 50$

$F_1W_5 = 140 - 50 + 72 - 56 = + 106$

$F_2W_1 = 96 - 72 + 56 - 40 = + 40$

$F_2W_3 = 80 - 44 + 70 - 40 + 56 - 72 = + 50$

$F_2W_4 = 88 - 90 + 70 - 40 + 56 - 72 = + 12$

$F_3W_2 = 110 - 56 + 40 - 70 = + 24$

$F_3W_5 = 96 - 70 + 40 - 56 + 72 - 50 = + 32$

As all the square evaluations are non-negative, the above solution is optimum solution.

Total transportation costs = ₹. 36,960

ILLUSTRATION 16:

The transportation cost per unit, availability of supply from the warehouses, requirements of market are given below:

Market

Warehouse	P	Q	R	S	Supply
A	6	3	5	4	22
B	5	9	2	7	15
C	5	7	8	6	8
Requirement	7	12	17	9	

The shipping clerk has worked out the following schedule from experience:

12 units from A to Q, 1 unit from A to R, 9 units from A to S, 15 units from B to R, 7units from C to P and 1 unit from C to R.

(i) Check and see if the clerk has the optimal schedule;

(ii) Find the optimal schedule and minimum total transportation cost;

(iii) If the clerk is approached by a carrier of route C to Q who offers to reduce his rate in the hope of getting more business, by how much the rate should be reduced before the clerk will offer him the business? **[ICWA-FINAL]**

[**Ans.:** (i) No. Total cost = ₹150, (ii) ₹149 (iii) ₹2 per unit.]

Solution:

(i) Schedule made by the shipping clerk is as follows:

	Market				
Warehouse	**P**	**Q**	**R**	**S**	**Supply**
A	6	3 (12)	5 (1)	4 (9)	22
B	5	9	2 (15)	7	15
C	5 (7)	7	8 (1)	6	8
Requirement	7	12	17	9	

Total Transportation costs = ₹. (3×12 + 5×1 + 4×9 + 2×15 + 5×7 + 8×1) = ₹. 150

Checking of optimal solution: Square evaluation of the unoccupied cells (Stepping Stone Method):

AP = 6 – 5 + 8 – 5 = + 3

BP = 5 – 2 + 8 – 5 = + 6

BQ = 9 – 3 + 5 – 2 = + 9

BS = 7 – 2 + 5 – 4 = + 6

CQ = 7 – 8 + 5 – 3 = = + 1

CS = 6 – 4 + 5 – 8 = - 1 (Negative)

As square evaluation shows negative result, the above solution is not optimum solution.

(ii) After making the adjustment, the following table gives the optimum solution:

	Market			
Warehouse	**P**	**Q**	**R**	**S**
A	6	3 (12)	5 (2)	4 (8)
B	5	9	2 (15)	7
C	5 (7)	7	8	6 (1)

Total Transportation costs = ₹. 149.

(iii) CQ = 7 – 3 + 4 – 6 = + 2

The rate should be reduced by at least ₹. 2 to offer him the business.

ILLUSTRATION 17:

The processing times for five jobs and their due dates are given for a single scheduling below:

Job	A	B	C	D	E
Processing time (Days)	9	7	5	11	6
Due dates (days)	16	20	25	15	40

The jobs may be sequenced according to any one of the following rules:

(a) Minimum processing time (MINPRT);

(b) First come first served (FCFS);

(c) Longest processing time (LPT);

(d) Earliest due dates (EDD).

For the above set of jobs, please compute the following:

(1) Total completion time;

(2) Average completion time;

(3) Average number of jobs in the system;

(4) Average job lateness.

Solution:

Minimum processing time rule

Job sequence	Process time (days)	Flow time (days)	Due date (days from now)	Job lateness (days late)
C	5	5	25	0
E	6	11	40	0
B	7	18	20	0
A	9	27	16	11
D	11	38	15	23
Total	38	99		34

(1) Total completion time = 38 days for all the jobs.

(2) Average completion time = Total flow times ÷ No. of jobs = (99 ÷ 5) days = 19.8 days.

(3) Average no. of jobs in the system = $\frac{\text{Total of flow times}}{\text{Total of process times}} = \frac{99}{38} = 2.61.$

The same result can be obtained as follows:

Process time	Jobs in sequence
First 5 days	5
Next 6 days	4
Next 7 days	3
Next 9 days	2
Final 11 days	1

The total process time for this sequence is 38 days.

Average jobs in the system = $\frac{5 \times 5 + 6 \times 4 + 7 \times 3 + 9 \times 2 + 11 \times 1}{38} = \frac{99}{38} = 2.61$

(4) Average job lateness = $\frac{\text{Total Job Lateness}}{\text{Total of process times}} = \frac{99}{38} = 6.8$ days.

The above rules are 'static' rules. A more dynamic rule is the "Minimum Critical Ratio" rule.

Critical Ratio = $\frac{\textbf{Time remaining for due date of the job}}{\textbf{Time needed to complete the job}} = \frac{\textbf{Time remaining}}{\textbf{Work remaining}}$

ILLUSTRATION 18:

Jobs waiting to be processed at the Milling Shop today (the July 23) are as follows:

Jobs	Due dates	Time needed to complete the job i.e., processing time (days)
A	July 31	9
B	August 2	6
C	August 16	24
D	July 29	5
E	August 30	30

Sequence the jobs at the Milling shop based on minimum critical ratio.

Solution:

Jobs	Due date	Time remaining from the due date in days (TR)	Time needed to complete the job in days (TN)	Critical Ratio $= \frac{TR}{TN}$	Ranking as per min. CR
A	July 31	8	9	0.89	1
B	August 2	10	6	1.67	5
C	August 16	24	24	1.00	2
D	July 29	6	5	1.20	3
E	August 30	38	30	1.27	4

As per the minimum critical ratio, the job sequence is as follows: A, C, D, E, B

ILLUSTRATION 19:

The following jobs have to be shipped a week from now(week has 5 working days):

Job	A	B	C	D	E	F
No. of days work remaining	4	5	8	7	6	3

Sequence the jobs according to priority established by:

(i) Least slack rule

(ii) Critical ratio rule

Solution:

	(i) Least slack rule		(ii) Critical ratio rule	
Job	**Slack (Days)**	**Ranking**	**Critical ratio**	**Ranking**
A	5 – 4 = 1	5	5/4 = 1.250	5
B	5 – 5 = 0	4	5/5 = 1.000	4
C	5 – 8 = -3	1	5/8 = 0.625	1
D	5 – 7 = -2	2	5/7 = 0.714	2
E	5 – 6 = -1	3	5/6 = 0.833	3
F	5 – 3 = 2	6	5/3 = 1.667	6

ILLUSTRATION 20:

CO. CLOUDY Ltd is setting an assembly line to produce 192 units per 8-hour shift. The work elements in terms of time and immediate predecessors are given below:

Work element	Time (sec.)	Immediate predecessors
A	40	None
B	80	A
C	30	D, E, F
D	25	B
E	20	B
F	15	B
G	120	A
H	145	G
I	130	H
J	115	C, I
Total	**720**	

What is the desired cycle time (or average cycle time)?

(i) What is the theoretical number of stations?

(ii) Use largest work element rule to work out a solution on a precedence diagram.

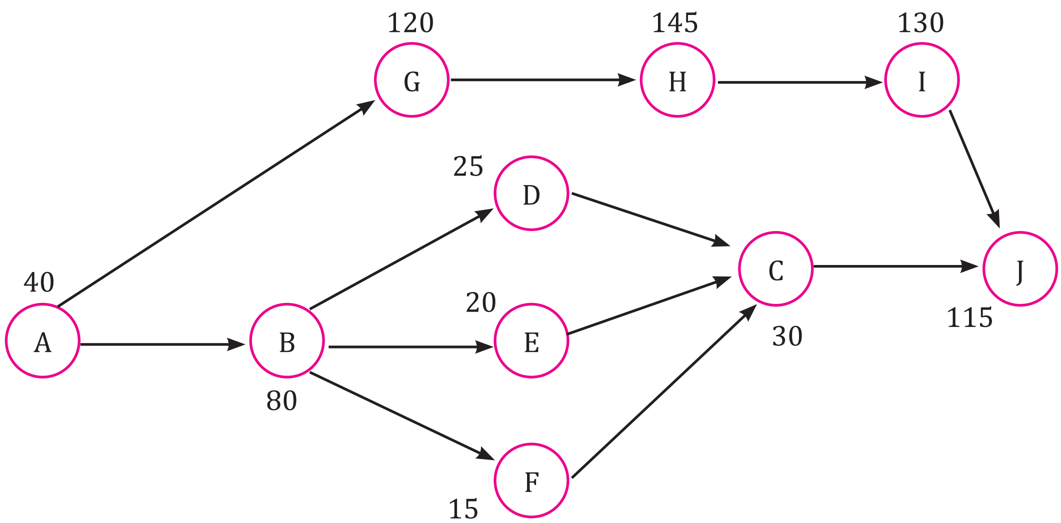
(iii) What are the efficiency and balance delay of the solution obtained?

Solution:

(i) Average cycle time (ACT) = (8 × 60 × 60 sec.) ÷ 192 units = 150 sec. per unit.

(ii) Theoretical number of work stations = 720 sec. ÷ 150 sec. = 4.8 i.e. 5.

(iii) Diagram [Flow Chart]



Work stations	Work elements	Cumulative time (sec.)	Idle time for station (Sec.)
W_1	A(40), B(80) & D(25)	145	5
W_2	G(120) & E(20)	140	10
W_3	H	145	5
W_4	I(130) & F(15)	145	5
W_5	C(30) & J(115)	145	5

(iv) Efficiency = $\frac{\sum t}{n \times ACT} \times 100 = \frac{720}{5 \times 150} \times 100 = 96\%$

Balance delay = 100% – 96% = 4%.

3 Productivity Management and Quality Management [Study Material - Module 5]

ILLUSTRATION 1:

In a particular plant of ANTIBIOTIC LIMITED, there are 20 workers manufacturing a single product and the output per month consisting of 25 days of that particular product is 4000.

(a) How much is the monthly productivity per worker?

(b) Compute the productivity per day per worker.

Solution:

(a) Monthly productivity per worker = $\dfrac{\text{Output}}{\text{Number of Workers}} = \dfrac{4{,}000 \text{ units}}{20}$ = 200 units.

(b) Productivity per day per worker = $\dfrac{\text{Output}}{\text{Number of man – days}} = \dfrac{4{,}000 \text{ units}}{20 \times 25}$ = 8 units

ILLUSTRATION 2:

The capital employed of BIG-TREE LIMITED is ₹. 500 crores, comprising ₹. 350 crores on net fixed assets and ₹. 150 crores in net working capital. The sales turnover is ₹. 2,000 crores and the net profit ratio is 16%.

