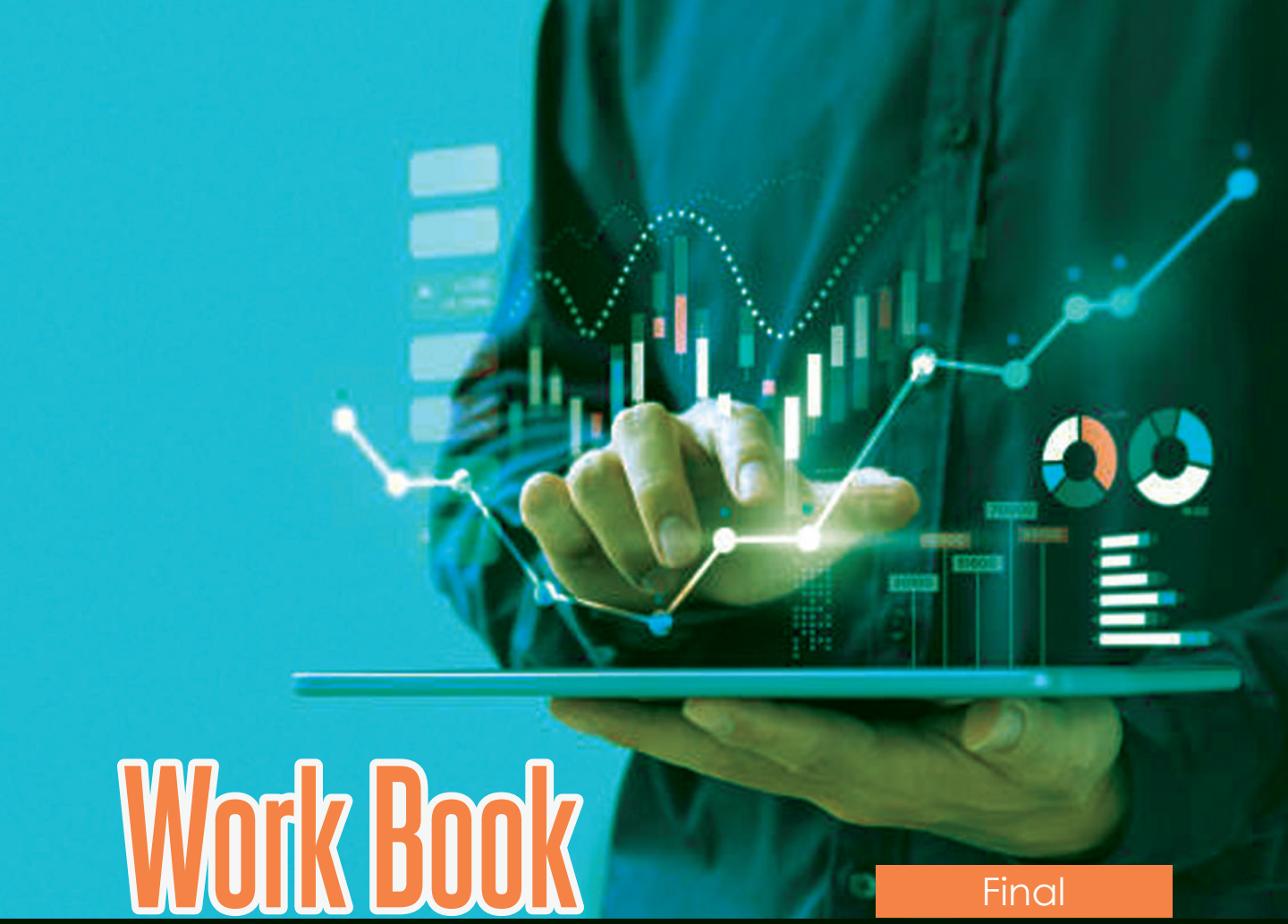

Work Book

Final

Strategic Cost Management

Paper 16

The Institute of Cost Accountants of India
Statutory Body under an Act of Parliament
www.icmai.in

WORKBOOK

Strategic Cost Management

FINAL

Paper 16

SYLLABUS 2022

The Institute of Cost Accountants of India

CMA Bhawan, 12, Sudder Street, Kolkata - 700 016

www.icmai.in

First Edition : March 2025

Published by :

Directorate of Studies
The Institute of Cost Accountants of India
CMA Bhawan, 12, Sudder Street, Kolkata - 700 016
studies@icmai.in

Preface

The landscape of professional education is undergoing a profound transformation, driven by the evolving demands of a globally integrated economy. In this dynamic environment, it is imperative to equip students not only with technical knowledge but also with the analytical skills and professional acumen essential for success.

Effective learning extends beyond theoretical understanding—it necessitates the development of strong conceptual foundations, critical thinking abilities, and disciplined study habits. These attributes are cultivated through continuous practice and engagement with thought-provoking academic material. To facilitate this process, the curriculum, instructional methods, and assessments must be designed to provide comprehensive, structured, and intellectually stimulating learning experiences.

Building on the success of the previous editions, we are pleased to present the new edition of our 'Workbook' in an e-distributed format. This edition has been meticulously developed to enhance students' comprehension and application of key concepts. Each chapter is structured to offer a seamless learning experience and integrating practical illustrations in a phased manner to align with the evolving regulatory framework.

We are confident that this new edition will continue to serve as a valuable academic resource, empowering students to achieve their professional aspirations with confidence and competence.
The Directorate of Studies, The Institute of Cost Accountants of India

The Directorate of Studies,

The Institute of Cost Accountants of India

INDEX

1 Decision Making Techniques [Study Material - Module 3]

ILLUSTRATION 1

Chung Lee Electronics Ltd. manufacturers 4 types of electronics products: 'A', 'B', 'C' and 'D' All these products have good market demand. The company supplied following data:

Particulars	Product 'A'	Product 'B'	Product 'C'	Product 'D'
1. Material Cost (₹./Unit)	64	72	45	56
2. Machining Cost (₹./Unit) @₹.8/hour	48	32	64	24
3. Variable Expenses (₹./Unit)	32	36	44	20
4. Selling Price (₹./Unit)	162	156	173	118
5. Market Demand (Number of Units)	52000	48500	26500	30000

Fixed cost varies according to production level (hours), as below:

Upto 150000 hours ₹.10 Lakhs
150001 – 300000 Hours ₹.10.50 Lakhs
300001 – 450000 Hours ₹.11.00 Lakhs
450001 – 600000 Hours ₹.11.50 Lakhs

Current production capacity is 498000 machine hours, that is not enough to fulfill the market demand. Production manager wants to increase the capacity. The capacity enhancement is likely to take some time and the Company wants to avail the market sentiment and cater to its requirement. It can subcontract a part of its production. Subcontract price per unit is offered as follows:

Product A – ₹.146/unit
Product B – ₹.126/unit
Product C – ₹.155/unit
Product D – ₹.108/unit

Please advise the management for quantities to be produced and or sub contracted, so as to achieve maximum possible profit. Show the quantum of profit earned.

Solution:

i. Statement of Ranking

Particulars	A	B	C	D
1. Subcontract Price (₹./Unit)	146	126	155	108
2. Material Cost (₹./Unit)	64	72	45	56
3. Machining Cost (₹./Unit) @₹.8/hour	48	32	64	24
4. Variable Expenses (₹./Unit)	32	36	44	20
5. Total Variable Cost	144	140	153	100
6. Saving in Manufacturing (1 – 5)	2	-14	2	8
7. Decision	Make	Buy	Make	Make
8. Machine Hours Required (₹./Unit) (3/8)	6	4	8	3
9. Saving (₹. Per Hours) (6/9)	0.33	N A	0.25	2.6
10. Ranking	II		III	I
11. Market Demand (Number of Units)	52000	48500	26500	30000
12. Hours Required for Full Demand (8 × 11)	312000	NIL	106000	90000

ii. Machine Hours Allocation

Product	150000 Hours		300000 Hours		450000 Hours		600000 Hours	
	Hours	Units	Hours	Units	Hours	Units	Hours	Units
D	90000	30000	90000	30000	90000	30000	90000	30000
A	60000	10000	210000	35000	312000	52000	312000	52000
C					48000	6000	198000	24750
Total	150000		300000		450000		600000	

iii. Profitability at Different Levels of Operations

Product	150000 Hours	300000 Hours	450000 Hours	600000 Hours
I	**Contribution from Own Production**			
D	30000×18 =540000	30000×18 =540000	30000×18 =540000	30000×18 =540000

Product	150000 Hours	300000 Hours	450000 Hours	600000 Hours
A	10000×18 =180000	35000×18 =630000	52000 × 18 = 936000	52000×18 = 936000
C			6000×20 = 120000	24750×20 = 495000
II	**Contribution from Subcontracted Sale**			
A	42000×16 =672000	17000×16 =272000		
B	48500×30 =1455000	48500×30 =1455000	48500×30 =1455000	48500×30 =1455000
C	26500×18 =477000	26500×18 =477000	20500×18 =369000	1750×18 =31500
Total Contribution	3324000	3374000	3420000	3457500
Fixed Cost	1000000	1050000	1100000	1150000
Profit	2324000	2324000	2320000	2307500

Decision: Operations at 1.50 Lakh and 3.00 lakh hours are equally profitable. Since the Company is already having 4.98 lakh hours, it would be advisable to operate at 3.00 lakh hours, to avoid market dependence to cater the demand for own products.

Working Note 1

Per Unit Contribution for Own Production

Product A= 162-64-48-32 =18

Product C= 173-45-64-44 =20

Product D= 118-56-24-20=18

Working Note 2

Per Unit Contribution for Sub Contracted Sale

Product A= 162-146 =16

Product B= 156-126 =30

Product C= 173-155 =18

ILLUSTRATION 2:

Leo Toys are the leading manufacturer and exporters of toys. They have been approached by a US company to supply a special toy. The order will be for 3000 pieces per month for a period of 3 years. Sale price will be ₹.200 per unit. Associated costs are:

a. Materials: ₹.60/unit

b. Labour: ₹.25/unit

c. Variable Overheads: ₹.20/unit

d. Primary Packing: ₹.15/unit

e. Secondary packing will be done in special boxes of 50 pieces each @₹.400 per box.

f. A special machine is to be imported for the purpose at the initial cost ₹.24 lakhs and duty @12% thereon. The expected life of machine is 3 years.

g. Annual fixed overheads are estimated to be ₹.4.00 lakhs with an annual increase of 6%.

You are required to prepare

i. Monthly and yearly profitability statement for first 2 years

ii. Compute monthly break even units for year 1

Solution:

i. Statement of Profitability for 2 Years (₹. Lakhs)

Particulars	Year 1		Year 2	
	Monthly	Annual	Monthly	Annual
A. Sale Revenue	6.00	72.00	6.00	72.00
B. Costs				
i. Material Cost	1.80	21.60	1.80	21.60
ii. Labour Cost	0.75	9.00	0.75	9.00
iii. Variable Cost	0.60	7.20	0.60	7.20
iv. Primary Packing	0.45	5.40	0.45	5.40
v. Secondary Packing	0.24	2.88	0.24	2.88
vi. Fixed Overheads	1.08	12.96	1.10	13.20
vii. Total Cost (i...vi)	4.92	59.04	4.94	59.28
Profit (A – B)	1.08	12.96	1.06	12.72

ii. Computation of BEP

Fixed Cost for the month = ₹.108000

Contribution per Unit = ₹.80

BEP Sale quantity = Fixed Cost / Contribution per unit = 108000/80 = 1350 unit

Secondary packing is to be considered for 1350/50 = 27 boxes

Additional Semi fixed cost = 27 × 400 = 10800

So revised BEP will be (108000+10800)/80 = 1485 units

For 1485 units we need 30 boxes, so BEP will be either 1500 units or 30 boxes.

Working Note 1

Computation of Fixed Cost: Year 1

Annual Fixed Overheads + Depreciation for new machine

= ₹.4.00 Lakhs+ (24 lakhs + Duty @12%)/3

= 4.00 + 8.96 = 12.96 Lakhs

Monthly Fixed Overheads = 12.96/12 = 1.08 Lakhs

Computation of Fixed Cost: Year 2

Annual Fixed Overheads + 6% annual increment +Depreciation for new machine

= 4.00 Lakhs + 6%of 4.00 Lakhs + 8.96 = 13.20 Lakhs

Monthly Fixed Overheads = 13.20/12 = 1.10 Lakhs

ILLUSTRATION 3

Geo life ltd is a home appliances manufacturer. The company is using JIT and there by carrying no inventory. Manufacturing is done in the small batches of 100 units each. Current batch size can be altered with significant cost implications. There is no batch size for sale. Following table shows the product wise details of 4 existing products:

Particulars	Product 'A'	Product 'B'	Product 'C'	Product 'D'	Total
Sale Quantity (Units)	2,00,000	2,60,000	1,60,000	3,00,000	9,20,000
Revenue (₹. Lakhs)	26.00	45.20	42.40	32.00	145.60
Material Cost (₹. Lakhs)	6.00	18.20	18.80	10.00	53.00

Particulars	Product 'A'	Product 'B'	Product 'C'	Product 'D'	Total
Labour Cost (₹. Lakhs)	8.00	20.80	12.80	12.00	53.60
Overheads(₹. Lakhs)	8.00	7.80	3.20	12.00	31.00
Profit/(Loss) (₹. Lakhs)	4.00	(1.60)	7.60	(2.00)	8.00
Machine Hours (Numbers)	4.00	3.90	1.60	6.00	15.50
Labour Hours (Numbers)	1.00	2.60	1.60	1.50	6.70

Since the company is concerned about the loss incurrence of two products. It has approached a Management Accountant to clear picture on its products and cost. He has submitted the following findings:

1. The overhead absorption rate of ₹.2 per machine hour has been used to allocate overheads into the above product costs.
2. Further analysis of overheads reveal that certain costs are related to number of machine hours utilized and batches produced. Some overheads are product specific that can be avoided if the product is discontinued. Other general fixed overheads cannot be curtailed. Summary of Overheads in ₹. Lakhs is given below:

   i. Overheads related to machine hours : ₹.6.20

   ii. Batch Related overheads : ₹.4.60

   iii. Product Specific Overheads A: 10.00
   B: 1.00
   C: 2.00
   D: 1.00 : ₹.14.00

   iv. General overheads : ₹.6.20

   v. Total Overheads : ₹.31.00

You are required to:

a. Prepare a profitability statement that is useful for decision making

b. Advise to the management on the basis of statement prepared

Solution:

Profitability Statement

Particulars	Product 'A'	Product 'B'	Product 'C'	Product 'D'	Total
Sale Quantity (Units)	2,00,000	2,60,000	1,60,000	3,00,000	9,20,000

Particulars	Product 'A'	Product 'B'	Product 'C'	Product 'D'	Total
Revenue (₹. Lakhs)	26.00	45.20	42.40	32.00	145.60
Material Cost (₹. Lakhs)	6.00	18.20	18.80	10.00	53.00
Labour Cost (₹. Lakhs)	8.00	20.80	12.80	12.00	53.60
Variable Overheads @0.40/machine hour (WN1)	1.60	1.56	0.64	2.40	6.20
Batch Cost @₹.50/batch (WN 2)	1.00	1.30	0.80	1.50	4.60
Total Variable & Semi variable Cost	16.60	41.86	33.04	25.90	117.40
Contribution	9.40	3.34	9.36	6.10	28.20
Avoidable Fixed Costs	10.00	1.00	2.00	1.00	14.00
General Fixed Costs					6.20
Net Profit					8.00

b. It is evident from the above statement that after making due allocation of overheads, scenario has changed and Product 'A' is not profitable. So its capacity can be diverted to some other product that has market demand.

Or some effective steps can be taken to make product 'A' profitable by increasing the sale price, if market permits or optimizing the cost.

ILLUSTRATION 4:

A company manufacturing agricultural machinery is preparing its budget for year 2025-26. An initial review shows that it will not be possible to manufacture all requirements for its 4 products 'A', 'B', 'C' & 'D' due to restricted availability of normal pressing capacity of 20000 hours.

The company can choose between the alternative courses of action given below to obtain the products in excess of available production capacity

i. to buy entirely from outside supplier

ii. to buy partially from outside supplier and make partially in second shift.

Standard cost in ₹. per unit is estimated as given below:

Cost Heads	Product 'A'	Product 'B'	Product 'C'	Product 'D'
Direct Material	18.50	13.00	12.50	22.00

Cost Heads	Product 'A'	Product 'B'	Product 'C'	Product 'D'
Direct Wages	5.00	4.00	11.00	20.00
Direct Expenses	5.00	10.00	5.00	30.00
Fixed Overheads	2.50	2.00	5.50	10.00
Total Production Cost	31.00	29.00	34.00	82.00
Requirement (in units)	2000	3500	1500	2800

Direct expenses relate to the use of metal press that costs ₹.5 per machine hour.

Fixed overheads are absorbed as a % of direct wages

Quotations obtained from outside suppliers are ₹.30, ₹.29.50, ₹. 26 and ₹.84 per unit for 'A', 'B', 'C' and 'D' respectively.

Second shift would increase wages by 25% and fixed overheads by ₹.250 for every 1000 hours or part thereof.

You are required to work out

a. the quantities to be produced in available 20000 machine hours.

b. Work out the viability of 2nd shift operations.

c. Workout the impact on profit as a result of decision at a & b above.

Solution:

a. Comparative Statement

Sr	Particulars	Product 'A'	Product 'B'	Product 'C'	Product 'D'
1	Machine Hours per Unit (Direct Expenses/5)	5/5=1	10/5 = 2	5/5 = 1	30/5 =6
2	Market Rate (₹. per unit)	30.00	29.50	26.00	84.00
3	Inhouse Production Cost (₹. per unit)	31.00	29.00	34.00	82.00
4	Cost Difference (₹. per unit) (2-3)	(1.00)	0.50	(8.00)	2.00
5	Difference per Machine Hour (4/1)	(1.00)	0.25	(8.00)	0.33

Since 'A' and 'C' are available at prices lower than own production cost, so these two items should be procured from outside.

Product 'D' is giving higher per hour cost saving, so complete market demand should be catered by using inhouse facility. Demand for product 'D' is 2800 units and per unit 6 hours are required. So, hours utilized for product of 'D' = 2800 × 6 = 16800.

Since only 20000 machine hours are available, remaining 3200 hours (20000- 16800) can be utilized for producing product 'B'.

Maximum number of units of 'B' to be produced inhouse = 3200/2 = 1600 units

Since total demand of 'B' is 3500 units, 1600 can be produced inhouse and remaining 1900 can be either procured from market or produced in second shift.

b. Viability of second shift operations:

Sr	Particulars	Product 'A'	Product 'B'	Product 'C'
1	Variable Cost (₹. per unit)	28.50	27.00	28.50
2	25% Increase in Wages (₹. per unit) (1 × 25%)	1.25	1.00	2.75
3	Total Variable Cost (₹. per unit) (1+2)	29.75	28.00	31.25
4	Additional Fixed Cost (₹. per unit) (Working Note 1)	0.25	0.526	0.33
5	Total Production Cost (₹. per unit) (3 +4)	30.00	28.526	31.58
6	Procurement Price (₹. per unit)	30.00	29.50	26.00
7	Cost Difference (₹. per unit) (5 – 6)	0	0.974	(5.58)
8	Number of units to be manufactured (Working Note 2)	200	1900	0
9	Savings for inhouse production	0	1850.60	0

Overall savings

1.	Product A & C to be procured from outside	Nil
2.	Product 'B' inhouse production	
	Shift 1 -1600 × 0.50	800
	Shift 2- 1900 × 0.974	1850
3.	Product 'D' Shift 1: 2800 × 2	5600
4.	Additional Saving in Fixed overheads if	
	200 units of 'A' are produced inhouse 200×0.25	50
	(Working Note 2)	
	Total Savings	8300

Conclusion: A total saving of ₹.8300 can be made by inhouse production of

2800 units of 'D' in shift 1,

1600 units of B in shift 1 & 1900 units in shift 2,

200 units of 'A' in shift 2.

Remaining units of A & C can be outsourced.

Working Note 1:

Additional Fixed Cost = ₹.250 per 1000 hours or fraction thereof worked =

250/1000 = 0.25 per hour.

For Product A = 250/1000 × 1= 0.25/unit

For Product B = Hours required for 1900 Units = 2 × 1900 = 3800,

So additional overheads cost will be – 250 × 4 = ₹.1000,

Cost per unit will be 1000/1900 = 0.526

For Product C = Hours required for 1500 Units = 1 × 1500 = 1500,

So additional overheads cost will be – 250 × 2 = ₹.500,

Cost per unit will be 500/1500 = 0.33

Working Note 2:

Since it is cost effective to produce Product B inhouse, we need to incur fixed overheads for 4000 hours. But for making of Product B hours utilized are only 3800 and 200 hours remain unutilised.

Since the saving in making of product A is zero, it is advisable to utilize additional 200 hours to produce Product A.

Saving in making of 200 units of Product A will be 200 × 0.25 = ₹. 50

ILLUSTRATION 5:

A company manufacturers 3 lakh units of product X & 2 lakh units of Product Y per annum. The following figures are extracted from its cost records

Sale Value	₹. 38.00
Direct Material	₹.7.00

Direct Labour	9.50
Factory overheads	9.50
Admin, S &D Overheads	6.00

50% of all overheads are fixed.

The selling price of X is ₹.6/u & Y ₹.10/unit. Direct Material & Labour ratio for X is 1:1.5 & for Y 1:1.25. For both the product selling price is 400% of direct labour. The factory overheads are charged in the ratio of direct labour. Admin, S&D are recovered at a flat rate of Re1/ unit of X & 1.50/unit of Y. 50% Overheads are variable.

Due to fall in demand of the above products, the company has a plan to diversify and make product Z using 40% of the present capacity. It has been estimated that for Z direct material and direct labour will be ₹.1.25 and 1.50 respectively. Other variable costs to be same as that of X and selling price of Z will be ₹. 7 per unit and production will be of 3 lakh units.

Assuming that balance 60% capacity is used for manufacturing of X & Y. Calculate,

a. present costs and profits

b. costs and profits after diversification is implemented

c. your recommendation for or against diversification.

Solution:

Statement of Contribution (₹. per unit)

Sr	Particulars	Product X	Product Y	Product Z
1	Selling Price	6.00	10.00	7.00
2	Direct Labour 25%×(1) for X & Y	1.50	2.50	1.50
3	Direct Material	1.00	2.00	1.25
4	Factory Overheads: Variable 50%	0.75	1.25	0.75
5	Admin & Selling Overheads: Variable 50%	0.50	0.75	0.50
6	Total Variable Cost	3.75	6.50	4.00
7	Contribution	2.25	3.50	3.00

a. Statement of Present Costs & Profits (in ₹. Lakhs)

Sr	Particulars	Product X	Product Y	Total
1	Sale Value	18.00	20.00	38.00
2	Direct Material	3.00	4.00	7.00

Sr	Particulars	Product X	Product Y	Total
3	Direct Labour	4.50	5.00	9.50
4	Variable Factory Overheads	2.25	2.50	4.75
5	Variable Admin & Selling Overheads	1.50	1.50	3.00
6	Total Variable Cost (2...5)	11.25	13.00	24.25
7	Contribution (1 – 6)	6.75	7.00	13.75
8	Fixed Factory Overheads			4.75
9	Fixed Admin & Selling Overheads			3.00
10	Profit (7 – 8 – 9)			6.00

b. Statement of Costs & Profits After Diversification (in ₹. Lakhs)

Sr	Particulars	Product X	Product Y	Product Z	Total
1	Sale Units (60% × existing units for X & Y)	180000	120000	300000	600000
2	Sale Value	10.80	12.00	21.00	43.80
3	Total Variable Cost	6.75	7.80	12.00	26.55
4	Contribution (2 – 3)	4.05	4.20	9.00	17.25
5	Fixed Overheads				7.75
6	Profit (4 – 5)				9.50

c. Recommendations:

It is recommended to diversify, as profit goes up by ₹.3.50 lakhs.

Further, it is advisable that if market permits, the sale of product 'Y' can be increased as it gives maximum contribution and sale of product 'X' can be reduced as it is giving lowest contribution.

ILLUSTRATION 6: RELEVANT COST

A company has 2,000 units of an obsolete item which are carried in inventory at the original purchase price of ₹30,000. If these items are reworked for ₹10,000, they can be sold for ₹18,000. Alternatively, they can be sold as scrap for ₹3,000 in the market.

In a decision model used to analyse the reworking proposal, the opportunity cost should be taken as relevant or not.

Advise the Company.

Solution:

Original price ₹ 30,000 is not relevant. It would be treated as a sunk cost

Further items which are reworked generates income of ₹ 18,000 is considered as relevant;

Cost of rework is ₹10,000 is considered as relevant;

Net inflow ₹ 8,000 (relevant cash in-flow)

The other alternative relevant cash flow is from sale as scrap = ₹3,000.

As reworked generates more relevant cash flow, we would not opt for sale of scrap. This would lead to loss of cash flow in context of sale of scrap, known as opportunity loss/cost. Thus we would consider sale of scrap as an opportunity loss/cost

Thus, opportunity cost should be taken as ₹ 3,000/-

Advise: The Company should opt for rework and sell. This will earn a cash profit of ₹.8000.

ILLUSTRATION 7: RELEVANT COST

The accountant of Zeta Ltd. has prepared the following estimate on the basis of which he has advised that a contract should not be accepted at the price offered. The estimate (₹) was as follows:

Material X in stock at original cost	₹1,50,000
Material Y on order at contract price	₹1,80,000
Material Z to be ordered at current price	₹ 3,00,000
Skilled Labour	₹ 5,40,000
Unskilled Labour	₹ 3,00,000
Supervisory Cost	₹ 1,00,000
General Overheads	₹ 1,80,000
Total Cost	₹ 17,50,000
Price offered	₹ 14,00,000
Net Loss (Price offered – Total Cost)	₹ 3,50,000

The following details are available about the cost components listed above

a. Material X is an obsolete material. It can be used on another product W, the material for which is available at ₹1, 35,000 (Material X requires some adaptation to be used which costs ₹15,000). It may take some time before W's order is confirmed. Until then storage will cost ₹12,000.

b. Material Y is ordered for some other product which is no longer required. It now has a residual value of ₹1, 25,000.

c. Skilled labour can work on other contracts which are presently operated by semiskilled labour at a cost of ₹4, 00,000

d. Unskilled labour are specifically employed for this contract.

e. Supervisory staff will remain whether or not the contract is accepted. Only two them can replace other positions where the salary is ₹50,000.

f. Overheads are charged at 33⅓% of skilled labour. Only ₹1, 00,000 would be avoidable.

You are required to answer the following questions using relevant cost approach:

(i) Relevant costs of material X, Y and Z

(ii) Relevant cost of labour-skilled and unskilled

(iii) Relevant cost of Supervisory cost and General overheads

(iv) If the contract is accepted, what would be the resulting financial impact on XYZ's profit?

Solution:

(i) Relevant costs of material X, Y and Z

Material X (Obsolete)

Material X in stock at original cost = ₹1, 50,000

Reuse Value = ₹1, 35,000

Adaptation Cost = ₹15,000

Storage Cost = ₹12,000

Relevant Cost of Material X = ₹ (1, 35,000 – 15,000 – 12,000) = ₹1, 08,000

Material Y (No longer required)

Material Y on order at contract price = ₹1, 50,000

Residual Value = ₹1, 25,000 Relevant Cost of Material Y = ₹1, 25,000

Material Z (To be ordered)

Material Z to be ordered at current price = ₹3, 00,000

Relevant Cost of Material Z = ₹3, 00,000

(ii) Relevant cost of labour-skilled and unskilled

Skilled Labour (Can replace unskilled labour) Cost of skilled labour = ₹5, 40,000

Replacement Cost (in place of unskilled labour) = ₹4, 00,000

Relevant Cost of Skilled Labour = ₹4, 00,000 [Lower of the above]

Unskilled Labour (Specifically Employed)

Cost of unskilled labour = ₹3, 00,000

Relevant Cost of Unskilled Labour = ₹3, 00,000

(iii) Relevant cost of Supervisory cost and General Overheads

Supervisory cost = ₹1, 00,000

Replacement Value for others = ₹50,000

Relevant Supervisory Cost = ₹50,000

Avoidable General Overheads = ₹1, 00,000

Relevant Costs of General Overheads = ₹1, 00,000

(iv) Computation of Financial Impact

Serial	Element	Amount in ₹
A	Price Offered	14,00,000
B	Relevant Costs	
	1. Material X	1,08,000
	2. Material Y	1,25,000
	3. Material Z	3,00,000
	4. Skilled Labour	4,00,000
	5. Unskilled Labour	3,00,000
	6. Supervisory Cost	50,000
	7. General Overheads	1,00,000
	8. Total (1 to 7)	13,83,000
C	Financial Impact (A – B)	17,000

Observation: Actually, there is no loss in accepting the order. However, if the contract is accepted, XYZ's profit will be increased by ₹17,000.

ILLUSTRATION 8: TARGET COSTING

Ashay Ltd. sells its product 'Beta' at ₹1,000 per unit. Due to competition, its competitors are likely to reduce price by 15%. B wants to respond aggressively by cutting price by 20% and expects that the present volume of 1,50,000 units p.a. will increase to 2,00,000 units. The Company wants to earn a 10% target profit on sales. Calculate per unit target cost for the product.

Solution:

Target selling price : ₹1,000 less 20%	₹ 800
Less: Target profit margin (10%)	₹ 80
Target costs per unit	₹ 720

ILLUSTRATION 9: TARGET COSTING

ABC Limited has a production capacity of 80,000 units and presently sells 20,000 units at ₹100 each. The demand is sensitive to Selling Price and it has been observed that for every reduction of ₹10 in Selling Price, the demand is doubled. Calculate:

i. the Target Cost at full capacity, if Profit Margin on Sale is 25%?

ii. the quantum of Cost Reduction at full capacity with same profit %, if 40% of Cost is variable,

iii. the maximum investment at full capacity, if desired Rate of Return is 16%,

Solution:

i. Target Cost at Full Capacity

Selling Price per unit	₹100	₹90	₹80
Demand(units)	20,000	40,000	80,000= Full Capacity

Target Cost per unit at Full Capacity = Sale Price less Profit Margin = ₹80 less 25% thereon = ₹60 p.u.

ii. Determination of Target Cost Reduction

(a) Since Present Price is `100 p.u. and Profit is 25% thereon, Present Cost p.u. = 75, of which 40% is variable. So, Fixed Cost is 60% of 75 = 45 p.u. So, Total Fixed Cost =	45 × 80,000 = `36 Lakhs
(b) Variable Cost at Full Capacity = (40% of `75 p.u.) × 80,000 units =	`24 Lakhs
(c) Estimated Cost at Full Capacity = Fixed Cost (constant at all levels) +Variable Cost (a + b)	`60 Lakhs
(d) Target Cost at Full Capacity = `60 p.u. for 80,000 units =	`48 Lakhs
(e)Cost Reduction Target = Estimated Cost - Target Cost = (c - d)	`12 Lakhs

iii. Computation of Investment required

(a) Profit at full capacity = 25% of ₹80 = ₹20 p.u.× 80,000 units	₹16 Lakhs
(b) Since ROCE desired is 16%, Maximum Required Investment = ₹16 Lakhs/ 16%	₹100 Lakhs

ILLUSTRATION 10: TARGET COSTING

You the manager of Zest Ltd. that is in the business of refining the edible oil and have recently come across a particular type of blended oil, which is being sold at substantially lower rate (by another company-ABC Ltd.) than the price charged by your company. The value chain for one MT of such oil for ABC Ltd is follows,

ABC Ltd sells this particular oil to the merchant at the rate of ₹.1,26,600 per MT. ABC Ltd pays for the freight which amounts to ₹. 300 per MT

Average returns and allowances amount to 4% of sales and approximately equal ₹.5000 per MT.

The value chain of your company, through which the oil reaches the ultimate customer is similar to that of ABC Ltd. However, your mill does not sell directly to the merchant, the latter receiving the oil from a huge distribution centre maintained by your company at Haryana. Shipment costs from the mill to the Distribution Center amount to ₹.110 per MT while the operating costs in the Distribution Center have been estimated to be ₹.250 per MT. The ROI required by the Distribution Center for the investments made amount to an estimated ₹.580 per MT.

You are required to compute the "Ex Factory Target Cost" for this particular type of blended oil for your company. You may assume that the return on the investment expected by your company equals ₹.5500 per MT of sale.

Solution:

Computation of Target Cost Per MT **(in ₹.)**

ABC Ltd selling price to the merchant		126600
Less: freight paid by ABC Ltd	300	
Less normal sales returns and allowances	5000	
Required ROI of Zest Ltd.	5500	10800
Total Target cost		115800
Less: Shipment cost Distribution Centre	110	
Operating cost in the Distribution Centre	250	
ROI required by Distribution centre	580	940
Ex Factory Target cost		114860

ILLUSTRATION 11: TRANSFER PRICING

Alpha Ltd. fixes the inter divisional transfer price for its products on the basis of cost plus estimated return of 28% on the investments in respective segment. The relevant portion of budget of Segment 'S' for current year is given. Determine the transfer price for Segment 'S' considering the fact that it is single product segment.

Serial	Particulars	Data
1	Fixed Assets	₹ 5.00 Lakhs
2	Current Assets (Other than debtors)	₹ 3.00 Lakhs
3	Sundry Debtors	₹ 2.00 Lakhs
4	Annual Fixed Cost	₹ 8.00 Lakhs
5	Variable Cost Per Unit of the product	₹ 10 per unit
6	Budgeted Volume of Production	4.00 Lakh units

Solution:

Computation of Transfer Price

Serial	Particulars	Data
1	Production Volume	400000 units
2	Total Variable Cost @₹.10/each	₹ 40.00 Lakhs
3	Fixed Cost pa	₹ 8.00 Lakhs
4	Total Cost	₹ 48.00 Lakhs
5	Return on Investment @28%	₹ 2.80 Lakhs
6	Required Revenue	₹ 50.80 Lakhs
7	Transfer Price (6/1)	₹ 12.70 per unit

Working Note:

Total Investment = Fixed Assets + Current Assets + Debtors = 5+3+2 = ₹ 10 Lakhs

ILLUSTRATION 12: TRANSFER PRICING

A Company has two divisions: LD and KD operating at 100% and 50% capacity respectively. LD produces LX & LY using the common labour force @ ₹.5 per hour. For next year it has budgeted capacity of 42000 labour hours and has commitment to sell 6000 kgs of LY and balance can be used for LD. Cost data is as below:

Particulars	LX (₹./Kg)	LY (₹./Kg)
Direct Materials	36	28
Direct Labour	30	20

Total Overheads amount to ₹.7.56 lakhs per annum relating to LX & LY in the proportion of direct wages incurred. At 100% capacity level ₹.4.20 lakhs are variable overheads. LD prices its products at 50% mark up of total cost.

KD wishes to buy 2000 kgs of LX for being processed into KX that fetches a sale price of ₹.300 per kg. Per kg direct material and labour cost is ₹ 30 and variable overheads ₹.4. The fixed costs of the division are ₹.1.00 lakh pa. Variable overheads of LD include ₹.4 towards selling and distribution expenses, that will be saved in case of sale to KD.

Prepare the profitability Statement in case of

i. LD transfers LX @ applicable to outside customers

ii. LD transfers LX @ applicable to outside customers considering impact of saving in S&D overheads

iii. LD transfers LX @marginal cost applicable to KD

Solution:

Computation of Unit Selling Price (₹.)

Sr	Particulars	LX	LY
1	Material Cost	36	28
2	Labour Cost	30	20
3	Variable Overheads @₹.10/hour (including S&D of ₹.4)	60	40
4	Total Variable Cost (1...3)	126	88
5	Fixed Overheads @₹.8/hour	48	32
6	Total Cost (4 + 5)	174	120
7	Mark up 50% (6 × 50%)	87	60
8	Selling Price (6 + 7)	261	180

i. Profit Statement: Transfer at Market Price

Particulars	Computation	LD	KD	Total
A. Revenue				
LX	3000×261	783000		783000
LY	6000×180	1080000		1080000
KX	2000×300		600000	600000
Total Revenue		1863000	600000	2463000

Particulars	Computation	LD	KD	Total
B. Costs				
LX – Market	1000×174	174000		174000
LX - Transfer	2000×170	340000		340000
LY - Market	6000×120	720000		720000
KX – Transferred Cost	2000×261		522000	522000
KX –Own Cost	2000×34		68000	68000
Fixed Cost			100000	100000
Total Costs		1234000	690000	1924000
Profit		**629000**	**-90000**	**539000**

ii. Profit Statement: Transfer at Market Price minus cost saving impact

Particulars	Computation	LD	KD	Total
A. Revenue				
LX Market	1000×261	261000		261000
LX Transfer	2000×257	514000		514000
LY	6000×180	1080000		1080000
KX	2000×300		600000	600000
Total Revenue		1855000	600000	2455000
B. Costs				
LX – Market	1000×174	174000		174000
LX - Transfer	2000×170	340000		340000
LY - Market	6000×120	720000		720000
KX – Transferred Cost	2000×257		514000	514000
KX –Own Cost	2000×34		68000	68000
Fixed Cost			100000	100000
Total Costs		1234000	682000	1916000
Profit		**621000**	**-82000**	**539000**

iii. Profit Statement: Transfer at Marginal Cost

Particulars	Computation	LD	KD	Total
A. Revenue				
LX Market	1000×261	261000		261000
LX Transfer	2000×122	244000		244000
LY	6000×180	1080000		1080000
KX	2000×300		600000	600000
Total Revenue		1585000	600000	2185000
B. Costs				
LX – Market	1000×174	174000		174000
LX - Transfer	2000×170	340000		340000
LY - Market	6000×120	720000		720000
KX – Transferred Cost	2000×257		244000	244000
KX –Own Cost	2000×34		68000	68000
Fixed Cost			100000	100000
Total Costs		1234000	412000	1646000
Profit		**621000**	**-82000**	**539000**

Learning Take: Change in Transfer price does affect Divisional Profit but Does not impact Overall profit of the Organisation.

Working Note 1:

Computation of Product Mix for next year

Product	Hours per Unit	Units	Total Hours
LY	20/5 = 4	6000	24000
LX	30/5 = 6	18000/6 = 3000	18000 (balance)
Total			**42000**

Working Note 2:

Computation of Overheads cost per hour

Variable Overheads per Hour = Total variable overheads/total hours = ₹.420000/42000 = ₹.10 per hour

Fixed Overheads = 336000/42000 = ₹.8 per hour

ILLUSTRATION 13: PRODUCT LIFE CYCLE COST

Ideas Furniture limited has designed a new range of prefab office furniture. It is ready to be launched in the market with a sale price of ₹.2.50 lakhs per set. An anticipated sale of 1000 sets is expected over a period of 3 years. Other relevant details are as follows:

1. Cost of Design and development	₹ 125 Lakhs
2. Variable Manufacturing Cost	₹ 1.10 lakh per Set
3. Brand Promotion Cost	₹ 200 lakhs
4. Selling & Marketing Expenses	₹ 5000 per set
5. Installation Expenses	₹ 25000 per set
6. Warranty Expenses	₹ 8900 per set
7. Administrative Expenses	₹ 50.00 Lakhs per annum

You are required to compute Product life cycle cost and related profits.

Solution:

Life Cycle Profit Statement **₹. In Lakhs**

Serial	Particulars	Amount
1	Revenue (1000×2.50)	2500
2	Design Development Cost	125
3	Brand Promotion	200
4	Variable Manufacturing Cost @ ₹110000/ Set	1100
5	Selling & Marketing Expenses @ ₹ 5000/Set	50
6	Installation Expenses @₹ 25000/Set	250
7	Warranty Expenses @₹ 8900/Set	89
8	Administrative Expenses (₹ 50 × 3 years)	150
9	Total Cost (2…..8)	1964
10	Profit (1 – 9)	536

ILLUSTRATION 14: USE OF PROBABILITY IN DECISION MAKING

A Company is selling a product @₹10 per unit in the market, its marginal cost is ₹.7.50 per unit and fixed cost ₹.58000 for the period. It has following demand probability:

Sales Volume (Units)	20000	22000	24000	26000	28000
Probability	0.10	0.25	0.20	0.30	0.15

You are required to Calculate the probability of

i. break even in the period

ii. earning profit of ₹.10000 in the period

Solution:

i. Probability of break even in the period

To calculate probability of attaining Break even sale, we first need to work out the Break even Point.

BEP = Fixed Cost/ Contribution per unit = 58000/2.50 = 23200 units

The probability that sales will be more than or equal to 23200 units is the probability that sales will be 24000, 26000 & 28000 units, i.e. 0.20+0.30+0.15 = 0.65 or 65% probability.

ii. Probability of earning profit of ₹.10000 in the period

Contribution needed = Fixed Cost + Desired profit = 58000+10000 = 68000

Sales volume to earn contribution of ₹.68000 = 68000/2.50 = 27200 units

The probability that sales will be more than or equal to 27200 units is the probability that sales will be 28000 units, i.e. 0.15 = 0.15 or 15% probability.

2 Activity Based Management and Just in Time (JIT) [Study Material - Module 4]

ILLUSTRATION 1

ABC Ltd is engaged in the production of three types of cookies: Ginger Honey, Blue Berry & Hot chocolate. The current annual sale of the Company is 50000 units, 20000 units & 60000 units at the rate of ₹.30, ₹.25 & ₹.20 each respectively. The annual operating capacity of these 3 products is 60500, 24200 & 72600 units.

The demand is price sensitive. 10% increase in demand is anticipated for decline of every rupee in sale price. The Company targets a profit of 20% on sales.

The management decides to switch to ABC. The identified activities and cost driver rates are given below:

Sr	Activity	Cost Rate
1	Ordering Cost	₹.800 per Order
2	Deliver Cost	₹.700 per delivery
3	Shelf Stocking	₹.199 per hour
4	Customer Support	₹.1.10 per unit sold

Product wise other information at 100% capacity utilization level is given below:

Sr.	Particulars	Ginger Honey	Blue Berry	Hot chocolate
1	Direct Material Per Unit (₹.)	10	8	7
2	Direct Labour Per Unit (₹.)	7	6	5
3	No. of Purchase Orders	35	30	15
4	No. of Deliveries	112	66	48
5	Shelf Stocking Hours	130	150	160

Under Traditional Costing System, customer support cost is charged at 30% of the prime cost.

Required:

1. Work out the reduced selling price to achieve 100% capacity utilization
2. Work out the Profit of all products at maximum capacity utilization using Traditional Costing method.
3. Work out the Profit of all products at maximum capacity utilization using ABC.

Solution:

1. Computation of Selling Price to achieve 100% Capacity Utilisation

Ginger Honey		Blue Berry		Hot chocolate	
Selling Price (₹.) (-) ₹1	**Quantity (Nos.) (+) 10%**	**Selling Price (₹.) (-) ₹1**	**Quantity (Nos.) (+) 10%**	**Selling Price (₹.) (-) ₹1**	**Quantity (Nos.) (+) 10%**
30	50000	25	20000	20	60000
29	55000	24	22000	19	66000
28	60500	23	24200	18	72600

2. Computation of Cost and Profits at 100% Capacity Utilisation in Traditional Cost Method

Particulars	Ginger Honey	Blue Berry	Hot chocolate
Production Quantity (Nos.)	60500	24200	72600
Selling Price (₹. per unit)	28.00	23.00	18.00
Direct Material (₹. per unit)	10.00	8.00	7.00
Direct Labour (₹. per unit)	7.00	6.00	5.00
Prime Cost (₹. per unit)	17.00	14.00	12.00
Customer Support Cost (30% of Prime Cost) (₹. per unit)	5.10	4.20	3.60
Total Cost (₹. per unit)	22.10	18.20	15.60
Total Cost at 100% Capacity Utilisation (₹. Lakhs)	13.37	4.40	11.33
Total Sales at 100% Capacity Utilisation (₹. Lakhs)	16.94	5.566	13.07
Total Profit at 100% Capacity Utilisation (₹. Lakhs)	3.57	1.166	1.74
Profit % to Sales	21.07%	20.95%	13.31%

3. Computation of Cost and Profits at 100% Capacity Utilisation in ABC Method

Particulars	Ginger Honey	Blue Berry	Hot chocolate
Production Quantity (Nos.)	60500	24200	72600
Selling Price (₹. per unit)	28.00	23.00	18.00
Direct Material (₹. per unit)	10.00	8.00	7.00
Direct Labour (₹. per unit)	7.00	6.00	5.00
Prime Cost (₹. per unit)	17.00	14.00	12.00
Total Prime Cost (₹. Lakhs)	10.285	3.388	8.712
Overheads (₹. Lakhs)			
Cost of Purchase Order @₹.800 each order	800 × 35 = 0.28	800 × 30 = 0.24	800 × 15 = 0.12
Cost of Delivery @₹.700 each Delivery	700 × 112 = 0.784	700 × 66 = 0.462	700 × 48 = 0.336
Shelf Stocking @₹.199 per hour	199 × 130 = 0.259	199 × 150 = 0.30	199 × 160 = 0.32
Customer Support @₹.1.10 per unit sold	1.10 × 60500 = 0.666	1.10 × 24200 = 0.266	1.10 × 72600 = 0.799
Total Overheads	1.99	1.268	1.575
Total Cost at 100% Capacity Utilisation (₹. Lakhs)	12.275	4.656	10.29
Total Sales at 100% Capacity Utilisation (₹. Lakhs)	16.94	5.566	13.07
Total Profit at 100% Capacity Utilisation (₹. Lakhs)	4.665	0.91	2.78
Profit % to Sales	27.54%	16.35%	21.27%

ILLUSTRATION 2:

Live Pure Limited has decided to analyse the profitability of its 5 new customers. It trades bottled water, per case purchase price is ₹.90 and listed Selling Price is ₹.108. The data pertaining to 5 customers is as below:

Sr	Particulars	Andrew	Steve	Chris	Donald	Elvis
1	Sales (No. of Cases)	4680	19688	136800	71550	8775

Sr	Particulars	Andrew	Steve	Chris	Donald	Elvis
2	Actual Selling Price	108	106.20	99	104.40	97.20
3	No. of Purchase Orders	15	25	30	25	30
4	No. of Customer Visits	2	3	6	2	3
5	No. of Deliveries	10	30	60	40	20
6	Distance Travelled per Delivery (Kms)	20	6	5	10	30
7	No. of Expedited Deliveries	0	0	0	0	1

Activities and their Cost Drivers rates are given below:

Sr	Activity	Cost Driver Rate
1	Order Taking	₹.750 per Purchase Order
2	Customer Visits	₹.600 per customer visit
3	Deliveries	₹.5.75 per km travelled
4	Product Handling	₹.3.75 per case sold
5	Expediting Deliveries	₹.2250 per expedited delivery

Required:

1. Compute customer wise profitability using ABC.
2. Considering supply constraint of 235000 cases, calculate the number of cases sold to each customer. The Company can consider to drop one customer completely. Give your justification.

Solution:

1. Computation of Customer Wise Profitability

Sr	Particulars	Andrew	Steve	Chris	Donald	Elvis
1	Sales (No. of Cases)	4680	19688	136800	71550	8775
2	Actual Selling Price per unit	108	106.20	99	104.40	97.20
3	Actual Revenue (₹.)	505440	2090866	13543200	7469820	852930
4	Cost of Purchases @₹.90 per unit (₹.)	421200	1771920	12312000	6439500	789750

Sr	Particulars	Andrew	Steve	Chris	Donald	Elvis
5	Gross Margin (₹.) (3-4)	84240	318946	1231200	1030320	63180
6	Operating Expenses (₹.)					
a	Orders Tracking @₹750 per order	750×15 =11250	750× 25 =18750	750× 30 =22500	750× 25 =18750	750× 30 =22500
b	Customer Visits @₹600 per visit	600×2 =1200	600× 3 =1800	600× 6 =3600	600× 2 =1200	600× 3 =1800
c	Delivery @₹.5.75 per Km	5.75×10×20 =1150	5.75× 30×6 =1035	5.75× 60×5 =1725	5.75×40×10 =2300	5.75×20×30 =3450
d	Product Handling @₹.3.75 per case sold	3.75×4680 = 17550	3.75× 19688 = 73830	3.75×136800 = 513000	3.75× 71550 = 268313	3.75× 8775 = 32906
e	Expedited Deliveries @₹.2250 each					2250
f	Total Operating Expenses (a...e)	31150	95415	540825	290563	62906
7	Profit (5-6)	53090	223531	690375	739757	274
8	% of Profit to Revenue	10.50	10.69	5.10	9.90	0.03
9	% of Total sales quantity	1.94	8.15	56.65	29.63	3.63

2. Dropping of customer should be a last resort available to the business. If any supply constraints are emerging in near future, the decision to drop a customer can be taken on 2 grounds

   a. Profit % earned on sale to a particular customer

   b. Future prospects of business growth and profitability increase for a particular customer

   Since Mr. Elvis is giving 0.03% of profit on sale, and taking only 3.63% of total quantity. So, he can be considered to be dropped.

ILLUSTRATION 3 (JIT)

Lintas Ltd. Wants to implement a JIT program, with the impact on 3 types of stocks as given below.,

Sr	Particulars	Present Situation	Proposed
1	Sales Value	₹.1200 Lakhs	Same at Present

Sr	Particulars	Present Situation	Proposed
2	Cost to Sales Value % Raw Material Conversion Cost	 40 30	 44 32
3	Stock Holding Raw Material WIP Finished Goods	 1 month 0.5 month 0.5 month	 25% less than Existing 50% less than Existing 40% less than Existing
4	Completion of WIP % Materials Conversion Cost	 90 75	 90 75
5	Fixed Cost for Raw Material Fixed Variable	 ₹.2.00 Lakhs ₹ 0.09 per Re of Stock held	 15% less than existing ₹ 0.05 per Re of Stock held
6	Fixed Cost for WIP Fixed Variable	 ₹.3.00 Lakhs ₹ 0.04 per Re of Stock held	 20% less than existing ₹ 0.02 per Re of Stock held
7	Fixed Cost for Finished Goods Fixed Variable	 ₹.2.50 Lakhs ₹ 0.02 per Re of Stock held	 40% less than existing ₹ 0.01 per Re of Stock held

Financial Charges 18% on Working capital

Find out the cost impact of implementation of JIT.