(a) Compute the capital productivity of the company for the year.

(b) What is the use of this productivity?

Solution:

(a) Capital productivity = $\dfrac{\text{Net Profit}}{\text{Capital employed}} \times 100 = \dfrac{2{,}000 \times 16\%}{500} \times 100 = 64\%$.

(b) This ratio of net profit to capital employed is a valuable means of measuring the performance of divisions, sections, plants, products and other components of a business.

ILLUSTRATION 3:

There are two industries SMALL LTD and BIG LTD manufacturing ball-bearings for machines. The standard time per piece is 10 minutes per worker. There are 24 workers in SMALL LTD and 25 workers in BIG LTD involved in the production process of ball-bearings. The output of two companies is 1,000 units and 1,500 units respectively per shift of 8 hours.

(a) Find the labour productivity of each per shift of 8 hours.

(b) What is the expected production of each industry per month of 25 working days?

(c) Compute the efficiency of both the companies.

Solution:

Particulars	SMALL LTD	BIG LTD
(a) Actual output (units) (P)	1,000	1,500
Total Labour-hours per shift (Q)	8 × 24 = 192	8 × 25 = 200
Labour productivity [P / Q]	5.21 units per LH	7.50 per LH
(b) Production per LH (units)	6	6
Expected production per month (units)	(6 × 8 × 25)×24 = 28,800	(6 × 8 × 25)×25 = 30,000
(c) Actual output per shift (units)	1,000	1,500
Standard output per shift of 8 hours	(6 × 8) × 24 = 1,152	(6 × 8) × 25 = 1,200
Efficiency [$\frac{\text{Actual Output}}{\text{Standard output}}$ ×100]	86.81%	125%

ILLUSTRATION 4:

Studies conducted by a management consultant in ABC Company indicate that annual office air-conditioning costs are 1% of the annual payroll for each 100 square meter of air-conditioned floor space per worker. Each sales personnel require an average of 200 square meter of floor space. The total annual costs include depreciation also. It is estimated that air-conditioning may boost output by 5% when it is effective. Air-conditioning will be effective between May and October only.

(a) Please find out the proposed productivity.

(b) Suggest whether the office should be air-conditioned or not.

Solution:

(a) Productivity = $\frac{\textbf{Output}}{\textbf{Input}}$ **× 100 = 100% (say)**

Proposed input = $100 + 1 \times \left(\frac{200}{100}\right) = 102$

Proposed output = $100 + \frac{6}{12} \times 5 = 102.5$

Therefore, Proposed Productivity = $\frac{\textbf{Output}}{\textbf{Input}} \times 100 = \frac{102.5}{102} \times 100 = 100.49\%$.

(b) The office should be air-conditioned as it increases the productivity.

ILLUSTRATION 5:

The following data is available for a manufacturing unit:

No. of operators	: 30
Daily working hours	: 8
No. of days per month	: 25
Standard production per month	: 600 units
Standard Labour hours per unit	: 8

The following information was obtained for January 2020:

Man-days lost due to absenteeism	: 60
Unit produced	: 480
Idle Time	: 552 man-hours

Find the following: —

(a) Percent absenteeism

(b) Efficiency of utilisation of labour

(c) Productive efficiency of labour

(d) Overall productivity of labour in terms of units produced per man per month.

Solution:

(a) Percent Absenteeism = $\frac{\text{Man – days lost due to absenteeism}}{\text{Total man – days}} \times 100 = \frac{60}{30 \times 25} \times 100 = 8\%$

(b)

Total man-hours (30 × 25 × 8)	= 6,000
Less: man-hours lost due to absenteeism (60 × 8)	= 480
Man-hours available	= 5,520

Less: Idle man-hours	= 552
Man-hours utilised	= 4,968

Efficiency of utilisation of labour = $\frac{4,968}{5,520} \times 100 = 90\%$

(c) Productive efficiency of labour = $\frac{\text{Standard man – hours for actual production}}{\text{Actual man – hours worked}} \times 100$

$$= \frac{480 \times 8}{4,968} \times 100 = 77.29\%$$

(d) Overall productivity of labour in terms of units produced per man per month

$$= \frac{\text{Actual production}}{\text{Man – month}} = \frac{480}{27.6} = 17.39 \text{ i.e., 17.39 units per man per month}$$

[Note: Man-days available – man-days lost due to absenteeism = 30 × 25 – 60 = 690

Therefore, man-month = $\frac{690}{25}$ = 27.6]

ILLUSTRATION 6:

Compute the productivity per machine hour with the following data. Also draw your interpretation.

Month	Number of machines involved	Working hours	Production units
April	400	220	99,000
May	550	180	1,00,000
June	580	220	1,25,000

Solution:

Month	Number of machines involved	Working hours	Machine hours	Production units	Productivity (Units per MH)
April	400	220	88,000	99,000	1.125
May	550	180	99,000	1,00,000	1.0101
June	580	220	1,27,600	1,25,000	0.9796

Formula used in the above table: Productivity per machine hour = $\frac{\text{Output}}{\text{Machine hours}}$

ILLUSTRATION 7:

An incentive scheme allows proportionate production bonus beyond 100% performance level.

Please calculate the amount of –

(i) Incentive bonus and

(ii) Total payment received by an operator on a particular day during which the following particulars apply:

Operation	: Assembling pocket transistor radio set
Work Content	: 30 Standard minutes per assembled set
Attended Time	: 8 Hours
Time spent on unmeasured work	: 2 Hours
Numbers of sets assembled during the day	: 15
Wage rate	: ₹75 per hour

(iii) What is the net labour productivity achieved by the operator during the day?

Solution:

Total standard minutes worked during the day = 30 × 15 = 450,

working time = 8 – 2 = 6 = 360 minutes.

Performance = (450 × 100) /360 = 125% i.e. incentive is payable on 25% which is above 100%

(i) Incentive bonus = 0.25 × 6 × ₹75 = ₹112.50 for six hours on measured work

(ii) Guaranteed wage for 8 hours = 8 × ₹75 = ₹600;

Total earnings for the days = ₹ (112.50 + 600) = ₹712.50.

(iii) Net labour productivity = $\frac{\text{Output}}{\text{Measured Man hours}} = \frac{15}{6}$ = 2.5 sets per hour.

ILLUSTRATION 8:

The following data is available for a machine in a manufacturing unit:

Hours worked per day	8
Working days per month	25
Number of operators	1

Standard minutes per unit of production :	
Machine time	22
Operator time	8
Total time	30

You are requested to answer the following:

(a) If plant is operated at 75% efficiency, and the operator is working at 100% efficiency, what is the output per month?

(b) If machine productivity is increased by 10% over the existing level, what will be the output per month?

(c) If operator efficiency is reduced by 20% over the existing level, what will be the output per month?

Solution:

Total man-hours per month = (8 × 25 × 1) × 60 = 12,000 minutes.

Output per month = 12,000 ÷ Actual time per unit.

	(a)	(b)	(c)
Machine time	22 ÷ 75% = 29.33	22 ÷ 110% = 20	22
Operator time	8.00	8	8 / 80% = 10
Actual time per unit (minutes)	37.33	30	32
Output per month (units)	**321.46**	**400**	**375**

ILLUSTRATION 9:

CO. JAI HO Ltd. requests you to construct both 'Mean' ($\overline{X}$) and 'Range' ($\overline{R}$) chart from the following data assuming each sub-group contains 4 samples:

Sub-group number	$\overline{X}$	$\overline{R}$	Sub-group number	$\overline{X}$	$\overline{R}$
1	6.36	0.10	11	6.32	0.18
2	6.38	0.18	12	6.30	0.10
3	6.35	0.17	13	6.34	0.11
4	6.39	0.20	14	6.39	0.14
5	6.32	0.15	15	6.37	0.17

Sub-group number	$\bar{X}$	$\bar{R}$	Sub-group number	$\bar{X}$	$\bar{R}$
6	6.34	0.16	16	6.36	0.15
7	6.40	0.13	17	6.35	0.18
8	6.33	0.18	18	6.35	0.13
9	6.37	0.16	19	6.34	0.18
10	6.33	0.13	20	6.34	0.16

The constant values are given below:

Sub-group size	A_2	D_3	D_4
2	1.88	0	3.27
3	1.02	0	2.57
4	0.73	0	2.28
10	0.31	0.22	1.78

Solution:

(i) $\sum \bar{X} = 127.03$,

Therefore, Central value $\left(\bar{\bar{X}}\right) = \frac{\sum \bar{X}}{n} = \frac{127.03}{20} = 6.351$, and $\bar{\bar{R}} = \frac{3.06}{20} = 0.153$

As the sub-group size is 4, the value of $\mathbf{A_2 = 0.73}$

$\therefore$ Upper Control Limit (UCL) = $\bar{\bar{X}} + A_2 \bar{\bar{R}}$ = 6.351 + 0.73 × 0.153 = 6.463

Central Value (CV) = 6.351

Lower Control Limit (LCL) = $\bar{\bar{X}} - A_2 \bar{\bar{R}}$ = 6.351 – 0.73 × 0.153 = 6.239

The Mean chart is drawn below:

UCL = 6.463

CV = 6.351

LCL = 6.239

(ii) For Range Chart, $D_3 = 0$ and $D_4 = 2.28$

Therefore,

UCL = $D_4 \bar{\bar{R}}$ = 2.28 × 0.153 = 0.349

LCL = $D_3 \bar{\bar{R}}$ = 0 × 0.153 = 0

The Range Chart can be shown as below:

UCL = 0.349

CV = 0.153

LCL = 0

ILLUSTRATION 10:

An Automatic Continuous Blending Process needs to be controlled for the acidity of the output, measured in pH. The pH levels are very fine-tuned and measured to a hundredth of a unit. The following 10 samples of three readings each were taken this morning when the process was running smoothly.

Sample No.	Values of pH			Sample No.	Values of pH		
1	5.32	5.29	5.38	**6**	5.34	5.27	5.29
2	5.28	5.41	5.40	**7**	5.35	5.33	5.31
3	5.33	5.37	5.30	**8**	5.27	5.38	5.36
4	5.40	5.29	5.32	**9**	5.40	5.41	5.38
5	5.31	5.40	5.39	**10**	5.33	5.37	5.42

Please design—(i) Sample means chart and (ii) Range chart for the statistical quality control of the process. Given, D3 = 0; D4 = 2.575.

Solution:

First calculate the sample means and Range for each of the samples and then calculate the mean of the sample means and the mean of the ranges for use in the design of the process control charts.