Solution:

A. Computation of Stocks

Sr	Particulars	Present Situation	Proposed
1	Sales Value	₹.1200 Lakhs	₹.1200 Lakhs
2	Raw Material Consumption	₹.480 Lakhs	₹.528 Lakhs
3	Conversion Expenses	₹.360 Lakhs	₹.384 Lakhs
4	Raw Material Stock	1 month = ₹. 480/12 × 1 = ₹.40 Lakhs	0.75 month = ₹. 528/12 × 0.75 = ₹.33 Lakhs

Sr	Particulars	Present Situation	Proposed
5	WIP Stocks	(480 × 90%+360 × 75%)/12 × 0.50 = ₹.29.25 Lakhs	(528 × 90%+ 384 × 75%)/ 12 × 0.25 = ₹. 15.90 Lakhs
6	Finished Goods Stocks	(480 + 360) /12 × 0.50 = ₹. 35 Lakhs	(528 + 384) /12 × 0.30 = ₹.22.80 Lakhs
7	Total Investment in Stocks	₹.104.25 Lakhs	₹.71.70 Lakhs

B. Computation of Stock holding Costs (₹ Lakhs)

Sr	Particulars	Present Situation	Proposed
1	Fixed Costs Raw Material WIP Finished Goods	 ₹.2.00 Lakhs ₹.3.00 Lakhs ₹.2.50 Lakhs	 ₹.1.70 Lakhs ₹.2.40 Lakhs ₹.1.50 Lakhs
2	Variable Costs Raw Material WIP Finished Goods	 40 × 0.09 = ₹.3.60 Lakhs 29.25 × 0.04 = ₹.1.17 Lakhs 35 × 0.02 = ₹.0.70 Lakhs	 33 × 0.05 = ₹.1.65 Lakhs 15.90 × 0.02 = ₹.0.32 Lakhs 22.80 × 0.01 = ₹.0.23 Lakhs
3	Interest Cost	104.25 × 18% = ₹.18.77 Lakhs	71.70 × 18% = ₹.12.91 Lakhs
4	Total Stock Holding Costs	₹.31.74 Lakhs	₹.20.71 Lakhs

Total impact on cost after implementing JIT is reduction in stock holding cost is ₹.11.03 Lakhs. So the Company should implement JIT.

ILLUSTRATION 4: BACK FLUSH ACCOUNTING

X Ltd. follows JIT System, it had following transactions for December 2024:

i. On 2nd December 2024: Raw material purchases ₹.5,00,000
ii. Direct Labour Cost ₹. 72,000
iii. Actual overhead costs ₹. 6,00,000
iv. Conversion costs applied ₹. 6,56,000

All materials purchased were consumed, production was complete with in month. No opening and closing stock of Finished goods. The difference between actual and applied costs is computed.

Pass necessary entries in traditional journal and Back flush Journal

Solution:

In the Books of X Ltd

Journal Entries (Traditional)

Date	Particulars		Debit	Credit
02.12.2024	Materials Account	Dr	5,00,000	
	To Accounts Payable			5,00,000
	(Being credit purchase of Raw material)			
02.12.2024	WIP Account	Dr	5,00,000	
	Raw Materials			5,00,000
	(Being materials issued to production)			
31.12.2024	WIP Account	Dr	72,000	
	Direct Wages Account			72,000
	(Being direct wages cost incurred for the month)			
31.12.2024	Overheads control Account	Dr	600,000	
	To Accounts Payable			6,00,000
	(Being overheads cost incurred for the month)			
31.12.2024	WIP Account	Dr	5,84,000	
	To Overheads control Account			5,84,000
	(Being Application of Overheads)			
31.12.2024	Finished Goods Account	Dr	11,56,000	
	To WIP Account			11,56,000
	(Being completion of goods)			
31.12.2024	Cost of Goods Sold Account	Dr	11,56,000	
	To Finished Goods Account			11,56,000
	(Being Cost of finished goods sold transferred)			
31.12.2024	Cost of Goods Sold Account	Dr	16,000	
	To Overheads Control Account			16,000
	(Being variance recognized)			

In the Books of X Ltd

Journal Entries (Backflush)

Date	Particulars		Debit	Credit
02.12.2024	Raw Material in Process Account	Dr	5,00,000	
	To Accounts Payable			5,00,000
	(Being credit purchase of Raw material)			
02.12.2024	Conversion Cost control Account	Dr	6,72,000	
	To Direct Wages Payable Account			72,000
	To Accounts Payable			6,00,000
	(Being Wages & overheads cost incurred)			
31.12.2024	Finished Goods Account	Dr	11,56,000	
	To Raw Material in Process Account			5,00,000
	To Conversion Cost Accounts			6,56,000
	(Being Completion of Goods)			
31.12.2024	Cost of Goods Sold Account	Dr	11,56,000	
	To Finished Goods Account			11,56,000
	(Being Cost of finished goods sold transferred)			
31.12.2024	Cost of Goods Sold Account	Dr	16,000	
	To Conversion Cost Account			16,000
	(Being variance recognized)			

Working Note: Difference between actual conversion cost & Applied Conversion cost

Actual Conversion cost = Labour Cost + Overheads cost = 72000 + 600000 = 672000

Actual conversion cost – Applied conversion cost = 672000 - 656000 =16000

ILLUSTRATION 5:

Data below relates to 2 products manufactured by Rely Ltd. :

Sr	Particulars	Product Alpha	Product Beta
1	Units Produced	40	40
2	Material Moves per product unit	12	28
3	Direct Labour Hours per unit	1740	1740

Budgeted material handling cost: ₹.348000

i. Determine material handling cost per unit of Product Alpha & Beta using volume based allocation method and ABC method

ii. Interpret the results in both computations of (i) above.

Solution:

i. Computation of Material Handling Cost

a. Volume based allocation method using Direct Labour Hours

Budgeted Direct Labour Hours = 40 × 1740 × 2 = 139200

Direct Labour Hour Rate = Budgeted Overheads/Budgeted Direct Labour Hours

= 348000/139200 = ₹.2.50 per Hour

Sr	Particulars	Product Alpha	Product Beta
1	Total Direct Hours Consumed Per Unit	1740	1740
2	Labour Rate Per Hour (₹.)	2.50	2.50
3	Material Handling Cost Per Unit (₹)	4350	4350

b. Activity based allocation method using Cost Driver

Cost driver for Material handling cost is Material moves per unit of product

Cost Driver Rate = Budgeted Overheads/Budgeted Material moves

= 348000/(12 × 40 + 28 × 40) = ₹.217.50 per Movement

Sr	Particulars	Product Alpha	Product Beta
1	Total Material Movements Per Unit	12	28
2	Cost Rate Per Hour (₹.)	217.50	217.50
3	Material Handling Cost Per Unit (₹)	2610	6090

ii. As we see that number of material movement is different in both the products, so its absorption rate needs to be proportionate. This does not reflect under traditional method of overheads absorption i.e. labour hour rate, as material handing cost absorbed per unit is same. Where as in ABC method material handling cost per unit is absorbed according to number of movements a product needs. This method is more justified to be used in such situations.

ILLUSTRATION 6:

Activity Centers and their related costs for a period are given below:

Sr	Particulars	Amount (₹)
1	Material Handling Cost	8000

Sr	Particulars	Amount (₹)
2	Machining Cost	5000
3	Assembling Cost	4800
4	Inspection Cost	1400

Other details are as follows:

Sr	Particulars	Numbers
1	Material Parts used	80000
2	Machine Hours utilised	150
3	Assembling Parts used	8000
4	Finished units produced	1000

Calculate Cost driver rates

i. Compute the cost of a component 'Delta' that uses 8 units of raw material costing ₹.200, remains 15 minutes on machines, uses 8 assembly parts that cost ₹.100, utilities consumed ₹.10 per unit.

Solution:

i. Computation of Cost Driver Rate:

Sr	Overheads Cost	Cost Driver	Cost Driver Rate (₹)
1	Material Handling Cost	Material Parts used	8000/80000=0.10
2	Machining Cost	Machine Hours utilised	5000/150=33.33
3	Assembling Cost	Assembling Parts used	4800/8000=0.60
4	Inspection Cost	Finished units produced	1400/1000=1.40

ii. Per Unit Cost of Product Delta

Sr	Particulars	Amount (₹.)
1	Material Cost	200.00
	Assembling Material Cost	100.00
	Utilities Consumed	10.00
	Material Handling Cost (8 × 0.10)	0.80
2	Machine Hours Cost (1/4×33.33)	8.33
3	Assembling Cost (8 × 0.60)	4.80

Sr	Particulars	Amount (₹.)
4	Inspection Cost (1 × 1.40)	1.40
5	Total Cost per Unit	325.33

ILLUSTRATION 7:

A firm of Cost Accountants offers 3 kinds of services viz. Audit, Taxation and Management Consultancy. Each service is a separate profit segment and is charged on billable hours @₹.500 per hour. For the year ending on 31st March 2025, the firm estimates the following direct and indirect expenses:

Sr	Expense Head	Amount (₹.)	Total Amount (₹.)
A	**Direct Costs**		
i	Audit	100.00	
ii	Taxation	100.00	
iii	Management Consultancy	50.00	250.00
B	**Indirect Costs**		
i	Planning & Review	7.50	
ii	Computer Processing	7.20	
iii	Professional Salaries	5.60	
iv	Professional Development	1.80	
v	Programming Costs	8.00	
vi	Office Expenses	4.90	
vii	Administrative Expenses	15.00	50.00
C	**Total**		**300.00**

Following additional information is available:

Sr	Particulars	Audit	Taxation	Management Consultancy
1	Billable hours	55000	35000	10000
2	EDP hours	5000	2500	500
3	Professional (Nos.)	30	16	10
4	Professional Development (₹)	57500	62500	60000

Sr	Particulars	Audit	Taxation	Management Consultancy
5	Programming hours	1250	500	2250
6	Space Occupied (Sft)	8000	4000	2000
7	No. of Clients	150	250	100

Till March 2024, firm was operating on traditional method, now they want to switch over to ABC method. You are required to

i. Work out cost driver rates & allocate in 3 segments

ii. Prepare comparative profitability statement in traditional method and on the basis of ABC.

iii. Give your opinion to improve the billable charges on the basis of ABC

Solution:

i. Computation of Cost Driver Rates & its Segment wise allocation

Sr	Expense Head	Cost Driver	Cost Driver Rate	Audit	Taxation	Management Consultancy	Total
			(₹.)	₹. Lakhs			
1	Planning & Review	Billable hours	7.50	4.125	2.625	0.750	7.50
2	Computer Processing	EDP hours	90.00	4.500	2.25	0.45	7.20
3	Professional Salaries	Professional (Nos.)	10000	3.00	1.60	1.00	5.60
4	Professional Development	Given (₹)		0.575	0.625	0.600	1.80
5	Programming Costs	Programming hours	200.00	2.50	1.00	4.50	8.00
6	Office Expenses	Space Occupied (Sft)	35.00	2.80	1.40	0.70	4.90
7	Administrative Expenses	No. of Clients	3000.00	4.50	7.50	3.00	15.00
8	Total			22.00	17.00	11.00	50

ii. Comparative Profitability Statement

Sr	Particulars	Conventional Method			ABC Method		
		Audit	Taxation	Management Consultancy	Audit	Taxation	Management Consultancy
1	Revenue	275.00	175.00	50.00	275.00	175.00	50.00
2	Direct Costs	100.00	100.00	50.00	100.00	100.00	50.00
3	Overheads	27.50	17.50	5.00	22.00	17.00	11.00
4	Profit	147.50	57.50	(5.00)	153.00	58.00	(11.00)

iii. Opinion on Billable Charges:

Under the ABC method total cost of 3 segments is ₹.122 lakhs, 117 Lakhs & 61 Lakhs respectively. Hence the billable charge should be revised accordingly. Existing margin of 40% of revenue to be maintained. Revised charges may be worked out as follows:

Sr	Particulars	Audit	Taxation	Management Consultancy
1	Total Cost	122.00	117.00	61.00
2	Margin	81.33	78.00	40.67
3	Target Revenue	203.33	195.00	101.67
4	Billable Hours	55000	35000	10000
5	Rate Per Hour	370.00	557.00	1017.00

Revised working suggests that charges for audit may be revised downward and management consultancy charges to be increased.

ILLUSTRATION 8: THROUGHPUT ACCOUNTING

CAT Co. makes a product using three machines – X, Y and Z. The per week capacity of each machine is 800, 600 & 500 units respectively.

The demand for the product is 1,000 units per week. For every additional unit sold per week, profit increases by ₹.50,000. CAT Co. is considering the following possible purchases (they are not mutually exclusive):

Purchase 1 Replace machine X with a newer model. This will increase capacity to 1,100 units per week and costs ₹.60 Lakhs.

Purchase 2 Invest in a second machine Y, increasing capacity by 550 units per week. The cost of this machine would be ₹.68 Lakhs.

Purchase 3 Upgrade machine Z at a cost of ₹.75 Lakhs, thereby increasing capacity to 1,050 units.

Required:

Which is CAT Co's best course of action under throughput accounting?

Solution:

Bottleneck resource in order of preference is firstly machine 'Z', secondly machine 'Y' and lastly machine 'X' because the no. of units are in that order in the existing capacity.

Particulars	X	Y	Z	Demand
Current capacity per week	800	600	500*	1,000
Buy Z	800	600*	1,050	1,000
Buy Z & Y	800*	1,150	1,050	1,000
Buy Z, Y & X	1,100	1,150	1,050	1,000*

* = bottleneck resource

All the three machines, to be purchased, in the above order to meet the existing demand.

ILLUSTRATION 9:

Super Industries Ltd. produces three products, X, Y and Z. The capacity of its plant is restricted by process alpha. Process 'Alpha' is expected to be operational for eight hours per day and can produce 1,200 units of X per hour, 1,500 units of Y per hour, and 600 units of Z per hour. Selling prices and material costs for each product are as follows

Product	Selling price ₹. per unit	Material cost ₹. per unit	Throughput contribution ₹. per unit
X	150	80	70
Y	120	40	80
Z	300	100	200

Conversion costs are ₹. 720,000 per day.

Required:

(i) Calculate the profit per day if daily output achieved is 6,000 units of X, 4,500 units of Y and 1,200 units of Z.

(ii) Calculate the TA ratio for each product.

(iii) In the absence of demand restrictions for the three products, advise the management of Super Industries Ltd on the optimal production plan.

Solution :

(a) Profit per day = throughput contribution – conversion cost

= [(₹.70 × 6,000) + (₹.80 × 4,500) + (₹.200 × 1,200)] – ₹.7,20,000

= ₹.3,00,000

(b) TA ratio = throughput contribution per factory hour / conversion cost per factory hour

Conversion cost per factory hour = ₹720,000 / 8 = ₹90,000

Product	Throughput contribution per factory hour	Cost per factory hour (₹.)	TA ratio
X	₹.70 × 1,200 = ₹.84,000	90,000	0.93
Y	₹.80 × 1,500 = ₹.120,000	90,000	1.33
Z	₹.200 × 600 = ₹.120,000	90,000	1.33

(c) An attempt should be made to remove the restriction on output caused by process Alpha's capacity. This will probably result in another bottleneck emerging elsewhere. The extra capacity required to remove the restriction could be obtained by working overtime, making process improvements or changes in product specifications. Until the volume of throughput can be increased, output should be concentrated upon products Y and Z (greatest TA ratios), unless there are good marketing reasons for continuing the current production mix.

Product X is losing money every time it is produced so, unless there are good reasons why it is being produced, for example it has only just been introduced and is expected to become more profitable, Super Industries Ltd should consider ceasing production of X.

ILLUSTRATION 10

A factory has a key resource (bottleneck) of Facility A which is available for 31,300 minutes per week. Budgeted factory costs and data on two products, X and Y, are shown below:

Product	Selling Price ₹. /Unit	Material Cost ₹. /Unit	Time in Facility A
X	50	35.00	5 minutes
Y	50	32.50	10 minutes

Budgeted factory costs per week: in ₹.

Direct labour	25,000
Indirect labour	12,500
Power	1,750

Depreciation	22,500
Space costs	8,000
Engineering	3,500
Administration	5,000

Actual production during the last week is 4,750 units of product X and 650 units of product Y. Actual factory cost was ₹.78,250. Calculate:

(i) Total budgeted factory costs

(ii) Budgeted Cost per Factory Minute

(iii) Return per Factory Minute for both products

(iv) TA ratios for both products

(v) Throughput cost per the week

(vi) Efficiency ratio & analyse it.

Solution:

(i) Total Budgeted Factory Costs = Total of all costs except materials.

= 25,000 + 12,500 + 1,750 + 22,500 + 8,000 + 3,500 + 5000.

= ₹.78,250

(ii) Budgeted Cost per Factory Minute = Total Factory Cost/Minutes available

= ₹.78,250 /31,300 = ₹.2.50

(iii) (a) Return per bottleneck minute for product X

= Selling Price - Material Cost / Minutes in bottleneck

= (50 – 35) /5 = ₹.3

(b) Return per bottleneck minute for product Y

= Selling Price - Material Cost / Minutes in bottleneck

= (50 – 32.5) / 10 = ₹.1.75

(iv) Throughput accounting (TA) Ratio for Product X = = (3 / 2.5) = ₹.1.2

Throughput Accounting (TA) Ratio for Product Y = = (1.75 / 2.5) = ₹.0.7

Based on the review of the TA ratios relating to two products, it is apparent that if we only make product Y, the enterprise would suffer a loss, as its TA ratio is less than 1. Advantage will be achieved, when product X is made.

(v) Standard minutes of throughput for the week: =

[4,750 × 5] + [650 × 10] = 23,750 + 6,500 = 30,250 minutes

Throughput cost per week: = 30,250 × ₹.2.5 per minutes = ₹.75,625

(vi) Efficiency % = (throughput cost / Actual TFC) % = (₹.75,625 / ₹.78,250) × 100 = 96.6%

The bottleneck resource of Facility A is available for 31,300 minutes per week but produced only 30,250 standard minutes. This could be due to: (a) The process of a "wandering" bottleneck causing facility A to be underutilized. (b) Inefficiency in facility A.

ILLUSTRATION 11: JIT

A Ltd has decided to adopt JIT policy, having following effects:

i. Capital cost of ₹.2.00 Lakhs to be incurred on machine modification. Additional annual operating cost is expected to be ₹.2.16 lakhs

ii. Raw material stock holding will reduce from ₹. 50.00 lakhs to 25.00 lakhs

iii. Rental expenditure of ₹. 0.75 lakhs for storage facility can be avoided

iv. Property tax and insurance can be saved to the extent of ₹.0.50 lakhs

v. 3 workers with monthly salary of ₹.10,000 can be utilised in other production facility of the company

vi. Additional inspection cost of ₹.0.30 lakh, and stock out cost of ₹.1.54 lakhs to be incurred

vii. The company gets 12% return on long term investments and assets to be depreciated @10%

Determine the financial impact of JIT policy and advise the management.

Solution:

Costs		**Benefits**	
Particulars	**Amount ₹. Lakhs**	**Particulars**	**Amount ₹. Lakhs**
Interest on Capital (₹.2.00 lakhs @12%)	0.24	Interest on working capital released (₹.25.00 @12%)	3.00
Additional Operating Cost	2.16	Rental Saved	0.75
Additional Inspection Cost	0.30	Property Tax & Insurance saved	0.50
Stock Out Cost	1.54	Salary cost saved (₹.10000 × 3 × 12)	3.60
Total Cost	4.24	Total Benefit	7.85
Net Benefit = 7.85 – 4.24 = 3.61			

Since, net saving of adopting JIT policy is ₹. 3.61 lakhs, this proposal should be adopted

Note: Depreciation being non cash expenditure, has been ignored.

ILLUSTRATION 1: MATERIAL VARIANCES

A company manufacturers two products X & Y by using same raw material named 'Polyfil'. 5 & 3 kgs of raw material is used for making 1 unit of X and Y respectively. Purchase price of Polyfil is ₹.10 per kg.

500 units of X and 200 units of Y were manufactured in a particular period. Actual 2700 & 700 kgs of Polyfil was consumed for the actual production of X & Y respectively at total cost of ₹.37400/-. Calculate all Material variances.

Solution:

i. Material Cost Variance = Standard Cost – Actual Cost
= 31000 – 37400 = 6400 (A)

ii. Material Price Variance = AQ (SR – AR)
= 2700+700 (10 – 11) = 3400 (A)

iii. Material Usage Variance = SR (SQ – AQ)
= 10 (3100 – 3400) = 3000 (A)

Reconciliation

MCV = MPV + MUV = 3400 (A) + 3000 (A) = 6400 (A)

Working Note:

i. Actual Rate = 37400/ (2700 + 700) = ₹.11 per kg

ii. Standard Material Quantity for Actual Production = 500 × 5 +200 × 3 = 3100 kgs

iii. Standard Material Cost = 3100 × 10 = 31000

iv. Actual Material Cost = 37400 (given)

ILLUSTRATION 2: LABOUR VARIANCES

A company manufacturers two products X & Y by using same grade of labour. 10 & 8 labour hours are used for making 1 unit of X and Y respectively. Labour rate is ₹.50 per hour.

500 units of X and 200 units of Y were manufactured in a particular period. Actual labour hours consumed for production of X & Y are 6000 and 2000 respectively at total cost of ₹.360000/-. Calculate all labour variances.

Solution:

i.	**Labour Cost Variance**	**= Standard Cost – Actual Cost**	
		= 330000 – 360000	= 30000 (A)
ii.	**Labour Rate Variance**	**= AH (SR – AR)**	
		= 6000+2000 (50 – 45)	= 40000 (F)
iii.	**Labour Efficiency Variance**	**= SR (SH – AH)**	
		= 50 (6600 – 8000)	= 70000 (A)

Reconciliation

LCV = LRV + LEV = 40000 (F) + 70000 (A) = 30000 (A)

Working Note:

i. Actual Rate = 360000/ (6000 + 2000) = ₹.45 per hour

ii. Standard labour hours for Actual Production = 500 × 10 +200 × 8= 6600 Hours

iii. Standard Labour Cost = 6600 × 50 = 330000

iv. Actual Labour Cost = 360000 (given)

ILLUSTRATION 3: MATERIAL USAGE VARIANCES

ABC Ltd. Is engaged in the production of a standard mix using 90 kgs of chemical X and 60 kgs of chemical Y. Standard price of chemical X & Y are ₹. 5 and 10 per kg respectively. The standard loss of production is 30%. The actual mixture and yield were as follows:

i. Actual yield 115 kgs.

ii. X 80 kgs @ ₹.4.50 per kg

iii. Y 70 kgs @ ₹. 8.00 per kg

Calculate all material variances

Solution:

i.	**Material Cost Variance**	**= Standard Cost of Actual Production – Actual Cost**	
		= 1150 – 920 = 230 (F)	
ii.	**Material Price Variance**	**= AQ (SR – AR)**	
	Chemical X	= 80 (5 - 4.50) = 40 (F)	
	Chemical Y	= 70 (10 – 8) = 140 (F)	
	Total Price Variance	= 180 (F)	

iii. Material Usage Variance = Material Yield Variance + Material Mix Variance

a. Material Yield Variance = Standard Yield Rate (Standard Yield – Actual Yield)
= 10 (105 – 115) = 100 (F)

b. Material Mix Variance = Standard Cost of (Standard Mix – Actual Mix)
= 1050 – 1100 = 50 (A)

c. Material Usage Variance = 100 (F) + 50 (A) = 50 (F)

Reconciliation

Material Cost Variance = Material Proce Variance + Material Usage Variance

230 (F) = 180 (F) = 50 (F)

Working Note:

1. Computation of Yield Rate

Standard Cost		Actual Cost	
Computation	**Amount**	**Computation**	**Amount**
X: 90 Kgs @ ₹5/kg	450	X: 80 Kgs @ ₹ 4.5/kg	360
Y: 60 Kgs @ ₹.10/kg	600	Y: 70 Kgs @ ₹. 8/kg	560
Total 150 Kgs	1050	Total 150 Kgs	920
Standard Loss @ 30%= 45 Kgs		Actual Loss (150 – 115) = 35 kgs	
Standard Yield = 105 kgs	1050	Actual Yield = 115 kgs	920
Standard Yield Rate (₹./kg) (1050/105)	10	Actual Yield Rate (₹./kg) (920/115)	8

2. Standard Cost of Standard Production = Standard Yield Rate × Actual Yield = 10 × 115=1150
3. Actual Cost of Actual Production = 920
4. Standard Cost of Standard Mix = ₹. 1050
5. Standard Cost of Actual Mix = Actual Qty × Standard Rate

X: 80 × 5 = 400

Y: 70 × 10 = 700

Total = 1100

ILLUSTRATION 4: SALE VARIANCES

A Company engaged in the production and sales of 3 products viz P, Q & R receives the following data of Sales Budget from its Marketing department for the month of December 2024:

Product	Sale Quantity	Sale Price (₹. per Unit)	Contribution Margin (₹. per Unit)
P	2000	12	6
Q	2000	8	4
R	2000	5	1

Actual Sales was as under

P: 1500 units for ₹.15,000

Q: 2500 units for ₹.17,500

R: 3500 units for ₹.21,000

You are required to calculate all Sales Variances

Solution:

Product	Budget			Actual		
	Quantity	Rate	Amount	Quantity	Rate	Amount
P	2000	12	24000	1500	10	15000
Q	2000	8	16000	2500	7	17500
R	2000	5	10000	3500	6	21000
Total	6000		50000	7500		53500

We also need to work out the Standard sales i.e. Actual quantity sold at Budgeted Price and Revised standard sales i.e. Actual quantity sold at standard price per unit of standard mix.

Product	Standard Sales		
	Actual Quantity	Budgeted Rates	Amount
P	1500	12	18000
Q	2500	8	20000
R	3500	5	17500
Total	7500		55500

Revised Standard Sales (RSS) = Actual quantity × Price per unit of standard mix

= 7500 × (50000/6000) = 62500

a. Sales Value Variance = Budgeted Sales – Actual Sales

= 50000 – 53500 = 3500 (F)

b. Sales Price Variance = Actual Quantity (Standard Rate – Actual Rate)

P:	= 1500 (12-10)	=	3000 (A)
Q:	= 2500 (8 – 7)	=	2500 (A)
R:	= 3500 (5 – 6)	=	3500 (F)
Total			2000 (A)

c. Sales Volume Variance = Standard Rate (Budgeted Quantity – Actual Quantity)

P:	= 12 (2000 -1500)	=	6000 (A)
Q:	= 8 (2000 – 2500)	=	4000 (F)
R:	= 5 (2000 – 3500)	=	7500 (F)
Total			5500 (F)

d. Sales Quantity Variance = Budgeted Sales - Revised Standard Sales

= 50000 – 62500 = 12500 (F)

e. Sales Mix Variance = Revised Standard Sales - Standard Sales

= 62500 – 55500 = 7000 (A)

Reconciliation:

i. Sales Value Variance = Sales Price Variance + Sales Volume Variance = 2000 (A) +5500(F) = 3500 (F)

ii. Sales Volume Variance = Sales Quantity Variance + Sales Mix Variance = 12500(F)+7000(A) = 5500 (F)

ILLUSTRATION 5: SALES MIX VARIANCES

You are a Management Consultant retained by a Company that operates on budgetary control system and works out variance on monthly basis. Data for a particular month is given below. Calculate relevant variances.

Product	Budget			Actual		
	Quantity Kgs	Rate (₹./kg)	Amount (₹. Lakhs)	Quantity Kgs	Rate (₹./kg)	Amount (₹. Lakhs)
P	50000	10.50	5.25	52000	11.00	5.72
Q	20000	12.00	2.40	16000	11.75	1.88
R	6000	15.00	0.90	5000	15.00	0.75
S	4000	16.00	0.64	5000	16.00	0.80
Total	80000		9.19	78000		9.15

Solution:

Product	Standard Sales		
	Actual Quantity	Budgeted Rates	Amount (₹. Lakhs)
P	52000	10.50	5.46
Q	16000	12.00	1.92
R	5000	15.00	0.75
	5000	16.00	0.80
Total	78000		8.93

Revised Standard Sales (RSS) = Actual quantity × Price per unit of standard mix

= 78000 × (9,19,000/80000) = 896025

a. Sales Value Variance = Budgeted Sales – Actual Sales

= 919000 – 915000 = 4000 (A)

b. Sales Price Variance = Actual Quantity (Standard Rate – Actual Rate)

P: = 52000 (10.5-11.00) = 26000 (F)

Q: = 16000 (12 – 11.75) = 4000 (A)

R: = 5000 (15 – 15) = 00

S: = 5000 (16 – 16) = 00

Total = 22000 (F)

c. Sales Volume Variance = Standard Rate (Budgeted Quantity – Actual Quantity)

P: = 10.5 (50000 -52000) = 21000 (F)

Q: = 12 (20000 – 16000) = 48000 (A)

R: = 15 (6000 – 5000) = 15000 (A)

S: = 16 (4000 – 5000) = 16000 (F)

Total = 26000 (A)

d. Sales Quantity Variance = Budgeted Sales - Revised Standard Sales

= 919000 – 896025 = 22975 (A)

e. Sales Mix Variance = Revised Standard Sales - Standard Sales

= 896025 – 893000 = 3025 (A)

Reconciliation:

iii. Sales Value Variance = Sales Price Variance + Sales Volume Variance

= 22000 (F) + 26000(A)

= 4000 (A)

iv. Sales Volume Variance = Sales Quantity Variance + Sales Mix Variance

= 22975(A) + 3025(A)

= 26000 (A)

ILLUSTRATION 6:

The financial results of Sell Well Ltd are given below. Price increase over one year period is estimated to be 10%. Management asks you to prepare a statement showing variation contributed by each factor.

₹ in Lakhs

Particulars	2022-23	2023-24
Materials Consumed	100000	132000
Wages	60000	66000
Variable Overheads	12000	14000
Fixed Overheads	20000	24000
Net Profit	8000	17000
Sale Value	200000	253000

Solution:

a. Ascertain sale price increase due to inflation and sale quantity increase over an year.

Sale of 2023-24 at 2022-23 prices	= 253000 × 100/110	= 230000
Sale in 2022-23		= 200000
Sale increase in quantity	= 30000, i.e. 15% increase over last year	
Add: 10% sale price increase over 2022-23	= 230000 × 10%	= 23000
Sale in 2023-24		= 253000

b. Ascertain Material consumption increase due to price and sale volume

Material consumed in 2024 at 2023 prices	= 132000 × 100/110	= 120000
Material Cost Variance	= 100000 – 132000	= 32000 (A)
Material Price Variance	= 132000 – 120000	= 12000 (A)
Material volume Variance	= 100000 × 15%	= 15000 (A)
Material Usage Variance		= 5000 (A)Balancing Figure

c. Ascertain Wages increase due to price and sale volume

Wages in 2024 at 2023 prices	= 66000 × 100/110	= 60000
Labour Cost Variance	= 60000 – 66000	= 6000(A)
Labour Rate Variance	= 60000 – 66000	= 6000(A)
Labour volume Variance	= 60000 × 15%	= 9000(A)
Labour Efficiency Variance		= 9000 (F) Balancing Figure
d. Variable Overheads Variance	= 12000 – 14000	= 2000 (A)
Volume Increase	= 12000 × 15%	= 1800(A)
Efficiency Variance		= 200 (A) Balancing Figure
e. Fixed Overheads Expenditure Variance	= 20000 – 24000	= 4000(A)

Statement of Variances **₹ in Lakhs**

Sr	Particulars	Total Variance	Price Variance	Volume Variance	Usage/ Efficiency Variance
1	Materials Consumed	32000 (A)	12000 (A)	15000 (A)	5000 (A)
2	Wages	6000 (A)	6000 (A)	9000 (A)	9000 (F)
3	Variable Overheads	2000 (A)		1800 (A)	200 (A)
4	Fixed Overheads	4000 (A)			4000 (A)
5	Total Cost	44000(A)	18000(A)	25800(A)	200 (A)
6	Sales	53000(F)	23000(F)	30000(F)	
7	Net Profit	9000(F)	5000(F)	4200(F)	200(A)

ILLUSTRATION 7: CASE STUDY

The executive of Something More Ltd. had several meetings and finalised the budget to be presented to Board of Directors in forthcoming Board Meeting. The budget envisaged an estimated profit of ₹.33 lakhs for the year.

On a scrutiny of the budget the Board felt that there is still a scope of profit improvement at least to the extent of 10% on budgeted figure.

In 2022-23, the total sales of the Industry were 10 lakh units, out of that the company's share was 1 lakh units. For the year 2023-24 the Sales Head had assumed the same total Industry market volume and Company's sales share. The Board directed that the industry volume and penetration be re-examined and a profit improvement plan to be submitted in consultation with other departments.

The plan submitted after due considerations embodied the following:

i. The total industry volume would grow in 2023-24 to 12 lakh units and the Company's share can be increased to 11%

ii. Sales mix will be changed from 50% of each size unit to 60% of the larger and 40% of the smaller with a contribution of ₹.11 & 9 respectively. The selling prices would be so raised that an additional contribution of ₹.0.50 per unit is available for all units sold.

iii. Additional Expenditure of ₹.50000 on advertisement and sales promotion, ₹.25000 on sales staff travelling, packaging design improvement ₹. 35000 will be incurred. A saving in sales office administrative expenses is anticipated to the extent of ₹.30000.

iv. Curtailing credit terms would result in saving of ₹.1.00 lakh and addition investment on inventory would cost ₹.70000 more. Borrowing cost is 18% pa.

You are required to draw the profit improvement plan in financial terms spelling out separately the increase or decrease in profit due to volume, sales mix, price, expenses and financing charges.

Solution:

A. Profit Improvement Plan

Particulars	Present	Budget	Variance
Sales Volume – Total Units	100000	132000	32000
Sales Volume – Larger Units	50000	79200	29200
Sales Volume – Smaller Units	50000	52800	2800
Contribution			
Larger Units	50000 × 11 = 550000	79200 × 11.5 = 910800	360800
Smaller Units	50000 × 9 = 450000	52800 × 9.5 = 501600	51600
Total Contribution	1000000	1412400	412400
Less: Increase in Expenses		50000 + 25000 + 35000 - 30000 = 80000	-80000
Add: Reduction in Financing Charges		(100000 - 70000) × 18% = 5400	5400
Net Contribution	1000000	1337800	337800

B. Increase in Profit Due to

1. Volume : Larger Size: 11(79200 – 50000) = 321200
   Smaller Size: 9 (52800 – 50000) = 25200
   Total = 346400

2. Price: 132000 units @ ₹.0.50 = 66000
3. Savings in Financing Charges = 5400
4. Decrease in Profit due to Expenses increase = 80000

Net Increase in Profit = 337800

C. Let us further analyse the increase in profit due to change in sales volume. As we know:

Sales Volume Variance = Sales Quantity Variance + Sales Mix Variance

Standard Contribution per unit of Standard Mix = 1000000/100000 = ₹.10/unit

Revised Standard Contribution = 132000 × 10 = 1320000

Standard Contribution of Actual Mix = Larger Size: 79200 × 11 = 871200

Smaller Size: 52800 × 9 = 475200

Total Standard Contribution of actual mix = 1346400

Sales Quantity Variance = Revised Standard Contribution – Standard Contribution

= 1320000 - 1000000 = 320000

Sales Mix Variance = Standard Contribution of Actual Mix – Revised Standard Contribution

= 1346400 – 1320000 = 26400

Reconciliation:

Volume Variance = Quantity Variance + Mix Variance = 320000 + 26400 = 346400

ILLUSTRATION 8: CASE STUDY

Net labour efficiency variance for punching department of a Company for the month of April 2024 was computed as follows:

Standard Labour Hours	28500
Actual Labour Hours	30200
Excess Hours	1700
Wage Rate per hour	₹. 50
Total Unfavourable Variance	₹. 85000

The factory superintendent is responsible for all production operations. He asked the punching department foreman who reports to him to explain the reasons for the excess labour hours consumed. Following analysis was submitted:

Sr	Particulars	Hours	Hours
A	Standard Labour Hours		28500
B	Excess Hours		2140
i	Trainee Operating Machines	48	
	Experienced operator instructing trainee	12	
	Rework time on rejected items	440	
	Rework time on job no. '007' as instructions of the supervisor not clear	80	
	Raw material was not as per quality standards	75	
	Work done on stand by machine as the regular one was overloaded	600	
	Extra set up time due to machine breakdown	15	
	Idle time, No production schedule	420	
	Unexplained	450	
C	Hours saved		440
	New tools used	400	
	Asst Foreman assigned for machine operation for 1 week	40	
D	Actual Working Hours		30200

Examine each item carefully and group them as responsibility of following officers with proper reason of assigning it:

a. Punching Department Supervisor

b. Factory Manager

c. Item not covered under the purview of both

Solution:

A. Items falling within the responsibility of Punching Department Supervisor

Sr	Particulars	No. of Hours	Reason
1	Rework time on items rejected during inspection	440	Due to inefficiency in the department

Sr	Particulars	No. of Hours	Reason
2	Extra Set up time due to machine breakdown	15	Preventive maintenance should have been undertaken to avoid this incurrence
3	Unexplained	450	Proper time booking is required
	Sub Total	905	
4	Less: Saving for deputing Asst Foreman to machine operation for 1 week	40	
5	Net Excess Hours	865	

B. Items falling within the responsibility of Factory Manager

Sr	Particulars	No. of Hours	Reason
1	Rework time on Job No.007	80	Instructions from Superintendent were not clear
2	Work done on standby machine	600	Proper loading of machine was not done
3	Idle Time	420	Proper work scheduling is is required to avoid such recurrence
	Sub Total	1100	
4	Less: Hours saved due to new tools used	400	Good initiative on the part of Factory
5	Net Excess Hours	700	

C. Items not falling within the purview of Both

Sr	Particulars	No. of Hours	Reason
1	Trainee Operating Machine	48	Management Policy
2	Experienced Operator Instructing Trainee	12	Management Policy
3	Raw material was not as per quality standards	75	Responsibility of the Purchase Department
	Sub Total	135	

Grand Total A + B + C = 1700 Hours

ILLUSTRATION 9:

A Ltd manufacturers metal furniture. Product 'Y' is one of the middle range products of the company that uses metal for frame and upholstery in plastic. The company follows standard Costing systems and works out variance analysis regularly and data remains quite confidential. During February 2024, a fire broke out in the office of the Company that destroyed all the documents related to budget and standards of the Company for year 2023-24.

All the operations data for the month of February 2024 is given below:

Particulars	Values
Production Materials	1500 standard units
Labour & Overheads	1200 standard units
Materials Issued in Kgs: Metal Plastic	 15000 3100
Material Cost in ₹. Lakhs Metal Plastic	 8250 3255
Direct Labour in ₹. Lakhs Direct Labour Hours Worked	19000 5000
Actual Overheads Fixed Variable	 6560 3950
Variances	
Metal Price: ₹.750 (Adverse)	Usage: Nil
Plastics Price: ₹.155 (adverse)	Usage ₹. 100 (Adverse)
Labour Rate: ₹.1000 (Favourable)	Efficiency ₹.800 (Adverse)
Variable Overheads: Total : ₹.350 (Adverse)	
Fixed Overheads: Budget: ₹.40 (Favourable)	Capacity: ₹.600 (Favourable)

Reconstruct the Standard Cost Sheet of the Product 'Y' and find out the standard cost per unit.

Solution:

A. Cost Sheet for Product 'Y'

Elements of Cost	Actual		Standard		
	Quantity	Amount	Quantity	Rate	Amount
Materials: Metal Price Variance (A) Usage Variance	15000 Unit	8250 (-)750 0	15000	0.50	7500
Plastic Price Variance (A) Usage Variance (A)	3100 U (-)100	3255 (-)155 (-)100	3000	1.00	3000
Labour Rate Variance (F) Efficiency Variance (A)	5000 H 200	19000 1000 800	4800	4	19200
Variable Overheads Total Variance (A)		3950 (-)350			3600
Fixed Overheads Budget Variance (F) Capacity Variance (F)		6560 40 600			7200
Total Cost					40500

B. Standard Cost Per unit of Product 'Y'

Elements of Cost	Quantity Per Unit	Rate (₹. Per Unit)	Amount (₹.)
Materials Metal Plastic	 15000/1500=10 Kgs 3000/1500 = 2 kgs	 0.50 per kg 1.00 per kg	 5.00 2.00
Direct Labour	4800/1200 = 4 hours	4.00 per Hour	16.00
Variable Overheads	3600/1200	₹.3.00 per unit	3.00
Fixed Overheads	7200 /1200	₹.6.00 per unit	6.00
Total Cost Per unit			32

ILLUSTRATION 10

A Ltd. uses standard Costing system for its widely sold product 'Alexa'. Standard Cost card is as follows:

Sr	Particulars	₹. Per Unit
1	Selling Price	120
2	Direct Material: 1 kg per unit	20
3	Direct Labour: 6 hours	48
4	Variable Overheads	24

Budgeted Sale & production is 50000 units for the period whereas actual production is 52000 units and actual sale is 51200 units.

Other actual information is Sale Value - ₹. 6133760
Direct Materials - ₹. 1065600
Direct Labour - 3.00 Lakhs Hours
- ₹. 2442000
Variable Overheads - ₹. 1228000

Calculate

i. Direct Labour Rate Variance,
ii. Direct Labour Efficiency Variance,
iii. Sales Volume Variance
iv. Sales Price Variance

Also comment on your findings in i. & ii above.

Solution:

i. Direct Labour Rate Variance = Standard Rate (SR) × Actual Hours produced (AHP) – Actual Rate (AR) × Actual Hours produced = 8 × 300000 – 2442000 = 42000 (A)

ii. Direct Labour Efficiency Variance = SR (Standard Hours for Actual Output – AHP)
= 8 ((6 × 52000) – 300000) = 96000 (F)

iii. Sales Volume Variance = Budgeted selling price (Actual Quantity – Budgeted Quantity)
= 120 × (51200 – 50000) = 144000 (F)

iv. Sales Price Variance = Actual Quantity × (Actual Selling Price – Budgeted Selling Price)
= 51200(119.80 - 120) = 10240(A)

Comments on i. & ii:

a. Labour efficiency variance of ₹. 96000 (F) represents cost savings due to time saved. On the other hand adverse Labour rate variance indicates extra wage burden due to rate increase.

b. Labour time saving will also lead to savings in the variable overheads and thereby contribution will also increase.

4 Linear Programming [Study Material - Module 6]

ILLUSTRATION 1: (PRODUCTION SCHEDULING PROBLEM)

A manufacturer can produce two different products, A and B, during a given time period. Each of these products requires four different manufacturing operations: Grinding, Turning, Assembly and Testing. The manufacturers' requirements for hours per unit of product are given below for A and B:

	A	B
Grinding	1	2
Turning	3	1
Assembly	6	3
Testing	5	4

The available capacities of these operations in hours for the given time period are: Grinding 30; Turning 60: Assembly 200; Testing 200. The Contribution to profit is ₹ 2 for each unit of A and ₹ 3 for each unit of B. The firm can sell all that it produces at the prevailing market price. Formulate the problem as a linear programming model to maximize profit.

Solution:

Step 1: Key Decision: Determining number of units of product A and B to be produced by the company

Step 2: Assumption: Let X_1 and X_2 be number of units that a company decides to produce of Product A and B respectively

Step 3: Objective Function: Here Objective is to maximize profit by selling A and B products

Maximize $Z = 2X_1 + 3X_2$

Step 4: Subject to Constraints:

a. $X_1 + 2X_2 \leq 30$ (Total no of hrs. available in Grinding)

b. $3X_1 + 1X_2 \leq 60$ (Total no of hrs. available in Turning)

c. $6X_1 + 3X_2 \leq 200$ (Total no of hrs. available in Assembly)

d. $5X_1 + 4X_2 \leq 200$ (Total no of hrs. available in Testing)

e. $X_1, X_2 \geq 0$ (Non Negative Function)

ILLUSTRATION 2: (DIET PROBLEM)

Vitamins V and W are found in two different foods viz. F_1 and F_2. One unit of food F_1 contains two units of Vitamin V and 3 units of Vitamin W. One unit of Food F_2 contains 4 units of Vitamin V and 2 units of Vitamin W. One unit of food F_1 and F_2 costs ₹ 3 and ₹ 2.5 respectively. The minimum daily requirements of vitamin V and W are 40 and 50 units respectively. If anything, more than daily minimum requirement of Vitamin V and W will not be absorbed. Formulate this as a linear programming problem.

Solution:

Step 1: Key Decision: Key decision is being determined in the summarized tabular form as under:

Decision Variables	Food	Units of Vitamins V	Units of Vitamins W	Cost per Unit
X_1	F_1	2	3	₹ 3
X_2	F_2	4	2	₹ 2.5
Daily requirement (minimum)		40	50	

Step 2: Assumption: Let X_1 and X_2 be no units of Food F_1 and F_2 respectively.

Step 3: Objective Function: Here Objective is to minimize the cost of Food F_1 and F_2

Minimize $Z = 3X_1 + 2.5X_2$

Step 4: Subject to Constraints:

a. $2X_1 + 4X_2 \geq 40$ (Daily minimum requirement of Units of Vitamin V)

b. $3X_1 + 2X_2 \geq 50$ (Daily minimum requirement of Units of Vitamin W)

c. $X_1, X_2 \geq 0$ (Non-Negative Function)

ILLUSTRATION 3: (STAFF MANAGEMENT PROBLEM)

A city Hospital has the following minimal daily requirement for nurses:

Period	Clock time	Minimal no of nurses required
1	6 am to 10 am	2
2	10 am to 2pm	7
3	2 pm to 6 pm	15
4	6 pm to 10 pm	8
5	10 pm to 2 am	20
6	2 am to 6 am	6

Nurses report to the hospital at the beginning of each period and work for 8 consecutive hours. The hospital wants to determine the no of nurses to be employed so that there will be sufficient number of nurses available for each period. Formulate this as a Linear Programming Problem by setting up appropriate constraints and objective function and do not solve.

Solution:

Step 1: Key Decision: Key decisions are already being determined in the summarized tabular form mentioned above.

Step 2: Assumption: Let X_1, X_2, X_3, X_4, X_5 and X_6 be the no of nurses

Step 3: Objective Function: Here Objective is to minimize the total no of nurses employed in the hospital

Minimize $Z= X_1 + X_2 + X_3 + X_4 + X_5 + X_6$

Step 4: Subject to Constraints:

a. $X_1 + X_2 \geq 7$*

b. $X_2 + X_3 \geq 15$*

c. $X_3 + X_4 \geq 8$*

d. $X_4 + X_5 \geq 20$*

e. $X_5 + X_6 \geq 6$*

f. $X_1 + X_6 \geq 2$*

g. $X_1 + X_2 + X_3 + X_4 + X_5 + X_6 \geq 0$ (Non-Negative Function)

* **Note:** Since each nurse must work for 8 consecutive hours the X_1 nurses who were employed during 1st period shall still be on duty when 2nd period starts. So, the minimal number of nurses required would be as per 2nd period, which would be:

$X_1 + X_2$ i.e. 7;

$X_2 + X_3$ i.e. 15;

$X_3 + X_4$ i.e. 8;

$X_4 + X_5$ i.e.20;

$X_5 + X_6$ i.e. 6;

$X_1 + X_6$ i.e. 2;

ILLUSTRATION 4:

A factory manufactures two articles A and B. To manufacture the article A, a certain machine has to work for 1.5 hours and craftsmen for 2 hours. To manufacture article B, the machine must work for

2.5 hours and craftsmen for 1.5 hours. In a week the factory can avail of 80 hours of machine time and that of 70 hours of craftsmen time. The profit of each article A is ₹ 5 and that of B is ₹ 4. If all the articles produced can be sold away, find how many each kind should be produced to earn the maximum profit per week. Formulate the problem as Linear programming problem.

Solution:

Step 1: Key Decision: Key decision is being determined in the summarized tabular form as under:

Decision Variables	Article	Hours on Machine	Hours on Craftsmen	Profit per Unit
X_1	A	1.5	2	₹ 5
X_2	B	2.5	1.5	₹ 4
Hours Available per week (maximum)		80	70	

Step 2: Assumption: Let X_1 and X_2 be no units of Article A and B respectively.

Step 3: Objective Function: Here Objective is to maximize the profit Article A and B respectively.

Maximize $Z = 5X_1 + 4X_2$

Step 4: Subject to Constraints:

a. $1.5X_1 + 2.5X_2 \leq 80$ (Maximum Hours Available per week on Machine)

b. $2X_1 + 1.5X_2 \leq 70$ (Maximum Hours Available per week on Craftsmen)

c. $X_1, X_2 \geq 0$ (Non-Negative Function)

ILLUSTRATION 5:

Production of a certain chemical mixture should contain 80 mg chlorides, 28 mg nitrates and 3 mg sulphate per kilogram. The company can use two substances and a base. Substances X contain 8 mg chlorides, 4 mg nitrates and 6 mg sulphates per gram. Substances Y contain 10 mg chlorides, 2 mg nitrates and 2 mg sulphates per gram. Both substances cost ₹ 20 per gram. It is required to produce the mixture using substances X and Y so that the cost is minimized. Formulate the problem as Linear programming problem.

Solution:

Step 1: Key Decision: Key decision is being determined in the summarized tabular form as under:

Decision Variables	Substance	Content (in mg)			Cost per Unit (In ₹)
		Chloride	Nitrate	Sulphate	
X_1	X	8	4	6	20

Decision Variables	Substance	Content (in mg)			Cost per Unit (In ₹)
		Chloride	Nitrate	Sulphate	
X_2	Y	10	2	2	20
Minimum Requirement		80	28	36	

Step 2: Assumption: Let X_1 and X_2 be no units of substances X and Y respectively.

Step 3: Objective Function: Here Objective is to minimize the cost of substances X and Y respectively.

Minimize $Z = 20X_1 + 20X_2$

Step 4: Subject to Constraints:

a. $8X_1 + 10X_2 \geq 80$ (Minimum Requirement of Content in mg of Chloride)

b. $4X_1 + 2X_2 \geq 28$ (Minimum Requirement of Content in mg of Nitrate)

c. $6X_1 + 2X_2 \geq 36$ (Minimum Requirement of Content in mg of Sulphate)

d. $X_1, X_2 \geq 0$ (Non-Negative Function)

ILLUSTRATION 6: (INVESTMENT PROBLEM)

Mr. Krishnamurthy, a retired government officer has received retirement benefits viz provident fund, gratuity etc. He is contemplating as to how much funds he should invest in various alternatives open to him as to maximize return on investment. The investment alternative are – government securities, fixed deposits of a public limited company, equity shares, time deposits in banks, national saving certificate and real estate. He has made a subjective estimate of the risk involved on a five point scale. The data on the return on investment, the number of years for which the funds will be blocked to earn this return on investment and the subjective risk involved are as follows:

Sr no	Investment Alternatives	Return	Number of Years	Risk
1	Government Securities	6%	15	1
2	Company Deposits	15%	3	3
3	Equity Shares	20%	6	7
4	Time Deposits	10%	3	1
5	National Savings Certificate	12%	6	1
6	Real Estate	25%	10	2

He was wondering what percentage of funds to invest to maximize return on his investment. He decided that the average risk should not be more than 4, and fund should not be locked up for more than 15 years. He should necessarily invest at least 30% in real estate. Formulate an LP Model for the problem.