Sample No.	Values of pH			Mean	Range
1	5.32	5.29	5.38	**5.33**	0.09
2	5.28	5.41	5.4	**5.36**	0.13
3	5.33	5.37	5.3	**5.33**	0.07
4	5.4	5.29	5.32	**5.34**	0.11
5	5.31	5.4	5.39	**5.37**	0.09
6	5.34	5.27	5.29	**5.30**	0.07
7	5.35	5.33	5.31	**5.33**	0.04
8	5.27	5.38	5.36	**5.34**	0.11

Sample No.	Values of pH			Mean	Range
9	5.4	5.41	5.38	**5.40**	0.03
10	5.33	5.37	5.42	**5.37**	0.09
Total				**53.47**	0.83

Mean of the sample means = $\frac{53.47}{10}$ = 5.347 = Central Value (CV)

Mean Chart:

For 3-sigma control limits of Mean,

UCL = 5.347 + 3 σ = 5.347 + 3 × 0.0293308 = 5.435;

LCL = 5.347 – 3 σ = 5.347   3 × 0.0293308 = 5.259.

Range chart: -

Central Value = $\frac{0.83}{10}$ = 0.083

UCL = $D_4\overline{R}$ = 2.575 × 0.083 = 0.2137;

LCL = $D_3\overline{R}$ = 0.

ILLUSTRATION 11:

Ten pieces of cloth out of different rolls of equal length contained the following number of defects:

3, 0, 2, 8, 4, 2, 1, 3, 7, 1.

Please prepare a C-chart and state whether the production process is in a state of statistical control.

Solution:

Here, 3-sigma central limit is to be formed as follows:

Central Value = $\overline{C} = \frac{(3+0+2+8+4+2+1+3+7+1)}{10}$ = 3.1

UCL = $\overline{C} + 3.\sqrt{\overline{C}}$ = 8.38;

LCL = $\overline{C} - 3.\sqrt{\overline{C}}$ = – 2.18 i.e.0

The C-Chart is shown as below:

UCL = 8.38

CV = 3.1

LCL = 0

ILLUSTRATION 12:

MR. MRITYUNJAY DUTTA THE GREAT Ltd. is monitored for quality as it arrives to be rolled on to a beam. Usually, a meter of the cloth is taken as a sample and checked for defects of different kinds such as-

(1) Knots,

(2) Oily or greasy patches or spots,

(3) Missed weave,

(4) Cotton seed shell particles.

The sample may have a number of these defects. For instance, if it has one knot, two oil marks and one missed weave, then the number of defects is counted as (1+ 2+ 1 = 4).

During the first shift on Thursday, 20 beams have been woven. The quality control record is given in the following table:

Sample no.	No. of Defects	Sample no.	No. of Defects	Sample no.	No. of Defects
1	4	8	2	15	2
2	9	9	1	16	2
3	3	10	9	17	1
4	12	11	12	18	3
5	5	12	9	19	1
6	3	13	3	20	4
7	2	14	9		

Please design an appropriate control chart. Was the process in control in the first shift on Thursday?

Solution:

Here ‘c’ chart should be drawn. $\bar{c}$ = Mean no. of defects = 4.8

For 3-s control limit, UCL = $\bar{c} + 3\sqrt{\bar{c}}$ and

LCL = $\bar{c} - 3\sqrt{\bar{c}}$.

Mean value = $\bar{c}$ = 4.8;

UCL = 11.4 &

LCL = 0 as control limit cannot be negative.

Note: The no. of defects in sample no. 4 and 11 exceeds the UCL, thus, these two samples should be ignored and the control limits should be recalculated.

Revised UCL = 10 & LCL = 0, as it cannot be negative.

ILLUSTRATION 13:

Draw the control chart for $\overline{X}$ (mean) and R (range) from the following data relating to 20 samples of size 5:

Sample No.	$\overline{X}$	R	Sample No.	$\overline{X}$	R
1	38.2	15	11	32.6	30
2	33.8	11	12	22.8	12
3	24.4	22	13	21.6	29
4	36.6	24	14	28.8	22
5	27.4	18	15	28.8	16
6	30.6	25	16	24.4	19
7	31.2	21	17	30.4	20
8	27.0	29	18	25.4	24
9	24.0	29	19	37.8	19
10	29.4	18	20	31.4	17

[For sample of size 5, d2 = 2.326; d3 = 0.864]

Solution:

For Mean Chart

Central line = $\overline{\overline{X}} = \frac{\sum \overline{X}}{N} = \frac{587}{20}$ = 29.35; $\sigma_P = \frac{\overline{R}}{d_2} = \frac{420 \div 20}{2.326} =$ 9.028375

Upper control limit (UCL) = $\overline{\overline{X}} + \frac{3.\sigma_P}{\sqrt{n}} =$ = 29.35 + 3 × (9.028375 ÷ $\sqrt{5}$) = 41.46

Lower control limit (LCL) = $\overline{\overline{X}} - \frac{3.\sigma_P}{\sqrt{n}} =$ 29.35 – 3 × (9.028375 ÷ $\sqrt{5}$) = 17.24

For Range chart

$$\bar{R} = \frac{\sum R}{N} = \frac{420}{20} = 21; \quad \sigma_R = \frac{\bar{R}}{d_2} \times d_3 \; = 9.028375 \times 0.864 = 7.800516$$

- UCL = $\bar{R} + 3.\sigma_R$ = 21 + 3 × 7.800516 = 44.40
- LCL = $\bar{R} - 3.\sigma_R$ = 21 – 3 × 7.800516 = – 2.40 i.e. Nil.

ILLUSTRATION 14:

The following table gives the result of inspection of 20 samples of 100 items each taken in 20 working days. Please draw a P – chart.

Sample No.	No. of defectives	Sample No.	No. of defectives
1	6	11	10
2	2	12	4
3	4	13	6
4	1	14	11
5	20	15	22
6	6	16	8
7	10	17	10
8	19	18	3
9	4	19	23
10	21	20	10

What conclusion can you draw from the above data? For revising the control limits what is to be done?

Solution:

Total no. items inspected = 20 × 100 = 2,000; total no. of defectives = 200

Average fraction defectives (i.e. proportion of defectives) = $\bar{p}$ = (200 ÷ 2,000) = 0.10

$1 - \bar{p}$ = 1 – 0.1 = 0.9

Central line (CL) = $\bar{p}$ = 0.1

$$UCL = \bar{p} + 3 \times \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.1 + 3 \times \sqrt{\frac{0.1 \times 0.9}{100}} = 0.19$$

$$LCL = \bar{p} - 3 \times \sqrt{\frac{\bar{p}}{n}(1-\bar{p})} = 0.1 - 3 \times \sqrt{\frac{0.1 \times 0.9}{100}} = 0.01$$

Conclusion:

Four samples numbered 5, 10, 15 & 19 lie outside the control limits of 0.01 to 0.19.

4 Project Management, Monitoring and Control [Study Material - Module 6]

NETWORK ANALYSIS
CPM (Critical Path Method)

PRACTICAL PROBLEMS

ILLUSTRATION 1:

The following table gives the activities in a construction project and other relevant information:

Activity	:	1—2	1—3	2—3	2—4	3—4	4—5
Duration (Days)	:	20	25	10	12	6	10

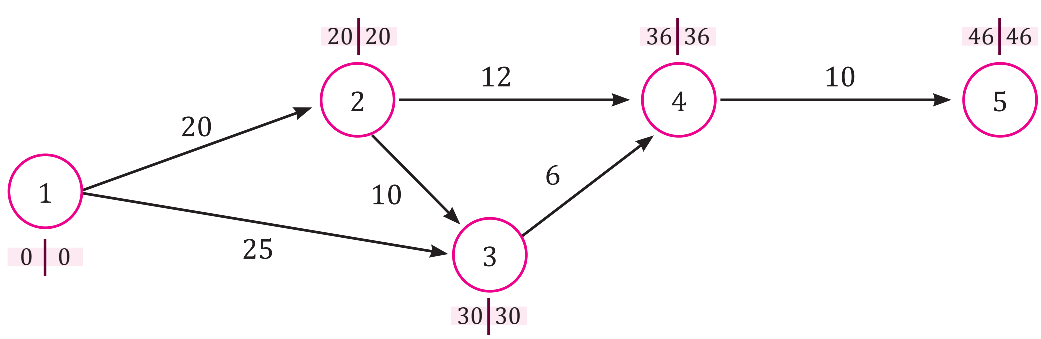
(a) Please draw the network diagram for the project.

(b) Find the critical path and normal duration of the project.

(c) Find the total float, free float and independent float for each activity.

Solution:

(a)



(b) Critical path: 1 – 2 – 3 – 4 – 5

Normal duration of the project: 46 days

(c) Computation of Floats:

Activity	Total Float = LF – ES - t	Free Float = TF – Head slack	Independent Float = FF – Tail slack
1 – 2	20 – 0 – 20 = 0	0 – 0 = 0	0 – 0 = 0
1 – 3	30 – 0 – 25 = 5	5 – 0 = 5	5 – 0 = 5
2 – 3	30 – 20 – 10 = 0	0 – 0 = 0	0 – 0 = 0
2 – 4	36 – 20 – 12 = 4	4 – 0 = 4	4 – 0 = 4
3 – 4	36 – 30 – 6 = 0	0 – 0 = 0	0 – 0 = 0
4 – 5	46 – 36 – 10 = 0	0 – 0 = 0	0 – 0 = 0

ILLUSTRATION 2:

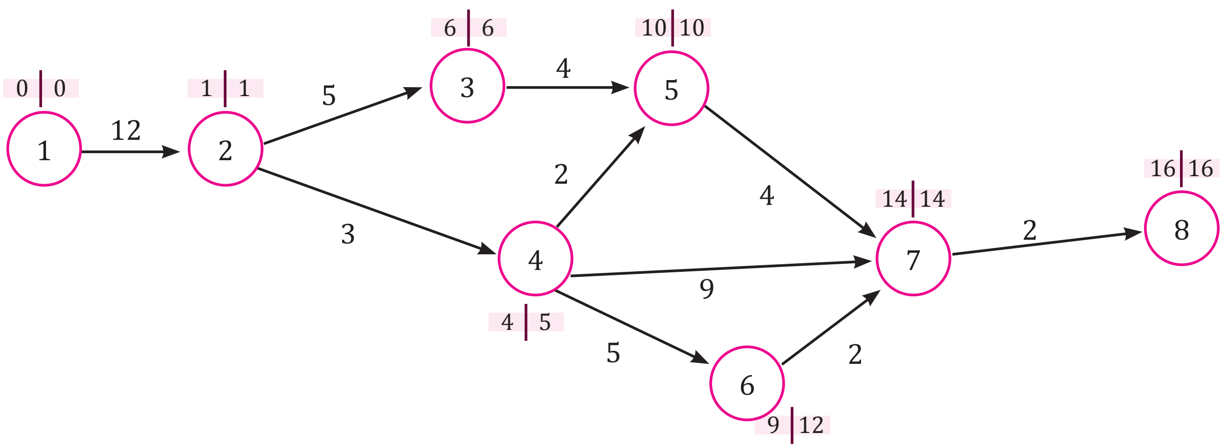
For the following activities of a repair works find—

(a) The critical path and normal duration of the project;

(b) The total, free & independent floats of the activities.