Solution:

Step 1: Key Decision: Key decision is being determined in the summarized tabular form as above.

Step 2: Assumption: Let X_1, X_2 X_3, X_4, X_5, and X_6 be the percentages of the total funds invested in government securities, fixed deposits of a public limited company, equity shares, time deposits in banks, national saving certificate and real estate respectively.

Step 3: Objective Function: Here Objective is to maximize total return on investment.

Maximize $Z = 0.06X_1 + 0.15X_2 + 0.20X_3 + 0.10X_4 + 0.12X_5, +0.25X_6$

Step 4: Subject to Constraints:

a. $1X_1 + 3X_2 + 7X_3 + 1X_4 + 1X_5, +2X_6 \leq 4$ (Average risk should not be more than 4)

b. $15X_1 + 3X_2 + 6X_3 + 3X_4 + 6X_5, +10X_6 \leq 15$ (Fund should not be locked up for more than 15 years)

c. $X_6 \geq 0.30$ (Invest at least 30% in real estate)

d. X_1, X_2, X_3, X_4, X_5, and $X_6 \geq 0$ (Non-Negative Function)

ILLUSTRATION 7:

A small scale industry manufactures electrical regulators, the assembly of which is being accomplished by a small group of persons. The number of workers employed cannot exceed 11 and their salary bill not more than ₹ 6000 per month. The male skilled workers are paid ₹ 600 per month while female of the same skill are paid ₹ 500 per month. Data collected on the performance of these workers indicate that a male member contributes ₹ 1000 per month to total return of the industry, while that of female member contributes ₹ 850 per month to total return of the industry. Formulate an LP Model for the problem.

Solution:

Step 1: Key Decision: Key decision is being determined in the summarized tabular form as under:

Decision Variables	Skilled Workers	Salary of Workers	Contribution Industry wise
X_1	Men	₹ 600 per month	₹ 1000 per month
X_2	Women	₹ 500 per month	₹ 850 per month
Maximum employee = 11	Maximum Salary Bill	₹ 6000 per Month	

Step 2: Assumption: Let X_1 and X_2 be the number of male and female workers to be employed in the industry respectively.

Step 3: Objective Function: Here Objective is to maximize contribution per month to total return of the industry.

Maximize $Z = 1000X_1 + 850X_2$

Step 4: Subject to Constraints:

a. $X_1 + X_2 \leq 11$ (Number of workers employed cannot exceed 11)

b. $600X_1 + 500X_2 \leq 6000$ (Salary bill not being more than ₹ 6000 per month)

i.e. $6X_1 + 5X_2 \leq 60$

c. $X_1, X_2 \geq 0$ (Non-Negative Function)

ILLUSTRATION 8: (DUAL PROBLEM)

Obtain the dual of:

Minimize $Z = 40 X_1 + 120 X_2$

Subject to:

$X_1 - 2X_2 \leq 8$

$3X_1 + 5X_2 = 90$

$15X_1 + 44X_2 \leq 660$

$X_1, X_2 \geq 0$

Solution:

Since the problem is of minimization, we will make all inequalities to be of "Greater than or equal to "type.

Primal problem

Minimize $Z = 40 X_1 + 120 X_2$

Subject to:

$-X_1 + 2X_2 \geq -8$

$3X_1 + 5X_2 \geq 90$

$-3X_1 - 5X_2 \geq -90$

$-15X_1 - 44X_2 \geq -660$

$X_1, X_2 \geq 0$

Dual problem

Maximize $Z^* = 8Y_1 + 90 Y_3 - 90Y_2 + 660 Y_4$

Subject to:

$-Y_1 + 3Y_2 - 3Y_3 - 15Y_4 \leq 40$

$2Y_1 + 5 Y_2 - 5Y_3 - 44Y_4 \leq 120$

$Y_1, Y_2, Y_3, Y_4 \geq 0$

ILLUSTRATION 9: (DUAL PROBLEM)

Obtain the dual of:

Maximize $Z = 3X_1 + 5X_2 + 7X_3$

Subject to:

$X_1 + X_2 + 3X_3 \leq 10$

$4X_1 - X_2 + 2X_3 \geq 15$

$X_1, X_2 \geq 0$, X_3 unrestricted variable

Solution: Since X_3 is unrestricted, it is replaced by $X_3' - X_3''$, where $X_3' \geq 0$; $X_3'' \geq 0$; Then the given problem becomes:

Maximize $Z = 3X_1 + 5X_2 + 7(X_3' - X_3'')$

Subject to:

$X_1 + X_2 + 3(X_3' - X_3'') \leq 10$

$4X_1 - X_2 + 2(X_3' - X_3'') \geq 15$

$X_1, X_2, X_3' - X_3'' \geq 0$

Since it is a maximization problem, the second constraint is multiplied by -1 on both sides to give

$-4X_1 + X_2 - 2(X_3' - X_3'') \leq -15$

Thus, here the given problem becomes:

Maximize $Z = 3X_1 + 5X_2 + 7X_3' - 7X_3''$

Subject to:

$X_1 + X_2 + 3X_3' - 3X_3'' \leq 10$

$-4X_1 + X_2 - 2X_3' + 2X_3'' \leq -15$

$X_1, X_2, X_3' - X_3'' \geq 0$

Let Y_1, Y_2 be the associated non negative dual variables. Then the dual of this problem is:

Minimize $Z^* = 10Y_1 - 15Y_2$

Subject to

$Y_1 - 4Y_2 \geq 3$

$Y_1 + Y_2 \geq 5$

$3Y_1 - 2Y_2 \geq 7$

$-3Y_1 + 2Y_2 \geq -7$ or $3Y_1 - 2Y_2 \leq 7$

$Y_1, Y_2 \geq 0$

ILLUSTRATION 10: (DUAL AND SIMPLEX PROBLEM)

Minimize $Z = 3X_1 + 5X_2$

Subject to:

$3X_1 + 2X_2 \geq 50$

$2X_1 + 4X_2 \geq 40$

$X_1, X_2 \geq 0$

Solve by dual and simplex method

Solution: The dual is given by:

Maximize $Z = 50Y_1 + 40Y_2$

Subject to:

$3Y_1 + 2Y_2 \leq 3$

$2Y_1 + 4Y_2 \leq 5$

$Y_1, Y_2 \geq 0$

The mathematical form of the dual with the simplex would be the:

Maximize $Z = 50Y_1 + 40Y_2 + 0S_1 + 0S_2$

Subject to:

$3Y_1 + 2Y_2 + 0S_1 + 0S_2 = 3$

$2Y_1 + 4Y_2 + 0S_1 + 0S_2 = 5$

$Y_1, Y_2 \geq 0$

Simplex Tableau I:

Cj	Product Mix	Quantity	50	40	0	0	Replacement Ratio
			Y_1	Y_2	S_1	S_2	
0	S_1	3	3*	2	1	0	1

Cj	Product Mix	Quantity	50	40	0	0	Replacement Ratio
			Y_1	Y_2	S_1	S_2	
0	S_2	5	2	4	0	1	2.5
	Zj	0	0	0	0	0	
	Cj- Zj	0	50	40	0	0	

Simplex Tableau II: (Introduce Y_1 and Drop S_1)

Cj	Product Mix	Quantity	50	40	0	0	Replacement Ratio
			Y_1	Y_2	S_1	S_2	
50	Y_1	1	1	2/3	1/3	0	3/2
0	S_2	3	0	8/3*	-2/3	1	9/8
	Zj	50	50	100/3	50/3	0	
	Cj- Zj	0	0	20/3	-50/3	0	

Simplex Tableau III: (Introduce Y_2 and Drop S_2)

Cj	Product Mix	Quantity	50	40	0	0
			Y_1	Y_2	S_1	S_2
50	Y_1	1/4	1	0	1/2	-1/4
40	Y_2	9/8	0	1	-1/4	3/8
	Zj	230/4	50	40	15	30/4
	Cj- Zj	0	0	0	-15	-30/4

* Cj- is in ₹.

- Since all numbers in index row are negative, the optimal solution is arrived at and is given by: $X_1 = 15$; $X_2 = 30/4$ and $Z = 230/4$.

5 Transportation [Study Material - Module 7]

ILLUSTRATION 1: (MINIMIZATION PROBLEM WITH NORTHWEST CORNER RULE)

A company has three production centers P1, P2 and P3. It sells goods at three selling centers S1, S2 and S3. The capacity of the production centers is 100, 200 and 300 units whereas the requirements of sales centers are 150, 250 and 200 respectively. The transportation costs (in ₹) from production centers to selling centers are given below:

From \ To	S1	S2	S3	Capacity (Supply)
P1	10	12	8	100
P2	15	12	16	200
P3	10	11	9	300
Requirement (Demand)	150	250	200	600

Solve this transportation problem by Northwest Corner Rule Method

Solution: Northwest Corner Rule Method

From \ To	S1	S2	S3	Capacity (Supply)
P1	100 10	12	8	~~100~~ 0
P2	50 15	150 12	16	~~200~~ ~~150~~ 0
P3	10	100 11	200 9	~~300~~ ~~200~~ 0
Requirement (Demand)	~~150~~ ~~50~~ 0	~~250~~ ~~100~~ 0	~~200~~ 0	600

Total Transportation Cost: (Unit × Per Unit Cost) = (100×10 + 50×15 + 150×12 + 100×11 + 200×9) = ₹6,450/-

ILLUSTRATION 2: (MINIMIZATION PROBLEM WITH NORTH WEST CORNER RULR)

Find the basic feasible solution of the following transportation problem by Northwest Corner Rule Method:

Warehouse / Factory	W1	W2	W3	W4	Capacity (Supply)
F1	21	16	25	13	**11**
F2	17	18	14	23	**13**
F3	32	27	18	41	**19**
Requirement (Demand)	**6**	**10**	**12**	**15**	**43**

Solution: Northwest Corner Rule Method

Warehouse / Factory	W1	W2	W3	W4	Capacity (Supply)
F1	6 21	5 16	25	13	~~11~~ 5 0
F2	17	5 18	8 14	23	~~13~~ 8 0
F3	32	27	4 18	15 41	~~19~~ ~~15~~ 0
Requirement (Demand)	~~6~~ 0	~~10~~ 5 0	~~12~~ 4 0	~~15~~ 0	43

Total Transportation Cost: (Unit × Per Unit Cost) = (6×21 + 5×16 + 5×18 + 8×14 +4×18 +15×41) = ₹1,095/-

ILLUSTRATION 3: (MINIMIZATION PROBLEM WITH LEAST COST METHOD)

Find the basic feasible solution of the following transportation problem by Least cost Method:

Warehouse / Factory	W1	W2	W3	W4	Capacity (Supply)
F1	21	16	25	13	**11**
F2	17	18	14	23	**13**
F3	32	27	18	41	**19**
Requirement (Demand)	**6**	**10**	**12**	**15**	**43**

Solution: Least Cost Method

Factory \ Warehouse	W1	W2	W3	W4	Capacity (Supply)
F1	21	16	25	11 13	~~11~~ 0
F2	1 17	18	12 14	23	~~13~~ ~~1~~ 0
F3	5 32	10 27	18	4 41	~~19~~ ~~9~~ 4 0
Requirement (Demand)	~~6~~ ~~5~~ 0	~~10~~ 0	~~12~~ 0	~~15~~ 4 0	43

Total Transportation Cost: (Unit × Per Unit Cost) = (11×13 +1×17 +12×14 +5×32 +10×27 +4×41) = ₹922/-

ILLUSTRATION 4: (MINIMIZATION PROBLEM WITH VOGEL'S APPROXIMATION METHOD)

Find the basic feasible solution of the following transportation problem by Vogel Approximation Method:

Factory \ Warehouse	W1	W2	W3	W4	Capacity (Supply)
F1	21	16	25	13	**11**
F2	17	18	14	23	**13**
F3	32	27	18	41	**19**
Requirement (Demand)	**6**	**10**	**12**	**15**	**43**

Solution: Vogel's Approximation Method (VAM):

Factory \ Warehouse		W1	W2	W3	W4	Capacity (Supply)	Column Penalty					
							1	2	3	4	5	6
F1		21	16	25	11 13	~~11~~ 0	3					
F2		6 17	3 18	14	4 23	~~13~~ ~~9~~ ~~3~~ 0	3	3	3	4		
F3		32	7 27	12 18	41	~~19~~ ~~7~~ 0	9	9	9	9		
Requirement (Demand)		~~6~~ 0	~~10~~ ~~3~~ 0	~~12~~ 0	~~15~~ ~~4~~ 0	43						
Row Penalty	1	4	2	4	10							
	2	15	9	4	18							
	3	15	9	4								
	4		9	4								
	5											
	6											

Total Transportation Cost: (Unit × Per Unit Cost) = (11×13 + 6×17 + 3×18 + 4×23 + 7×27 + 12×18) =₹796/-

Learning Take: For a given problem, total cost is minimum with Vogel's Approximation Method

ILLUSTRATION 5: (MINIMIZATION PROBLEM WITH VAM).

Find the basic feasible solution of the following transportation problem:

Warehouse \ Stores	I	II	III	IV	Capacity (Supply)
A	7	3	5	5	**34**
B	5	5	7	6	**15**
C	8	6	6	5	**12**
D	6	1	6	4	**19**
Requirement (Demand)	**21**	**25**	**17**	**17**	**80**

Solution: We will solve this problem by Vogel Approximation Method, as the method is not mentioned. We understand that transportation solution obtained under VAM is minimum.

Vogel Approximation Method:

Warehouse \ Stores		I	II	III	IV	Capacity (Supply)	Column Penalty					
							1	2	3	4	5	6
A		6 7	6 3	17 5	5 5	~~34~~ ~~28~~ ~~11~~ ~~5~~ 0	2	2	2	2	2	2
B		15 5	5	7	6	~~15~~ 0	1	1				
C		8	6	6	12 5	~~12~~ 0	1	1	1	1	1	1
D		6	19 1	6	4	~~19~~ 0	3					
Requirement (Demand)		~~21~~ 6 0	~~25~~ 6 0	~~17~~ 0	~~17~~ ~~12~~ 0	80						
Row Penalty	1	1	2	1	1							
	2	2	2	1	1							
	3	1	3	1	0							
	4	1		1	0							
	5	1			0							
	6											

Total Transportation Cost: (Unit × Per Unit Cost) = (6×7 + 6×3 + 17×5 + 5×5 + 15×5 + 12×5 + 19×1) = **₹324/-**

ILLUSTRATION 6: (MAXIMIZATION PROBLEM WITH VAM):

A firm owns warehousing facilities at 4 places (W1, W2, W3, W4) and it has manufacturing plants at 3 places (P1, P2, P3). Table below shows the net profit for each unit, along with manufacturing plants daily production (supply) and facilities daily requirements (demand). Find the optimal schedule to maximize the profit.

Destination / Sources	W1	W2	W3	W4	Capacity (Supply)
P1	12	15	6	25	200
P2	8	7	10	15	500
P3	12	6	10	20	300
Requirement (Demand)	180	320	100	400	1000

Solution: The above problem is balanced as total supply = Total demand =1000

Since, it is a maximization problem, first step is to convert this problem into minimization problem. To convert this problem into minimization one, a Relative Loss Matrix is to be prepared.

To convert this profit matrix into relative loss matrix, all elements of the profit matrix need to be subtracted from maximum is element. In given problem 25 is the highest value of profit matrix, so each cell needs to be deducted from 25. We get the matrix below:

Relative Loss Matrix

Destination / Sources	W1	W2	W3	W4	Capacity (Supply)
P1	13	10	19	0	200
P2	17	18	15	10	500
P3	13	19	15	5	300
Requirement (Demand)	180	320	100	400	1000

Reduced Row Matrix (from Relative Loss Matrix)

Destination / Sources	W1	W2	W3	W4	Capacity (Supply)
P1	13	10	19	0	200
P2	7	8	5	0	500
P3	8	14	10	0	300
Requirement (Demand)	180	320	100	400	1000

Reduced Column Matrix (from Relative Loss Matrix)

Destination / Sources	W1	W2	W3	W4	Capacity (Supply)
P1	0	0	4	0	200
P2	4	8	0	10	500
P3	0	9	0	5	300
Requirement (Demand)	180	320	100	400	1000

Modified Relative Loss Matrix

*Adding Row and Column reduction operation in each of the cells we would get:

Destination / Sources	W1	W2	W3	W4	Capacity (Supply)
P1	13	10	23	0	200
P2	11	16	5	10	500
P3	8	23	10	5	300
Requirement (Demand)	180	320	100	400	1000

We will solve this problem by Vogel Approximation Method if the method is not mentioned.

Vogel Approximation Method:

Destination / Sources	W1	W2	W3	W4	Capacity (Supply)	Row Penalties			
P1	13	10	23	200 0	~~200~~ 0	10	-	-	-
P2	80 11	320 16	100 5	10	~~500~~ ~~400~~ ~~320~~ 0	5	5	5	6
P3	100 8	23	10	200 5	~~300~~ ~~100~~ 0	3	3	3	2
Requirement (Demand)	~~180~~ ~~80~~ 0	~~320~~ 0	~~100~~ 0	~~400~~ ~~200~~ 0	1000				
Column Penalties	3	6	5	5					
	3	7	5	5					
	3	-	5	5					
	3	-	5	-					

Now the optimal solution using the Profit Matrix would be as mentioned below:

Destination / Sources	W1	W2	W3	W4	Capacity (Supply)
P1	12	15	6	200 25	200
P2	80 8	320 7	100 10	15	500
P3	100 12	6	10	200 20	300
Requirement (Demand)	180	320	100	400	1000

Total Maximum Profit = 200×25 + 80×8 + 320×7 + 100×10 + 100×12+ 200×20= ₹ 14,080/-

ILLUSTRATION 7: (MINIMIZATION AND UNBALANCED PROBLEM):

A Company has received a contract to supply gravel for three new construction projects located in towns A, B and C. Construction engineers have estimated the required amounts of gravel which are needed at these construction projects.

Project Location	Weekly requirements (Truck Loads)
A	144
B	204
C	82

The company has three gravel pits located in towns W, X and Y respectively. The gravel required by the construction projects can be supplied with these three plants. The amount of gravel which can be supplied by these plants is as follows:

Plant	Quantity Available (Truck Loads)
W	152
X	164
Y	154

The company has computed the delivery cost from each plant to each project site. These costs (in rupees) are shown in the following table:

Plant	Cost per Truck Load		
	A	B	C
W	8	16	16
X	32	48	32
Y	16	32	48

Schedule shipment from each plant to each project in such a manner to minimize the total transportation cost within the constraints imposed by plant capacities and projects requirements. Find the minimum cost.

Solution:

In the given problem, Quantity Available ≠ Quantity Required, hence the problem is an unbalanced problem. First of all, problem needs to be balanced by adding a dummy row or column. Balanced matrix will be as follows:

Plant	Project Location (Cost per Truck Load)				Availability (Truck Loads)
	A	B	C	Dummy Column	
W	8	16	16	0	**152**
X	32	48	32	0	**164**
Y	16	32	48	0	**154**
Requirement (Truck Loads)	**144**	**204**	**82**	**40**	**470**

Now, the initial basic feasible solution to be obtained by using the Vogel Approximation Method i.e. the VAM Method given as below:

Plant	Project Location (Cost per Truck Load)				Availability (Truck Loads)	Column Penalty					
	A	B	C	Dummy Column							
W	~~8~~	152~~16~~	~~16~~	~~0~~	~~152~~ ~~0~~	8	8				
X	~~32~~	42 ~~48~~	82 ~~32~~	40 ~~0~~	~~164~~ ~~124~~ ~~42~~ ~~0~~	32	0	0	16		
Y	144 ~~16~~	10 ~~32~~	~~48~~	~~0~~	~~154~~ ~~10~~ ~~0~~	16	16	16	16		
Requirement (Truck Loads)	~~144~~ ~~0~~	~~204~~ ~~52~~ ~~42~~ ~~0~~	~~82~~ ~~0~~	~~40~~ 0	470						
Row Penalty	8	16	16	0							
	8	16	16								
	16	16	16								
		16	16								

Total Minimum cost of transportation = 144×16 + 152×16 +42×48 +10×32 +82×32+40×0 = ₹ 9696/-

ILLUSTRATION 8: (MAXIMIZATION AND UNBALANCED PROBLEM):

ABC Enterprises has three plants manufacturing dry cells, located at different locations. Production cost differs from plant to plant. There are five sales offices of the company located in different regions of the country. The sales prices can differ from region to region. The shipping cost from each plant to each sales office and other data are given by following table:

Production Cost and Capacity of the plants

Plant number	Maximum production Capacity	Production Cost ₹
1	150 units	20
2	200 units	22
3	125 units	18

Shipping Cost per unit from different plants to various Sales Officers

From Plants	To Sales Officers at				
	Northern Region	Southern Region	Western Region	Eastern Region	Capital City
1	1	1	5	9	4
2	9	7	8	3	6
3	4	5	3	2	7

Demand and sales Prices for different plants to various Sales Officers

From Plants	To Sales Officers at				
	Northern Region	Southern Region	Western Region	Eastern Region	Capital City
Demand	**80**	**100**	**75**	**45**	**125**
Selling Price	**30**	**32**	**31**	**34**	**29**

Find the production and distribution schedule most profitable to the company by way of initial basic feasible solution.

Solution:

In order to solve this transportation problem, the profit matrix needs to be prepared, where: Profit = Sale Price – Production cost – shipping Cost

Thus, to transport one unit of dry cell from each of the three plants to each of the five sales offices, the following matrix is obtained:

Profit Matrix

From Plants	To Sales Officers at					Production Capacity (in units)
	Northern Region	Southern Region	Western Region	Eastern Region	Capital City	
1	9	11	6	5	5	150
2	-1	3	1	9	1	200
3	8	9	10	14	4	125
Demand (in units)	80	100	75	45	125	475 / 425

The objective of the company is to maximize the profit. To solve this maximization problem, let us convert it into minimization problem, by subtracting all the elements of the above pay – off matrix from the highest pay – off, i.e. 14. Thus, we have:

Relative Loss Matrix

From Plants	To Sales Officers at					Production Capacity (in units)
	Northern Region	Southern Region	Western Region	Eastern Region	Capital City	
1	5	3	8	9	9	**150**
2	15	11	13	5	13	**200**
3	6	5	4	0	10	**125**
Demand (in units)	**80**	**100**	**75**	**45**	**125**	**475** / **425**

From Plants	To Sales Officers at						Production Capacity (in units)	Row Penalty					
	Northern Region	Southern Region	Western Region	Eastern Region	Capital City	Dummy Column							
1	50 ~~5~~	100 ~~3~~	8	9	9	0	~~150~~ ~~50~~ 0	3	2	2	2	4	
2	25 ~~15~~	11	13	5	125 ~~13~~	50 ~~0~~	~~200~~ ~~150~~ ~~125~~ 0	5	6	2	2	2	2
3	5 ~~6~~	5	75 ~~4~~	45 ~~0~~	10	0	~~125~~ ~~80~~ ~~5~~ 0	0	4	1	1	4	4
Demand (in units)	~~80~~ ~~30~~ ~~25~~ 0	~~100~~ 0	~~75~~ 0	~~45~~ 0	~~125~~ 0	~~50~~ 0	475 / 475						

From Plants	To Sales Officers at						Production Capacity (in units)	Row Penalty					
	Northern Region	Southern Region	Western Region	Eastern Region	Capital City	Dummy Column							
	1	2	4	5	1	0							
	1	2	4	5	1								
	1	2	4		1								
Column Penalty	1	2			1								
	1				1								
	9				3								

Since the demand ≠ production capacity, this problem is an unbalanced problem. Hence a dummy sales office is added with cost equal to zero for all plants and demand equal to 50 units. Now, apply Vogel's Approximation Method to the resultant balanced matrix for finding the basic feasible solution.

Total Maximum profit in the production and the distribution schedule= 50×9 +100×11+ 25×(1) +125×1 +5×8 +75×10 +45×14+ 50×0 =₹ 3070/-

ILLUSTRATION 9: (MINIMIZATION AND UNBALANCED PROBLEM):

Consider the following data for the transportation problem:

Factory	Distribution			Supply to be exhausted
	(1)	(2)	(3)	
A	5	1	7	**10**
B	6	4	6	**80**
C	3	2	5	**15**
Demand	**75**	**20**	**50**	**145** / **105**

Since there is not enough supply, some of the demands at the three destinations may not be satisfied. For the unsatisfied demands, let the penalty costs be Rupees 1, 2 and 3 for destinations (1), (2) and (3) respectively. Find the optimum allocation that minimizes the transportation and penalty costs.

Solution:

Since the Demand ≠ Supplies, this problem is an unbalanced problem. Hence a dummy factory is added with cost equal to penalty for all distribution destinations and supplies equal to 40 units. Now, apply Vogel's Approximation Method to the resultant balanced matrix for finding the basic feasible solution with matrix given below:

Factory	Distribution			Supply to be exhausted
	(1)	(2)	(3)	
A	5	1	7	**10**
B	6	4	6	**80**
C	3	2	5	**15**
Dummy Row	1	2	3	**40**
Demand	**75**	**20**	**50**	**145**

The initial solution is obtained below by Vogel's Approximation method:

Factory	Distribution			Supply to be exhausted	Row Penalty			
	(1)	(2)	(3)					
A	5	10 1	7	10 0	4			
B	20 6	10 4	50 6	80 30 20 0	2	2	2	6
C	15 3	2	5	15 0	1	1	1	3
Dummy Row	40 1	2	3	40 0	1	1	1	1
Demand	~~75~~ ~~55~~ ~~40~~ 0	~~20~~ ~~10~~ ~~0~~	~~50~~ 0	145				
Column Penalty	2	1	2					
	2	0	2					
	2	0						
	2							

Total Minimum Cost of transportation and the penalty costs = 10×1 +20×6 +10×4 +50×6 +15×3 + 40×1 =₹ 555/-

ILLUSTRATION 10: (MINIMIZATION PROBLEM AND VAM)

A company has three warehouses W1, W2, and W3. It is required to deliver a product from these warehouses to three customers A, B and C. The warehouses have the following units in stock:

Warehouse	W1	W2	W3
No. of units	65	42	43

Customer requirements are:

Customer	A	B	C
No. of units	70	30	50

The table below shows the costs of transporting one unit from warehouse to the customer

		Warehouse		
		W1	**W2**	**W3**
Customer	A	5	7	8
	B	4	4	6
	C	6	7	7

Find the optimum transportation route by way of initial basic feasible solution.

Solution:

Let us formulate the given problem into a transportation problem as given below:

		Warehouse			**Requirement**
		W1	**W2**	**W3**	
Customer	A	5	7	8	**70**
	B	4	4	6	**30**
	C	6	7	7	**50**
Stock		65	42	43	**150**

Since stock is equal to customer requirements (i.e. 150 units), we note that the above transportation problem is a balanced one and it is a minimization problem. We shall now apply Vogel's Approximation Method to find an initial basic feasible solution

		Warehouse			**Requirement**	**Column Penalty**		
		W1	**W2**	**W3**				
Customer	A	65 5	5 7	8	~~70 5~~ 0	2	2	1
	B	4	30 4	6	~~30~~ 0	0		
	C	6	7 7	43 7	~~50 7~~ 0	1	1	0
Stock		~~65~~ 0	~~42 37 7~~ 0	~~43~~ 0	150			
Row Penalty		1	3	1				
		1	0	1				
			0	1				

Total Minimum Cost of the transporting one unit from warehouse to the customer = 65×5 + 5×7+ 30×4 + 7×7 + 43×7 = ₹ 830/-

6

Assignment
[Study Material - Module 8]

ILLUSTRATION 1 (MINIMIZATION PROBLEM) :

Consider the problem of assigning five operations to five machines. The assignment cost is given below:

		Operators				
		I	**II**	**III**	**IV**	**V**
Machines	**A**	10	5	13	15	16
	B	3	9	18	3	6
	C	10	7	2	2	2
	D	5	11	9	7	12
	E	7	9	10	4	12

Solution:

Step 1: Row-reduction

		Operators				
		I	**II**	**III**	**IV**	**V**
Machines	**A**	5	0	8	10	11
	B	0	6	15	0	3
	C	8	5	0	0	0
	D	0	6	4	2	7
	E	3	5	6	0	8

Step 2: Column-reduction

		Operators				
		I	**II**	**III**	**IV**	**V**
Machines	**A**	5	0	8	10	11
	B	0	6	15	0	3
	C	8	5	0	0	0
	D	0	6	4	2	7
	E	3	5	6	0	8

Step 3: Assignment

		Operators				
		I	II	III	IV	V
Machines	A	5	0	8	10	11
	B	0	6	15	0	3
	C	8	5	0	0	0
	D	0	6	4	2	7
	E	3	5	6	0	8

Here the number of rows as well as columns is 5, and number of assignments is 4, hence the solution is not optimal. Let's go to next step:

Step 4: Improved Matrix

The smallest uncovered element is 3. Subtract 3 from all uncovered elements. Add 3 to all elements lying at the intersection of two lines. Then proceed with the new matrix.

		Operators				
		I	II	III	IV	V
Machines	A	5	0	5	10	8
	B	0	6	12	0	0
	C	8	5	0	0	0
	D	0	6	1	2	4
	E	3	5	3	0	5

Here the number of rows or column is 5, and number of assignments is also 5, hence the solution is optimal

Step 5: Final assignment

Machines	Operators	Cost (₹)
A	II	₹ 5
B	V	₹ 6
C	III	₹ 2
D	I	₹ 5
E	IV	₹ 4
Total		**₹ 22 (being Minimum Cost)**

ILLUSTRATION 2 (MINIMIZATION PROBLEM):

A Computer Centre has got four expert programmers. The centre needs four application programs to be developed. The head of the Computer Centre, after studying carefully the programs to be developed, estimates the computer time in minutes required by the respective experts to develop the application programs as follows:

		Programmers			
		A	**B**	**C**	**D**
Application Programs	1	120	100	80	90
	2	80	90	110	70
	3	110	140	120	100
	4	90	90	80	90

Solution:

Step 1: Row-reduction

		Programmers			
		A	**B**	**C**	**D**
Application Programs	1	40	20	0	10
	2	10	20	40	0
	3	10	40	20	0
	4	10	10	0	10

Step 2: Column-reduction

		Programmers			
		A	**B**	**C**	**D**
Application Programs	1	30	10	0	10
	2	0	10	40	0
	3	0	30	20	0
	4	0	0	0	10

Step 3: Assignment

		Programmers			
		A	**B**	**C**	**D**
Application Programs	~~1~~	~~30~~	~~10~~	~~0~~	~~10~~
	2	0	10	40	0
	3	0	30	20	0
	4	0	0	0	10

Here the number of rows or columns is 4, and number of assignments is also 4, hence the solution is optimal

Step 4: Final optimal assignment (in minimum time)

Application Programs	Programmers	Computer time (in minutes)
1	C	80
2	A	80
3	D	100
4	B	90
Total		350 Minutes

ILLUSTRATION 3 (MINIMIZATION PROBLEM):

A company is producing a single product and is selling through five agencies situated in different cities. Suddenly, there is demand of products in another five cities not having any agency of the company. The company is faced with the problem of deciding on how to assign the existing agencies to dispatch the product to needy cities in such a way that the travelling distance is minimized. The distance (in kms) between the surplus and deficit cities are given in the following distance matrix. Determine the optimal assignment schedule.

		Deficit Cities				
		I	II	III	IV	V
Surplus Cities	A	160	130	175	190	200
	B	135	120	130	160	175
	C	140	110	155	170	185
	D	50	50	80	80	110
	E	55	35	70	80	105

Solution:

Step 1: Row-reduction

		Deficit Cities				
		I	II	III	IV	V
Surplus Cities	A	30	0	45	60	70
	B	15	0	10	40	55
	C	30	0	45	60	75
	D	0	0	30	30	60
	E	20	0	35	45	70

Step 2: Column-reduction

		Deficit Cities				
		I	**II**	**III**	**IV**	**V**
Surplus Cities	**A**	30	0	35	30	15
	B	15	0	0	10	0
	C	30	0	35	30	20
	D	0	0	20	0	5
	E	20	0	25	15	15

Step 3: Assignment

		Deficit Cities				
		I	**II**	**III**	**IV**	**V**
Surplus Cities	**A**	30	0	35	30	15
	~~**B**~~	~~15~~	~~0~~	~~0~~	~~10~~	~~0~~
	C	30	0	35	30	20
	~~**D**~~	~~0~~	~~0~~	~~20~~	~~0~~	~~5~~
	E	20	0	25	15	15

Here the number of rows or columns is 5, and number of lines are 3, hence the solution is not optimal

Step 4: Improved Matrix

.The smallest uncovered element is 15. Subtract 15 from all uncovered elements. Add 15 to all elements lying at the intersection of two lines. Then proceed with the new matrix.

		Deficit Cities				
		I	**II**	**III**	**IV**	**V**
Surplus Cities	~~**A**~~	~~15~~	~~0~~	~~20~~	~~15~~	~~0~~
	~~**B**~~	~~15~~	~~15~~	~~0~~	~~10~~	~~0~~
	C	15	0	20	15	5
	~~**D**~~	~~0~~	~~15~~	~~20~~	~~0~~	~~5~~
	~~**E**~~	~~5~~	~~0~~	~~10~~	~~0~~	~~0~~

Here the number of rows or column is 5, and no of the lines are also 5, hence the solution is optimal

Step 5: Final assignment

Surplus City (Route)	Deficit City (Route)	Distance in (Kms)
A	V	200
B	III	130
C	II	110
D	I	50
E	IV	80
Total		**570 km**

ILLUSTRATION 4 (MINIMIZATION PROBLEM):

An automobile dealer wishes to put four repairmen to four different jobs. The repairmen have somewhat different kinds of skills, and they exhibit different levels of efficiency from one job to another. The dealer has estimated the number of man hours that would be required for each job man combination. This is given in the matrix from in the following table.

		Job			
Man		**A**	**B**	**C**	**D**
	1	5	3	2	8
	2	7	9	2	6
	3	6	4	5	7
	4	5	7	7	8

Find the optimal assignment that will result in minimum man hours needed.

Solution:

Step 1: Row-reduction

		Job			
Man		**A**	**B**	**C**	**D**
	1	3	1	0	6
	2	5	7	0	4
	3	2	0	1	3
	4	0	2	2	3

Step 2: Column-reduction

		Job			
		A	**B**	**C**	**D**
Man	1	3	1	0	3
	2	5	7	0	1
	3	2	0	1	0
	4	0	2	2	0

Step 3: Assignment

		Job			
		A	**B**	**C**	**D**
Man	1	3	1	0	3
	2	5	7	0	1
	~~3~~	~~2~~	~~0~~	~~1~~	~~0~~
	~~4~~	~~0~~	~~2~~	~~2~~	~~0~~

Here the number of rows or columns is 4

And no of lines are 3

Here the solution is not optimal

Step 4: Improved Matrix

.The smallest uncovered element is 1. Subtract 1 from all uncovered elements. Add 1 to all elements at the intersection of two lines. Then proceed with the new matrix.

		Job			
		A	**B**	**C**	**D**
Man	1	2	0	0	2
	2	4	6	0	0
	3	2	0	2	0
	~~4~~	~~0~~	~~2~~	~~3~~	~~0~~

OR

		Job			
		A	**B**	**C**	**D**
Man	1	2	0	0	2
	2	4	6	0	0
	3	2	0	2	0
	4	0	2	3	0

Here the number of rows or columns is 4

And the number of the lines are 4

Here the solution is optimal

Step 5: Final assignment

Man	**Job**	**Man Hrs. needed**
1	B	3
2	C	2
3	D	7
4	A	5
Total		**17 Hrs.**

OR

Man	**Job**	**Man Hrs. needed**
1	C	2
2	D	6
3	B	4
4	A	5
Total		**17 Hrs.**

Learning Tip: Some problems may have 2 or more set of solutions.

ILLUSTRATION 5 (MINIMIZATION PROBLEM):

A trip from Madras to Bangalore takes six hours by bus. A typical timetable of the bus service in both directions is given below:

Departure from Madras	Route Number	Arrival at Bangalore	Arrival at Madras	Route Number	Departure from Bangalore
06.00	a	12:00	11:30	1	05:30
07:30	b	13:30	15:00	2	09:00
11:30	c	17:30	21:00	3	15:00
19:00	d	01:00	00:30	4	18:30
00:30	e	06:30	06:00	5	00:00

The cost of providing this service by the transport company depends upon the time spent by the bus crew (driver and conductor) away from their places in addition to service times. There are five crew members. There is a constraint that every crew should be provided with more than 4 hours of rest before the return trip again and should not wait for more than 24 hours for the return trip. The company has residential facilities for the crew of Madras as well as at Bangalore. Find Which line of service relate to which other line so as to reduce the waiting time to the minimum.

Solution:

As the service time is constant for each line it does not appear directly in the computation. If the entire crew resides at Madras then the waiting times in hours at Bangalore for different route connections are given in the following Table:

Route	1	2	3	4	5
a	17.5	21	∞	6.5	12
b	16	19.5	∞	5	10.5
c	12	15.5	21.5	∞	6.5
d	4.5	8	14	17.5	23
e	23	∞	8.5	12	17.5

∞ is an infeasible assignment

If the crew are assumed to reside at Bangalore, then the waiting times of the crew in hours at Madras for different route combinations are given in the following table:

Route	1	2	3	4	5
a	18.5	15	9	5.5	∞
b	20	16.5	10.5	7	∞

Route	1	2	3	4	5
c	∞	20.5	14.5	11	5.5
d	7.5	∞	22	18.5	13
e	13	9.5	∞	∞	18.5

As the crew can be asked to reside either at Madras or at Bangalore, the minimum waiting time from the above operation can be computed for different route combinations by choosing the minimum of the two waiting times. This is presented on the following table. The asterisk marked waiting times indicates that the crew are based at Madras, otherwise they are based at Bangalore.

Route	1	2	3	4	5
a	17.50*	15	9	5.50	12*
b	16*	16.5	10.50	5*	10.50*
c	12*	15.50*	14.50	11	5.50
d	4.50*	8*	14*	17.50*	13
e	13	9.50	8.50*	12*	17.50*

*The layover time marked with star (*) denote that the crew is based in Madras, otherwise based in Bangalore.

Step 1: Row-reduction:

Route	1	2	3	4	5
a	12*	9.50	3.50	0	6.50*
b	11*	11.50	5.50	0*	5.50*
c	6.50*	10*	9.00	5.50	0
d	0*	3.50*	9.50*	13*	8.50
e	4.50	1	0*	3.50*	9*

Step 2: Column-reduction:

Route	1	2	3	4	5
a	12*	8.50	3.50	0	6.50*
b	11*	10.50	5.50	0*	5.50*
c	6.50*	9*	9.00	5.50	0
d	0*	2.50*	9.50*	13*	8.50
e	4.50	0	0*	3.50*	9*

Step 3: Assignment

Route	1	2	3	4	5
a	12*	8.50	3.50	0	6.50*
b	11*	10.50	5.50	0*	5.50*
c	6.50*	9*	9.00	5.50	0
d	0*	2.50*	9.50*	13*	8.50
e	4.50	0	0*	3.50*	9*

Here the number of rows or columns is 5

And no of lines are 4

Here the solution is not optimal

Step 4: Improved Matrix

.The smallest uncovered element is 2.50. Subtract 2.50 from all uncovered elements. Add 2.5 to all elements lying at the intersection of two lines. Then proceed with the new matrix.

Route	1	2	3	4	5
a	12*	6	1	0	6.50*
b	11*	8	3	0*	5.50*
c	6.50*	6.50*	6.50	5.50	0
d	0*	0*	7*	13*	8.50
e	7	0	0*	6*	11.50*

Here the number of rows or columns is 5

And no of lines are 4

Here the solution is not optimal

Step 5: Improved Matrix 2

The smallest uncovered element is 1. Subtract 1 from all those elements which are not covered. Add 1 all elements which are at the intersection of two lines. Then proceed with the new matrix.

Route	1	2	3	4	5
a	11*	5	0	0	6.50*
b	10*	7	2	0*	5.50*
c	5.50*	5.50*	5.50	5.50	0
d	0*	0*	7*	14*	9.50
e	7	0	0*	7*	12.50*

Here the number of rows or columns is 5

And no of lines are 5

Here the solution is now optimal

Step 6: Final assignment

Routes to be paired	Residence of the Crew	Waiting Time
a-3	Bangalore	9.00 Hours
b-4	Madras	5.00 Hours
c-5	Bangalore	5.50 Hours
d-1	Madras	4.50 Hours
e-2	Bangalore	9.50 Hours
Total Waiting Time		**33.50 Hours**

The minimum total waiting time is thus 33.5 hours

ILLUSTRATION 6 (MAXIMIZATION PROBLEM):

XYZ Corporation has four plants, each of which can manufacture any one of the four products. The production costs differ from one plant to another as do sales revenue. Given the revenue and the cost data below, obtain which product each plant should produce to maximize the profit.

Plant	Sales Revenue (₹.'000) Product			
	1	2	3	4
A	50	68	49	62
B	60	70	51	74
C	55	67	53	70
D	58	65	54	69

Plant	Production Cost (₹.'000) Product			
	1	2	3	4
A	49	60	45	61
B	55	63	45	69
C	52	62	49	68
D	55	64	48	66

Solution:

Since we are given with the sale revenue and the production cost, lets work out the profit.

Profit Matrix

Plant	Profit Matrix (Sales Revenue-Production Cost) (₹.'000) Product			
	1	2	3	4
A	1	8	4	1
B	5	7	6	5
C	3	5	4	2
D	3	1	6	3

Step 2: Relative Loss Matrix (Conversion of Maximization into the Minimization Problem): To solve Maximisation problem, we need to convert the problem into minimisation problem and then work out the optimum solution. A relative loss matrix is obtained by deducting each cell from the highest number of the matrix. 8 is the highest number of the matrix, by reducing all elements from 8 we would get:

Plant	Relative Loss Matrix (₹.'000) Product			
	1	2	3	4
A	7	0	4	7
B	3	1	2	3
C	5	3	4	6
D	5	7	2	5

Step 3: Row Reduction:

Plant	(₹.'000) Product			
	1	2	3	4
A	7	0	4	7
B	2	0	1	2
C	2	0	1	3
D	3	5	0	3

Step 4: Column Reduction:

Plant	(₹.'000) Product			
	1	2	3	4
A	5	0	4	5
B	0	0	1	0
C	0	0	1	1
D	1	5	0	1

Step 5: Assignment

Plant	(₹.'000) Product			
	1	2	3	4
A	5	0	4	5
B	0	0	1	0
C	0	0	1	1
D	1	5	0	1

Here the number of rows or columns is 4

And no of lines are 4

Here the solution is being optimal

Step 6: Final assignment

Plant	Product	Maximum Profit (in ₹ '000)
A	2	8,000
B	4	5,000
C	1	3,000
D	3	6,000
Total Maximum Profit of ₹		**22,000**

ILLUSTRATION 7 (UNBALANCED PROBLEM):

Solve the following unbalanced assignment problem of minimizing total time for doing all the jobs:

Operator		Jobs				
		1	**2**	**3**	**4**	**5**
	A	6	2	5	2	6
	B	2	5	8	7	7
	C	7	8	6	9	8
	D	6	2	3	4	5
	E	9	3	8	9	7
	F	4	7	4	6	8

Solution: Step 1: Conversion of Unbalanced into Balanced Problem

By adding a dummy column or row, we can balance the matrix and obtain the solution. In the given

problem number of operators are more than the number of jobs available. So, a dummy job is created to make the matrix balanced.

		Jobs					
Operator		1	2	3	4	5	6 (Dummy Column)
	A	6	2	5	2	6	0
	B	2	5	8	7	7	0
	C	7	8	6	9	8	0
	D	6	2	3	4	5	0
	E	9	3	8	9	7	0
	F	4	7	4	6	8	0

Step 2: Row Reduction

		Jobs					
Operator		1	2	3	4	5	6 (Dummy Column)
	A	6	2	5	2	6	0
	B	2	5	8	7	7	0
	C	7	8	6	9	8	0
	D	6	2	3	4	5	0
	E	9	3	8	9	7	0
	F	4	7	4	6	8	0

Step 3: Column Reduction

		Jobs					
Operator		1	2	3	4	5	6 (Dummy Column)
	A	4	0	2	0	1	0
	B	0	3	5	5	2	0
	C	5	6	3	7	3	0
	D	4	0	0	2	0	0
	E	7	1	5	7	2	0
	F	2	5	1	4	3	0

Step 4: Assignment

		Jobs					
Operator		1	2	3	4	5	6 (Dummy Column)
	~~A~~	~~4~~	~~0~~	~~2~~	~~0~~	~~1~~	~~0~~
	~~B~~	~~0~~	~~3~~	~~5~~	~~5~~	~~2~~	~~0~~
	C	5	6	3	7	3	0
	~~D~~	~~4~~	~~0~~	~~0~~	~~2~~	~~0~~	~~0~~
	E	7	1	5	7	2	0
	F	2	5	1	4	3	0

Here the number of rows or columns is 6

And no of lines are 4

Here the solution is not optimal

Step 5: Improved Matrix

.The smallest uncovered element is 1. Subtract 1 from all uncovered elements. Add 1 to all elements lying at the intersection of two lines. Then proceed with the new matrix.

		Jobs					
Operator		1	2	3	4	5	6 (Dummy Column)
	~~A~~	~~4~~	~~0~~	~~2~~	~~0~~	~~1~~	~~1~~
	B	0	3	5	5	2	1
	C	4	5	2	6	2	0
	~~D~~	~~4~~	~~0~~	~~0~~	~~2~~	~~0~~	~~1~~
	E	6	0	4	6	1	0
	F	1	4	0	3	2	0

Here the number of rows or columns is 6

And no of lines are 6

Here the solution is optimal

Step 6: Final assignment

Operator	Jobs	Minimum Time (in Hrs)
A	4	2 Hrs

Operator	Jobs	Minimum Time (in Hrs)
B	1	2 Hrs
C	6	0 Hrs
D	5	5 Hrs
E	2	3 Hrs
F	3	4 Hrs
Total Minimum Time		**16 Hrs**

ILLUSTRATION 8 (UNBALANCED PROBLEM):.

In the modification of a plant layout of a factory four new machines M1, M2, M3 and M4 are to be installed in a machine shop. There are five vacant places A, B, C, D and E available. Because of limited space, machine M2 cannot be placed at C and M3 cannot be placed at A. The Cost of locating of 4 machines to places A to E in rupees lakhs is shown below. Find optimal assignment schedule.

	A	B	C	D	E
M1	9	11	15	10	11
M2	12	9	-	10	9
M3	-	11	14	11	7
M4	14	8	12	7	8

Solution:

Step 1: Conversion of Unbalanced into Balanced Problem

	A	B	C	D	E
M1	9	11	15	10	11
M2	12	9	∞	10	9
M3	∞	11	14	11	7
M4	14	8	12	7	8
MD (Dummy Row)	0	0	0	0	0

Step 2: Row Reduction

	A	B	C	D	E
M1	0	2	6	1	2
M2	3	0	∞	1	0
M3	∞	4	7	4	0
M4	7	1	5	0	1
MD (Dummy Row)	0	0	0	0	0

Step 3: Column Reduction

	A	B	C	D	E
M1	0	2	6	1	2
M2	3	0	∞	1	0
M3	∞	4	7	4	0
M4	7	1	5	0	1
MD (Dummy Row)	0	0	0	0	0

Step 4: Assignment

	A	B	C	D	E
M1	0	2	6	1	2
M2	3	0	∞	1	0
M3	∞	4	7	4	0
M4	7	1	5	0	1
MD (Dummy Row)	0	0	0	0	0

Here the number of rows or columns is 5

And no of lines are 5

Here the solution is optimal.

Step 5: Final assignment

Machines	Vacant Places	Minimum Cost in ₹ Lakhs
M1	A	9
M2	B	9
M3	E	7
M4	D	7
MD	C	-
Total Minimum Cost		**32**

ILLUSTRATION 9 (MINIMIZATION PROBLEM):

Find the optimal solution for the assignment problem with the following cost matrix:

		Area			
		I	II	III	IV
Salesmen	A	11	17	8	16
	B	9	7	12	6
	C	13	16	15	12
	D	14	10	12	11

Solution:

Step 1: Row Reduction:

		Area			
		I	II	III	IV
Salesmen	A	3	9	0	8
	B	3	1	6	0
	C	1	4	3	0
	D	4	0	2	1

Step 2: Column Reduction:

		Area			
		I	II	III	IV
Salesmen	A	2	9	0	8
	B	2	1	6	0
	C	0	4	3	0
	D	3	0	2	1

Step 3: Assignment:

		Area			
		I	II	III	IV
Salesmen	A	2	9	0	8
	B	2	5	6	0
	C	0	4	3	0
	D	3	0	2	1

Here the number of rows or columns is 4

And no of lines are 4

Here the solution is being optimal.

Step 4: Final assignment

Salesmen	Area	Minimum Cost
A	III	8
B	IV	6
C	I	13
D	II	10
Total Minimum Cost		**37**

ILLUSTRATION 10 (MINIMIZATION PROBLEM):

Find the optimal solution for the assignment problem with the following cost matrix:

Salesmen	Sales Territories (Cost in ₹ Lakhs)				
		I	II	III	IV
	A	30	25	26	28
	B	26	32	24	20
	C	20	22	18	27
	D	23	20	21	19

Solution:

Step 1: Row Reduction:

	Sales Territories (Cost in ₹ Lakhs)				
Salesmen		I	II	III	IV
	A	5	0	1	3
	B	6	12	4	0
	C	2	4	0	9
	D	4	1	2	0

Step 2: Column Reduction:

	Sales Territories (Cost in ₹ Lakhs)				
Salesmen		I	II	III	IV
	A	3	0	1	3
	B	4	12	4	0
	C	0	4	0	9
	D	2	1	2	0

Step 3: Assignment:

	Sales Territories (Cost in ₹ Lakhs)				
		I	II	III	IV
Salesmen	A	3	0	1	3
	B	4	12	4	0
	C	0	4	0	9
	D	2	1	2	0

Here the number of rows or columns is 4

And no of lines are 3

Here the solution is not optimal.

Step 4: Improved Matrix

The smallest uncovered element is 1. Subtract 1 from all uncovered elements. Add 1 to all elements lying at the intersection of two lines. Then proceed with the new matrix.

	Sales Territories (Cost in ₹ Lakhs)				
		I	II	III	IV
Salesmen	A	2	0	0	3
	B	3	12	3	0
	C	0	5	0	10
	D	1	1	1	0

Here the number of rows or columns is 4

And no of lines are 3

Here the solution is not optimal.

Step 5: Improved Matrix 2

The smallest uncovered element is 1. Subtract 1 from all uncovered elements. Add 1 to all elements lying at the intersection of two lines. Then proceed with the new matrix.