Activity	Duration (Days)	Activity	Duration (Days)
1—2	1	4—6	5
2—3	5	4—7	9
2—4	3	5—7	4
3—5	4	6—7	2
4—5	2	7—8	2

Solution:



Here, the Critical path is 1—2—3—5—7—8

Normal duration of the project is 16 days.

(b) Computation of Floats:

Activity	Total Float = LF – ES - t	Free Float = TF – Head slack	Independent Float = FF – Tail slack
1—2	0	0	0
2—3	0	0	0
2—4	5 – 1 – 3 = 1	1 – 1 = 0	0 – 0 = 0
3—5	0	0	0
4—5	10 – 4 – 2 = 4	4 – 0 = 4	4 – 1 = 3
4—6	12 – 4 – 5 = 3	3 – 3 = 0	0 – 1 = - 1 i.e., 0
4—7	14 – 4 – 9 = 1	1 – 0 = 1	1 – 1 = 0
5—7	0	0	0
6—7	14 – 9 – 2 = 3	3 – 0 = 3	3 – 3 = 0
7—8	0	0	0

Note: All the floats of the critical activities are zero.

ILLUSTRATION 3:

A new project has the following activities—

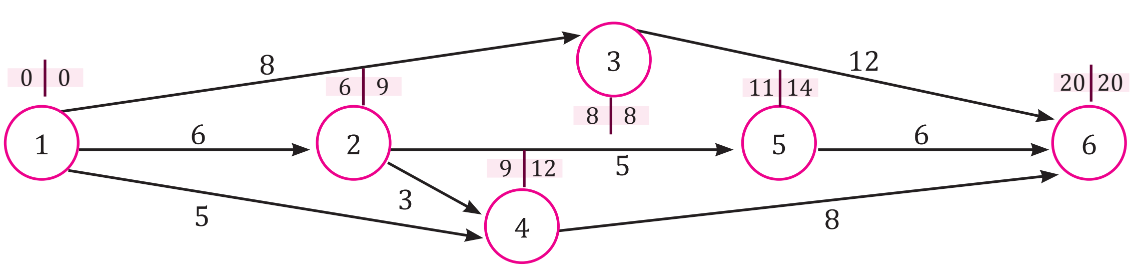
Activity	:	1—2	1—3	1—4	2—4	2—5	3—6	4—6	5—6
Duration (Days)	:	6	8	5	3	5	12	8	6

(a) Please draw the network diagram;

(b) Find out the critical path and normal duration of the project;

(c) Compute the floats of the activities.

Solution:



(c) Computation of Floats:

Activity	Total Float = LF – ES - t	Free Float = TF – Head slack	Independent Float = FF – Tail slack
1—2	9 – 0 – 6 = 3	3 – 3 - 0	0 – 0 = 0
1 – 3	0	0	0
1 – 4	12 – 0 – 5 = 7	7 – 3 = 4	4 – 0 = 4
2 – 4	12 – 6 – 3 = 3	3 – 3 = 0	0 – 3 = - 3 i.e., 0
2 – 5	14 – 6 – 5 = 3	3 – 3 = 0	0 – 3 = - 3 i.e., 0
3 – 6	0	0	0
4 – 6	20 – 9 – 8 = 3	3 – 0 = 3	3 – 3 = 0
5 – 6	20 – 11 – 6 = 3	3 – 0 = 3	3 – 3 = 0

ILLUSTRATION 4:

Draw the network and find the critical path from the following data:

Activity	Predecessor	Duration (weeks)	Cost (₹)
A	NIL	2	50
B	NIL	4	50
C	A	1	40
D	B	2	100
E	C, D	3	100
F	E	2	60

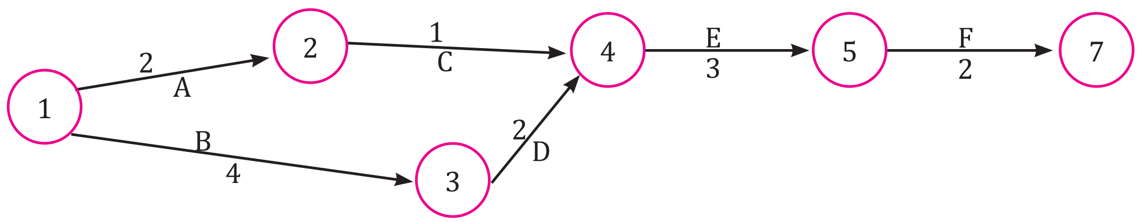
(a) Draw the network diagram and find out the critical path of the project.

(b) What is the normal duration of the project?

(c) Draw a Gantt chart for early start schedule.

Solution:

(a)



Paths	Duration (Weeks)
1 — 2 — 4 — 5 — 7	8
1 — 3 — 4 — 5 — 7	11

Critical path is the path with longest duration of the project.

Here, Critical path is 1—3—4—5—7

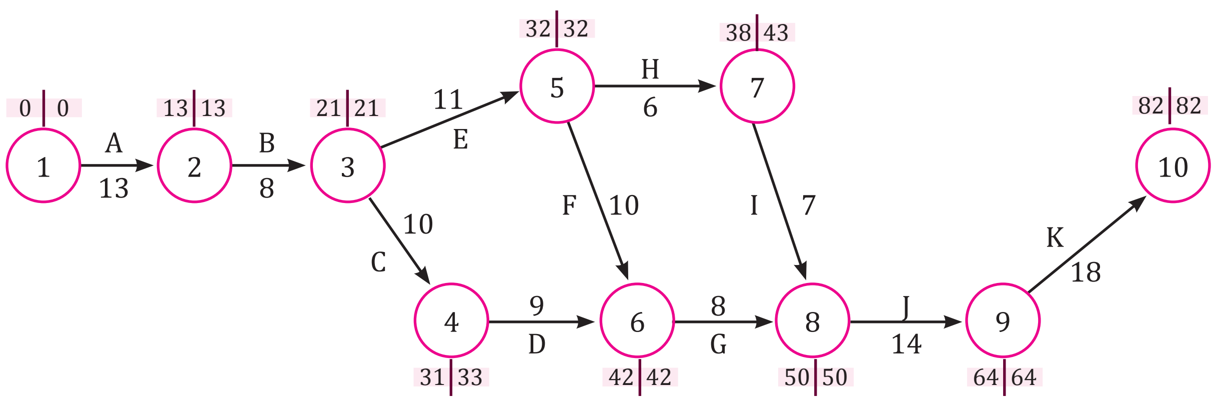
(b) The normal duration of the project is 11 weeks.

ILLUSTRATION 5:

Draw network diagram from following activities and find critical path and slacks of the activities:

Job	: A	B	C	D	E	F	G	H	I	J	K
Days	: 13	8	10	9	11	10	8	6	7	14	18
Immediate Predecessor	: A	B	C	B	E	D, F	E	H	G, I	J	

Solution:



(b) Computation of Floats:

Activity	Total Float = LF – ES - t	Free Float = TF – Head slack	Independent Float = FF – Tail slack
1—2	0	0	0
2—3	0	0	0
3 – 4	33 – 21 – 10 = 2	2 – 2 = 0	0 – 0 = 0

Activity	Total Float = LF – ES - t	Free Float = TF – Head slack	Independent Float = FF – Tail slack
3 – 5	0	0	0
4 – 6	42 – 31 – 9 = 2	2 – 0 = 2	2 – 2 = 0
5 – 6	0	0	0
5 – 7	43 – 32 – 6 = 5	5 – 5 = 0	0 – 0 = 0
6 – 8	0	0	0
7 – 8	50 – 38 –7 = 5	5 – 0 = 5	5 – 5 = 0
8 – 9	0	0	0
9 – 10	0	0	0

Note: All the floats of the critical activities are zero.

ILLUSTRATION 6:

A company manufacturing plant and equipment for chemical processing is in the process of quoting a tender called by a public sector undertaking. Delivery data once promised is very crucial and penalty clause is applicable. The winning of tender also depends on how soon the company can deliver the goods. Project Manager has listed down the activities in the project as under:

Activity	Predecessor	Duration (Week)
A	--	3
B	--	4
C	A	5
D	A	6
E	C	7
F	D	8
G	B	9
H	E, F, G	3

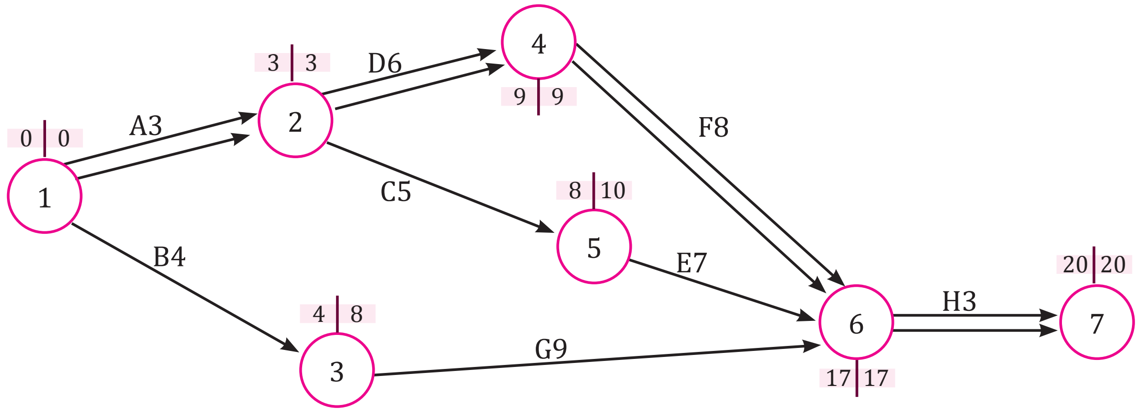
(a) Please calculate the delivery week from the date of acceptance of quotation.

(b) Find out the Total float, Free float and Independent float for each activity.

(c) Do you think the float of critical activity is always nil? Why?