	Sales Territories (Cost in ₹ Lakhs)				
		I	II	III	IV
Salesmen	A	2	0	0	4
	B	2	11	2	0
	C	0	5	0	11
	D	0	0	0	0

Here the number of rows or columns is 4

And no of lines are 4

Here the solution is optimal.

Step 6: Final assignment

Salesmen	Sales Territories	Minimum Cost in ₹ Lakhs
A	II	25
B	IV	20
C	III	18
D	I	23
Total Minimum Cost in ₹ Lakhs		**86**

Learning Tip: There are two more set of assignments possible for this set of problem, Students can try finding them.

ILLUSTRATION 11 (MAXIMIZATION PROBLEM):

A captain of a cricket team has to allot five middle batting positions to five batsman. The average runs scored by each batsmen at these positions are as follows:

	Playing Positions					
Batsmen		**III**	**IV**	**V**	**VI**	**VII**
	A	40	40	35	25	50
	B	42	30	16	25	27
	C	50	48	40	60	50
	D	20	19	20	18	25
	E	58	60	59	55	53

Solution:

Step 1: Relative Loss Matrix (Conversion of Maximization into the Minimization Problem)
Since 60 is the Maximum number of the matrix, to get the relative loss matrix, each cell of the matrix is to be reduced from 60 and we would get:

	Playing Positions					
Batsmen		**III**	**IV**	**V**	**VI**	**VII**
	A	20	20	25	35	10
	B	18	30	44	35	33
	C	10	12	20	0	10
	D	40	41	40	42	35
	E	2	0	1	5	7

Step 2: Row Reduction:

		Playing Positions				
		III	**IV**	**V**	**VI**	**VII**
Batsmen	A	10	10	15	25	0
	B	0	12	26	17	15
	C	10	12	20	0	10
	D	5	6	5	7	0
	E	2	0	1	5	7

Step 3: Column Reduction:

		Playing Positions				
		III	**IV**	**V**	**VI**	**VII**
Batsmen	A	10	10	14	25	0
	B	0	12	25	17	15
	C	10	12	19	0	10
	D	5	6	4	7	0
	E	2	0	0	5	7

Step 4: Assignment

		Playing Positions				
		III	**IV**	**V**	**VI**	**VII**
Batsmen	A	10	10	14	25	0
	B	0	12	25	17	15
	C	10	12	19	0	10
	D	5	6	4	7	0
	E	2	0	0	5	7

Here the number of rows or columns is 5

And no of lines are 4

Here the solution is not optimal.

Step 5: Improved Matrix

The smallest uncovered element is 4. Subtract 4 from all uncovered elements. Add 4 to all elements lying at the intersection of two lines. Then proceed with the new matrix.

	Playing Positions					
Batsmen		III	IV	V	VI	VII
	A	10	6	10	25	0
	B	0	8	21	17	15
	C	10	8	15	0	10
	D	5	2	0	7	0
	E	6	0	0	9	11

Here the number of rows or columns is 5

And no of lines are 5

Here the solution is being optimal.

Step 6: Final assignment

Player (Batsman)	Playing Positions	Maximum Total Average Expected Runs
A	VII	50
B	III	42
C	VI	60
D	V	20
E	IV	60
Maximum Total Average Expected Runs		**232**

ILLUSTRATION 12 (MAXIMIZATION PROBLEM):

Four jobs can be processed on four different machines, one job on one machine. Resulting profits vary with assignments which are given below in ₹. Lakhs. Find the optimum assignment of jobs to machines and the corresponding profit.

	Machines				
Jobs		A	B	C	D
	I	42	35	28	21
	II	30	25	20	15
	III	30	25	20	15
	IV	24	20	16	12

Solution:

Step 1: Relative Loss Matrix (Conversion of Maximization into the Minimization Problem)

Deducting all cells by the largest number of the matrix 42, we would get:

		Machines			
		A	**B**	**C**	**D**
Jobs	I	0	7	14	21
	II	12	17	22	27
	III	12	17	22	27
	IV	18	22	26	30

Step 2: Row Reduction:

		Machines			
		A	**B**	**C**	**D**
Jobs	I	0	7	14	21
	II	0	5	10	15
	III	0	5	10	15
	IV	0	4	8	12

Step 3: Column Reduction:

		Machines			
		A	**B**	**C**	**D**
Jobs	I	0	3	6	9
	II	0	1	2	3
	III	0	1	2	3
	IV	0	0	0	0

Step 4: Assignment

		Machines			
		A	**B**	**C**	**D**
Jobs	I	0	3	6	9
	II	0	1	2	3
	III	0	1	2	3
	IV	0	0	0	0

Here the number of rows or columns is 4

And no of lines are 2

Here the solution is not optimal.

Step 5: Improved Matrix

The smallest uncovered element is 1. Subtract 1 from all uncovered elements. Add 1 to all elements lying at the intersection of two lines. Then proceed with the new matrix.

		Machines			
		A	**B**	**C**	**D**
Jobs	I	0	2	5	8
	II	0	0	1	2
	III	0	0	1	2
	IV	1	0	0	0

Here the number of rows or columns is 4

And no of lines are 3

Here the solution is not optimal.

Step 6: Improved Matrix 2

The smallest uncovered element is 1. Subtract 1 from all uncovered elements. Add 1 in all elements lying at the intersection of two lines. Then proceed with the new matrix.

		Machines			
		A	**B**	**C**	**D**
Jobs	I	0	2	4	7
	II	0	0	0	1
	III	0	0	0	1
	IV	2	1	0	0

Here the number of rows or columns is 4

And no of lines are 4

Here the solution is being optimal.

Step 7: Final assignment

Jobs	Machines	Total Maximum Profit in ₹. Lakhs
I	A	42
II	B	25
III	C	20
IV	D	12
Maximum Total Profit		**99**

7 Simulation [Study Material - Module 10]

ILLUSTRATION 1:

The following demand pattern was observed for 100 days for a tour operator engaged in the business of renting cars:

No. of Cars	5	7	10	15
No. of Days	20	30	40	10

The random numbers are 88, 76, 10, 05, 23.

You are required to simulate demand of cars for 5 days.

Solution:

Statement of the Random Numbers

No. of Cars	No. of Days	Probability	Cumulative Probability	Random No. Range
5	20	0.20	0.20	0 – 19
7	30	0.30	0.50	20 – 49
10	40	0.40	0.90	50 – 89
15	10	0.10	1.00	90 – 99
Sub Total	**100**	**1.00**		

Statement of Simulation

Day	Random No.	Demand
1	88	10
2	76	10
3	10	5
4	05	5
5	23	7

ILLUSTRATION 2:

A bakery keeps stock of a popular brand of cookies. Previous experience shows the daily demand pattern for the item with associated probabilities as given:

Demand (daily) (Nos)	0	10	20	30	40	50
Probability	0.01	0.20	0.15	0.50	0.12	0.02

Use the following sequence of random numbers to simulate the demand for next 10 days. Also find out the average demand per day

Random Numbers: 25, 39, 65, 76, 12, 05, 73, 89, 19, 49

Solution:

Daily demand	**Probability**	**Cumulative Probability**	**Random No. Range**
0	0.01	0.01	-
10	0.20	0.21	1-20
20	0.15	0.36	21-35
30	0.50	0.86	35-85
40	0.12	0.98	86-97
50	0.02	1.00	97-99

Stimulated demand for next 10 days

Day	Random no.	Demand
1	25	20
2	39	30
3	65	30
4	76	30
5	12	10
6	05	10
7	73	30
8	89	40
9	19	10
10	49	30

Average Demand per day = 240 / 10 = 24 Units

ILLUSTRATION 3:

A trader deals in a perishable commodity, the daily demand and supply of which are random variables. Records of 500 trading days are shown as follows:

SUPPLY		DEMAND	
Tons Available	**Number of Days**	**Tons Demanded**	**Number of Days**
10	40	10	50
20	50	20	110
30	190	30	200
40	150	40	100
50	70	50	40

The trader buys and sells commodity @ ₹ 20 per kg and ₹ 30 per kg respectively. If it remains at the end of the day, it has no saleable value. The loss through unsatisfied demand is ₹. 7 per kg.

Given are random numbers, simulate six-day trading:

(31) (18), (63) (84), (15) (79), (07) (32), (43) (75), (81) (27)

Use the random numbers alternatively i.e. first pair (31) to simulate supply and second pair (18) to simulate demand. Also work out the total profit for 6 days.

Solution:

Statement of the Random Numbers

SUPPLY					DEMAND				
Qty (MT)	**Number of Days**	**Probability**	**Cum Prob**	**Random No**	**Qty. (MT)**	**Number of Days**	**Probability**	**Cum Prob**	**Random No**
10	40	40/500 =0.08	0.08	0-7	10	50	50/500 =0.10	0.10	0-9
20	50	50/500 =0.10	0.18	8-17	20	110	110/500 =0.22	0.32	10-31
30	190	190/500 =0.38	0.56	18-55	30	200	200/500 =0.40	0.72	32-71
40	150	150/500 =0.30	0.86	56-85	40	100	100/500 =0.20	0.92	72-91
50	70	70/500 =0.14	1.00	86-99	50	40	40/500 =0.08	1.00	92-99
Total	**500**	**1.00**			**Total**	**500**	**1.00**		

Statement of Simulation

SUPPLY			DEMAND			Cost (₹)	Revenue (₹)	Shortage Loss (₹)	Profit/ (Loss) (₹)
Day	Random Number	Tons Available	Day	Random Number	Tons Demanded				
1	31	30	1	18	20	600	600	-	-
2	63	40	2	84	40	800	1200	-	400
3	15	20	3	79	40	400	600	140	60
4	07	10	4	32	30	200	300	140	(40)
5	43	30	5	75	40	600	900	70	230
6	81	40	6	27	20	800	600	-	(200)
Total						**3400**	**4200**	**350**	**450**

Total Profit for 6 days will be ₹. 450/-

ILLUSTRATION 4:

A production line turns out about 50 trucks per day, fluctuations occur for many reasons. The production can be described by a probability distribution as follows:

Production / Day	Probability	Production per day	Probability
45	0.03	51	0.15
46	0.05	52	0.10
47	0.07	53	0.07
48	0.10	54	0.05
49	0.15	55	0.03
50	0.20		

Finished Trucks are transported by train at the end of the day. If the train capacity is only 51, what will be the average number of trucks waiting to be shipped and what will be the average number of empty spaces on the train?

Random numbers given are 37, 35,63, 25, 50, 71, 95 & 16

Devise simulation model to solve this problem.

Solution:

Statement of the Random Numbers

Production per Day	Probability	Cumulative Probability	Random Numbers
45	0.03	0.03	0-2

Production per Day	Probability	Cumulative Probability	Random Numbers
46	0.05	0.08	3-7
47	0.07	0.15	8-14
48	0.10	0.25	15-24
49	0.15	0.40	25-39
50	0.20	0.60	40-59
51	0.15	0.75	60-74
52	0.10	0.85	75-84
53	0.07	0.92	85-91
54	0.05	0.97	92-96
55	0.03	1	97-99

Statement of Simulation

Random No	Daily Production	Trucks left Overnight	Total Trucks Waiting	Trucks Shipped	Empty Places
37	49	0	49	49	2
35	49	0	49	49	2
63	51	0	51	51	0
25	49	0	49	49	2
50	50	0	50	50	1
71	51	0	51	51	0
95	54	0	54	51	0
16	48	3	51	51	0

ILLUSTRATION 5:

A confectioner sells confectionery items. Past data of demand per week in hundred kilograms with frequency is given below:

Demand/Week	0	5	10	15	20	25
Frequency	2	11	8	21	5	3

Using the following sequence of random numbers, generate the demand for the next 15 weeks. Also find out the average demand per week.

Random number given are 36, 54, 18, 93, 25, 80, 35, 59, 38, 81, 94, 56, 67, 66, 85.

Solution:

Table – I: Statement of the Random Numbers

Random No. Range Table for demand				
Demand per week	**Frequency (f)**	**Probability (p = f ÷ ∑f)**	**Cumulative Probability**	**Range of Random Nos.**
0	2	0.04	0.04	0-3
5	11	0.22	0.26	4-25
10	8	0.16	0.42	26-41
15	21	0.42	0.84	42-83
20	5	0.10	0.94	84-93
25	3	0.06	1	94-99
	∑f = 50	1		

Table-II: Statement of Simulation

Simulated Values for next 15 weeks		
Weeks	**Random Numbers**	**Demand**
1	36	10
2	54	15
3	18	5
4	93	20
5	25	5
6	80	15
7	35	10
8	59	15
9	38	10
10	81	15
11	94	25
12	56	15
13	67	15
14	66	15
15	85	20
Total	**-**	**210**

Average weekly demand = 210/15 = 14

ILLUSTRATION 6:

A process involves the production of a particular component that is installed into an end product. Past observation has indicated that the average time for the production of component is 4 minutes and for installation is 3 minutes. But fluctuations about the average do occur and probability distribution has been derived from past observation:

Production		Installation	
Minutes	**Probability**	**Minutes**	**Probability**
2	0.10	2	0.30
3	0.25	3	0.45
4	0.40	4	0.15
5	0.10	5	0.10
6	0.10		
7	0.05		

You are required to simulate 10 arrivals of completed production on the current system using the two-digit random numbers:

For Production: 20, 74, 94, 22, 93, 45, 44, 16, 04, 32,

For Installation: 03, 62, 61, 89, 01, 27, 49, 50, 90, 98.

Work out the idle time also.

Solution:

Statement of the Random Numbers

Production				Installations			
Minutes	**Probability**	**Cumulative Probability**	**Random Numbers**	**Minutes**	**Probability**	**Cumulative Probability**	**Random Numbers**
2	0.10		0-9	2	0.30		0-29
3	0.25		10-34	3	0.45		30-74
4	0.40		35-74	4	0.15		75-89
5	0.10		75-84	5	0.10		90-99
6	0.10		85-94	-	-		-
7	0.05		95-99	-	-		-

Statement of Simulation

Sr. no	Random Number	Production Time	Production Time Cumulative	Installation				Idle Time
		Minutes		Random No	Time	Start	End	Minutes
1	20	3	3	3	2	3	5	3
2	74	4	7	62	3	7	10	2
3	94	6	13	61	3	13	16	3
4	22	3	16	89	4	16	20	0
5	93	6	22	1	2	22	24	2
6	45	4	26	27	2	26	28	2
7	44	4	30	49	3	30	33	2
8	16	3	33	50	3	33	36	0
9	4	2	35	90	5	36	41	0
10	32	3	38	98	5	41	46	0
								14

Total idle time will be 14 minutes.

ILLUSTRATION 7:

A company trading in motor vehicle spares wishes to determine the levels of stock it should carry for the items in its range. Demand is not certain and there is a lead time for stock replenishment. The following information is obtained:

Demand (Units per Day)	Probability
3	0.10
4	0.20
5	0.30
6	0.30
7	0.10

Carrying costs (per unit per day) = ₹ 0.20

Ordering costs (per order) = ₹ 500

Lead time for replenishment (in days) = 3

Stock in the beginning is 20 units. Carry out the simulation run for ten days. Further order 15 units when present inventory plus any outstanding order falls below 15 units. The sequence of random

numbers is 0,9,1,1,5,1,8,6,3,5 using the first number for day one. Calculation should include the total cost of operating this inventory for ten days,

Solution:

Statement of the Random Numbers

Demand	Probability	Cumulative probability	Random Numbers
3	0.10	0.10	0
4	0.20	0.30	1-2
5	0.30	0.60	3-5
6	0.30	0.90	6-8
7	0.10	1	9

Statement of Simulation

Days	Random Number	Simulated Demand	Opening Stock	Closing Stock	Orders Placed	Orders Received	Average Stock
1	0	3	20	17			18.5
2	9	7	17	10	Yes		13.5
3	1	4	10	6			8
4	1	4	6	2			4
5	5	5	2	(3)	Yes		1
6	1	4	12	8		Yes	10
7	8	6	8	2			5
8	6	6	2	(4)	Yes		1
9	3	5	11	6		Yes	8.5
10	5	5	6	1			3.5
			Total		**3**	**2**	**73**

Total Cost for 10 Days period:

Ordering Cost = No of orders × Cost per order = 3 × 500 = 1500

Inventory Carrying Cost = Average inventory × Cost per unit = 73 × ₹.20 per unit = ₹ 1460

Total cost of inventory holding is ₹ 2960 (₹.1500 + ₹.1460)

ILLUSTRATION 8:

A Small retailer has studied the weekly receipts and payments over the past 200 weeks and has developed the following set of information:

Weekly Receipts ₹	Probability	Weekly Payments ₹	Probability
4000	0.20	5000	0.30
6000	0.30	7000	0.40
8000	0.40	9000	0.20
13000	0.10	11000	0.10

Using the following set of random numbers, simulate the weekly pattern of receipts and payments for the 12 weeks of the next quarter, assuming further that the beginning bank balance is ₹.10000. What is the estimated balance at the end of the 12-weekly period? What is the highest weekly balance during the quarter? What is the average weekly balance for the quarter?

Random Numbers

For Receipts	04	92	39	56	18	76	33	44	70	73	25	23
For payments	62	97	31	33	4	89	49	29	89	19	72	99

According to the given information, the random number interval is assigned to both the receipts and the payments.

Solution:

Statement of the Random Numbers

Receipts ₹	Probability	Cumulative Probability	Random Numbers
4000	0.20	0.20	0-19
6000	0.30	0.50	20-49
8000	0.40	0.90	50-89
13000	0.10	1.00	90-99

Payment ₹	Probability	Cumulative Probability	Random Numbers
5000	0.30	0.30	0-29
7000	0.40	0.70	30-69
9000	0.20	0.90	70-89
11000	0.10	1.00	90-99

Simulation of Data for a period of 12 weeks

Week	Random No. for receipt	Expected Receipt (₹)	Random No. for payment	Expected Payment (₹)	Week end Balance (₹)
Opening Balances					10000
1	04	4000	62	7000	7000 (10000 + 4000-7000)

Week	Random No. for receipt	Expected Receipt (₹)	Random No. for payment	Expected Payment (₹)	Week end Balance (₹)
2	92	13000	97	11000	9000
3	39	6000	31	7000	8000
4	56	8000	33	7000	9000
5	18	4000	04	5000	8000
6	47	6000	89	9000	5000
7	33	6000	49	7000	4000
8	44	6000	29	5000	5000
9	70	8000	89	9000	4000
10	73	8000	19	5000	7000
11	25	6000	72	9000	4000
12	23	6000	99	11000	(1000)

Estimated balance at the end of the 12th week = ₹. (1,000)

Highest balance = ₹. 9,000

Average balance during the quarter = 69,000/12 = ₹.5,750

ILLUSTRATION 9:

The following information is available:

Arrival of patients		Services	
Inter-arrival time (minutes)	Probability	Inter-Service time (minutes)	Probability
2	0.20	4	0.25
4	0.24	6	0.34
6	0.28	8	0.26
8	0.18	10	0.15
10	0.10		

The following random numbers are to be used for the simulation of arrival and service patterns:

Arrival	740	225	906	048	421
Services	402	183	706	923	638

Required:

(i) Find out the average time spent by the patient in the queue by simulation. Assume that the time

starts at 6:00 a.m. and that there is only one counter and there is no time gap between finishing with one patient and starting the next patient if the next patient is already in the queue.

(ii) A second counter is to be set up if the probability of waiting beyond 3 minutes exceeds 40% or if the average waiting time of a patient exceeds 5 minutes if there is a wait. Should the second counter be set up? Substantiate based on the simulation results

Solution:

Statement of the random numbers

Arrivals				Service			
Minutes	**Probability**	**Cumulative Probability**	**Random No.**	**Minutes**	**Probability**	**Cumulative Probability**	**Random No.**
2	0.20	0.20	000-199	4	0.25	0.25	000-249
4	0.24	0.44	200-439	6	0.34	0.59	250-589
6	0.28	0.72	440-719	8	0.26	0.85	590-849
8	0.18	0.90	720-899	10	0.15	1.00	850-999
10	0.10	1.00	900-999	-	-	-	-

Statement of Simulation

Sr. No	Random No Arrival	Time	Entry time In queue	Service Start time	Random No For Service	Service Time	Service End Time	Waiting Time of Patient	Idle Time of Centre
1	740	8	6:08	6:08	402	6	6:14	-	8
2	225	4	6:12	6:14	183	4	6:18	2	-
3	906	10	6:22	6:22	706	8	6:30	-	4
4	048	2	6:24	6:30	923	10	6:40	6	-
5	421	4	6:28	6:40	638	8	6:48	12	-

Probability of waiting time being more than 3 minutes = 2/5 = 40%

Hence is not exceeding 40%.

Average waiting time of a patient = Total of waiting time column / 3 instances of waiting = 20/3 = 6.67

Hence the second counter should be set up based on simulated results.

ILLUSTRATION 10:

The management of a company is considering a new product. Fixed cost of the project is ₹.4000/-. Three factors are uncertain viz the selling price, variable cost and sales quantity. The management has gathered the following data for these three factors:

Selling Price (₹)	Probability	Variable Cost (₹.)	Probability	Sales Volume (Nos)	Probability
3	0.2	1	0.3	2000	0.3
4	0.5	2	0.6	3000	0.3
5	0.3	3	0.1	5000	0.4

Following Sequence of random numbers is to be considered for simulating the average profit of the project on the basis of 10 trials.

81, 32, 60, 4, 46, 31, 67, 25, 24,10, 40, 02, 39, 68, 08, 59, 66, 90, 12, 64, 79, 31, 86, 68, 82, 89, 25, 11, 98, 16

Solution:

Statement of the random numbers

	Probability	Cumulative Probability	Random Numbers
Selling Price (₹)			
3	0.2	**0.2**	**00 -19**
4	0.5	**0.7**	**20 – 69**
5	0.3	**1.0**	**70 – 99**
Variable Cost (₹.)			
1	0.3	**0.3**	**00 – 29**
2	0.6	**0.9**	**30 - 89**
3	0.1	**1.0**	**90 – 99**
Sales Volume (Nos)			
2000	0.3	**0.3**	**00 – 29**
3000	0.3	**0.6**	**30 – 59**
5000	0.4	**1.0**	**60 - 99**

Statement of Simulation

Sr	Random Number	Selling Price (₹)	Random Number	Variable Cost (₹.)	Random Number	Sales Volume (Nos)	Simulated Profit (₹.) (C-E)×G-4000
A	**B**	**C**	**D**	**E**	**F**	**G**	**H**
1	81	5	32	2	60	5000	11000

Sr	Random Number	Selling Price (₹)	Random Number	Variable Cost (₹.)	Random Number	Sales Volume (Nos)	Simulated Profit (₹.) (C-E)×G-4000
A	B	C	D	E	F	G	H
2	04	3	46	2	31	3000	-1000
3	67	4	25	1	24	2000	2000
4	10	3	40	2	02	2000	-2000
5	39	4	68	2	08	2000	0
6	59	4	66	2	90	5000	6000
7	12	3	64	2	79	5000	1000
8	31	4	86	2	68	5000	6000
9	82	5	89	2	25	2000	2000
10	11	3	98	3	16	2000	-4000
							21000

Average Profit = 21000/10 = ₹. 2100

8 Business Application of Maxima and Minima [Study Material - Module 13]

ILLUSTRATION 1:

If $y = x^3 - 9x^2 + 15x + 20$, then find the points where y has maximum and minimum values.

Solution:

$dy / dx = 3x^2 - 18x + 15$

As $dy / dx = 0$ according to the first order condition,

$3x^2 - 18x + 15 = 0$

Or $3x^2 - 15x - 3x + 15 = 0$

Or $3x(x-5) - 3(x-5) = 0$

Or $3(x-3)(x-5) = 0$

Or $(x-1)(x-5) = 0$

Or $x = 1, x = 5$

$d\,2y / dx^2 = 6x - 18$

Putting $x = 1$, we get $d\,2y / dx^2 = -12 < 0$

Putting $x = 5$, we get $d\,2y / dx^2 = 12 > 0$

So, the function is maximum at $x = 1$ and minimum at $x = 5$

ILLUSTRATION 2:

The demand (rides per day) of roller coaster ride in an entertainment park in one of the metro cities is given by equation $q = -450P + 41500$, where p = Price per ride in ₹. What price should have been charged to maximize the total revenue?

Solution:

Demand $q = -450p + 41500$

Price per Ride = P

Total Revenue (R) = Demand (q) × Price per Ride (P)

$R = (-450p + 41500) \times P$

$R = -450P^2 + 41500P$

Differentiating both sides with respect to P

$dR/dP = -900P + 41500$

As per the necessary condition (First Order of derivative)

$dR/dP = -900P + 41500=0$

$-900P = -41500$

$P = 41500/900 = 46.11$

To ascertain whether the value of "P" corresponds to "Maxima", we have to check the sufficient condition (Second order of derivative):

$dR/dP = -900P + 41500$

$d^2R/dP^2 = -900$

$d^2R/dP^2 = -900 <0$; It means there exists maxima

The price to be charged to maximize the revenue ₹46.11/- per Ride

Total Revenue $= -450P^2 + 41500P$

$=-450 (46.11)^2 + 41500 \times 46.11$

$=-450 (2126.13) + 1913565$

$=-956759 + 1913565$

=₹ 9, 56, 806/-

ILLUSTRATION 3:

A firm has the Cost Function $C = x^3/3 – 7x^2 + 111x + 50$ and Demand function $x = 100-p$. Determine the Equilibrium Output, Price and Profit earned.