Solution:

(a)



(b) Computation of Floats:

Activity	Total Float = LF – ES - t	Free Float = TF – Head slack	Independent Float = FF – Tail slack
A	0	0	0
B	8 – 0 – 4 = 4	4 – 4 = 0	0 – 0 = 0
C	10 – 3 – 5 = 2	2 – 2 = 0	0 – 0 = 0
D	0	0	0
E	17 – 8 – 7 = 2	2 – 0 = 2	2 – 2 = 0
F	0	0	0
G	17 – 4 – 9 = 4	4 – 0 = 4	4 – 4 = 0
H	0	0	0

(c) Yes, the floats of the critical activities are always zero. The critical activities are the activities with longest duration when all the activities relating to that point is completed. If float is considered that means if rest is taken, then the duration of the project will be delayed.

ILLUSTRATION 7:

A project consists of the following activities and following three-time estimates (days)—

Activity	Optimistic time (t_o)	Pessimistic time (t_p)	Most likely time (t_m)
1—2	3	15	6
1—3	2	14	5

Activity	Optimistic time (t_o)	Pessimistic time (t_p)	Most likely time (t_m)
1—4	6	30	12
2—5	2	8	5
2—6	5	17	11
3—6	3	15	6
4—7	3	27	9
5—7	1	7	4
6—7	2	8	5

(a) Draw the network diagram;

(b) Determine the expected task times and their variances;

(c) Find the critical path;

(d) Compute the floats of the activities;

(e) What is the probability that the project will be completed by 27 days?

Solution:

First, we have to compute the expected time and variance of each activity applying the following formula:

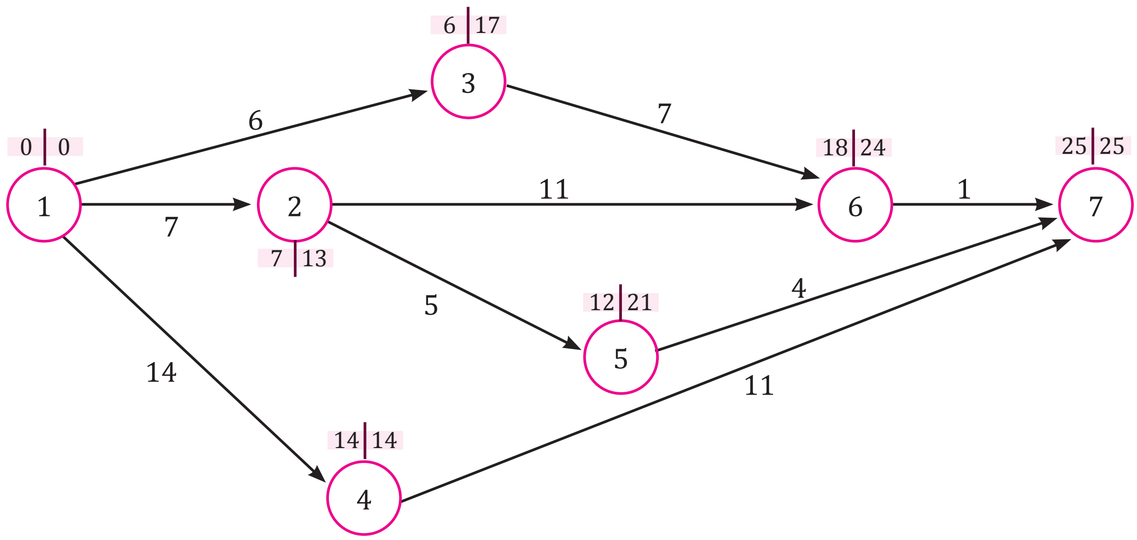
$$\text{Expected time} = \frac{(1 \times \text{Optimistic time} + 1 \times \text{Pessimistic time} + 4 \times \text{Most likely time})}{6}$$

$$\text{Variance} = \left(\frac{\text{Pessimistic Time - Optimistic Time}}{6}\right)^2$$

Activity	Expected time	Variance
1—2	7	4
1—3	6	4
1—4	14	16
2—5	5	1
2—6	11	4
3—6	7	4
4—7	11	16
5—7	4	1
6—7	5	1

Solution:

(a), (b) & (c)



The critical path is 1 – 4 – 7 and the normal project duration is 25 days.

(d) Computation of Floats:

Activity	Total Float = LF – ES - t	Free Float = TF – Head slack	Independent Float = FF – Tail slack
1 – 2	13 – 0 – 7 = 6	6 – 6 = 0	0 – 0 = 0
1 – 3	17 – 0 – 6 = 11	11 – 11 = 0	0 – 0 = 0
1 – 4	0	0	0
2 – 5	21 – 7 – 5 = 9	9 – 9 = 0	0 – 6 = - 6 i.e., 0
2 – 6	24 – 7 – 11 = 6	6 – 6 = 0	0 – 6 = - 6 i.e., 0
3 – 6	24 – 6 – 7 = 11	11 – 6 = 5	5 – 11 = - 6 i.e., 0
4 – 7	0	0	0
5 – 7	25 – 12 –4 = 9	9 – 0 = 9	9 – 9 = 0
6 – 7	25 – 18 – 1= 6	6 – 0 =6	6 – 6 = 0

(e) Normal duration = $\bar{x}$ = 25 days. x = required duration of the project

$Z = \frac{x - \bar{x}}{\sigma}$; where, = Standard deviation of the project = $\sqrt{16+16} = \sqrt{32} = 5.657$

$$P(x \le 27) = P\left(Z \le \frac{(27-25)}{5.657}\right) = P(z \le 0.35)$$

= 0.50 + 0.1368 [From Normal Distribution Table] = 0.6368 i.e., 63.68%.

ILLUSTRATION 8:

A company manufacturing plant and equipment for chemical processing is in the process of quoting a tender called by a public sector undertaking. Delivery data once promised is very crucial and penalty clause is applicable. The winning of tender also depends on how soon the company is able to deliver the goods. Project Manager has listed down the activities in the project as under:

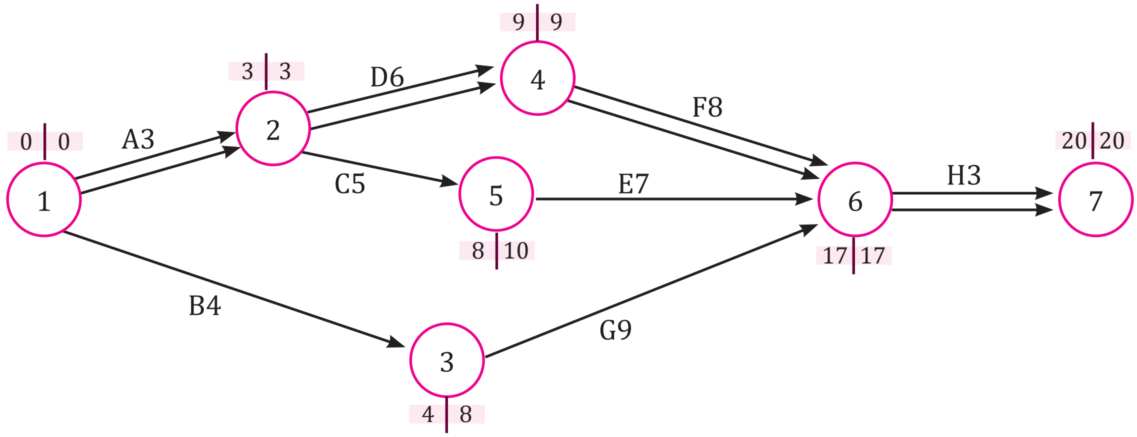
Activity	Predecessor	Optimistic time (week)	Most likely time (week)	Pessimistic time (week)
A	--	1	3	5
B	--	2	4	6
C	A	3	5	7
D	A	5	6	7
E	C	5	7	9
F	D	6	8	10
G	B	7	9	11
H	E, F, G	2	3	4

(a) Please calculate the delivery week from the date of acceptance of quotation.

(b) Find out the Total float, Free float and Independent float for each activity.

Solution:

Activity	Expected Duration (Week)
A	3
B	4
C	5
D	6
E	7
F	8
G	9
H	3



Computation of Floats:

Activity	Total Float = LF – ES - t	Free Float = TF – Head slack	Independent Float = FF – Tail slack
A	0	0	0
B	8 – 0 – 4 = 4	4 – 4 = 0	0 – 0 = 0
C	10 – 3 – 5 = 2	2 – 2 = 0	0 – 0 = 0
D	0	0	0
E	17 – 8 – 7 = 2	2 – 0 = 2	2 – 2 = 0
F	0	0	0
G	17 – 4 – 9 = 4	4 – 0 = 4	4 – 4 = 0
H	0	0	0

ILLUSTRATION 9:

A project is composed of 7 activities whose time estimates are listed in the time below—

Activity	Optimistic time (t_o)	Pessimistic time (t_p)	Most likely time (t_m)
1—2	1	7	1
1—3	1	7	4
1—4	2	8	2
2—5	1	1	1
3—5	2	14	5

Activity	Optimistic time (t_o)	Pessimistic time (t_p)	Most likely time (t_m)
4—6	2	8	5
5—6	3	15	6

(a) Draw the network diagram;

(b) Determine the expected task times and their variances;

(c) Find the critical path;

(d) Compute the floats of the activities;

(e) What is the probability that the project will be completed—

(i) At least 4 weeks earlier than expected time?

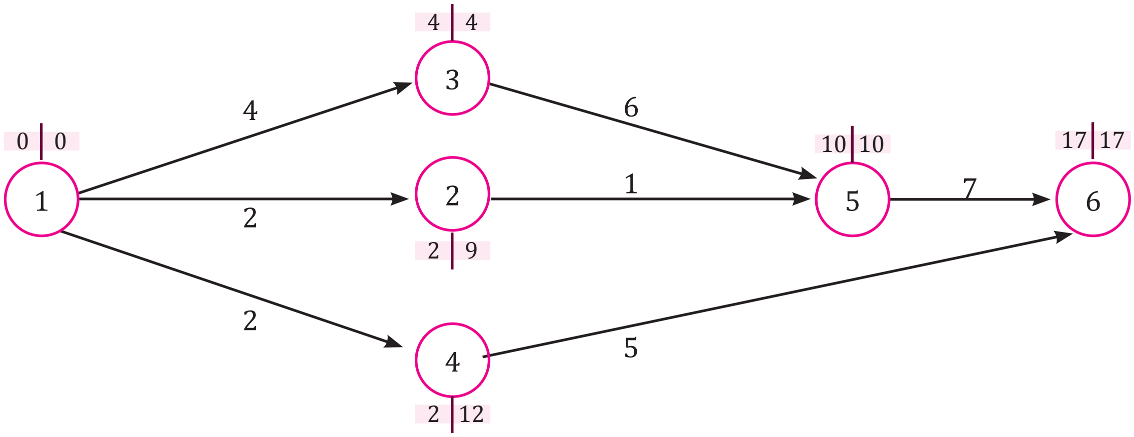
(ii) No more than 4 weeks later than expected time?

(f) If the project due date is 19 weeks, what is the probability of not meeting the due date?

Solution:

(a), (b) and (c):

Activity	Expected time (weeks)	Variance
1—2	2	1
1—3	4	1
1—4	2	1
2—5	1	0
3—5	6	4
4—6	5	1
5—6	7	4



The critical path is 1 – 3 – 5 – 6 and the normal project duration is 17 weeks.

(d) Computation of Floats:

Activity	Total Float = LF – ES - t	Free Float = TF – Head slack	Independent Float = FF – Tail slack
1 – 2	9 – 0 – 2 = 7	7 – 7 = 0	0 – 0 = 0
1 – 3	0	0	0
1 – 4	12 – 0 – 2 = 10	10 – 10 = 0	0 – 0 = 0
2 – 5	10 – 2 – 1 = 7	7 – 0 = 7	7 – 7 = 0
3 – 5	0	0	0
4 – 6	17 – 2 – 5 = 10	10 – 0 = 10	10 – 10 = 0
5 – 6	0	0	0

(e) Variance of the project = 1 + 4 + 4 = 9

Therefore, Standard deviation of the project = σ = $\sqrt{9}$ = 3

(i) $P(x \leq 13) = P\left(Z \leq \frac{(13-17)}{3}\right) = P(z \leq -1.33)$

= 0.50 - 0.4082 [From Normal Distribution Table] = 0.0918 i.e., 9.18%

(ii) $P(x \leq 21) = P\left(Z \leq \frac{(21-17)}{3}\right) = P(z \leq 1.33)$

= 0.50 + 0.4082 [From Normal Distribution Table] = 0.9082 i.e., 90.82%.

ILLUSTRATION 10:

Following data are available from the records of a construction project:

Activity	Normal		Crash	
	Time (Days)	Cost (₹)	Time (Days)	Cost (₹)
1—2	8	100	6	200
1—3	4	150	2	350
2—4	2	50	1	90
2—5	10	100	5	400

Activity	Normal		Crash	
	Time (Days)	Cost (₹)	Time (Days)	Cost (₹)
3—4	5	100	1	200
4—5	3	80	1	100

Indirect cost is ₹70 per day.

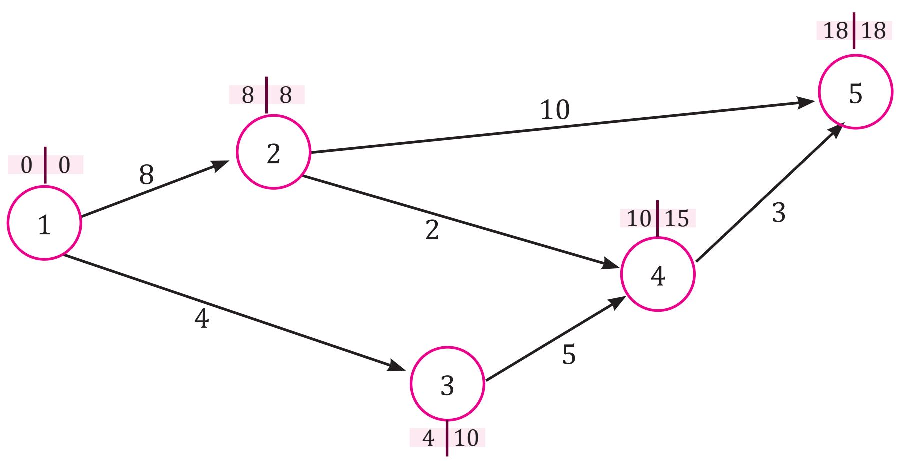
(a) Draw the network diagram;

(b) Find out the critical path and determine the normal project duration and associated project cost.

(c) Crash the activities systematically and find out the optimum duration, crash duration and their associated costs.

Solution:

(a)



(b) Critical path: 1 – 2 – 5

Normal duration of the project: 18 days

Project cost for 18 days = Direct costs + Indirect costs = ₹. (580 + 18 × 70) = ₹. 1,840

(c) Table 1: Slope = $\frac{\text{Crash Cost – Normal Cost}}{\text{Normal Duration – Crash Duration}}$

Activity	Slope (₹.)	Days that can be crashed			
1 – 2	50	2	0		
1 – 3	100	2			

Activity	Slope (₹.)	Days that can be crashed			
2 – 4	40	1			
2 – 5	60	5		1	
3 – 4	25	4			
4 – 5	10	2		0	

Table 2:

Paths	Duration (Weeks)				
1 – 2 – 5	18	16	12	11	
1 – 2 – 4 – 5	13	11	11	10	
1 - 3 – 4 – 5	12	12	12	11	

Table 3: Schedule of time and associated cost of the project:

Activities crashed	Duration (days)	Total costs (₹)
NIL	18	1,840
1 – 2 by 2 days	16	1,840 + (50 – 70) × 2 = 1,800
2 – 5 by 4 days	12	1,800 + (60– 70) × 4 = 1,760
2 – 5 & 4 – 5 by 1 day each	11	1,760 + (60 + 10 – 70) × 1= 1,760

ILLUSTRATION 11:

The normal and crash duration with costs for various activities relating to a repair work are given below:

Activity	Normal		Crash	
	Time (Days)	Cost (₹)	Time (Days)	Cost (₹)
1—2	6	4,000	2	12,000
1—3	8	3,000	3	6,000
2—4	7	2,800	4	4,000
3—4	12	9,000	8	11,000
4—6	3	10,000	1	13,000
5—6	5	4,900	2	7,000

Activity	Normal		Crash	
	Time (Days)	Cost (₹)	Time (Days)	Cost (₹)
3—5	7	1,800	3	5,000
5—7	11	6,600	5	12,000
6—7	10	4,000	6	8,400

Indirect cost is ₹2,000 per day.

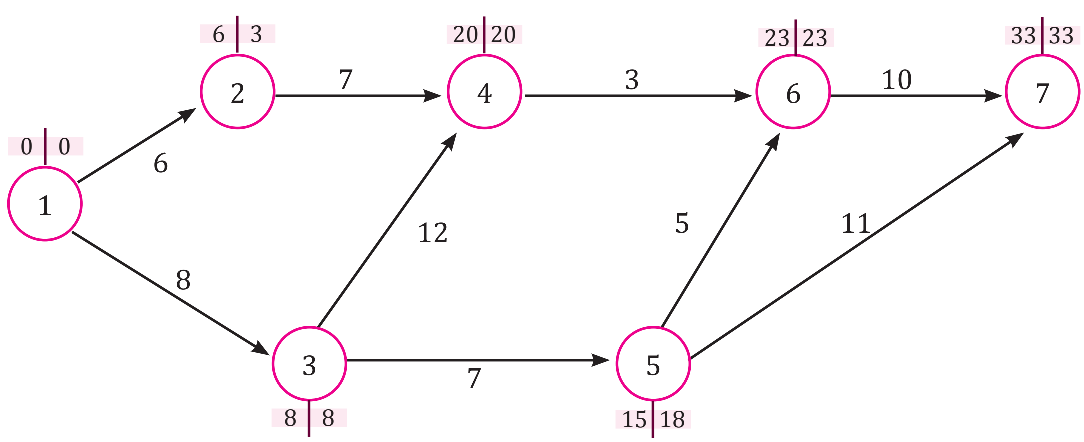
(a) Draw the network diagram;

(b) Find out the critical path and determine the normal project duration and associated project cost.

(c) Crash the activities systematically and find out the optimum duration, crash duration and their associated costs.

Solution:

(a)



Critical path is 1 – 3 – 4 – 6 – 7 and normal project duration is 33 days.

(c) Table 1: Slope = $\frac{\text{Crash Cost} - \text{Normal Cost}}{\text{Normal Duration} - \text{Crash Duration}}$

Activity	Slope (₹)	Days that can be crashed						
1 – 2	2,000	4						
1 – 3	600	5		1	0			
2 – 4	400	3			2		1	
3 – 4	500	4	1				0	

Activity	Slope (₹)	Days that can be crashed						
4 – 6	1,500	2						0
5 – 6	700	3						
3 – 5	800	4					3	1
5 – 7	900	6						
6 – 7	1,100	4				0		

Table 2:

Paths	Duration (Weeks)						
1 – 2 – 4 – 6 – 7	26	26	26	25	21	20	18
1 – 3 – 4 – 6 – 7	33	30	26	25	21	20	18
1 - 3 – 5 – 6 – 7	30	30	26	25	21	20	18
1 – 3 – 5 – 7	26	26	22	21	21	20	18

Table 3: Schedule of time and associated cost of the project:

Activities crashed	Duration (days)	Total costs (₹)
NIL	33 (normal)	46,100 + 33 × 2,000 = 1,12,100
3 – 4 by 3 days	30	1,12,100 + (500 – 2,000) × 3 = 1.07,600
1 – 3 by 4 days	26	1,07,600 + (600 – 2,000) × 4 = 1,02,000
1 – 3 & 2 – 4 by 1 day each	25	1,02,000 + (600 + 400 – 2,000) × 1 = 1,01,000
6 – 7 by 4 days	21	1,01,000 + (1,100 – 2,000) × 4 = 97,400
2 – 4, 3 – 4 & 3 – 5 by 1 day each	20 (Optimum)	97,400 + (400 + 500 + 800 – 2,000) × 1 = 97,100
4 – 6 & 3 – 5 by 2 days each	18 (Minimum)	97,100 + (1,500 + 800 – 2,000) × 2 = 97,700

ILLUSTRATION 12:

A Marketing Manager wants to launch a new product. He furnishes the following estimates of time and cost of the project:

Activity	Normal		Crash	
	Time (Days)	Cost (₹)	Time (Days)	Cost (₹)
1—2	6	1,400	4	1,900
1—3	8	2,000	5	2,800
2—3	4	1,000	2	1,500
2—4	3	800	2	1,400
3—4	Dummy	NIL	Dummy	NIL
3—5	6	900	3	1,600
4—6	10	2,500	6	3,500
5—6	3	500	2	800

Indirect cost for the project is ₹300 per day.

(a) Draw the network diagram.