Solution:

Demand function is $x = 100 – p$ or, $p = 100 - x$

So, Total Revenue = TR = p.x or, $TR = (100 – x) x$ Or, $TR = 100x – x^2$

Also Profit = Total Revenue – Cost

Or, $\pi = TR – C$

Or, $\pi = (100x - x^2) - (x^3/3 - 7x^2 + 111x + 50)$

Or, $\pi = - x^3/3 + 6x^2 - 11x - 50$

Differentiating both sides with respect to x

we have $d/dx (\pi) = - x^2 + 12x - 11$..... (1)

As per the necessary condition of maximization we have $d /dx (\pi) = 0$

Or, $- x^2 + 12x - 11 = 0$ Or, $(x - 1) (x - 11) = 0$

So the critical values are x = 1 and x = 11

Now differentiating both sides of (1) we have $d^2 /dx^2 (\pi) = - 2x + 12$

When x = 1 then $d^2 /dx^2 (\pi) = - 2.1 + 12 = 10 > 0$

So by the sufficient condition of 2nd Order Derivative test there is a minima at x = 1

When x = 11 then $d^2 /dx^2 (\pi) = -2.11 + 12 = - 10 < 0$

So by the sufficient condition of 2nd Order Derivative test there is a maxima at x = 11

Thus Profit (π) is Maximum when x = 11 units.

This is the required Equilibrium Output.

Equilibrium Price = p Equilibrium = [100 – x]at x = 11 = 100 – 11 = ₹89

Equilibrium Profit = (π)Max = $[- x^3/3 + 6x^2 - 11x - 50] = - (11)3/3 + 6(11)2 - 11.11 - 50 =$ ₹111.33

[**Note** – The equilibrium output can be determined by using the relation MR = MC. Subsequently this value of output can be substituted in the Demand and Profit functions to obtain Equilibrium Price and Profit.]

ILLUSTRATION 4:

Assume the Cost in Rupee term for manufacturing x number of a product per day is $C(x) = 14400 + 550x + 0.01x^2$. Suggest the no. of units of the product that should be manufactured per day so that the Average Cost is minimum. Also find the Average Cost and the total cost at this level of production.

Solution:

Cost function is given to be $C(x) = 14400 + 550x + 0.01x^2$

So Average Cost function = C(x) / x Or,

$AC(x) = (14400 + 550x + 0.01x^2) /x$ Or,

$AC(x) = 14400/x + 550 + 0.01x$ This is the Objective function which has to be minimized.

Differentiating both sides of the above function with respect to 'x' we get $\boldsymbol{d} / \boldsymbol{dx}$ [AC(x)] = 14400/ x^2 + 0.01..... (i)

As per the necessary condition of optimization, $\boldsymbol{d}$/[AC(x)] = 0

or, -14400/x^2 + 0.01

or, 0.01x^2 = 14400

or, x^2 = 14400/ 0.01

Or, x = ± √1440000 Or, x = ± 1200

But x being the quantity cannot be negative. Hence x = 1200

To ascertain whether this value of x corresponds to minima, we have to take help of the sufficient condition mentioned above.

Again differentiating both sides of 9i) with respect to 'x' we get, d^2 / dx^2 [AC(x)] = 28800/x^3

For x = 1200, the value of 2nd order Derivative is d^2 dx2 [AC (1200)] = 28800/(1200)3 = 1.67 ×10-5 > 0

So there exist a Minima to the Objective Function at x = 1200

Hence 1200 units should be produced per day to minimize the Average Cost.

At this level of production,

Average Cost = [AC(X)] at x = 1200

= 14400/1200 + 550 + 0.01 × 1200

= ₹. 574 per unit.

Also at this level of production,

Total Cost = [C(x)] at x = 1200

= 14400 + 550 × 1200 + 0.01 × 12002

= ₹. 6, 88,800/-

ILLUSTRATION 5:

A company produces two products x and y. The total Profit (in ₹ '000) earned by the company is expressed algebraically by the function ∏ = 100x – x2 – 2xy + 200y – 3y2. Critically assess the Profit maximizing quantities of the products. Also determine the maximum profit with justification in support of your determined value.

Solution:

Profit function is given as: ∏ = 100x – x2 – 2xy + 200y – 3y2

Differentiating the function partially with respect to x we get,

∏x = 100 – 2x – 2y..... (I)

Also differentiating the function partially with respect to y we get

∏y = -2x + 200 – 6y.....(II)

To determine the Critical Point, we have ∏x = 0 and ∏y = 0

So, 100 – 2x – 2y = 0 Or. x + y = 50..... (1)

and -2x + 200 -6y = 0 or, x + 3y = 100..... (2)

(2) – (1) gives, 2y = 50 or, y = 25

Putting y = 25 in (1) we get x = 25

Thus Critical Point is (25, 25)

To check whether this point is a local Maxima, we have to find out the values of the 2nd Order Partial Derivatives at this point.

Again differentiating (I) partially with respect to x we get ∏xx = -2 Or, A = -2 (Let) Or, A<0

Similarly differentiating (II) partially with respect to y we get ∏yy = -6 Or, C = -6 (Let) or, C<0

Also differentiating (I) partially with respect to y we get ∏xy = -2 Or, B = -2 (Let)

So D = AC – B2 = (-2) × (-6) – (2)2 = 8 >0

Hence D> 0 and A, C <0

Thus there is a local Maxima at the already determined Critical Point (25, 25)

Required Profit maximizing quantities of the products are x = 25 units and y = 25 units.

Also Maximum profit = Value of the function ∏ at x = 25 & y = 25

= 100 ×25 – 252 – 2 ×25 ×25 + 200 × 25 - 3 × 252

= ₹. 3,750 (₹000)

ILLUSTRATION 6:

A manufacturing company selling two wheelers (x) and three wheelers (y) has the following function px = 40-0.02x-0.01y and py = 80-0.06y-0.01x. Find the revenue maximizing levels of output and price of the two wheelers as well as the three wheelers. What is the maximum total

revenue? The prices are in ₹ '000. Ensure the total revenue obtained is maximum by 2nd order derivative test?

Solution:

Here we are solving the total revenue function

The price function for two wheelers px is = 40 - 0.02x - 0.01y

The price function for three wheelers py is = 80 - 0.06y - 0.01x.

And the demand function of two and three wheelers be x and y respectively.

Thus the total revenue would be calculated as

TR = px × x + py × y

TR= (40-0.02x - 0.01y) × x + (80 - 0.06y - 0.01x.) × y

= $40x – 0.02x^2 – 0.01xy + 80y - 0.06y^2 -0.01xy$

= $40x – 0.02x^2 – 0.02xy + 80y - 0.06y^2$

The total revenue function would be $40x – 0.02x^2 – 0.02xy + 80y - 0.06y^2$

Differentiating the function partially with respect to x we get,

dR/dP = 40 - 0.04x - 0.02y

Differentiating the function partially with respect to y we get,

dR/dP = -0.02x + 80 - 0.12y

As per the necessary condition (First Order of derivative)

We have to determine the Critical Point, we have ∏x = 0 and ∏y = 0

40 - 0.04x - 0.02y = 0

-0.04x - 0.02y = -40

0.04x + 0.02y = 40 1

-0.02x + 80 - 0.12y = 0

-0.02x - 0.12y = -80

0.02x + 0.12y = 80 2

Multiplying equation 1 by 6

0.24x + 0.12y = 240.....3

Subtracting equation 2 from 3

0.24x + 0.12y = 240

(-) 0.02x + 0.12y = 80

0.22x = 160

x = 727

Putting x = 727 in equation 1 we will get

0.04(727) + 0.02y = 40

29.08 + 0.02y = 40

0.02y = 10.92

y = 546

The critical point of (x.y) will be (727,546)

To check whether this point is a local Maxima, we have to find out the values of the 2nd Order Partial Derivatives at this point.

Again differentiating (I) partially with respect to x we get ∏xx = -0.04 Or, A = -0.04 (Let) Or, A<0

Similarly differentiating (II) partially with respect to y we get ∏yy = -0.12 Or, C = -0.12(Let) or, C<0

Also differentiating (I) partially with respect to y we get ∏xy = -0.02 Or, B = -0.02 (Let)

So D = AC – B2 = (-0.04) × (-0.12) – (0.02)2 = 0.0044>0

Hence D> 0 and A, C <0

Thus there is a local Maxima at the already determined Critical Point (727,546)

Required Profit maximizing quantities of the products are x = 727 units and y = 546 units.

Price = px= 40 - 0.02x - 0.01y

= 40 - 0.02 × 727-0.01 × 546

= ₹20,000

Price = py = 80 - 0.06y - 0.01x.

= 80 - 0.06 × 546 - 0.01 × 727

= ₹ 40,000

Also Maximum Revenue= Value of the function ∏ at x = 727 & y = 546

= px × x + py × y

= ₹20,000 × 727 + ₹40,000 × 546

= ₹3, 63, 80,000/-

ILLUSTRATION 7:

The total cost function y for x units is given by y = 3x (x + 7 /x + 5) + 5. Prove that the marginal cost decreases continuously as the output increases.

Solution:

To prove the marginal cost decreases continuously as the output increases, we should prove dy/dx is positive.

y = 3x (x + 7 /x + 5) + 5

= 3x ((x + 5) +2 /x + 5) + 5

= 3x (x + 5/ x + 5 +2 /x + 5) + 5

= 3x (1 +2 /x + 5) + 5

= 3 (x +2x /x + 5) + 5

dy/dx = 3 d/dx (x +2x /x + 5) + d/dx 5

= 3(1+2 d/dx (x /x+5)) +0

= 3(1+2 (x+5-x/x+52))

= 3(1+2 (5/x+52))

= 3(1+10/x+52), which is positive

Hence proved that the marginal cost decreases continuously, if the output increases.

9 Business Forcasting Models - Time Series and Regression Analysis [Study Material - Module 14]

ILLUSTRATION 1: REGRESSION

The following data shows the exports of raw cotton and the value of imports of manufactured goods into India for 7 years:

Exports	42	44	58	55	89	98	60
Imports	56	49	53	58	67	76	58

Ascertain the regression equation of imports on exports and estimate the import when the export in a particular year was to the value of ₹ 70 crores.

Solution:

Regression equation of Y on X is Y = a + bx

Calculations for the regression equation:

Exports (X)	**Imports (Y)**	**X^2**	**XY**
42	56	1764	2352
44	49	1936	2156
58	53	3364	3074
55	58	3025	3190
89	67	7921	5963
98	76	9604	7448
60	58	3600	3480
446	417	31214	27663

$\sum Y = Na + b\sum X$

$\sum XY = a\sum X + b\sum X^2$

Substituting the values, we have

417= 7a +446b ..... (i)

27663 =446a +31214b ..... (ii)

Multiplying equation (i) by 63.71 we get

26567 = 446a + 28415b ..... (iii)

Subtracting equation (iii) from (ii)

27663 = 446a + 31214b (ii)

26567 = 446a + 28415b (iii)

1096 = 2799b

b =1096/2799 = 0.39

Substituting b = 0.39 in equation (i)

417 = 7a + 446b

417 = 7a + 446(0.39)

417 =7a + 173.94

417-173.94 = 7a

243.06 = 7a

a = 243.06/7 = 34.72

Thus we will get a = 34.72, b = 0.39

Thus Substituting i and ii we will get a = 34.72, b = 0.39

Hence Y=34.72+0.39X when X (export) = 70 crores, Y = 62 crores (import)

ILLUSTRATION 2:

The following table shows the number of motor registrations in a certain territory for a term of 5 years and the sale of motor tyre by a firm in that territory for the same period.

Year	Motor Registration	No of Tyres sold
1	600	1250
2	630	1100

Year	Motor Registration	No of Tyres sold
3	720	1300
4	750	1350
5	800	1500

Find the regression equation to estimate the sale of tyres when motor registration is known. Estimate sale of tyres when registration is 850.

Solution: Let us denote the number of motor registrations by the random variable X and the number of tyres sold by the variable Y. Then we must find regression equation of Y on X i.e. Y= a + Bx

Calculations for the regression equation:

X	Y	dx =X- $\overline{X}$ x-700	dy =Y- $\overline{Y}$ y-1300	dx^2	dxdy
600	1250	-100	-50	10000	5000
630	1100	-70	-200	4900	14000
720	1300	20	0	400	0
750	1350	50	50	2500	2500
800	1500	100	200	10000	20000
∑X 3500	∑Y 6500	∑dx =0	∑dy =0	$\sum dx^2$ =27800	∑dxdy=41500

Now we will calculate the necessary values:

$\overline{X}$ = (600 + 630 + 720 + 750 + 800) / 5 = 700

$\overline{Y}$ = (1,250 + 1,100 + 1,300 + 1,350 + 1,500) / 5 = 1,300

SSXX = (600 - 700)^2 + (630 - 700)^2 + (720 - 700)^2 + (750 - 700)^2 + (800 - 700)^2 = 50,000

SSYY = (1,250 - 1,300)^2 + (1,100 - 1,300)^2 + (1,300 - 1,300)^2 + (1,350 - 1,300)^2 + (1,500 - 1,300)^2 = 100,000

SSXY = (600 - 700)(1,250 - 1,300) + (630 - 700)(1,100 - 1,300) + (720 - 700)(1,300 - 1,300) + (750 - 700)(1,350 - 1,300) + (800 - 700)(1,500 - 1,300) = -25,000

b = -25,000 / 50,000 = -0.5

a = 1,300 - (-0.5 × 700) = 1,650

The estimated regression equation is:

Y = 1,650 - 0.5X

To estimate the sale of tyres when motor registration is 850, we can substitute X = 850 into the regression equation:

Y = 1,650 - 0.5 × 850 = 1,950 - 425 = 1,225

Therefore, the estimated sale of tyres when motor registration is 850 is 1,525.

ILLUSTRATION 3:

The following information was available in the case of two being variable rainfall (X) and yield of paddy (Y):

Particulars	Yield per acre (kg)	Annual Rainfall (cm)
Mean	970	18
Standard Deviation	38	2

Corelation coefficient r = 0.6

Find the line of regression of Y on X and obtain most likely yield of paddy when annual rainfall is 20 cm.

Solution: In the usual notations, we have $\overline{X}$ = 18, $\overline{Y}$ = 970, σy = 38, r = 0.6

Equation on line of regression of Y on X is:

$Y - \overline{Y} = r\ \sigma y/\ \sigma x\ (X- \overline{X})$

Y - 970 = 0.6 × 38/2 (X-18)

Y = 11.4X - 205.2 + 970 = 764.8 + 11.4X

If X = 20 cm (annual rainfall), Y would be 764.8 + 11.4 × 20 = 922.8 per acre (being most likely yield of paddy)

Where Y = a + bx.

ILLUSTRATION 4: TIME SERIES

A medium-sized manufacturing organization has the following sales analysis for the period 1977-86 inclusive:

Year	Sales (₹ Million)	Year	Sales (₹ Million)
1977	15.3	1982	20.9

Year	Sales (₹ Million)	Year	Sales (₹ Million)
1978	14.6	1983	22.3
1979	16.8	1984	20.0
1980	17.3	1985	23.1
1981	17.2	1986	24.5

Calculate:

(i) The trend line using the least squares equation

(ii) The trend values for 1977 and 1986 and explain the meaning of these results.

Solution:

(i) Here n = 10 (even no), Thus here arithmetic mean (X*) would be 1981.5 i.e. interval (average) between years 1977 to 1986, where such values are in units of 6 Months i.e. ½ year. If the Linear trend is Y = a + bX, the normal equations for determining the constants a and b are:

*X = t = 1/2 (1981+1982) / 1/2(Interval) = 2 (t-1981.5)

$\sum Y = Na + b\sum X$

$\sum XY = a\sum X + b\sum X^2$

Calculations for fitting Linear Trend

Year (t)	Sales (Y)	X	X^2	XY
1977	15.3	-9	81	-137.7
1978	14.6	-7	49	-102.2
1979	16.8	-5	25	-84.0
1980	17.3	-3	9	-51.9
1981	17.2	-1	1	-17.2
1982	20.9	1	1	20.9
1983	22.3	3	9	66.9
1984	20.0	5	25	100.0
1985	23.1	7	49	161.7
1986	24.5	9	81	220.5
	$\sum Y=192$	$\sum X=0$	$\sum X^2=330$	$\sum XY=177$

Substituting the values in the normal equations, we get

192 = 10a + b × 0

a = 192/10 = 19.2

and 177 = a × 0 + 330b

b = 177/330 = 0.536

The fitted linear equation in Y = a + bX ,trend equation is therefore would be

Y = 19.2 + 0.536X

(ii) The trend values for 1977 and 1986 would be:

Y 1977 = 19.2 + 0.536* -9 (x = -9) = 14.376 (Million ₹)

Y 1986 = 19.2 + 0.536* 9 (x = 9) = 24.024 (Million ₹)

*The meaning of these results the trend line underlying long-term movement in sale values when the random disturbances are smoothed out.

ILLUSTRATION 5:

Determine the sales for the year 1984 by the least square method:

Year	Sales of Refrigerators
1978	100
1980	110
1981	130
1982	125
1983	160

Solution:

Let the straight-line trend be Y = a + bX, where Origin is year 1981 of July and x unit is 1year. The normal equations for estimating a and b are:

$\sum Y = Na + b\sum X$

$\sum XY = a\sum X + b\sum X^2$

Calculations for fitting Linear Trend

Year (x)	Value (Y)	$\overline{X}$ =X - 1981	X^2	XY
1978	100	-3	9	-300
1980	110	-1	1	-110
1981	130	0	0	0

Year (x)	Value (Y)	$\bar{X}$ =X - 1981	X^2	XY
1982	125	1	1	125
1983	160	2	4	320
	∑Y = 625	∑X = -1	$\sum X^2 = 15$	∑XY = 35

Substituting the values in the normal equations, we get

625 = 5a-b ..... (i)

35 = -a+15b ..... (ii)

Multiplying equation (ii) by -5

-175 = 5a + (-75b)

-175 = 5a -75b .....(iii)

Subtracting equation (iii) from (i) we will get:

625 = 5a - b ..... (i)

-175 = 5a - 75b .....(iii)

800 = 74b

b = 800/74 = 10.8

Substituting b = 10.8 in equation (i)

625 = 5a - b

625 = 5a - 10.8

625 + 10.8 = 5a

635.8 = 5a

127 = a

Thus we will get b = 10.8 and a = 127

Therefore Substituting (i) and (ii) we get b = 10.8 and a = 127

The fitted linear equation in Y=a+bx, trend equation is therefore would be

Y= 127 + 10.8X

The trend values or sale value for year 1984 obtained by putting x = +3 in trend equation:

Y 1984 = 127 + 10.8 × 3(x=3) = 159.40

ILLUSTRATION 6: 3 YEARS MOVING AVERAGES

Determine the trend values by the moving average method: (Take the 3 yearly cycles)

Year	Sales in Millions
1980	412
1981	438
1982	446
1983	454
1984	470
1985	483
1986	490

Solution:

Computation of the Moving Averages

Year	Sales in Millions	Columns of differences	Three Year Moving Totals	Three Year Moving Average
1980	412			
1981	438	454 - 412=42	1296	1296/3 =432
1982	446	470- 438 =32	1338	1338/3= 446
1983	454	483 – 446 =37	1370	1370/3=457
1984	470	490 - 454 =36	1407	1407/3=469
1985	483		1443	1443/3=481
1986	490			

ILLUSTRATION 7: 4 YEARS MOVING AVARAGES

Determine the trend values by the moving average method: (Take the 4 yearly cycles)

Year	Tonnage of goods carried
1975	2204
1976	2500
1977	2360
1978	2680

Year	Tonnage of goods carried
1979	2424
1980	2634
1981	2904
1982	3098
1983	3172
1984	2952
1985	3248
1986	3172

Solution:

Computation of the 4 Yearly Moving Averages

Year (1)	Tonnage of goods carried (2)	Columns of differences (3)	Four Year Moving Totals (not centered) (4)	Two Year Moving Totals (centered) (5)	Two Year Moving Averages (not centered) (6) = (5)/8
1975	2204				
1976	2500				
		2424-2204=220	9744		
1977	2360			19708	2463.50
		2634-2500=134	9964		
1978	2680			20062	2507.75
		2904-2360=544	10098		
1979	2424			20740	2592.50
		3093-2680=418	10642		

Year (1)	Tonnage of goods carried (2)	Columns of differences (3)	Four Year Moving Totals (not centered) (4)	Two Year Moving Totals (centered) (5)	Two Year Moving Averages (not centered) (6) = (5)/8
1980	2634			21702	2712.75
		3172-2424 =748	11060		
1981	2904			22868	2858.50
		2952-2634= 318	11808		
1982	3098			23934	2991.75
		3248-2901=344	12126		
1983	3172			24596	3074.50
		3172-3098=74	12470		
1984	2952			25014	3126.75
1985	3248	-	12544		
1986	3172				

ILLUSTRATION 8: EXPONENTIAL SMOOTHING

A firm uses simple exponential smoothing with alpha of 0.1 to forecast demand. The forecast for the week of February 1 was 1000 units, whereas actual demand turned out to be 900 units.

(a) Forecast the demand for the week of February 8.

(b) Assume that the actual demand during the week of February 8 turned out to be 1010 units. Forecast the demand for the week of February 15. Continue forecasting through March 15, assuming that subsequent demands were 1032, 976, 934,1008 and1020 units.

Solution:

St = St-1+alpha (Xt-St-1)

=1000+0.1(900-1000) = 990 units

Week	Demand (Xt)	Old Forecast (St-1)	Forecast Error (Xt-St-1)	Correction Alpha (Xt-St-1)	New Forecast St
(1)	(2)	(3)	(4)	(5)	(6) = (3) +(5)
Feb 1	900	1000	-100	-10	990
Feb 8	1010	990	20	2	992
Feb 15	1032	992	40	4	996
Feb 22	976	996	-20	-2	994
Mar 1	934	994	-60	-6	988
Mar 8	1008	988	20	2	990
Mar 15	1020	990	20	2	992

ILLUSTRATION 9:

XYZ Co unloaded quantities of a particular chemical from ships during 1985 to 1986 as shown in the following table:

Year	Quarter	Tonnage Unloaded	Forecast
1985	I	180	175
	II	168	?
	III	159	?
	IV	175	?
1986	I	190	?
	II	205	?
	III	180	?
	IV	182	?
1987	I	?	?

Solution:

St = St-1+ alpha (Xt-St-1)

Year	Quarter	Forecast
1985	I	175
	II	175+0.10 (180-175) =175.50
	III	175.50+0.10 (168-175.50) =174.75
	IV	174.75+0.10 (159-174.75) =173.18
1986	I	173.18+0.10 (175-173.18) =173.36
	II	173.36+0.10 (190-173.36) =175.02
	III	175.02+0.10 (205-175.02) =178.02
	IV	178.02+0.10 (180-178.02) =178.22
1987	I	178.22+0.10 (182-178.22) =178.60

ILLUSTRATION 10:

Use an exponential smoothing technique to compute forecasts for the following time series data under two situations when smoothing constant is 0.3 and when smoothing constant is 0.7, Which forecast will you accept and why?

Period	1	2	3	4	5	6	7	8	9	10
Observation	27	30	32	31	28	27	30	33	33	31

Solution:

The basic exponential smoothing model is St = St-1+alpha (Xt-St-1)

Where St= the smoothed value of the time series at time t

0 is smoothing constant ($0<\alpha<1$),

Xt is the actual time series value at time t

St-1 is smoothed value of the series at time t-1.

Since no past smoothed value or forecast for first time is given, we assume that the smoothed value of the time series for first period is equal to the actual first value of the time series. Thus, the table given below gives forecasts for the time series under two situations when $\alpha = 0.3$ and $\alpha = 0.7$.

Period	Xt	et (Yt-St-1)	α et (α=0.3)	St	et^2	et (Yt-St-1)	α et (α=0.7)	St	et^2
1	27	0	0	27	0	0	0	27	0
2	30	3	0.9	27.9	9	3	2.1	29.1	9
3	32	4.1	1.23	29.1	16.8	2.9	2.03	31.13	8.41
4	31	1.9	0.57	29.7	3.61	-0.1	-0.07	31.0	0.01
5	28	-1.7	-0.51	29.2	2.89	-3.0	-2.1	28.9	9
6	27	-2.2	-0.66	28.5	4.84	-1.9	-1.33	27.6	3.61
7	30	1.5	0.45	29.0	2.25	2.4	1.68	29.3	5.76
8	33	4.0	1.20	30.2	16	3.7	2.59	31.9	13.69
9	33	2.8	0.84	31	7.84	1.1	0.77	32.7	1.21
10	31	0	0	0	0	-1.7	-1.19	31.5	2.89
Total					**63.27**				**53.58**
		Variance = 63.27/9 = 7.03				**Variance = 53.58/9 = 5.95**			

*Since the variance for the case α=0.3 is more than that of for α=0.7. We will choose the forecast with α=0.7 as less the variance, less would be the risk in forecast for the smoothing constant time series.

The Institute of Cost Accountants of India

Statutory Body under an Act of Parliament

www.icmai.in

Headquarters:
CMA Bhawan; 12, Sudder Street; Kolkata - 700016
Ph: +91-33-2252-1031/34/35/1602/1492/1619/7373/7143

Delhi Office:
CMA Bhawan; 3, Institutional Area; Lodhi Road; New Delhi - 110003
Ph: +91-11-24666100/24622156/57/58; 24666124/129

E-mail: studies@icmai.in

Follow us on

Behind Every Successful Business Decision, there is always a CMA