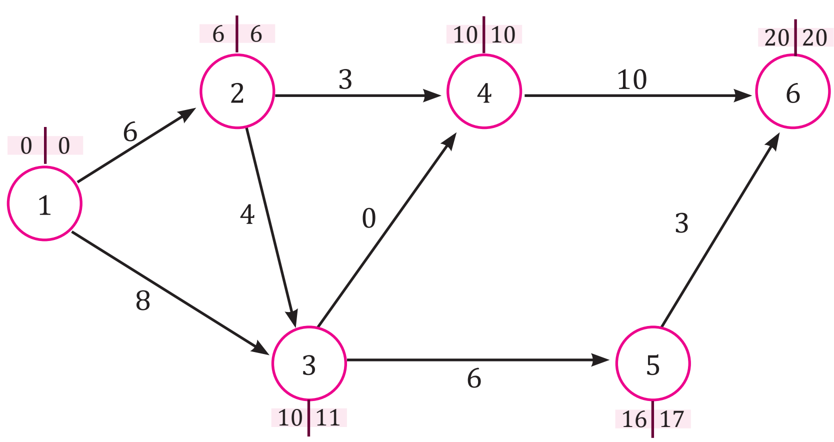
(b) What is the normal duration and cost of the project ?

(c) If all activities are crashed, what will be the project duration and cost of the project ?

(d) Please find out the optimum duration and corresponding cost of the project.

Solution:

(a)



(b) Normal duration of the project is 20 days.

Total cost of the project = Direct costs + Indirect costs = 9,100 + 20 × 300 = 15,100.

(c) & (d):

Table 1:

Activity	Slope (₹)	Days that can be crashed				
		1st	2nd	3rd	4th	5th
1 – 2	250	2	0			
1 – 3	267	3				1
2 – 3	250	2				0
2 – 4	600	1				
3 – 4	0	0				
3 – 5	233	3			0	
4 – 6	250	4		3	0	
5 – 6	300	1				

Table 2:

Paths	Duration (Weeks)				
1 – 2 – 4 – 6	19	17	16	13	13
1 – 2 – 3 – 4 – 6	20	18	17	14	13
1 – 2 – 3 – 5 – 6	19	17	17	14	13
1 – 3 – 4 – 6	18	18	17	14	13
1 – 3 – 5 – 6	17	17	17	14	13

Table 3: Schedule of time and associated cost of the project:

Activities crashed	Duration (days)	Total costs (₹.)
NIL	20 (Normal)	15,100
1 – 2 by 2 days	18	15,100 + (250 – 300) × 2 = 15,000
4 – 6 by 1 day	17 (Optimum)	15,000 + (250 – 300) × 1 = 14,950 =
3 – 5 & 4 – 6 by 3 days each	14	14,950 + (233 + 250 – 300) × 3 = 15,499
1 – 3 & 2 – 3 by 1 day each	13 (Minimum)	15,499 + (267 + 250 – 300) × 1 = 15,716

5 Economies of Maintenance & Spares Management [Study Material - Module 7]

ILLUSTRATION 1:

A workshop has 25 identical machines. The failure pattern of the machine is given below:-

Elapsed time after maintenance attention (month)	Probability of failure
1	0.12
2	0.15
3	0,23
4	0.25
5	0.10
6	0.05

The workshop incurs ₹. 3,250 to attend and rectify a failed machine.

Please compute the yearly cost of servicing the broken-down machines.

Solution:

Expected life of the machines (i.e., Expected time before failure)

$= (1 \times 0.12 + 2 \times 0.15 + 3 \times 0.23 + 4 \times 0.25 + 5 \times 0.10 + 6 \times 0.05) = 2.91$ Months.

$\therefore$ Yearly cost of servicing $= 25 \times \left(\frac{12}{2.91}\right) \times$ ₹ 3,250 = ₹ 3,35,052

ILLUSTRATION 2:

The breakdown probability of an equipment is given below:

Month	1	2	3	4	5
Probability of failure	0.05	0.15	0.30	0.30	0.20

There are 50 such equipments in the plant. The cost of individual preventive maintenance is ₹1,500 per equipment and the cost of breakdown maintenance is ₹3,000 per equipment.

Which is the most suitable maintenance policy in this – preventive maintenance or breakdown maintenance ?

[Ans.: MTTF = 3.45 Months; mean number of breakdown per month = $\frac{50}{3.45}$ = 14.49.

Averaage individual breakdown maintenance cost per month = $\frac{50}{3.45}$ × ₹ 3,000 = ₹43,478.26.

Average cost of individual preventive maintenance policy:

	Average cost per month (₹)
Every 1 month	78,750
Every 2 months	46,154
Every 3 months	40,909
Every 4 months	41,538

Therefore, individual preventive maintenance policy after every 3 months is the most economical option]

ILLUSTRATION 3:

A plant is utilizing 6 types of machines. If one machine breaks down, the entire assembly line comes to a halt unless the breakdown machine is repaired or replaced. It takes 5 hours to replace or repair a machine out of which 1 hour is variable. The mechanic's wage rate is ₹250 per hour. Repair material cost is ₹2,000 and downtime cost is ₹1,000 per hour.

Which of the two maintenance strategies should the company adopt?—

(a) Replace only the defective machine;

(b) Replace the defective machine as also all other machines.

Solution:

Particulars	Strategy 1	Strategy 2
Total downtime hours	4 + 1× 1 = 5	4 + 6 × 1 =10
Downtime costs @ ₹. 1,000 per hour	₹. 5,000	₹. 10,000
Mechanic's wages @ ₹250 per hour	₹. 1,250	₹. 2,500
Material costs @ ₹ 2,000 per machine	₹. 2,000	₹. 12,000
Total costs	₹. 8,250	₹. 24,500

Recommendation:

Strategy 1 is more economical than Strategy 2. Thus Strategy 1 should be adopted.]

ILLUSTRATION 4:

COOL-BREEZE LTD, a laptop assembling company is experiencing the following number of breakdowns of laptop for over the past 24 months:

Number of breakdowns of laptop	0	1	2	3	4
Number of months that occurred	4	7	9	3	1

Each break down of laptop costs the company an average of ₹3,200. For a cost of ₹2,500 per month, preventive maintenance can be carried out to limit the breakdowns to an average of one laptop per month.

Which policy is suitable for the firm – Breakdown maintenance policy or Preventive maintenance policy?

Solution:

Breakdown Maintenance Policy:

Expected number of breakdowns per month = $\frac{(0\times4+1\times7+2\times9+3\times3+4\times1)}{24}$ = 1.583.

Expected Breakdown cost per month = Expected no. of breakdowns per month × cost of each breakdown = 1.583 × ₹3,200 = ₹5,065.60.

Preventive Maintenance Policy:

Average cost of one breakdown/month	=	₹ 3,200
Maintenance contract cost/month	=	₹ 2,500
Preventive maintenance cost per month	=	₹ 5,700

Recommendation: Preventive maintenance policy is suitable for the company.

ILLUSTRATION 5:

The main shaft of an equipment has a very high reliability of 0.98. The equipment comes from Canada and has a high downtime cost associated with the failure of this shaft. This is estimated at ₹300 lakh as the costs of sales lost and other relevant costs. However, this spare is quoted at ₹8.25 lakh at present.

Should the shaft spare be procured along with the equipment and kept or not?

Solution:

The expected cost of down-time = (Probability of failure) × (Cost when break-down occurs)

= (1 - 0.98) × (₹ 300 lakh) = ₹6 lakh.

Therefore, expected cost of downtime (₹. 6 lakh) is less than the cost of shaft spare (₹. 8.25 lakh).

Recommendation: The company should not buy the shaft spare along with the equipment.

ILLUSTRATION 6:

Product A has a Mean Time Between Failures (MTBF) of 50 hours and has a Mean Time To Repairs (MTTR) of 10 hours. Product B has a MTBF of 60 hours and has a MTTR of 5 hours.

You are requested to calculate—

(i) Which product has the higher reliability?

(ii) Which product has greater maintainability?

(iii) Which product has greater availability?

Solution:

(i) Product B, with higher MTBF (i.e., 60 hours) than Product A (i.e., 50 hours), is more reliable since it has lesser chance of failure during servicing.

(ii) By MTTR we mean the time taken to repair a machine and put it into operation. Thus, Product B, with lesser MTTR (i.e., 5 hours) than Product A (i.e., 10hours), has greater maintainability.

(iii) Availability of a machine/product = $\frac{\text{MTBF}}{(\text{MTBF} + \text{MTTR})} \times 100$

Therefore, Availability of Product A = $\frac{50}{(50+10)} \times 100 = 83.33\%$

Availability of Product B = $\frac{60}{(60+5)} \times 100 = 92.31\% > 83.33\%$ of Product A.

Hence, Product B has more availability.

ILLUSTRATION 7:

CO. DILLI KA LADDU LTD. purchases a machine for ₹ 65,000. The following table gives the running costs per year and resale value of the machine in different years:

Year	1	2	3	4	5	6	7	8
Running Costs (₹)	14,000	15,000	17,000	20,000	24,000	28,000	33,000	39,000
Resale Value (₹)	40,000	30,000	22,000	17,000	13,000	10,000	10,000	10,000

Please find out the optimum replacement period.

Solution:

Year	Running Costs	Cumulative Running Costs	Capital Costs	Total Costs	Average Annual Costs
[A]		[B]	[C]	[B + C] = [D]	[D / A]
1	14,000	14,000	25,000	39,000	39,000
2	15,000	29,000	35,000	64,000	32,000
3	17,000	46,000	43,000	89,000	29,667
4	20,000	66,000	48,000	1,14,000	28,500
5	24,000	90,000	52,000	1,42,000	28,400
6	28,000	1,18,000	55,000	1,73,000	28,833
7	33,000	1,51,000	55,000	2,06,000	29,429
8	39,000	1,90,000	55,000	2,45,000	30,625

Recommendation: The machine should be replaced at the end of 5th year.

ILLUSTRATION 8:

The following table gives the age of cars and its annual maintenance costs of STRATEGY LTD:

Age of cars in years (x)	2	4	6	8
Maintenance costs in thousand ₹ (y)	10	20	25	30

Required:

(a) Fit a regression equation of y on x;

(b) Estimate: (i) maintenance cost for age of cars of 10 years.

(ii) Age of cars in years for maintenance costs of ₹. 50,000

[**Ans.:** (a) y = 5 + 3.25x; (b) maintenance cost for age of cars of 10 years is ₹. 37,500 (ii) Age of cars in years for maintenance costs of ₹. 50,000 is 13.85 years (approx.).]

Solution: (a)

Age of cars (X)	Maintenance costs (Y)	X^2	XY
2	10	4	20
4	20	16	80
6	25	36	150

Age of cars (X)	Maintenance costs (Y)	X^2	XY
8	30	64	240
20	85	120	490

Let the Trend equation be

$Y = a + b.X$

$XY = aX + b.X^2$

$\sum Y = 4a + b.\sum X$

$\sum XY = a.\sum X + b.\sum X^2$

Putting the values from the above table:

$85 = 4a + 20b,$ i.e., $425 = 20a + 100b$

$490 = 20a + 120b,$

Subtracting the above two equations, we get

$20b = 65,$ i.e., $b = 3.25$

$\therefore 85 = 4a + 20 \times 3.25,$ $a = \dfrac{85 - 65}{4} = 5$

Therefore, the Trend equation is

$Y = 5 + 3.25X$

ILLUSTRATION 9:

MR. GREG CHAPPEL, a taxi owner estimates from his past records that the costs per year for operating a taxi are as follows:

Age	1	2	3	4	5
Operating Costs (₹)	1,00,000	1,20,000	1,50,000	1,80,000	2,00,000

After 5 years, the operating costs become ₹. 60,000K, where K = 6, 7, 8, 9, 10; K denotes age in years.

The purchase price of a new taxi is ₹. 6,00,000. The resale value of the taxi decreases by 10% of purchase price each year. Please compute the optimum replacement policy.

Solution:

Year [A]	Running Costs	Cumulative Running Costs [B]	Capital Costs [C]	Total Costs [B + C] = [D]	Average Annual Costs [D / A]
1	1,00,000	1,00,000	60,000	1,60,000	1,60,000
2	1,20,000	2,20,000	1,20,000	3,40,000	1,70,000
3	1,50,000	3,70,000	1,80,000	5,50,000	1,83,333
4	1,80,000	5,50,000	2,40,000	7,90,000	1,97,500
5	2,00,000	7,50,000	3,00,000	10,50,000	2,10,000
6	3,60,000	11,10,000	3,60,000	14,70,000	2,45,000
7	4,20,000	15,30,000	4,20,000	19,50,000	2,78,571
8	4,80,000	20,10,000	4,80,000	24,90,000	3,11,250

Recommendation: The taxi should be replaced after each 1 year as it will be cheaper.

ILLUSTRATION 10:

MR. SACHIN TENDULKAR, the great cricketer furnishes the following data about a new generation car:

Year of service	1	2	3	4	5
Year-end trade-in-value (₹)	1,90,000	1,05,000	60,000	50,000	50,000
Annual operating costs (₹)	1,50,000	1,80,000	2,10,000	2,40,000	2,70,000
Annual Maintenance costs (₹)	30,000	40,000	60,000	80,000	1,00,000

The purchase price is ₹.3,50,000 and the trade-in-value drops as time passes until it reaches a constant value of ₹.50,000. Please determine the proper length of service before the car should be replaced.

Year [A]	Running Costs	Cumulative Running Costs [B]	Capital Costs [C]	Total Costs [B + C] = [D]	Average Annual Costs [D / A]
1	1,80,000	1,80,000	1,60,000	3,40,000	3,40,000
2	2,20,000	4,00,000	2,45,000	6,45,000	3,22,500
3	2,70,000	6,70,000	2,90,000	9,60,000	3,20,000

Year	Running Costs	Cumulative Running Costs	Capital Costs	Total Costs	Average Annual Costs
[A]		[B]	[C]	[B + C] = [D]	[D / A]
4	3,20,000	9,90,000	3,00,000	12,90,000	3,22,500
5	3,70,000	13,60,000	3,00,000	16,60,000	3,32,000

Recommendation: The car should be replaced after 3rd year.

ILLUSTRATION 11:

MR. DING DONG BING BONG gives the following data relating to machine A, whose purchase price is ₹1,00,000:

Year	1	2	3	4	5	6	7
Operating costs (₹)	15,000	19,000	23,000	29,000	36,000	45,000	55,000
Resale value (₹)	50,000	25,000	12,500	6,000	4,000	4,000	4,000

(a) Please compute the optimum period for replacement.

(b) When machine A is 2 years old, machine B which is a new model for the same usage is available. The optimum period of replacement is 4 years with an average cost of ₹.36,000. Should we change machine A with that of machine B? If so, when?

Solution:

Year	Resale value (₹)	Depreciation (₹)	Operating cost (₹)	Total cost (₹)	Cumulative cost (₹)	Average cost per year (₹)
1	50,000	50,000	15,000	65,000	65,000	65,000
2	25,000	25,000	19,000	44,000	1,09,000	54,500
3	12,500	12,500	23,000	35,500	1,44,500	48,167
4	6,000	6,500	29,000	35,500	1,80,000	45,000
5	4,000	2,000	36,000	38,000	2,18,000	43,600
6	4,000	Nil	45,000	45,000	2,63,000	43,833
7	4,000	Nil	55,000	55,000	3,18,000	45,429

As the average cost per year is minimum in the 5th year, the optimum period of replacement of machine A is 5th year.

(b) When the machine A is 2 years old, the calculation is done as follows:

Year	Depreciation	Operating costs	Total costs	Cum. Costs	Avg. cost per year
3	12,500	23,000	35,500	35,500	35,500
4	6,500	29,000	35,500	71,000	35,500
5	2,000	36,000	38,000	1,09,000	36,333
6	Nil	45,000	45,000	1,54,000	38,500
7	Nil	55,000	55,000	2,09,000	41,800

Machine A should be changed with machine B when machine A is 4 years old, since the average cost per year for Machine A in 5th year is ₹36,333 which is more than ₹36,000.

ILLUSTRATION 12:

CO. NON-STOP LTD. supplies the following information:

- The cost of a new machine is ₹5,00,000.
- The maintenance cost during the n-th year is given as $R_n = 50{,}000 \times (n - 1)$.
- Cost of capital is 5%.

Please compute after how many years it will be economical to replace the machine by a new one.

Solution:

Year	Maintenance cost	Discount factor	Discounted maintenance cost	Discounted total cost (₹)	Discounted. average. cost p.a. (₹)
1	0	1.0000	0	5,00,000	5,00,000
2	50,000	0.9523	47,615	5,47,615	2,80,497
3	1,00,000	0.9070	90,700	6,38,315	2,23,242
4	1,50,000	0.8638	1,29,570	7,67,885	2,06,249
5	2,00,000	0.8227	1,64,540	9,32,425	2,05,118
6	2,50,000	0.7835	1,95,875	11,28,300	2,11,716

The machine should be replaced after 5th year.

ILLUSTRATION 13:

MISS SLIGHT CONFUSION is considering purchasing a machine for her own factory. Relevant data about alternative machines are as follows:

Particulars	Machine A	Machine B	Machine C
Present investment (₹)	1,00,000	1,20,000	1,50,000
Total annual costs (₹)	20,000	15,000	12,000
Life	10	10	10
Salvage value (₹)	5,000	10,000	12,000

As an advisor to the buyer, you have been requested to select the best machine, considering 12% rate of return. Given that PVIF (12%, 10th year) = 0.322 & PVIFA (12%, 10 years) = 5.650.

[**Ans.:** Machine B should be selected.]

Solution:

Particulars	Machine A	Machine B	Machine C
Present investment (₹) PV of annual costs for the life of machine	1,00,000 20,000 × 5.650 = 1,13,000	1,20,000 15,000 × 5.650 = 84.750	1,50,000 12,000 × 5.650 = 67,800
PV of Total costs (₹) Less: PV of salvage value	2,13,000 5,000 × 0.322 = 1,610	2,04,750 10,000 × 0.322 = 3,220	2,17,800 12,000 × 0.322 = 1,932
PV of Net Total costs	2,11,390	2,01,530	2,15,868

Recommendation: Machine B should be selected as it is cheaper.

ILLUSTRATION 14:

The management of HOTEL KOLKATA INTERNATIONAL is considering the period of replacement of light bulbs fitted in its rooms. There are 500 rooms in the hotel and each room has 6 bulbs. The management is now following the policy of replacing the bulbs as they fail at a total cost of ₹30 per bulb. The management feels that this cost can be reduced to ₹10 per bulb by adopting the periodic replacement policy. On the basis of the information given below, evaluate the alternative and make a recommendation to the management:

Months of use	1	2	3	4	5
Percent of bulb failing by that month	10	25	50	80	100

Solution:

For solving this problem, the following assumptions are made:

- Bulbs that fail during a month are replaced just before the end of that month,
- The actual percentage of failures during a month as for a subpopulation of bulbs with the same age is the same as the expected percentage of failures during the month of that subpopulation.

The probability of failures can be arranged as follows:

End of month	1	2	3	4	5
Percentage of failures	10	15	25	30	20

Let N_0 represents the number of original bulbs and Fi denotes the number of bulbs failed and replaced at the end of the i-th month. i = 1,2,3,4,5.

$N_0 = 500 \times 6 = 3,000$

$F_1 = N_0P_1 = 3,000 \times 0.1 = 300.$

$F_2 = N_0P_2 + F_1P_1 = 3,000 \times 0.15 + 300 \times 0.10 = 480.$

$F_3 = N_0P_3 + F_1P_2 + F_2P_1 = 3,000 \times 0.25 + 300 \times 0.15 + 480 \times 0.10 = 843.$

$F_4 = N_0P_4 + F_1P_3 + F_2P_2 + F_3P_1 = 3,000 \times 0.30 + 300 \times 0.25 + 480 \times 0.15 + 843 \times 0.10 = 1,131.$

$F_5 = N_0P_5 + F_1P_4 + F_2P_3 + F_3P_2 + F_4P_1 = 3,000 \times 0.20 + 300 \times 0.30 + 480 \times 0.25 + 843 \times 0.15 + 1,131 \times 0.10 = 1,049.$

Note: F2: 15% of the original set of 3,000 bulbs and 10% of the bulbs replaced at the end of 1st month as at the end of second month, these are one month old.

F3: 25% of the original set of 3,000 bulbs, 10% of the bulbs replaced at the end of 2nd month as these are one month old and 15% of the bulbs replaced at the end of 1st month as these are 2 months old.

And so on.

Group Replacement policy:

End of month	No. of bulbs failed	Cumulative no. of bulbs failed	Cost of replacement @ ₹30 per bulb	Total cost	Average cost per month
1	300	300	9,000	39,000	39,000
2	480	780	23,400	53,400	26,700
3	843	1,623	48,690	78,690	26,230
4	1,131	2,754	82,620	1,12,620	28,155
5	1,049	3,803	1,14,090	1,44,090	28,818

The minimum average cost per under group replacement is ₹.26,230 at the end of 3rd month.

Individual Replacement policy:

Expected life of each bulb = 1 × 0.10 + 2 × 0.15 + 3 × 0.25 + 4 × 0.30 + 5 × 0.20 = 3.35 months.

Average no. of failure and replacement per month = 3,000 ÷ 3.35 = 896.

Average cost of individual replacement = 896 × ₹ 30 = ₹ 26,880.

Comment: The average cost of group replacement is lower than individual replacement. Hence, it is cheaper to have a group replacement after every 3rd month and individually any that fails before that.

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